Slow transient pressure regulation in water distribution systems

Olivier Piller, Guillaume Gancel, Marco Propato

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Olivier PILLER, Guillaume GANCEL and Marco PROPATO
Hyd. and Civ. Eng. Res. Unit, CEMAGREF, Bordeaux, France
{olivier.piller, guillaume.gangel& marco.propato} @bordeaux.cemagref.fr

Abstract

An optimal control problem is formulated for pressure regulating devices using a slow transient model (rigid water column) for water distribution networks. Pipe flows and node heads are sought to (1) minimize deviations from pressure targets at pressure control valves and (2) to satisfy the slow transient state equations (conservation of mass and energy). The continuous control variable is the degree of closing of pressure regulating devices ranging from zero (fully open) to one (closed). The control variable time derivative is bounded as well. As in conventional steady state analysis, it is possible to reduce the state equation system so that it is loop-based. Sensitivity equations with respect to the control variable are derived from the state equations. An efficient method is presented to numerically integrate these stiff ordinary differential equations in a coupled manner. The control variables are then updated using Levenberg-Marquardt iterations. Finally, feasibility of the method and efficiency of the proposed algorithm are proved on study cases and real networks.

Keywords

Water Distribution Networks, Pressure Regulation, Control, Sensitivity Equations

1. INTRODUCTION

Slow transient (or rigid column) has been recently introduced in contrast to fast transient (water hammer analysis). In slow transient, valves are opened or closed slowly and inertia terms in the state equations do not take into account of immediate system response.

Next we provide a brief review of solution approaches to steady state without pressure control device. Within the framework of a problem aimed at minimizing energy, Collins *et al.* [1] were the first to model the operation of a Water Distribution Network (WDN) for steady state analysis. Partially closed valves, small diameter pipes and overestimation of consumption lead to stiff numerical problems. In these cases, a suitable stepsize [2] for the minimization of the objective function is required to prevent non-convergence of solutions. Prior to this time, the path method [3], simultaneous path methods [4], the linear method [5] and simultaneous nodes methods [6] were primarily employed. The major solution algorithms [7, 8] that derived from optimisation were found in the course of the following decade. A synthesis and a definitive conclusion were stated in the mid 1990's [9].

Nowadays, all the hydraulic software programs on the market or in public domain, include Pressure Regulating Devices (PRDs), but problems are nonetheless reported. The three most common Control Valves are Pressure Sustaining Valves (PSVs), Pressure Reducing Valves (PRVs) and Flow Control Valves (FCVs). Modelling such hydraulic devices using a discrete control problem formulation gives rise to numerical solution problems: searching for the states of the devices (open, closed, normal operation) can lead, even in simple configurations, either to problems of convergence or to incorrect solutions. "Clearly, more research work is needed to develop improved algorithms for solving systems with multiple pressure regulating devices" [10]. Recently, two specific formulations have been proposed for steady state operation of the PRDs and PSVs. The first one [11] adjusts the local head loss coefficients to satisfy the optimality of a least-squares problem where the residuals are the differences between the computed head and the head target. The numerical procedure has to be completed by minimizing partial least-squares without open or closed control devices that satisfy the optimality conditions. In a different manner, [12] have introduced a separate convex minimization problem for any pressure control device related to a set pressure node. The control variables are the generated local head losses and the corresponding partial gradient is energy balancing between a reference head node and the set pressure node. The necessary and sufficient optimality conditions for a Nash-Equilibrium are then simultaneously solved.

Nevertheless, steady state analysis has some limitations because in reality WDNs usually present gradually varied flow due to water demand and/or tank level fluctuations, or due to slow valve or pump operation. Existing EPS hydraulic models are not able to accurately reproduce the impacts of these slow transient phenomena [13]. In rigid column or slow transient analysis the inertial forces are included while the compressibility effects of both the fluid and pipe walls are neglected. A much larger time step can been used than in water hammer analysis and one and only one flow and tension can been defined for each pipe so that it

results in fair computational efforts. To support the last point, suitable graph theoretical tools can been introduced [14, 15].
A new algorithm is presenting for the solution of slow transient equations. Moreover, PRDs are incorporated in slow transient equations. To authors knowledge this is the first attempt. To keep the same graph for all the time of integration, the closure of valve is solved through a continuous formulation, a significant benefit for numerical calculation with respect to previous approach.
In this paper, we first recall the slow transient hydraulic balancing equations and give our solution algorithm. Then, we indicate how to take the pressure regulating devices into account. Next, we derive the sensitivity equations with respect to the control variable. Finally, results are presented and commented.

