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Abstract

This paper presents an averaged model and nonlinear observer for an on/off pneumatic actuator. The actuator is composed of two chambers and four on/off solenoid valves. The averaged model is elaborated which has the advantage of using only one continuous input instead of four binary inputs. Based on this new model, a sliding mode observer is designed using the piston's position and the pressure measurements in one of the chambers to estimate the piston's velocity and the pressure in the other chamber. The finite time convergence of the observer is proven and its performance is verified in closed loop on an experimental benchmark.

1 Introduction

Pneumatic actuators are widely used in the automation of different industrial production lines, robotics and also in medical applications. These systems present many advantages: reliability, velocity, low cost and effort/input energy ratio, which allow to build light and powerful actuators. Unfortunately, the major difficulty concerning pneumatic actuators is their nonlinear dynamics due to air compressibility, friction forces and mainly the nonlinear behavior of air flow rate through the power modulators. Hence, the design of a control strategy is not simple. In order to have high performance in terms of positioning, it is possible to use proportional servo-valves instead of the traditional on/off solenoid valves. These servo-valves deliver an air flow rate, depending on the control voltage and the upstream pressure, ranging from 0% to 100% contrary to the traditional on/off valves which deliver either 0% or 100% of the available mass flow rate depending on the binary input voltage of the valves. Many control strategies and techniques have been elaborated for this type of actuators: PID [1], fuzzy control [2], sliding modes [3], backstepping [4], adaptive control [5], neural networks [6]. However, servo valves are still far less common than solenoid on/off valves because of their price and the high expertise needed to fully exploit them.

To overcome these drawbacks, recently improved solenoid valves could be used instead of the servo-valves. However, precise control is difficult due to the inputs of these on/off solenoid valves. From an operating point of view, the pneumatic actuators equipped with these kind of components belong to the class of the switching systems with nonlinear dynamics. Traditionally, there is no general theory able to characterize the stability of such systems. Fortunately, with the recent development of high frequency switching on/off valves, it is possible to handle new perspectives in terms of modeling and control.

A significant amount of high quality research has been addressed in this direction using averaging techniques. In power electronics, the concept of switching systems with discrete inputs is well known. Averaging techniques are used to create one unique model with one single continuous input. In fluid power systems, solenoid valves are being controlled with a Pulse Width Modulation (PWM) controller to determine its input: 1 or 0. One can describe the equivalent continuous dynamics of PWM controlled nonlinear systems by transforming the system, originally discontinuous and possibly non-affine in the input, into an equivalent system that is both continuous and affine in the control input, *i.e.* transforms the system to nonlinear control canonical form [7].

Many works show the interest of using on/off valves: [7–15] to control the position of the pneumatic actuators. In order to design these control strategies, one needs to have access to all the state variables of the actuator: position, velocity and the pressures of the two chambers. However, in order to reduce the price, decreasing number of sensors is required. Therefore, the estimation of the non measured variables is essential. In the literature, there are some works related to the observation of pneumatic systems with servo-valves: [16] presents the synthesis of pressure observers based on Lyapunov theory. The authors of [17] propose two observers using the position of the piston and the pressure in one chamber. Nevertheless, the observer design for pneumatic systems with on/off valves is a standard solution in automotive applications and particularly in the control of clutch systems. In [18], the authors propose a reduced order observer to estimate the pressure to have an internal pressure controller in order to control the clutch position. In [19], the authors consider the design of a full sate observer for electro pneumatic clutch actuator with position sensor. This observer is used in [20] to obtain a dual-mode switched controller for the clutch actuator position. Nevertheless, this observer uses the input of the on/off solenoid valve. As it is mentioned earlier, there is no general theory to guarantee the observer stability of such systems since they use switched models in their design. Therefore, it's important to establish a model for which the stability can be guaranteed. In this context, averaging techniques will be used to obtain such a model.

The authors of [14, 15] presented an output averaged model for an on/off pneumatic actuator equipped with four solenoid valves which is the same as the experimental benchmark considered in this paper. Unfortunately, the elaborated output averaged model can only be used for position control strategies and it can not be used for observer design. One of the main contribution of this paper is the extension of the model elaborated in [14, 15] to a full state averaged model to describe the behavior of the on/off pneumatic systems. A nonlinear sliding mode observer is designed [21] using only two outputs. The convergence of the observation error towards zero is proven using the averaged model and Lyapunov theory. Furthermore, the performance of this observer is verified experimentally to control the position of the actuator using sliding mode strategy [22, 23].

This paper is organized as follows: Section 2 presents the modeling of a pneumatic actuator with four on/off solenoid valves. The result is a switching model with four binary inputs. In Section 3, a full state averaged model is elaborated using techniques from power electronics. Section 4 shows the design of sliding mode observer with a finite time convergence proof. This observer is used to calculate a control input to track a desired position trajectory. The experimental results are shown in Section 5.

2 Modeling

The pneumatic system (Fig.1) is a one degree of freedom actuator composed of two pneumatic chambers. Every chamber is connected to a pressure source and to an exhaust pressure using two on/off solenoid valves. Each valve i is controlled with a discrete binary input U_i.
Let us assume that [13]:

- The only considered friction forces are viscous;
- The air is a perfect gas and its kinetic energy is negligible in each chamber;
- The pressure, the temperature and the density of the air are homogeneous in each chamber;
- The evolution of the air inside each chamber is polytropic;
- The supply and the exhaust pressure are constant and there is no leakage.

