Limits and rules of use of a dynamic flux tube model

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***Abstract* — The simulation of electromagnetic devices remains an essential research tool for optimization. Nowadays, the miniaturization of devices leads to increase the supply frequency, in this case the material is hardly sollicitated. An accurate description of dynamical material law must be introduced in the magnetic circuit representation. Our team has already created a dynamic behavioural magnetic model which lumps together all dynamic effects developped in the circuit. The main assumption of this model is to consider that all magnitudes are homogeneous in the cross section. The aim of this paper is to analyse this assumption and to define a validity domain and rules of use.**

I. INTRODUCTION

Nowadays, the miniaturization of electromagnetic devices leads to increase the fundamental supply frequency, moreover, thes systems are usually fed by static converters. Thus the magnetic materials of these devices are hardly stressed due to the fast dynamic working conditions.

The design of these devices requires simulation tools, which need to take into account accurately both the description of the geometry of the system and dynamical material laws. A 3D field calculation including a dynamic realistic material law would lead to a prohibitive calculation time and numerical difficulties ; at present time, it hardly seems possible with standard computers. Some authors [1] [2] consider dynamical effects due to the material in a 2D field calculation considering a modified law of the material.

Our laboratory has developped a magnetic dynamical flux tube model [3]. This model points out the dynamical behaviour of the material and considers a simple geometry (flux tube with a constant cross-section). The association of different flux tubes allows to simulate a real industrial device [4]. This model has already been effectively used to represent different industrial devices. Nethertheless, the main assumption of the model which is to lump together the different dynamical effects has not yet been tested.

The purpose of this paper, is to analyse in details the main assumtion of this model and to define its validity domain and its rules of use.

II. DESCRIPTION OF THE DYNAMICAL FLUX TUBE MODEL

The model allows to obtain the dynamic behaviour of a magnetic circuit with a constant cross-section, where the anisotropy of the material is neglected. The different dynamic effects developped in the circuit (wall motion, macroscopic eddy currents) are considered in this model by a single representation. The dynamic behaviour of the circuit is described by a first order differential equation which can be represented by a bloc-diagram in Fig. 1

The quantity B_a is the average flux density in the cross-section, H_{dyn}, is the excitation field applied at the surface, and $H_{stat}\left(B_a\right)$, is a fictitious static excitation field value

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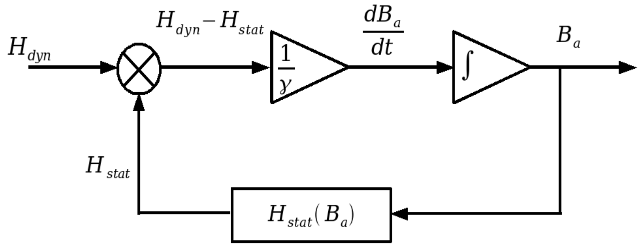

Fig. 1. dynamic bloc-diagram of the flux tube model

which corresponds to a given value of B_a.The magnetization history of the material can be taken into account by considering an hysteretic static model for $H_{stat}\left(B_a\right)$. The parameter γ is a dynamic behavioural parameter. Its value has to be fitted by comparing simulated and measured dynamic loops. This model presents several advantages :

- it requires only one parameter γ (apart those required to model the static hysteresis), supposed to be independant of the waveform and velocity of the excitation
- it is a time domain model
- the calculation time is short
- it is reversible ($B_a\left(H_{dyn}\right)$ or $H_{dyn}\left(B_a\right)$
- it can be easily introduced in many kinds of softwares (circuit type, design, simulation…

The main assumption of this model is to consider the homogeneity of the phenomena in the cross-section of the flux tube. This model has been tested for different materials and devices. For the representation of ferrite components, this model allows to obtain accurate results, in that the assumption of homogeneity holds with good approximation. For other materials, where the conductivity leads to a field diffusion across the section, the model provides quite satisfying results.

III. VALIDITY OF THE MAIN ASSUMPTION

The dynamic flux tube model lumps together all dynamic effects developped in the circuit. The value of the dynamic parameter γ, determine the width of the dynamic loop of the material. This value depends on the thickness of the circuit, the resistivity ρ of the material, the wall motions, the static hysteresis phenomenon and the permeability of the material.

In the aim to simplify the problem, we limit our study to simple geometries : we consider torus samples where the anisotropy phenomenon is negligible[1].

