HAL open science

Probability distribution dynamics explaining agent model convergence to extremism

Guillaume Deffuant, G. Weisbuch, B. Edmonds, C. Hernandez, K. Troitzsch

► **To cite this version:**

Guillaume Deffuant, G. Weisbuch, B. Edmonds, C. Hernandez, K. Troitzsch. Probability distribution dynamics explaining agent model convergence to extremism. Social Simulation, technologies, advances and new discoveries, Information Science Reference, pp.43-60, 2008, 978-1-59904-522-1. hal-02591572

HAL Id: hal-02591572

https://hal.inrae.fr/hal-02591572v1

Submitted on 15 May 2020

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Social Simulation:
Technologies, Advances, and New Discoveries

Bruce Edmonds
Manchester Metropolitan University, UK

Cesáreo Hernández
University of Valladolid, Spain

Klaus G. Troitzsch
Universität Koblenz-Landau, Germany

Information Science REFERENCE
INFORMATION SCIENCE REFERENCE
Hershey · New York

Acquisitions Editor: Kristin Klinger
Development Editor: Kristin Roth
Senior Managing Editor: Jennifer Neidig
Managing Editor: Sara Reed
Copy Editor: Michael Goldberg
Typesetter: Jamie Snavely
Cover Design: Lisa Tosheff
Printed at: Yurchak Printing Inc.

Published in the United States of America by
Information Science Reference (an imprint of IGI Global)
701 E. Chocolate Avenue, Suite 200
Hershey PA 17033
Tel: 717-533-8845
Fax: 717-533-8661
E-mail: cust@igi-global.com
Web site: http://www.igi-pub.com/reference

and in the United Kingdom by
Information Science Reference (an imprint of IGI Global)
3 Henrietta Street
Covent Garden
London WC2E 8LU
Tel: 44 20 7240 0856
Fax: 44 20 7379 0609
Web site: http://www.eurospanonline.com

Copyright © 2008 by IGI Global. All rights reserved. No part of this publication may be reproduced, stored or distributed in any form or by any means, electronic or mechanical, including photocopying, without written permission from the publisher.

Product or company names used in this set are for identification purposes only. Inclusion of the names of the products or companies does not indicate a claim of ownership by IGI Global of the trademark or registered trademark.

Library of Congress Cataloging-in-Publication Data

Social simulation : technologies, advances and new discoveries / Bruce Edmonds, Cesareo Hernandez & Klaus G. Troitzsch, editors.

p. cm.

Summary: "This book, a reference survey of social simulation work comprehensively collects the most exciting developments in the field. Drawing research contributions from a vibrant community of experts on social simulation, it provides a set of unique and innovative approaches, ranging from agent-based modeling to empirically based simulations, as well as applications in business, governmental, scientific, and other contexts"--Provided by publisher.

Includes bibliographical references and index.

ISBN-13: 978-1-59904-522-1 (hardcover)

ISBN-13: 978-1-59904-524-5 (ebook)

1. Social sciences--Computer simulation. 2. Social interaction--Simulation methods. 3. Social exchange--Simulation methods. I. Edmonds, Bruce. II. Hernandez, Cesareo. III. Troitzsch, Klaus G.

H61.3.S625 2008

300.1'13--dc22

British Cataloguing in Publication Data
A Cataloguing in Publication record for this book is available from the British Library.

All work contributed to this book set is new, previously-unpublished material. The views expressed in this book are those of the authors, but not necessarily of the publisher.

Chapter IV
Probability Distribution Dynamics Explaining Agent Model Convergence to Extremism

Guillaume Deffuant
Laboratoire d'Ingénierie des Systèmes Complexes, France

Gérard Weisbuch
Laboratoire de Physique Statistique, France

ABSTRACT

This chapter studies continuous opinion models with extremists, and we use probability distribution models which approximate the behaviour of agent-based models in order to explain their attractor patterns. The probability distribution is defined on a discrete grid in the opinion/uncertainty space. We compute the equations of probability flows between each of the sites of the grid for different variants of the opinion influence model (bounded confidence, relative agreement, and two others). The simulations show that the probability distribution models yield attractor patterns very similar to those obtained with the agent-based models. Moreover, a study of the probability distribution evolution helps to better understand the process of convergence to single and double extreme attractors observed in agent-based models.