The complete modeling could be divided into three stages: Piston's dynamics, the air evolution in each chamber and the valve dynamics. Using Newton's second law and the

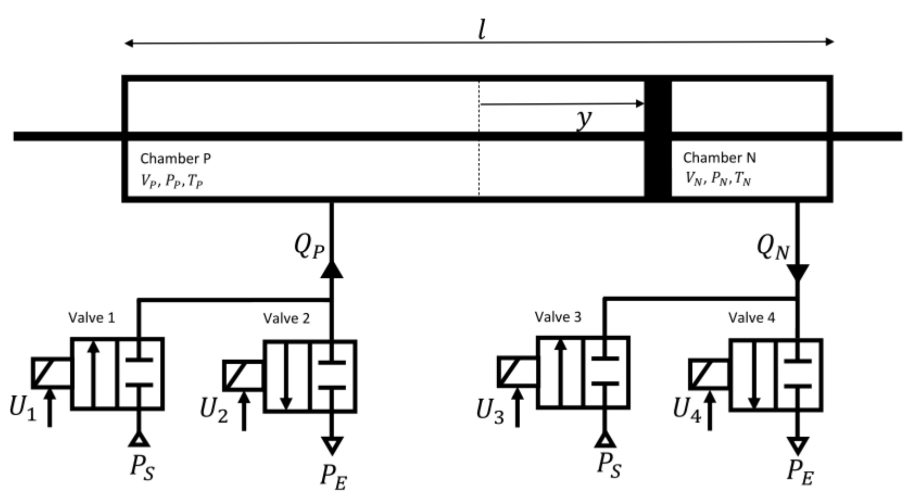

Figure 1: Pneumatic system with four on/off valves

first principle of thermodynamics, the piston's dynamics and the air evolution inside each pneumatic chamber are given by [14]:

$$\begin{cases} m\ddot{y} = \ (A_P P_P - A_N P_N) - b_v \dot{y} \\ \dot{P}_P = \dfrac{k}{(l/2+y)A_P}(rTQ_P - A_P P_P \dot{y}) \\ \dot{P}_N = \dfrac{k}{(l/2-y)A_N}(rTQ_N + A_N P_N \dot{y}) \end{cases} \tag{1}$$

where l is the total length, y is the piston's position and m is its mass. b_v is the viscosity coefficient. P_P, A_P and Q_P (respectively P_N, A_N and Q_N) are the air pressure, the cylinder's area and the input mass flow rate in chamber P (respectively chamber N). Please note that A_P and A_N are explicitly made equal. r is the perfect gas constant and k is the polytropic constant. T is the air temperature. Q_P and Q_N are the input mass flow rate in each chamber. They are given by:

$$\begin{cases} Q_P = U_1 Q(P_S, P_P) - U_2 Q(P_P, P_E) \\ Q_N = U_3 Q(P_S, P_N) - U_4 Q(P_N, P_E) \end{cases} \tag{2}$$

where U_i with $i = \{1, 2, 3, 4\}$ is the binary input voltage of valve i. P_S and P_E are the supply and the exhaust pressures respectively. The air flow inside each valve depends on the upstream pressure P_u and temperature T_u and the downstream pressure P_d and temperature T_d. It depends also on the valve's characteristics like the mass flow rate constant C_{val} and the critical pressure ratio b_{crit} [14]:

$$Q(P_u, P_d) = C_{val} P_u \sqrt{T/T_u} \begin{cases} \sqrt{1 - \left(\dfrac{\dfrac{P_d}{P_u} - b_{crit}}{1 - b_{crit}} \right)^2} & \text{if } \dfrac{P_d}{P_u} > b_{crit} \\ 1 & \text{if } \dfrac{P_d}{P_u} \le b_{crit} \end{cases} \tag{3}$$

The global model of (1) becomes:

$$\begin{cases} \ddot{y} = \dfrac{1}{m}\left(A_P P_P - A_N P_N - b_v \dot{y}\right) \\ \dot{P}_P = -k \dfrac{P_P}{l/2+y}\dot{y} + krT \dfrac{U_1 Q(P_S, P_P) - U_2 Q(P_P, P_E)}{(l/2+y)A_P} \\ \dot{P}_N = k \dfrac{P_N}{l/2-y}\dot{y} + krT \dfrac{U_3 Q(P_S, P_N) - U_4 Q(P_N, P_E)}{(l/2-y)A_N} \end{cases} \tag{4}$$

Each binary input voltage U_i can be in two situations either "on" or "off". Hence, every chamber can be in one of the following three states:

- Pressurizing: connected to the supply pressure (for the chamber P: $U_1 = 1$, $U_2 = 0$);
- Venting: connected to the exhaust pressure ($U_1 = 0$, $U_2 = 1$);
- Close: both of the valves are closed ($U_1 = 0$, $U_2 = 0$).