2. NETWORK EQUATIONS WITHOUT PRD

We give a briefly review of the slow transient hydraulic equations that can be derived from local partial differential equation Saint-Venant conservative equations for water hammer analysis assuming the fluid incompressible [16]. A great advantage is that it is possible to define the same cross-sectional flow rate along the pipe. Moreover, integrating the continuity equation on volumes surrounding nodes leads to Kirchoff's current laws. Similarly, by integrating momentum equation over the head between the start and the end node of a pipe and summing over loops or paths between head source nodes, results in Kirchoff's voltage laws where the head losses play the role of the voltage drops.

2.1 Slow transient mass and energy balancing

The WDN is represented by a graph, i.e., a set of pipes (or links) and nodes (reservoirs, free surface tanks and junctions). For each pipes $i = 1,...,a$, we search for the flow rate q_i and, for each junction node $j = 1,...,n$ we search for the total head: h_j. The initial levels at free surface tanks $h_f(0)$ are known. The full temporal profiles of head at the r resources nodes: $h_r(t)$ are also given (a constant value if a reservoir). Likewise, demand d_j at junction node j is considered as being known and a differentiable function of time.
The problem of water network equilibrium consists of solving for q, h_f, t and h in the system [15]:

$$\begin{cases} Aq = -d \\ S_f \dot{h}_f + A_f q = 0_f \\ t = {}^tAh + {}^tA_f h_f + {}^tA_r h_r = \left(h_{start(i)} - h_{end(i)}\right) \\ L\dot{q} - t + \xi(q) = 0_a \end{cases}$$

where the overdot denotes time derivatives, A is the incidence matrix reduced to the junction nodes, ${}^tA$ the matrix transpose, A_f (resp. A_r) the incidence matrix reduced to the free surface tank nodes (resp. to the reservoir nodes), S_f the inertia rate of tank is the diagonal matrix of tank cross-sectional area, L the inertia rate of pipes is the diagonal matrix defined by the pipe length divided by the product of the cross-sectional area and the acceleration of gravity, $\xi(q)$ the vector of friction head loss through the pipes and t is a tension, drop of head through the pipes.
The three first equations are linear and express conservation of mass and energy. The last, non-linear equation links head loss with flow.

2.2 The slow transient state equation

Finding a minimum spanning tree permits to define loop flow rate and to reduce the junction mass balance. Thus, previous algebro-differential system can be rewritten in a Cauchy problem form:

$$\begin{cases} L_c\dot{q}_c = M_0\left(-\xi(q) + {}^tA_f h_f + {}^tA_r h_r - L\dot{q}_d\right) \\ S_f \dot{h}_f = -A_f q \end{cases}$$
$$\text{with } q = {}^tM_0 q_c + q_d;\ q_d = \begin{pmatrix} -A_{tree}{}^{-1}d \\ 0_{a-n} \end{pmatrix};\ q_c(0) = q^0 \text{ and } h_f(0) = h_f^0 \text{ (IC)} \tag{1.1}$$

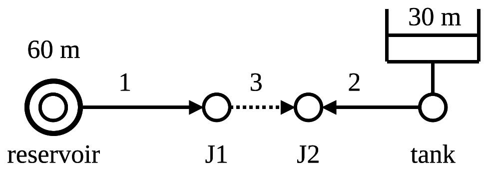
with q_c the loop flow rate vector, M_0 the fundamental loop matrix, $L_c = M_0 L\, {}^tM_0$ the inertia rate of loops, and q_d the flow rate gotten from the demands: d at junction nodes while making tree ascent. System (1.1) consists of the minimum set of independent equations and constitutes the slow transient state equation.
The head at junction nodes can be solved from the linear system:

$$\left(AL^{-1}\,{}^{t}A\right)h(t) = AL^{-1}\left(\xi(q) - {}^{t}A_r h_r - {}^{t}A_f h_f\right) - \dot{d} \tag{1.2}$$

There is no need to search for loops and paths between source nodes because M_0 can be derived directly from the tree/cotree decomposition:

$$M_0 = \left[-\,{}^{t}\left(A_{tree}{}^{-1}A_{cotree}\right) \;\vdots\; I_{a-n} \right] \text{ and } M_0\,{}^{t}A = 0_{a-n,n}$$

An example can be derived from Fig. 1. J1 and J2 are two junction nodes. Incidence matrices are independent of the states of valves (closed, open) and they reflect the structural link topology of the network. Also closed valve links need not to be eliminated as it is done in [15].