Consider the formula (1) associated to the dynamic flux tube model.

$$H_{dyn} - H_{stat}(B_a) = \gamma . \frac{dB_a}{dt} \quad (1)$$

This formula can be compared with the expression (2) defined in the case of a magnetic lamination if the skin effect, saturation and edge effects are negligible.

$$H_{tot} = H_{stat}(B_a) + \frac{\sigma . d^2}{12} . \frac{\partial B_a}{\partial t} \quad (2)$$

Where : H_{tot} = excitation field at the lamination surface when eddy currents are induced in the cross section, d = thickness of the lamination, σ = conductivity of the material, B_a = averaged magnetic flux density over the thickness. In both cases, the homogeneity of magnetic data assumption is assumed. An analogy between (1) and (2) allows to estimate the value of the parameter γ :

$$\gamma = \frac{\sigma . d^2}{12} \quad (3)$$

In the aim to validate on the one hand the estimation of γ and on the other hand the lumped model, we carry out successively tests on four samples numbered 1 to 4.

A. Sample tests n1

We consider a toroidal sample made of NiFe(50/50) main characteristics are reported in the table I. The value of ρ is given by the manufacturer and μ_r is estimated considering the static characteristic $H_{stat}(B_a)$ in its linear part. Due to the large thickness of the torus, the weak resistivity and its high relative permeability, eddy currents are not negligible in this sample. A sinusoidal excitation field H is imposed at the surface of the sample.

TABLE I. SAMPLE DATA

D_{out} (mm)	D_{in} (mm)	thickness (mm)	ρ $(\Omega.m)$	μ_r
18.8	9.9	1.1	48.10^{-8}	100000

The table II regroups for different working frequencies (25Hz, 50Hz, 100Hz and 150Hz) :

- the skin depth δ :

$$\delta = \sqrt{\frac{\rho}{\mu_r . f}} \quad (4)$$

- the relative error ε_1 between the areas of the simulated and measured loops (which is representative of the losses)
- the quadratic error ε_2 defined by the formula (5).

$$\varepsilon_2 = \sqrt{\frac{1}{n} \sum_{i=1}^{n} B_{simulated}(i) - B_{measured}(i)} \quad (5)$$

We observe that the quadratic error ε_2 increases with the frequency. This result agrees with the indication given by

TABLE II. SAMPLE N1 RESULTS

f (Hz)	25	50	100	150
δ (mm)	0.22	0.16	0.11	0.09
ε_1 (%)	19	4.2	23	36.7
ε_2 (%)	14.5	22.7	29.8	37.2

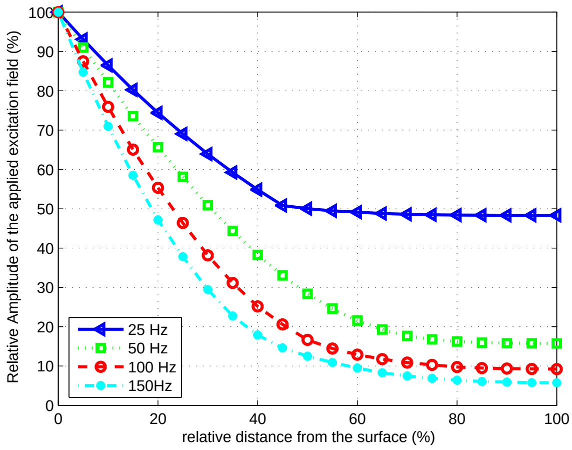

Fig. 2. Normalized amplitude of the excitation field H versus the relative thickness

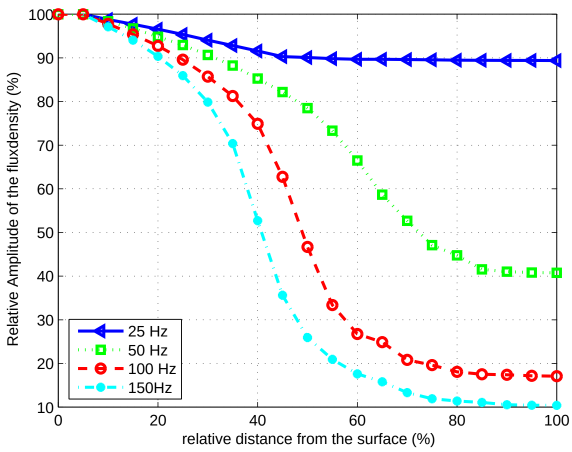

Fig. 3. Normalized amplitude of the flux density B versus the relative thickness

the skin depth value : in fact when δ becomes comparable to the size of the section, the assumption of homogeneity doesn't hold. However, we point out that the classical formula (4) has been obtained for *linear materials* and with a *semi-infinite plane conductor* : hence it provides a very approximate result with our samples (for instance see [5] for analytical computations of δ with different geometries). Local information inside the cross section is not available. So as to obtain this information, we use a numerical tool based on the magnetic field diffusion [6]. The figure (2) shows the simulated excitation field H, normalized with respect of its intensity at the surface, as a function of its relative position in the thickness of the lamination ($H(0)$ = field at the surface, $H(100\%)$ = field in the middle of the thickness) for different frequencies. One observes that even at 25 Hz the excitation field H is not uniform ; however, the skin effect is less important than the prediction obtained by (4)

In the same way, the figure (3) shows the normalized flux density as a function of the relative geometric position through the thickness. One sees that the saturation phenomenon tends to homogeneize the flux density through the thickness of the lamination. Hence, the validity of the flux tube model is enlarged.