From these three states, nine modes can be obtained for the two chambers. It has been proved in [14] and [15] that only seven modes can be used depending on the input vector $U = (U_1 \;\; U_2 \;\; U_3 \;\; U_4)^T$. For each mode j, the input vector $U^{(j)}$ is associated such that:

$$\begin{array}{ll} \text{Mode 1: } U^{(1)} = (0 \;\; 0 \;\; 0 \;\; 0)^T & \text{Mode 2: } U^{(2)} = (1 \;\; 0 \;\; 0 \;\; 0)^T \\ \text{Mode 3: } U^{(3)} = (0 \;\; 1 \;\; 0 \;\; 0)^T & \text{Mode 4: } U^{(4)} = (0 \;\; 0 \;\; 0 \;\; 1)^T \\ \text{Mode 5: } U^{(5)} = (0 \;\; 0 \;\; 1 \;\; 0)^T & \text{Mode 6: } U^{(6)} = (1 \;\; 0 \;\; 0 \;\; 1)^T \\ \text{Mode 7: } U^{(7)} = (0 \;\; 1 \;\; 1 \;\; 0)^T & \end{array} \tag{5}$$

3 Averaged model

For power converters, the averaged model can be obtained using the different dynamics of the system undergoing pulse-width modulation of the switched inputs. The PWM duty cycle will act as a continuous input for the averaged model. The average modeling consists of evaluating all the different modes of the switching system in order to obtain one equivalent model for all the p modes:

$$F = (F^{(1)} \;\; \dots \;\; F^{(p)})^T$$

Due to the variation of the input injected into the system, it can switch between several modes within a PWM period. For each mode $F^{(j)}$, a duty ratio $d^{(j)}$ is associated:

$$D = (d^{(1)} \;\; \dots \;\; d^{(p)})^T$$

The total duration of all the duty ratios $d^{(j)}$ is equal to the PWM period *i.e.* D can be normalized: $\|D\|_1 = 1$. Then, the averaged model will represent all the modes $F^{(j)}$ within a switching period PWM:

$$\dot{X}_a = \sum_{j=1}^{p} d^{(j)} F^{(j)} \tag{6}$$

where X_a represents the averaged state vector.

The averaged model presented in [14,15] uses three mode-model (mode 1 is for no actuation, mode 6 corresponds to the movement in positive direction while mode 7 is for the negative direction) to control the position of the piston. In these works, there is no need to average all the state variables, only the output y is averaged and replaced by y_a. However, this model is not suitable for state observer design. There is a need to elaborate a state averaged model.

To obtain a state averaged model, all the states dynamics must be explicitly given with the binary discrete input U_i. Model presented in (4) becomes:

$$\begin{cases} \dddot{y} = -\dfrac{k}{m}\left(\dfrac{A_P P_P}{l/2+y} + \dfrac{A_N P_N}{l/2-y}\right)\dot{y} - \dfrac{b_v}{m}\ddot{y} \\ \quad + \dfrac{krT}{m}\dfrac{U_1 Q(P_S, P_P) - U_2 Q(P_P, P_E)}{(l/2+y)} \\ \quad - \dfrac{krT}{m}\dfrac{U_3 Q(P_S, P_N) - U_4 Q(P_N, P_E)}{(l/2-y)} \\ \dot{P}_P = -k\dfrac{P_P}{l/2+y}\dot{y} + krT\dfrac{U_1 Q(P_S, P_P) - U_2 Q(P_P, P_E)}{(l/2+y)A_P} \\ \text{with: } P_N = \dfrac{A_P P_P - \ddot{y} - b_v \dot{y}}{A_N} \end{cases} \tag{7}$$

As it can be seen in (7), the dynamics $\dddot{y}$ and $\dot{P}_P$ change according to the input $U^{(j)}$ (Mode j) of (5). Hence, let us introduce the following notations:

$$\begin{array}{ll} x_1 = y & f_1 = x_2 \\ x_2 = \dot{y} & f_2 = x_3 \\ x_3 = \ddot{y} & f_3^{(j)} = \dddot{y}|_{U^{(j)}} \\ x_4 = P_P & f_4^{(j)} = \dot{P}_P|_{U^{(j)}} \end{array}$$

where the dynamics $f_3^{(j)}$ and $f_4^{(j)}$ correspond to the dynamics $\dddot{y}$ and $\dot{P}_P$ evaluated for the input $U^{(j)}$. The model (4) can be written as:

$$\dot{X}(t) = \quad F^{(j)}(X(t), U^{(j)}) \qquad j = 1, \ldots, 7 \tag{8}$$

where $X = (x_1\ x_2\ x_3\ x_4)^T$ and $F^{(j)} = (f_1\ f_2\ f_3^{(j)}\ f_4^{(j)})^T$. Averaging techniques consists of mixing all the p modes of the on/off actuator by introducing one single continuous input to create an equivalent continuous input similar to the one in servo-valves. The use of the averaged model is justified by the high switching frequency of the on/off valves used in this paper, *i.e.* their dynamics are much faster than the other dynamics. Then, the state averaged model using only three modes (mode 1, 6 and 7) can be given by:

$$\dot{X}_a = d^{(1)} F^{(1)} + d^{(6)} F^{(6)} + d^{(7)} F^{(7)} \tag{9}$$

where the vector $X_a = (x_{1a}\ x_{2a}\ x_{3a}\ x_{4a})^T$ corresponding to the vector $(y_a\ \dot{y}_a\ \ddot{y}_a\ P_{P_a})^T$ is the averaged state variable vector.