$$A = \begin{pmatrix} -1 & 0 & \vdots & 1 \\ 0 & -1 & \vdots & -1 \end{pmatrix}$$
$$A_r = \left(1 \quad 0 \;\vdots\; 0\right)$$
$$A_f = \left(0 \quad 1 \;\vdots\; 0\right)$$
$$M_0 = \left(1 \quad -1 \;\vdots\; 1\right)$$

$$q_1 = -q_2 = q_3$$
$$S_f \dot{h}_f = q_3$$
$$t_1 = h_r - h_1$$
$$t_2 = h_f - h_2$$
$$t_3 = h_1 - h_2$$
$$L_c \dot{q}_3 = h_r - h_f - (r_1 + r_2 + r_3) q_3 \left|q_3\right|$$

Fig. 1: example of slow transient formulation

Local existence and uniqueness result from the continuity of the functions involved. Besides, we can globalize this existence because we work with bounded energy. As a Lyapunov function that decreases strictly with the trajectory can be identified [14], steady state solutions are also globally asymptotically stable equilibrium points.

2.3 Solution algorithm

With an implicit scheme, perturbation of the systems could be observed with delay if the time step is too large. However, implicit schemes are unconditionally stable. That is the reason why semi-implicit methods have been preferred.
We rewrite the system (1.1) as below:

$$R\dot{x} = f(t, x)$$

Application of θ–scheme gives:

$$\frac{1}{\delta t} R(x^{k+1} - x^k) = \theta f(t^{k+1}, x^{k+1}) + (1-\theta) f(t^k, x^k)$$

The first-order Taylor expansion around (t^k , x^k) is:

$$f(t^{k+1}, x^{k+1}) = f(t^k, x^k) + \partial_x f(t^k, x^k)(x^{k+1} - x^k) + \delta t \partial_t f(t^k, x^k)$$

We finally get the semi-implicit scheme:

$$\begin{bmatrix} M_0\left(\delta t^{-1} L + \theta D_k\right){}^{t}M_0 & -\theta M_0\,{}^{t}A_f \\ \theta A_f\,{}^{t}M_0 & \delta t^{-1} S_f \end{bmatrix} \begin{pmatrix} q_c^{k+1} - q_c^k \\ h_f^{k+1} - h_f^k \end{pmatrix} = \begin{bmatrix} M_0\left(-\xi(q^k) + {}^{t}A_r h_r + {}^{t}A_f (h_f + \delta t \theta \dot{h}_f) - L(\dot{q}_d + \delta t \theta \ddot{q}_d)\right) \\ -A_f q^k \end{bmatrix}$$

This system can be solved in efficient manner with the same tips used in steady state cases: the matrix to inverse is very sparse and its structure is not time-dependant.
Rosenbrock method [17] is a generalisation of Runge Kutta that uses the Newton method and solves four linear systems equivalent to the previous one. To overcome numerical instabilities due to valve and pump operations and conditions (stiff phenomena), estimation of the truncating error between the third-order and fourth-order solutions controls the time step. Compared to the well-known Runge Kutta Felhberg scheme, the Rosenbrock method is faster and more accurate for solving stiff problems.
For numerical reasons, working in l/s rather than in m^3/s improves the conditioning of the system since friction coefficients are smaller one million times.

3. ADDING PRESSURE CONTROL VALVES

Next, we propose a problem formulation to describe valve operation that is independent of the device state (open, closed or normal). The solution algorithm uses equation system (1.1) to update the flows and the heads.

3.1 Functions and applications of the Pressure Regulating Devices

When required, the PRV operates to reduce pressure downstream from a pipe at higher pressure. PRVs regulate multilevel networks, to supply a low service network from a high service network, to protect a delicate sector from dangerous pressure levels, to protect the installations in customer homes or to save water by lowering pressure etc. In contrast, PSVs maintain pressure above a pre-set value upstream the device. For example, they are used to raise hydraulic grade lines to allow supply to high-level sectors.
We thus add a PRV (or a PSV) on a short link i with respect to a target pressure and therefore the piezometric head at the downstream end node: i_D (or upstream end: i_U, respectively):

$$h_{i_D} \le h_{i_D}^{max} \text{ for the PRV or } h_{i_U} \ge h_{i_U}^{min} \text{ for the PSV}$$