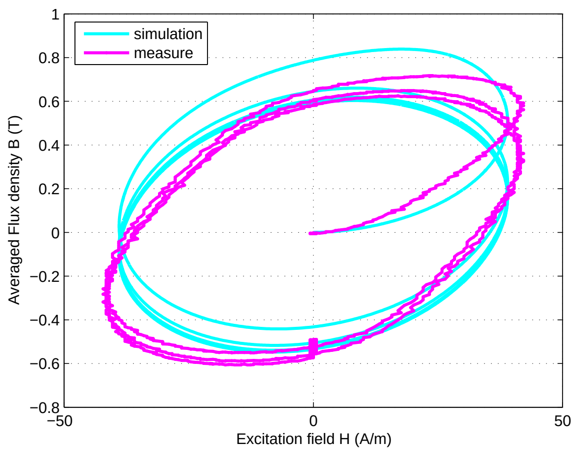

Fig. 4. Measured and simulated loops

A last observation concerns the comparison between both errors ε_1 and ε_2 carried out for 50Hz frequency operation where a discrepancy appears. The figure 4 shows the simulated and measured loops for this frequency. Both loops have nearly the same area but are hardly different. These different results bring out several preliminary conclusions :

- If this model provides accurate results on the estimation of the area of the loop, we must be more careful concerning the estimation of the waveform when a great heterogeneity exits in the cross section.
- The skin depth provides information about the validity domain of the tube flux model
- The saturation phenomenon tends to homogeneize the flux density distribution, and thus enlarges the limits of use of our model.

B. Sample test n2

The second sample is a stack of rings (thickness 0.2mm) of SiFe(3%). The material resistivity given by the manufacturer

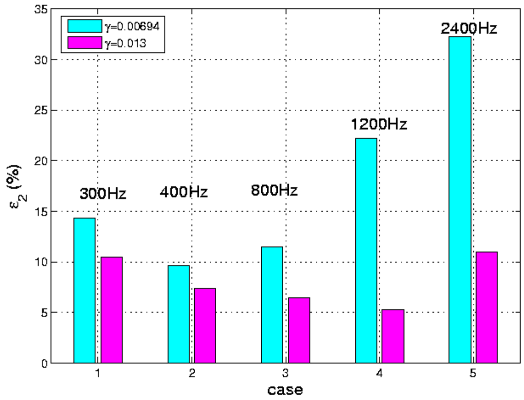

Fig. 5. Quadratic error

is $48.10^{-8}\Omega.m$, its relative permeability μ_r estimated from the linear part of the quasi-static characteristic is about 6000. Conversely to the previous sample, few eddy currents can be induced in the circuit (thin ring) : by using (4), one sees that the skin effect can be neglected until at least 1500 Hz.

First, we analyse the results provided by the model considering the value of $\gamma = 6.94.10^{-3}$ computed by (4). We carry out some 300Hz to 2400Hz simulations with an imposed excitation field H at the surface. The figure 5 shows the quadratic error ε_2 computed for each frequency. An important error is observed, and can not be explainde by the skin effect, which is negligible until at least 1500Hz frequency. Therefore, we carried out other simulations by taking the value of γ which minimizes the quadratic error ε_2 at the frequency of 800 Hz. The obtained value of γ is now 0.013. The quadratic error ε_2 is reduced for all the considered frequencies (not only at 800Hz), as shown in figure 5.