Remark 3.1 *Unlike the Takagi-Sugeno fuzzy modeling or the T-S modeling [24, 25], averaging techniques consist in combining the different existing discontinuous modes of a system. T-S modeling approximates a nonlinear system by an interpolation of several local linear models and by combining them using activation functions. Therefore, T-S modeling can be seen as creating and combing artificial modes to approximate the nonlinear behavior of system. However, the relevance of the fuzzy model depends on its ability to represent the actual system. The averaging technique used in this paper is another alternative to represent a switched system with different discontinuous modes and combine them using the duty ratios to represent the real switched system as in (9). Therefore, unlike T-S methods, averaging techniques do not create any artificial modes.*

Since modes 1 and 6 correspond to the actuation in the positive direction, from the chamber P to the chamber N (see Fig. 1), the modes 1 and 6 can be modulated together while mode 7 is not used. For the negative direction, from the chamber N to the chamber P, one can modulate the modes 1 and 7 while mode 6 is not used. A new and a continuous input u is introduced to modulate modes 1 and 6 together and modes 1 and 7 together. For the positive direction, u is positive and its value corresponds to the duty ratio of mode 6. For the negative direction, u is negative and its absolute value corresponds to the duty ratio of mode 7. Hence the following modulation (switching) scheme between the coefficients $d^{(j)}$ and the input u is proposed:

- **For the positive direction**: $d^{(1)}$ and $d^{(6)}$ are mixed $d^{(1)} = 1 - |u|$ and $d^{(6)} = |u|$ (while $d^{(7)} = 0$)

- **For the negative direction**: $d^{(1)}$ and $d^{(7)}$ are mixed $d^{(1)} = |u|$ and $d^{(7)} = 1 - |u|$ (while $d^{(6)} = 0$)

The averaged model of these three modes can be given by:

$$\dot{X}_a = F^{(1)}(X_a) + \begin{cases} (F^{(6)}(X_a) - F^{(1)}(X_a))u & \text{if} \quad u \geq 0 \\ (F^{(1)}(X_a) - F^{(7)}(X_a))u & \text{if} \quad u < 0 \end{cases}$$

Hence, the state averaged model is given by:

$$\begin{cases} \dot{x}_{1_a} = x_{2_a} \\ \dot{x}_{2_a} = x_{3_a} \\ \dot{x}_{3_a} = \varphi_1(X_a) + g_1(X_a)u \\ \dot{x}_{4_a} = \varphi_2(X_a) + g_2(X_a)u \end{cases} \tag{10}$$

with:

$$\varphi_1(X_a) = -\frac{k}{m}\left(\frac{A_P x_{4_a}}{l/2 + x_{1_a}} + \frac{A_N \psi(X_a)}{l/2 - x_{1_a}}\right) x_{2_a} - \frac{b_v}{m} x_{3_a}$$

$$g_1(X_a) = \frac{krT}{m}\begin{cases} \left(\dfrac{Q(P_S, x_{4_a})}{(l/2 + x_{1_a})} + \dfrac{Q(\psi(X_a), P_E)}{(l/2 - x_{1_a})}\right) & \text{if } \ u \geq 0 \\ \left(\dfrac{Q(x_{4_a}, P_E)}{(l/2 + x_{1_a})} + \dfrac{Q(P_S, \psi(X_a))}{(l/2 - x_{1_a})}\right) & \text{if } \ u < 0 \end{cases}$$

$$\varphi_2(X_a) = -k \frac{x_{4_a}}{l/2 + x_{1_a}} x_{2_a}$$

$$g_2(X_a) = krT \begin{cases} krT \dfrac{Q(x_{4_a}, P_E)}{(l/2 + x_{1_a})A_P} & \text{if } u \geq 0 \\ krT \dfrac{Q(P_S, x_{4_a})}{(l/2 + x_{1_a})A_P} & \text{if } u < 0 \end{cases}$$

and: $\psi(X_a) = (A_p x_{4_a} - x_{3_a} - b_v x_{2_a})/A_N$.

Remark 3.2 *System (10) describes the equivalent nonlinear dynamics of the on/off solenoid valve pneumatic system (8) where the initial state vector X is averaged. Similarly to the servo valve pneumatic actuators, a part of the dynamics of (10) depends on the sign of the input u; if $u \geq 0$, chamber P is charging while chamber N is discharging and vice versa. Unlike power converters as SEPIC, which has two modes and a unique averaged model, pneumatic actuators have seven modes, which makes the elaboration of a unique averaged model similar to those of power converters impossible.*

Remark 3.3 *Unlike the switched model (4) with four binary discrete inputs the averaged model (10) has the advantage of using only one continuous input $u \in [-1, 1]$. The dynamics $g_1(X_a)$ and $g_2(X_a)$ correspond to the variation of actuation between the modes $F^{(6)}$ and $F^{(7)}$, which performed gradually and smoothly unlike the switched model (4) where the switch are instantaneous.*

The state averaged model of (10) belongs to the class of dynamical systems given by:

$$\dot{X}_a = AX_a + \Phi(X_a) + \mathcal{G}(X_a)u \qquad\qquad u \in [-1, 1] \tag{11}$$

with:

$$A = \begin{pmatrix} A_1 & 0 \\ 0 & A_2 \end{pmatrix} \quad A_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad A_2 = 0$$

$$\Phi(X_a) = \begin{pmatrix} x_2 \\ x_3 \\ \varphi_1(X_a) \\ \varphi_2(X_a) \end{pmatrix} \quad \mathcal{G}(X_a) = \begin{pmatrix} 0 \\ 0 \\ g_1(X_a) \\ g_2(X_a) \end{pmatrix}$$

Compared to the output averaged model in [13, 14] the new model (11) is a full state averaged model. It can be used to satisfy control objectives but also for designing observers. In the next section, the averaged model (11) will be used to design a sliding mode observer.

4 Sliding mode Observer

4.1 Observability

The observability of a system expresses the possibility to reconstruct its initial conditions, with nothing else than the measure of its inputs and outputs. The observability analysis allows to determine the minimum number of ouputs to have the observability of a system [26]. Let us consider the following general form of nonlinear system:

$$(\Sigma) : \begin{cases} \dot{X} = f(X, u) \\ Y = h(X) \end{cases} \tag{12}$$

Where $X(t) \in \mathcal{V}$ (an open set of $\mathbf{R}^n$), the input $u(t) \in \mathcal{U}$ where $\mathcal{U}$ is measurable set of $\mathbf{R}^m$.