If S is the selection matrix of the s nodes with pressure settings, the complete set of constraints is written in matrix form:

$$Sh \le h_{set}$$

When in use, PRDs create a local head loss that depends on the status of the network, stabilizing the network as closely as possible to the selected values h_{set} (set pressure + ground level). It is therefore added to the friction loss $\xi(q)$ for edges with PRDs:

$$\xi^{STAB}(\lambda, q) = B(q).\lambda \text{ with } B(q) = \Lambda.{}^tS_q,\ \lambda_i = \frac{8\lambda_i^*}{g\pi^2\phi_i^4},\ i=1,...,s$$

where λ_i is a local head loss parameter with λ_i^* the dimensionless parameter in the local head loss $\lambda_i^* V_i^2 / 2g$ and ϕ_i the diameter of the pipe i, $\Lambda = \mathrm{diag}\left(q_i.\left|q_i\right|\right)$ and S_q is the matrix for selecting the edges equipped with a PRD. To account for flow reversal, a check valve is added to prevent back-flow, otherwise a local head loss occurs.

3.2 Least-squares minimization

A separate non-linear least-squares problem is introduced for each PRD. The optimal PRD state operation is found by minimizing the squared deviation from the PRD head target:

$$\begin{aligned} &\text{minimize} \quad c_i(\lambda_i) = \tfrac{1}{2}\left(Sh(\lambda) - h_{set}\right)_i^2 \\ &\text{subject to} \quad 0 \le \lambda_i \le \lambda_i^{max} \text{ primal feasibility (PF) condition} \end{aligned} \tag{1.3}$$

with $\lambda \to h(\lambda)$ defined implicitly by the system (1.2) coupled with (1.1). λ^{max} may be chosen to have all its components equal to 25,000,000 that corresponds to a local penalty head loss of 25m if the flow rate reaches 0,001 l/s. The existence of a global minimum is ensured by the continuity of c on a non-empty compact set (Weierstrass theorem). Using previous example network, Fig.2 shows the dependence of c as a function of lambda for different PRV pressure settings:

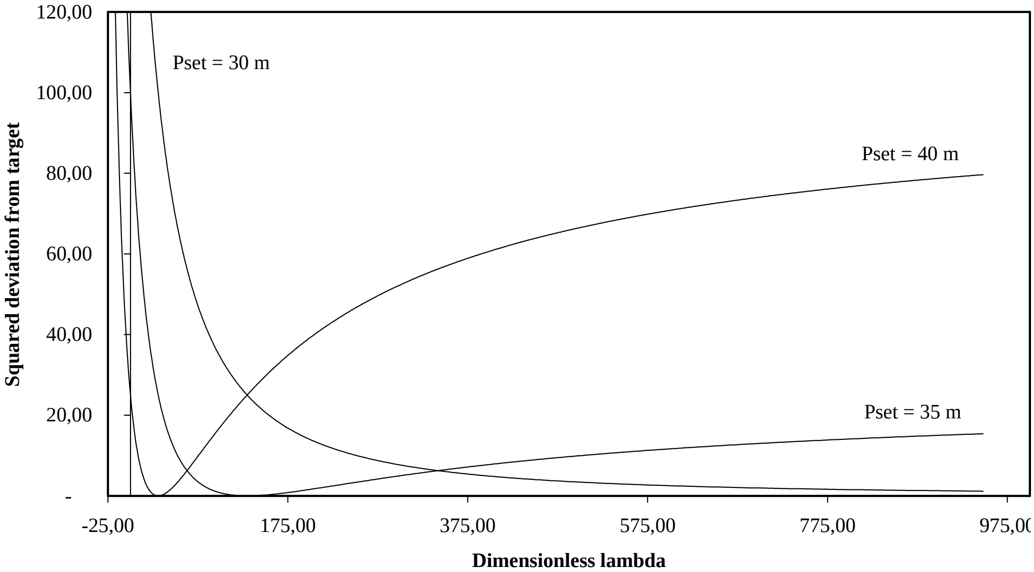


Fig.2 : General form of criterion $c(\lambda)$ to be minimised

Note that the 3 curves are smooth, have no more than one minimum and no other extremum. Thus, a gradient method is suitable to identify the optimal device state that minimizes problem (1.3). With direct method, it is necessary to derive sensitivity equations:

$$R\partial_\lambda \dot{x} = \frac{\partial f}{\partial x}\partial_\lambda x + \frac{\partial f}{\partial \lambda}, \text{ with } \partial_\lambda x(0) = 0$$
$$\partial_\lambda h = \left(AL^{-1}\,{}^tA\right)^{-1} AL^{-1}\left[D(q)\,{}^tM_0\partial_\lambda q_c + B(q) - {}^tA_f\partial_\lambda h_f\right] \qquad (1.4)$$