INTRODUCTION

In recent years, we proposed several models of opinion dynamics to model the influence of individuals with an extreme opinion in a population of moderate agents (Deffuant et al., 2002; Amblard & Deffuant, 2004; Weisbuch et al., 2004). These models rely on several general hypotheses about the agent behaviour:

- Agents opinions vary continuously between some bounds.

Copyright © 2008, IGI Global, distributing in print or electronic forms without written permission of IGI Global is prohibited.

- An agent might change its own opinion under the influence of other agents with opinions which are not too far from its own opinion.
- There are extremist agents, that are very convinced of their opinion (they do not change easily), and their opinion is extreme (at the bounds or very close to the bounds).

Deffuant (2006) performed a systematic comparison between model variants, on several network topologies:

- Bounded confidence model (BC) (Hegselman & Krause, 2002; Deffuant et al., 2000; Weisbuch et al., 2002; Urbig, 2003; Urbig & Lorenz, 2004). In this model, each agent has a threshold in addition to its opinion (sometimes interpreted as an uncertainty), which limits the range of opinions of those agents it interacts with. The extremists are initialised with an opinion equal to one of the bounds, and a very low uncertainty. In this model, each agent has a threshold in addition to its opinion (sometimes interpreted as an uncertainty), which limits the range of opinions of those agents it interacts with. The extremists are initialised with an opinion equal to one of the bounds, and a very low uncertainty.
- Gaussian bounded confidence (GBC), in which the opinion and uncertainty influence depend on a Gaussian function of the difference of opinions.
- Relative agreement (RA) model (Deffuant et al., 2002), in which the influence takes into account the interlocutor uncertainty compared with the overlap between both segments of opinions.
- Gaussian bounded confidence model (GBCU), in which we multiply the influence by a Gaussian function of the interlocutor uncertainty.

The objective of this chapter is to integrate the corresponding master equations for the probability distribution, thus gaining some insight into the behaviour of the agent-based models. This approach can sometimes be seen as an alternative to agent-based models, as practiced in sociodynamics (Weidlich & Haag, 1999). However, in several cases it can be used as a complement to agent-based models, in order to give a more precise and systematic understanding of their behaviour; this was particularly the case for binary or discrete states models (Edwards et al., 2003; Deffuant & Huet, 2006), but also for the bounded confidence model with continuous opinion (Ben Naim et al., 2003). In the latter case, the master equation approach requires one to discretise the continuous opinion. It can be also considered as a distribution model of the discrete opinion version proposed by Stauffer et al. (2004). In the present chapter, after checking that the distribution model gives a good prediction of the agent-based model attractor, we study in more detail the convergence process of the distribution model. This allows us to draw some conclusions about this process, which are also valid for the agent-based model.

In Section 2 we recall the definitions of the different agent-based models and their convergence types. In Section 3 we derive the master equations for the probability distribution and apply them to our particular case. In Section 4 we compare patterns of attractors of the distribution in the parameter space with those obtained from the agent-based models. In Section 5 we study more closely the evolution of the distribution in single and double extreme convergences in order to better understand these dynamics. The final section provides some points of discussion and conclusions.

THE AGENT-BASED MODELS

Common Features

The considered models share the following aspects:

- The population includes N individuals, each having a continuous opinion and a continuous uncertainty.
- The moderate agents are initialised with opinions uniformly distributed between –1 and +1, with uncertainty U.
- The population includes a proportion p_e of extremists. Half of the extremists are initialised with opinion –1, and the other half with opinion +1, all with uncertainty u_e.
- The interactions take place in randomly chosen pairs of connected individuals.

Moreover, for all the models, when an individual with opinion x and uncertainty u meets another individual of opinion x' and uncertainty u', the modifications of the individuals' uncertainties and opinions follow a common scheme. They depend on a kernel function of x, x', u, and u'. Let k be this function:

$$\begin{aligned} x &:= x + \mu . k(x, x', u, u').(x' - x) \\ x' &:= x' + \mu . k(x', x, u', u).(x - x') \end{aligned} \qquad (1)$$

$$\begin{aligned} u &:= u + \mu . k(x, x', u, u').(u' - u) \\ u' &:= u' + \mu . k(x', x, u', u).(u - u') \end{aligned} \qquad (2)$$

In the next paragraph, we list the variants of the models corresponding to different choices for k.