Definition 4.1 *The system (12) is said to be observable if for any initial conditions X_0 and $\bar{X}_0$, there exists a control input u such that the corresponding outputs trajectories $h(X_0(t))$ and $h(\bar{X}_0(t))$ are not equal for all time $t \in [t_0; t_0 + T_h]$ with t_0 an initial instant and $T_h > 0$ a time horizon [27].*

Thus, the concept of observability is based on the possibility to distinguish two distinct initial conditions using two different outputs. It can easily proved that system (11) could be observable using only two outputs $y_{1_a} = x_{1_a}$ and $y_{2_a} = x_{4_a}$ [17]. Hence, system (11) can be rewritten as:

$$\begin{cases} \dot{X}_a = AX_a + \Phi(X_a) + \mathcal{G}(X_a)u \\ Y_a = CX_a \end{cases} \tag{13}$$

with:

$$C = \begin{pmatrix} C_1 & 0 \\ 0 & C_2 \end{pmatrix}, C_1 = (1 \quad 0 \quad 0) \text{ and } C_2 = 1$$

4.2 Sliding Mode Observer Design

Given a switched pneumatic model that can be averaged and rewritten as the model (13), consider the following standard sliding mode observer [21]:

$$\begin{cases} \dot{\widehat{X}}_a = A\widehat{X}_a + \Phi(\widehat{X}_a) + \mathcal{G}(\widehat{X}_a)u \\ \qquad -K(\widehat{Y}_a - Y_a) - Lsign(\widehat{Y}_a - Y_a) \\ \widehat{Y}_a = C\widehat{X}_a \end{cases} \tag{14}$$

where:

$$K = \begin{pmatrix} K_1 & 0 \\ K_2 & 0 \\ K_3 & 0 \\ 0 & K_4 \end{pmatrix}, \quad L = \begin{pmatrix} L_1 & 0 \\ L_2 & 0 \\ L_3 & 0 \\ 0 & L_4 \end{pmatrix}$$

K is chosen in order to make $A - KC$ Hurwitz.

System (14) is an observer for system (13) where the observation errors converge toward zero in finite time if L_1, L_2, L_3 and L_4 satisfy the following conditions:

$$\begin{cases} L_1 > \max |e_2| \\ L_2 > 0 \\ -\dfrac{\vartheta - \sqrt{|\zeta|}}{P_3} L_1 < L_3 < -\dfrac{\vartheta + \sqrt{|\zeta|}}{P_3} L_1 \\ L_4 > \max |\delta_2| \end{cases} \tag{15}$$

with:

$$\zeta = -\frac{4L_2P_2P_3}{L_1}\frac{\delta_1}{e_2^2 + e_3^2}e_3 \quad \vartheta = -\left(P_2 + \frac{\delta_1}{e_2^2 + e_3^2}e_2P_3\right)$$

where P_2 and P_3 are strictly positive scalars. and $e_2 = \widehat{x}_{2_a} - x_{2_a}$, $e_3 = \widehat{x}_{3_a} - x_{3_a}$. δ_1 and δ_2 are given by:

$$\delta_1 = \varphi_1(\widehat{X}_a) - \varphi_1(X_a) + (g_1(\widehat{X}_a) - g_1(X_a))u$$

$$\delta_2 = \varphi_2(\widehat{X}_a) - \varphi_2(X_a) + (g_2(\widehat{X}_a) - g_2(X_a))u$$

Proof The proof is based on Lyapunov theory. System (13) is observable using two outputs y_{1_a} and y_{2_a} which give two accessible observation errors $e_1 = \widehat{x}_{1_a} - x_{1_a}$ and $e_4 = \widehat{x}_{4_a} - x_{4_a}$.

The other errors, e_2 and e_3, are given by: $e_2 = \widehat{x}_{2_a} - x_{2_a}$ and $e_3 = \widehat{x}_{3_a} - x_{3_a}$. The error dynamics are given by

$$\left\{\begin{array}{l} \dot{e}_1 = e_2 - K_1 e_1 - L_1 \mathrm{sign}(e_1) \\ \dot{e}_2 = e_3 - K_2 e_1 - L_2 \mathrm{sign}(e_1) \\ \dot{e}_3 = \delta_1 - K_3 e_1 - L_3 \mathrm{sign}(e_1) \\ \dot{e}_4 = \delta_2 - K_4 e_4 - L_4 \mathrm{sign}(e_4) \end{array}\right. \tag{16}$$

In order to make e_1 converge toward zero, one needs to define a sliding surface $s_1 = e_1$. Then, a Lyapunov function $V_1 = \dfrac{1}{2}e_1^2$ can be chosen. The convergence of the estimation error to zero is obtained if $\dot{V}_1 < 0$. Hence, L_1 has to satisfy:

$$\left\{\begin{array}{lll} L_1 > \ \ e_2 - K_1 e_1 & \text{if} & e_1 > 0 \\ L_1 > -e_2 + K_1 e_1 & \text{if} & e_1 < 0 \end{array}\right.$$