This gives us the expression of the gradient of c:

$$\nabla_{\lambda_i} c_i(\lambda) = {}^tJ_i\left(Sh(\lambda) - h_{set}\right)_i \text{ where } J_i = \left(S\partial_{\lambda_i} h\right)_i$$

We will now write the first order optimality conditions which lead to the Karush, Kuhn and Tucker equations: The objective function c_i being C^1, $\exists \eta_i^{inf}, \eta_i^{inf} \geq 0$ /

$$J_i\left(Sh(\hat{\lambda}) - h_{set}\right)_i - \eta_i^{inf} + \eta_i^{sup} = 0 \text{ dual feasibility } \textbf{(DF)} \text{ conditions}$$

$$\eta_i^{inf}\hat{\lambda}_i = 0 \text{ and } \eta_i^{sup}\left(\hat{\lambda}_i - \lambda_i^{max}\right) = 0 \text{ complementary slackness } \textbf{(CS)} \text{ conditions}$$

For each PRD a separate least-squares problem is to be solved in parallel. This idea meet the one of [12] but with different objective functions. We do not need to define s paths between a reference node and the s head targets.

3.3 Choice of a solution algorithm

The solution method used is a slight modification (projection method) of the Levenberg-Marquardt algorithm described in [9], which accounts for all the constraints. The iteration formula is as follows:

$$\lambda^{k+1} = \lambda^k - \left[{}^tJ_kJ_k + e_k \text{diag}\left({}^tJ_k.J_k + I_s\right)\right]^{-1} {}^tJ_k\left(Sh(\lambda^k) - h_{set}\right)$$

where $J_k = \text{diag}[J_i(x^k, \lambda^k)], i = 1, ..., s$ and e_k is the damping factor. The value of e_k is increased if the primal feasibility conditions (PF) are not respected, if ${}^tJ_k.J_k$ is an ill-conditioned matrix or if there is no descent. The state sensitivity equations (1.4) are solved using the same algorithm for the state equations: the matrix to inverse and the linear systems being unchanged.

4. RESULTS

For illustration, we will first consider the simple example network in Figure 3 proposed by [10].

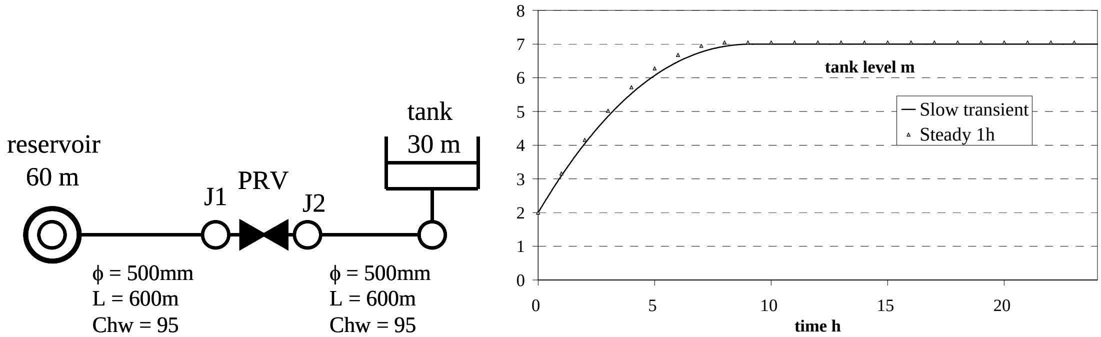


Fig 3: Illustrative example

The ground levels in J1 and J2 are null. The target head at node J2 is 35m. The initial level of water in the tank is 2 m. As a result, the tank has to fill up to 35m to meet this target. The PRV that was initially active closes itself at the end of 8h50mn. We started from initial steady state conditions. The flow rate in the PRV decreases from 322 l/s to 0.001 l/s and the PRV local head loss raise from 20 m to 25m. An interesting result is that working in l/s and choosing a sufficiently large head loss coefficient (from 250,000 to 250 millions) is a good trade off between numerical accuracy of the solution and the well conditioning of the system. The algorithm was tested on several dozen-test sets and more experiment results will be later presented.

5. CONCLUSIONS AND PERPECTIVES

We have presented hydraulic modelling of pressure control devices in slow transient. The control variable is the local head loss coefficient produced by the PRD. A least-squares problem to minimize deviation between predicted and target head is proposed to identify the state of each control device. Equal importance is given to each objective function.