Variants of the Kernel Function

Bounded Confidence (BC)

The initial bounded confidence model including only equations (1) was modified, for instance in Weisbuch et al.(2005), to include equations (2) on the uncertainties. In this case, the kernel function is independent of u', it is a Heaviside function of the difference in opinions:

$$\begin{aligned} k(x - x', u) &= 1 \quad \text{if } |\text{x-x'}| < u \\ k(x - x', u) &= 0 \quad \text{otherwise} \end{aligned} \qquad (3)$$

Gaussian Bounded Confidence (GBC)

In this case, the kernel function has the form of a Gaussian function, and is also independent of u':

$$k(x - x', u) = \exp\left(-\left(\frac{x - x'}{u}\right)^2\right) \qquad (4)$$

Relative Agreement (RA)

The relative agreement was proposed in Deffuant et al. (2002), and it introduces a new assumption: individuals take into account the uncertainty of their interlocutor, such that interlocutors with a low uncertainty (high confidence) tend to be more influential than those with a high uncertainty. The rules use v, the size of the overlap between segments $[x\text{-}u, x\text{+}u]$ and $[x'\text{-}u', x'\text{+}u']$.

$$v = \min(x + u, x' + u') - \max(x - u, x' - u')$$

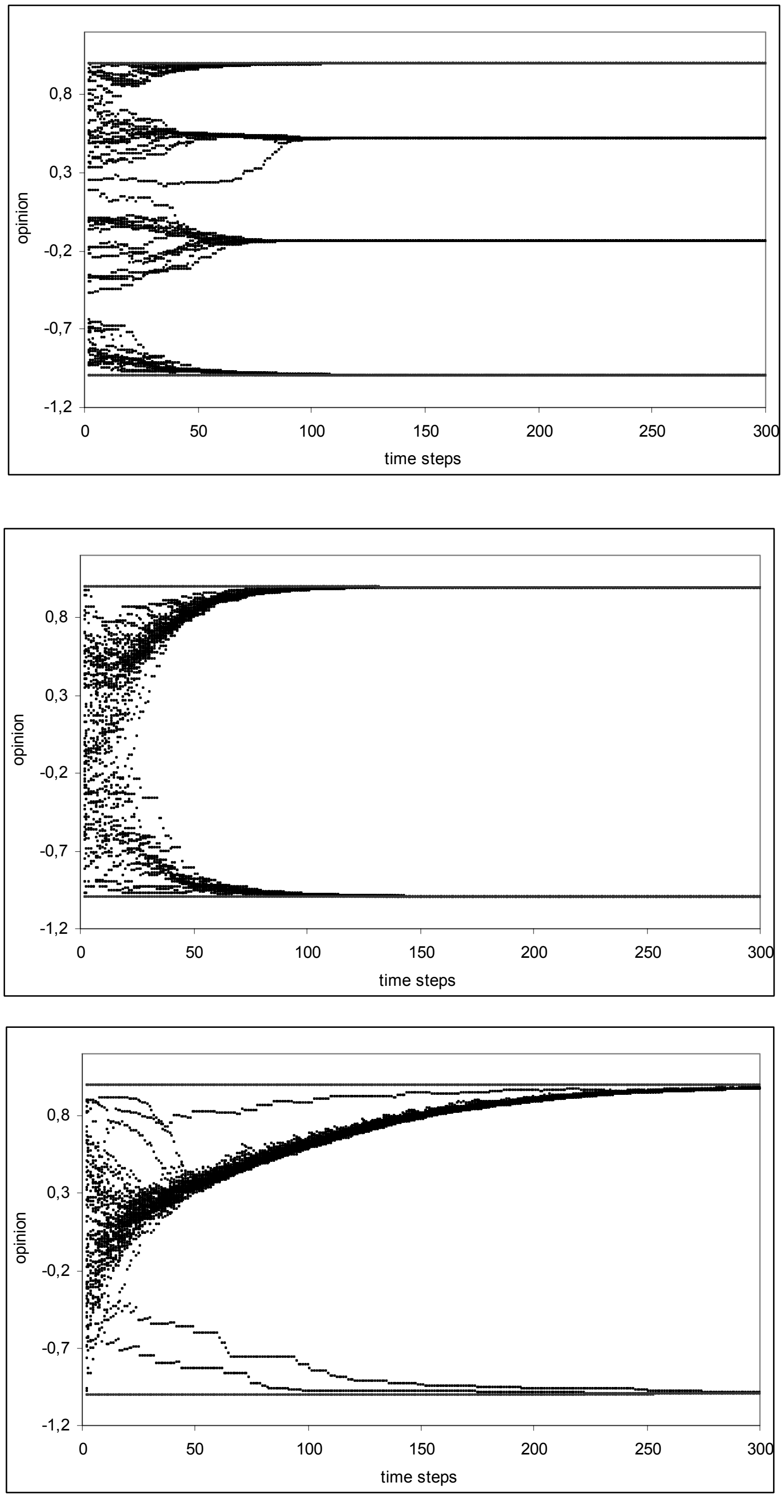
$$\begin{aligned} k(x, x', u, u') &= \frac{v}{2u'} \quad \text{if} \quad v > 0 \\ k(x, x', u, u') &= 0 \text{ otherwise} \end{aligned} \qquad (5)$$