Both conditions can be satisfied by estimating the worst case scenario [26] $i.e.$ considering the maximum value of e_2 that can occur. Hence, both condition are satisfied if:

$$L_1 > \max |e_2|$$

Then, the sliding surface s_1 is attractive and an ideal sliding motion takes place on this surface, $e_1 = 0$ and $\dot{e}_1 = 0$. Using (16), one can write: $\mathrm{sign}(e_1) = e_2/L_1$. The error dynamics becomes

$$\left\{\begin{array}{l} \dot{e}_1 = 0 \\ \dot{e}_2 = e_3 - L_2 e_2/L_1 \\ \dot{e}_3 = \delta_1 - L_3 e_2/L_1 \end{array}\right.$$

Then, the dynamics of e_2 and e_3 can be written as:

$$\underbrace{\begin{pmatrix} \dot{e}_2 \\ \dot{e}_3 \end{pmatrix}}_{\dot{e}_{23}} = \underbrace{\begin{pmatrix} -L_2/L_1 & 1 \\ -L_3/L_1 & 0 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} e_2 \\ e_3 \end{pmatrix}}_{e_{23}} + \underbrace{\begin{pmatrix} 0 \\ \delta_1 \end{pmatrix}}_{\Upsilon}$$

The convergence of e_{23} toward zero is ensured if $\dot{V}_2 < 0$, where $V_{23} = e_{23}^T P e_{23}$ and P is a matrix given by $P = \begin{pmatrix} P_2 & 0 \\ 0 & P_3 \end{pmatrix}$. Recall that P_2 and P_3 are strictly positive scalars. Then, the derivative of V_2 with respect to time is given by:

$$\dot{V}_{23} = -e_{23}^T \underbrace{\left(-PA - A^T P - \frac{e_{23}\Upsilon^T P}{e_{23}^T e_{23}} - \frac{P\Upsilon e_{23}^T}{e_{23}^T e_{23}} \right)}_{M} e_{23}$$

To make $\dot{V}_{23} < 0$, one needs to guarantee that M is strictly positive. M is given by:

$$M = \begin{pmatrix} 2\dfrac{L_2 P_2}{L_1} & \dfrac{L_3}{L_1}P_3 - P_2 - \dfrac{e_2 \delta_1 P_3}{e_2^2 + e_3^2} \\ P_3 \dfrac{L_3}{L_1} - P_2 - \dfrac{P_3 \delta_1 e_2}{e_2^2 + e_3^2} & -2\dfrac{P_3 \delta_1 e_3}{e_2^2 + e_3^2} \end{pmatrix}$$

using Schur's Lemma [28], M is positive definite if and only if:

$$\begin{cases} 2\dfrac{L_2P_2}{L_1} > 0 \\ -4\dfrac{P_3\delta_1 e_3}{e_2^2+e_3^2}\dfrac{L_2P_2}{L_1} - \left(\dfrac{L_3}{L_1}P_3 - P_2 - \dfrac{e_2\delta_1 P_3}{e_2^2+e_3^2}\right)^2 > 0 \end{cases}$$

Hence, the conditions on L_2 and L_3 given in (15) are obtained and the errors e_2 and e_3 converge toward zero. Finally for the remaining error e_4, similarly to e_1, a Lyapunov function $V_4 = \dfrac{1}{2}e_4^2$ could be defined. In order to make $\dot{V}_4 < 0$, L_4 must satisfy:

$$\begin{cases} L_4 > \quad \delta_2 - K_4 e_4 & \text{if} \quad e_4 > 0 \\ L_4 > -\delta_2 + K_4 e_4 & \text{if} \quad e_4 < 0 \end{cases}$$

Both conditions can be satisfied if $L_4 > \max|\delta_2|$. $\square$

Remark 4.1 *The proof above gives conditions on L in order to make the observation errors converge towards zero. Please note that those conditions hold even with the switching in $\mathcal{G}(\widehat{X}_a)$ by considering the worst case scenario in both cases: $u \geq 0$ or $u < 0$ i.e. considering the worst estimation of δ_1 and δ_2 in both cases. The proof thus holds for any input $u \in [-1, 1]$ of the averaged model (10).*

5 Experimental Results

The observer of (14) is used in closed loop *i.e.* its variables $\hat{X}_a$ are used in the control strategy. Then, the control input u is used to calculate the different coefficients $d^{(1)}$, $d^{(6)}$ and $d^{(7)}$. Finally, using a PWM technique on each valve i, the different inputs U_i are generated. The pneumatic solenoid valves (Matrix model GNK821213C3K) used to control the air flow have switching times of approximately 1.3 ms (opening time) and 0.2 ms (closing time). With such fast switching times, the on/off valves are appropriate for the purposes of control and observation. The controller and the observer are implemented using a dSPACE board (DS1104), running at a sampling rate of 500 Hz. This sampling rate has been chosen according to the open/close bandwidth of the valves opening and closing time smaller than 1.3 ms and to enable an acceptable tracking response.

The benchmark used to validate the observer has three sensors: a position sensor (y) and two pressure sensors (P_P and P_N). However, the observer (14) only uses the measures of y and P_P to estimate the velocity $\dot{y}$ and the pressure P_N. The measurement the pressure P_N is used neither in the observer nor in the control strategy. It is only used to validate the estimation of the pressure P_N given by the observer. The benchmark parameters are summarized in TABLE 1.