Loop flow rates and heads at free surface tanks constitute the state vector. In contrast to previous model formulation, here the incidence matrices are considered independent of time; this is a significant benefit for the numerical efforts necessary to identify the state vector. Moreover, incidence matrices remain constant also when valves are closed. This result is obtained by giving a continuous penalty to local head losses.

The solution algorithm for the state equation is a three order method of Newton type with time step control. The sensitivity equations are solved with the same algorithm in a parallel manner to update the gradients of the objective functions. A direct descent method of Levenberg-Marquard type is used.

An attractive perspective for future work is to include pressure driven models that account for water consumption decrease at nodes with insufficient pressure.

References

[1]. Collins, M., *et al.*, "Solving the Pipe Network Analysis Problem using Optimization Techniques", *Management Science*, Vol. 24, No. 7, 1978, pp. 747-760.

[2]. Piller, O., Brémond, B. and Poulton, M. "Least Action Principles Appropriate to Pressure Driven Models of Pipe Networks". in *World Water and Environmental Resources Congress (EWRI03)*, ASCE, Philadelphia (PA), US, June 22-26, 2003.

[3]. Cross, H., "Analysis of Flow in Networks of Conduits or Conductors". 1936, Bulletin No. 286, University of Illinois Engineering Experimental Station.

[4]. Epp, R. and Fowler, A.G., "Efficient Code for Steady-State Flows in Networks", *Journal of the Hydraulics Division*, Vol. 96, No. HY1, 1970, pp. 43-56.

[5]. Wood, D.J. and Charles, C.O.A., "Hydraulic Network Analysis Using Linear Theory", *Journal of the Hydraulics Division*, Vol. 98, No. HY7, 1972, pp. 1157-1170.

[6]. Chandrashekar, M. and Stewart, K.H., "Sparsity Oriented Analysis of Large Pipe Networks", *Journal of the Hydraulics Division*, Vol. 101, No. HY4, 1975, pp. 341-355.

[7]. Carpentier, P., Cohen, G. and Hamam, Y. "Water Network Equilibrium, Variational Formulation and Comparison of Numerical Algorithms". in *EURO VII*, Bologna, IT, June 16-20, 1985.

[8]. Todini, E. and Pilati, S. "A Gradient Projection Algorithm for the Analysis of Pipe Networks". in *Computer Applications for Water Supply and Distribution*, Research Study Press Ltd., Leicester Polytechnic, 1987.

[9]. Piller, O., "Modeling the behavior of a network - Hydraulic analysis and a sampling procedure for estimating the parameters", PhD thesis from the University of Bordeaux, defended on 03 February 1995, 288 pages (in French), Bordeaux.

[10]. Simpson, A. "Modeling of Pressure Regulating Devices: The last major Problem to be Solved in hydraulic Simulation". in *Water Resources Planning and Management Conference*, ASCE, Tempe (AZ), US, June 6-9, 1999.

[11]. Piller, O. and Brémond, B. "Modeling of Pressure Regulating Devices: A Problem now Solved". in *World Water & Environmental Resources congress, EWRI01*, ASCE, Orlando (FL), US, May 20-24, 2001.

[12]. Deuerlein, J., Cembrowicz, R.G. and Dempe, S. "Hydraulic Simulation of Water Supply Networks Under Control". in *World Water and Environmental Resources Congress (EWRI05)*, ASCE, Anchorage (AK), US, May 15-19, 2005.

[13]. Choi, D.-Y., Yoon, J.-H. and Woo, H.-M. "Water Quality Modeling under Gradually Varied Flows in Distribution Systems". in *World Water & Environmental Resources congress, EWRI01*, ASCE, Orlando (FL), US, May 20-24, 2001.

[14]. Onizuka, k., "System Dynamics Approach to Pipe Network Analysis", *Journal of Hydraulic Engineering*, Vol. 112, No. 8, 1986, pp. 728-747.

[15]. Shimada, M., "Graph-Theoretical Model for Slow Transient Analysis of Pipe Networks." *Journal of Hydraulic Engineering*, Vol. 115, No. N°9, 1989, pp. 1165-1183.

[16]. Tonnet, N., "Adaptative Schemes for Solving Slow Transients in Drinking Water Distribution Network", MS thesis from the MATMECA engineering school, defended on September 2003, 43 pages (in French), Bordeaux.

[17]. Shampine, L.F., "Solving ODEs and DDEs with residual control", *Applied Numerical Mathematics*, Vol. 52, No. 1, 2005, pp. 113-127.