Hence, the formula (4) is no more applicable. By considering the assumption of magnetic losses separation [7], the excitation field H_{dyn} at the surface of the sample can be decomposed by the sum of different terms : $H_{stat}(Ba)$ due to the static law of the material, H_{edd} due to the eddy currents, and H_{exc} due to the effects of wall motions.

$$H_{dyn} = H_{stat}(Ba) + H_{edd} + H_{exc} \qquad (6)$$

We now compare (1) to (6), by using formula (2), so as to obtain the following expression for H_{exc} :

$$H_{dyn} = \left(\gamma - \frac{\sigma.d^2}{12}\right).\frac{dB_a}{dt} \qquad (7)$$

The dynamic flux tube model lumps different dynamic effects, which are represented by a sole formulation. The representation of dynamical effects associated with the wall motions has a similar formulation as those linked to eddy currents. This result has already been validated in previous works [8] to simulate ferrite circuits. In this kind of material, dynamic effects due to the wall motions are dominant, and the flux tube model gives accurate results. The assumption of magnetic losses separation together with the identification of the parameter γ allows to specify the different energy dissipations (W/kg) associated respectively with eddy currents and to wall motions.

$$P_{edd} = \frac{\sigma.d^2}{12}\oint\frac{dB_a}{dt}.dB_a \qquad (8)$$

$$P_{exc} = \left(\gamma - \frac{\sigma.d^2}{12}\right) \oint \frac{dB_a}{dt}.dB_a \quad (9)$$

Many simulations using the dynamical flux tube model have been carried out by using the same value of $\gamma = 0.013$. The contribution of each kind of losses cannot be measured separately. So as to obtain such a comparison, we use the results obtained with the diffusion model (*diff*) [6]. Different simulations have been carried out by imposing a sinusoidal excitation field H at the surface of the sample, for the range of frequencies (800 Hz - 2400 Hz). The results are regrouped in the table III. P_T are the total losses, P_{hyst} are the static losses, P_{edd} are the losses due to eddy currents and P_{exc} are the excess losses.

TABLE III. SAMPLE N2 RESULTS

f (Hz)	**800**		**1200**		**2400**	
model	*DFTM*	*Diff*	*DFTM*	*Diff*	*DFTM*	*Diff*
P_T (mW/kg)	0.63	0.58	0.75	0.7	0.96	0.89
P_{hyst} (mW/kg)	0.32	0.38	0.32	0.4	0.25	0.37
P_{edd} (mW/kg)	0.16	0.14	0.21	0.2	0.35	0.32
P_{exc} (mW/kg)	0.16	0.06	0.21	0.1	0.35	0.2

These results lead to different investigations :

- P_{hyst} are important in this range of frequencies, thus an accurate static hystersis model $H_{stat}\,(Ba)$ must be used.
- By considering (8) and (9) formulas, the power losses due to eddy currents and to the wall motions are proportional to the coefficients $\sigma.d^2/12$ and $\left(\gamma - \sigma.d^2/12\right)$
- The dynamic flux tube model allows to obtain accurate results, until the skin effect is not dominant.

C. Sample tests n3 and n4

These tests have been carried out on rings made of CrNiFe. The resistivity is $\rho = 94.10^{-8}\Omega.m$, the relative permeability related to the linear part of the static first magnetization is about 50000. The manufacturer produces two series of rings with different thicknesses: $d =$ 0.2 mm and 0.6 mm. Manufacturing precautions are taken into account in the aim to ensure the same magnetic properties of both ring series. So as to avoid the static hysteresis phenomenon, we carry out simulations limited to the first magnetization. For the previous sample, we obtain the value of γ which minimizes the quadratic error ε_2 for the frequency 50 Hz. The obtained results are regrouped in table IV.

The optimal value of γ depends upon the thickness of the ring. This result agrees with the formula (4). Conversely, we observe that the parameter $\gamma - \frac{\sigma.d^2}{12}$ seems to be constant.

This parameter is characteristic of dynamic effects due to wall motions. This result is coherent, in that wall motions are linked to the structure of the material, and are independant of the geometry of the sample.

TABLE IV. SAMPLE N3 AND 4 RESULTS

d (mm)	**0.2**	**0.6**
ε_2 (%)	1.77	4.37
γ	0.05	0.033
$\frac{\sigma.d^2}{12}$	0.00354	0.0319
$\gamma - \frac{\sigma.d^2}{12}$	0.00146	0.0011
$\frac{2.\delta}{d}$	3.08	1

IV. CONCLUSION

The limits of a dynamic flux tube model is analysed by considering tests (simulations + measurements) on 4 different samples. This model lumps dynamic effects, which are represented by a same formulation. It gives accurate results when the skin effect is negligible, and when a weak heterogeneity of magnetic data exists. Nevertheless, the saturation phenomenon enlarges the validity domain of the model. The value of γ can be decomposed into two terms : the first one is linked to eddy currents, and is given by the classic expression depending of σ and d (2). The second one seems to be a constant value, depending upon the structure of the material, and not upon the geometry. This decomposition based on the assumption of losses contributions, allows to obtain separately an estimation of the losses linked respectively to the static hysteresis, to eddy currents and to wall motion effects. The estimation of γ is still empiric; we are working on this subject.

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