The authors of [14, 15] used the output averaged model to control the position of the actuator y with a sliding mode control strategy [22, 23]. The objective is to make the piston's position y track a desired sinusoidal trajectory $y_d(t)$. Using the sliding mode control strategy, the control input u is given by:

Table 1: Benchmark parameters

Parameter	Values
m: Total mass of the load and the piston	0.9 kg
b_v: Viscous friction coefficient	50 N.s.m^{-1}
l: Total length of the chamber	0.075 m
A_P: Piston cylinder area in chamber P	1.81×10^{-4} m ^{2}
A_N: Piston cylinder area in chamber N	1.81×10^{-4} m ^{2}
k: Polytropic constant	1.2
r: Perfect gas constant	286.68 J.(kg.K)$^{-1}$
T: Air temperature	296 K
P_S: Source pressure	3.01 bar
P_E: Exhaust pressure	1.01 bar
b_{crit}: Critical pressure ratio	0.493
C_{val}: Mass flow rate constant	3.4×10^{-9} kg.(s.Pa)$^{-1}$

$$u = \begin{cases} \dfrac{\widetilde{u} - K_c sign(s_p)}{\dfrac{krT}{m}\left(\dfrac{Q(P_S, \hat{x}_{4_a})}{(l/2 + \hat{x}_{1_a})} + \dfrac{Q(\psi(\hat{X}_a), P_E)}{(l/2 - \hat{x}_{1_a})}\right)} & \text{if } \widetilde{u} \geq 0 \\ \dfrac{\widetilde{u} - K_c sign(s_p)}{\dfrac{krT}{m}\left(\dfrac{Q(\hat{x}_{4_a}, P_E)}{(l/2 + \hat{x}_{1_a})} + \dfrac{Q(P_S, \psi(\hat{X}_a))}{(l/2 - \hat{x}_{1_a})}\right)} & \text{if } \widetilde{u} < 0 \end{cases}$$

Where: $\widetilde{u}$ is the equivalent control, K_c is a time variant gain to ensure robustness (See [14] for more details), s_p is the sliding surface, it is given by:

$$s_p = \left(\frac{d}{dt} + \lambda\right)^3 \int_0^\tau e_p d\tau$$

where λ is a positive tunning parameter and e_p is the position tracking error: $e_p = y_d - \hat{x}_{1_a}$. The equivalent control $\widetilde{u}$ is given by:

$$\widetilde{u} = \dddot{y}_d + \frac{k}{m}\left(\frac{A_P \hat{x}_{4_a}}{l/2 + \hat{x}_{1_a}} + \frac{A_N \psi(\hat{X}_a)}{l/2 - \hat{x}_{1_a}}\right)\hat{x}_{2_a}$$
$$+ \frac{b_v}{m}\hat{x}_{3_a} - 3\lambda \ddot{e}_p - 3\lambda^2 \dot{e}_p - \lambda^3 e_p$$

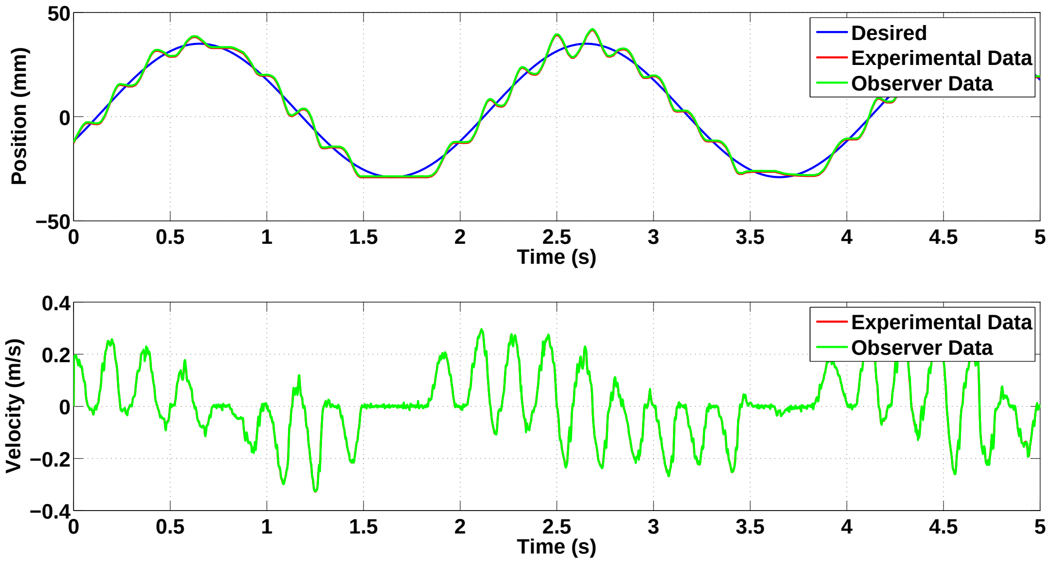

Figure 2: Real and estimated position and velocity

The constraints (15) give sufficient conditions for the system (14) to be a finite convergence time observer for system (13). Please note that they do not offer a direct method to calculate L. The constraints (15) include the observation errors e_2 and e_3 which are not available. However, in practice, one can use experimental data to estimate the worst case scenario *i.e.* the worst estimation of e_2 and e_3 in order to satisfy the constraints.

The parameters K and L of the observer (14) and λ of the sliding mode strategy are:

$$K = \begin{pmatrix} 100 & 0 \\ 1840 & 0 \\ 12000 & 0 \\ 0 & 10 \end{pmatrix} \quad L = \begin{pmatrix} 5 & 0 \\ 2 & 0 \\ 15 & 0 \\ 0 & 20 \end{pmatrix} \qquad \lambda = 60$$

Fig.2 and Fig.3. show the estimation of the averaged state variable corresponding to y, $\dot{y}$, P_P and P_N. y and P_P are measured directly and used in the sliding mode observer: their estimations are close to the experimental values. For the estimated velocity $\hat{x}_{2_a}$, it is close to the velocity calculated using a robust differentiation algorithm [29]. For the pressure P_N, there is a slight error in the estimation.

