Modélisation de processus géostatistiques sur des arbres : application aux réseaux de drainage

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Geostatistical modelling of spatial processes on trees: applications to drainage networks.

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The aim of this study is to model spatial variation of hydraulic characteristics of artificial drainage networks in small rural catchments. The use of geostatistics appeared clearly as adequate since these networks were spatially structured and showed stochastic variation. However, the extension of geostatistical concepts from the euclidean space in two or three dimensions to tree shaped support is not direct and raised several theoretical questions.

We first introduce notations considering both the topological representation of the network that is a tree in the graph theory terminology and the geometrical set of points that is the support of the studied characteristics. Then bases of geostatistics were revisited and usual hypotheses redefined in this new context.

For example, stationarity that is usually defined as the invariance by translation of the spatial distribution on a subset of points cannot be extended since the translation is no more naturally defined in an oriented tree. As well isotropy has to be redefined from the tree topology and does not seems anymore relevant in the applied context of drainage network. Similar difficulties appear for the covariance function modelling. The matrix of covariance for any set of points cannot be explicitly expressed because it is closely linked to point locations in the tree topology. The model is expressed through a covariance function between upstream-downstream points assuming a conditional independence between branches that are not in an upstream-downstream relationship. From this model, a specific simulation procedure was then implemented, based on sequential conditional simulation of branches using the conditional independence between parts of the tree structure. Several drift models are also proposed considering the tree geometrical characteristics and the measured variable properties.

On a small rural catchment in the South-East of France, an eleven kilometer long artificial network, mainly composed of ditches was studied and the experimental variogram was computed and fitted for the sampled characteristic "width" which was spatially structured.

Figure 1 : Roujan catchment area ($1km^2$) and its artificial drainage network

Monte-Carlo tests on stationarity were developed using the previous simulation procedure, and a drift model was then fitted and tested.

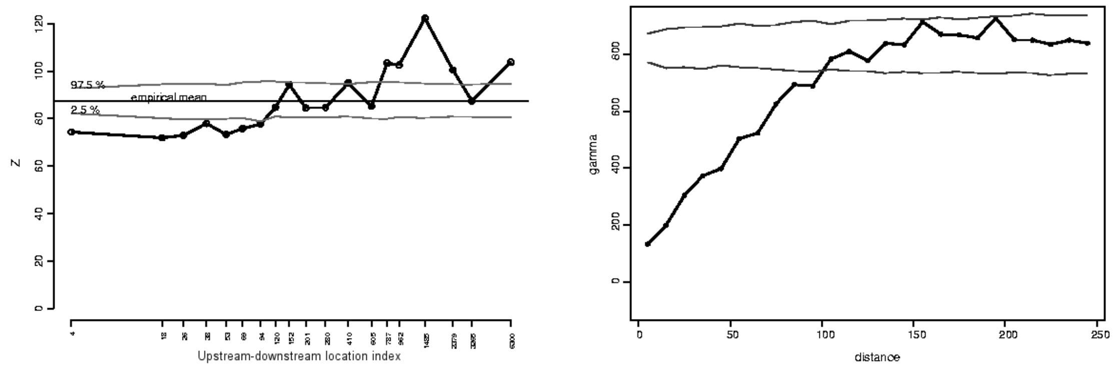

Figure 2 : experimental drift and its confidence band (left), experimental variogram and its confidence band (right)on the ``width'' variable

Further modelling was then introduced using spatial point processes, in order to build more realistic spatial processes of network characteristics than multigaussian distribution, based on an extension of boolean models and tessellation models.

These first results allow us to extend the concept of spatial structure analysis to a directed tree structure, with useful applications in drainage network characterization for hydrological modelling.

References :

Audergon, J., P. Monestiez, R. Habib, 1993. Spatial dependences and sampling in fruit tree : a new concept for spatial prediction in fruit studies. *Journal of Horticultural Science*, **68**, 99-112.

Gottschalk, L. 1993, Interpolation of runoff applying objective methods. *Stochastic Hydrology and Hydraulics*, **1**, 269-281.

Lagacherie, P., O. Diot, N., Domange, V., Gouy, C., Floure, C., Kao, R., Moussa, J., Robbez-Masson, and V. Szleper, 2003, An indicator approach for describing the spatial variability of human-made stream network in regard with herbicide pollution in cultivated watershed. (submitted)

Lantuejoul, Ch. 2002, Geostatistical simulation, model and algorithms. Springer.

Monestiez, P., R. Habib, and J. Audergon, 1989. Estimation de la covariance et du variogramme pour une fonction aleatoire a support arborescent : application à l'etude des arbres fruitiers. In *Geostatistics,* M.Armstrong (ed.), Kluwer Academic Publishers, Vol. 1, 39-56.

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