Fig.4 shows the estimation errors. The y estimation error does not exceed 0.1 mm. The

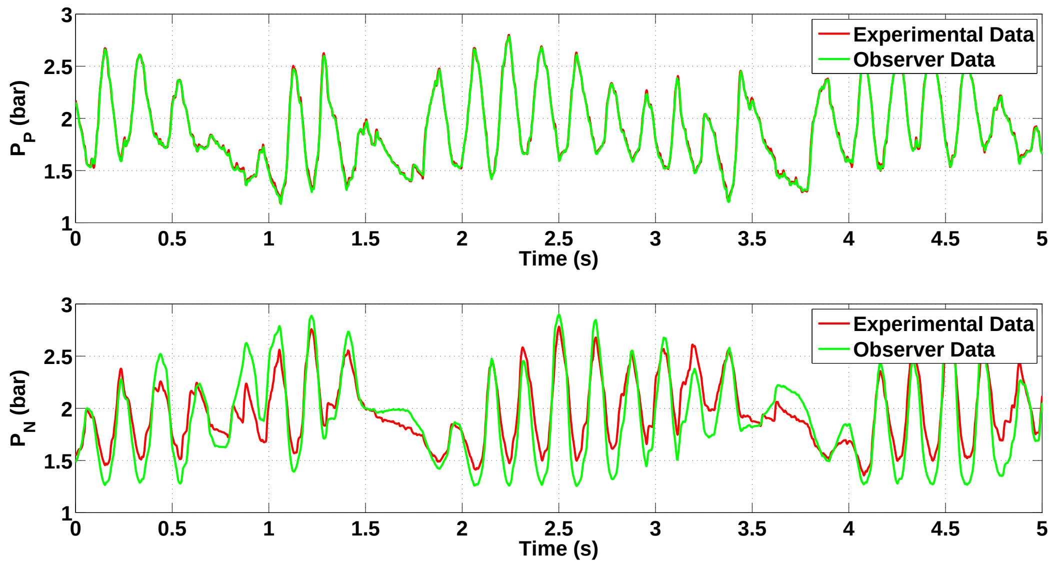

Figure 3: Real and estimated pressures

velocity estimation error does not exceed 0.03 m/s. While the estimation error of P_P is less than 0.1 bar for values reaching 3 bar, the estimation error of P_N is much more important and it can reach 0.5 bar. Hence, the sliding mode observer can achieve good performance with small estimation errors. Furthermore, it is clear that the control objective which is to track y_d is satisfied as it is shown in Fig.2 with small oscillations of the measured position around y_d. However, this is due the dry frictions which were not considered in the modeling step in the design of the control strategy.

6 Conclusion

In this paper, an averaged state model is proposed for a pneumatic actuator with four on/off solenoid valves. The advantage of this averaged model is to have one continuous input instead of having four binary inputs. This model can be easily used to design control

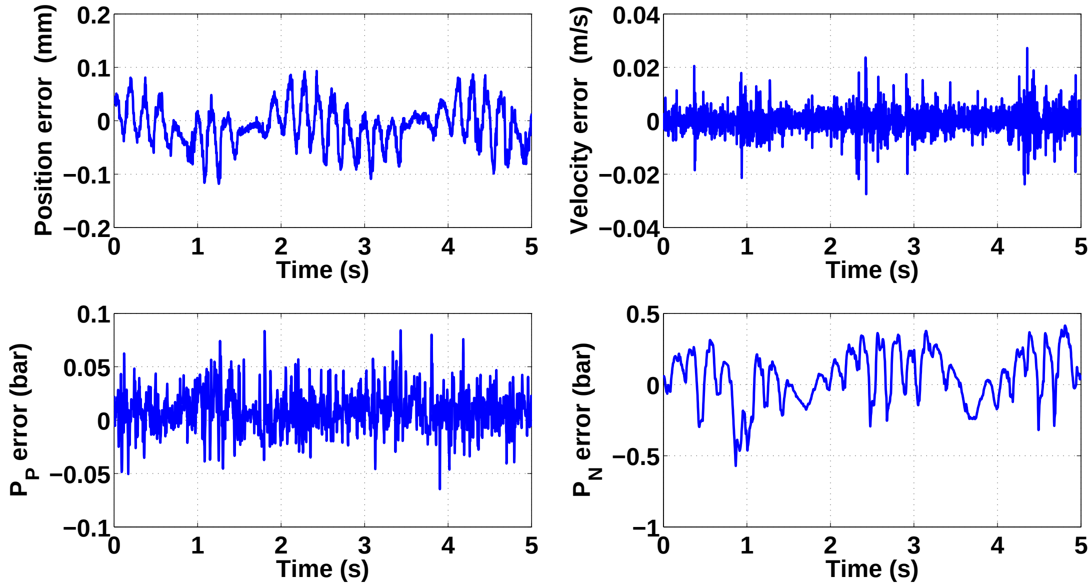

Figure 4: Estimation errors

strategies, observers or fault detection. Based on this model and Lyapunov theory, a sliding mode observer is designed. The estimation errors of this nonlinear observer have the property to converge towards zero in a finite time if the correction gains are well chosen. This observer has been validated experimentally in closed loop to track a desired trajectory for the actuator position. The estimation errors are small and the control objective is satisfied. The improvement of the estimation quality and the design of other types of observers are proposed as perspective work.

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