The value of this function is 1 when the segment $[x'\text{-}u', x'\text{+}u']$ is totally included in the segment $[x\text{-}u, x\text{+}u]$, otherwise, it is lower than 1.

Gaussian Bounded Confidence with Influence of Uncertainty (GBCU)

This model expresses the same assumption as the RA model, considering that low uncertainty

Figure 1. The three attractors. Time plot of the agents opinion for N = 50, 2 extremists at +1 and −1, ue = 0.01 (extremist opinions are represented in grey). Top: U = 0.3, RA model. Central attractor. Middle: U = 0.9, RA model. Double extreme attractor. Bottom: U = 1.3, GBCU model. Single extreme attractor.



gives more influence than high uncertainty. This is expressed as follows:

$$k(x-x',u,u') = \exp\left(-\left(\frac{x-x'}{u}\right)^2\right).\exp\left(-\left(\frac{u}{u'}\right)^2\right) \quad (6)$$

With this kernel, if u is much smaller than u', then the change is necessarily small.

Attractor Types

These attractors depend upon the values of the parameters and we would like to compare the patterns of attractor in the parameter space for the different variants of the model.

THE PROBABILITY DISTRIBUTION MODEL

Initialisation of the Distribution

Rather than following individual opinion trajectories, we use the master equation describing the evolution of the joint probability distribution of opinion and uncertainty. We here generalize the approach taken by Ben Naim et al. (2003) by taking into account the uncertainties. Therefore, we consider a probability distribution on a grid defined on the compact [-1,+1] × [u_e, U]. We thus cut the opinion and uncertainty intervals into pieces of size δx, by discretising the opinion and uncertainty segments:

$$x(i) = -1 + i.\delta x, \quad \text{for} \quad i = 0,...,i_m.$$
$$u(j) = u_e + j\delta x, \quad \text{for} \quad j = 0,...,j_m. \quad (7)$$

The integer values i_m and j_m are the maximum indices, given the value of δx.

The probability distribution $\rho(i,j)$ represents the probability that an agent of the population has its opinion x and its uncertainty u such that:

$$x \in [(x(i) - \frac{\delta x}{2}, x(i) + \frac{\delta x}{2}] \quad (8)$$

$$u \in [(u(j) - \frac{\delta x}{2}, u(j) + \frac{\delta x}{2}] \quad (9)$$

Distribution ρ is initialized with:

$$\rho(0,0) = (1+\Delta)\frac{p_e}{2}, \rho(i_m,0) = (1-\Delta)\frac{p_e}{2}$$
$$\rho(i,j_m) = (1-p_e)/(i_m-1), \text{ for } i = 1,...,i_m-1$$
$$\rho(i,j) = 0, \text{ otherwise.}$$
$$(10)$$

Parameter Δ allows us to introduce a small asymmetry in the initial distribution, giving, e.g., a highter initial density to the negative extreme.

Computation of the Master Equation

The principle of the model's dynamics is to compute the flows of distribution from one site (i,j) to any other site (k,l), and to sum them up to compute the distribution change (the update of the distribution is parallel).

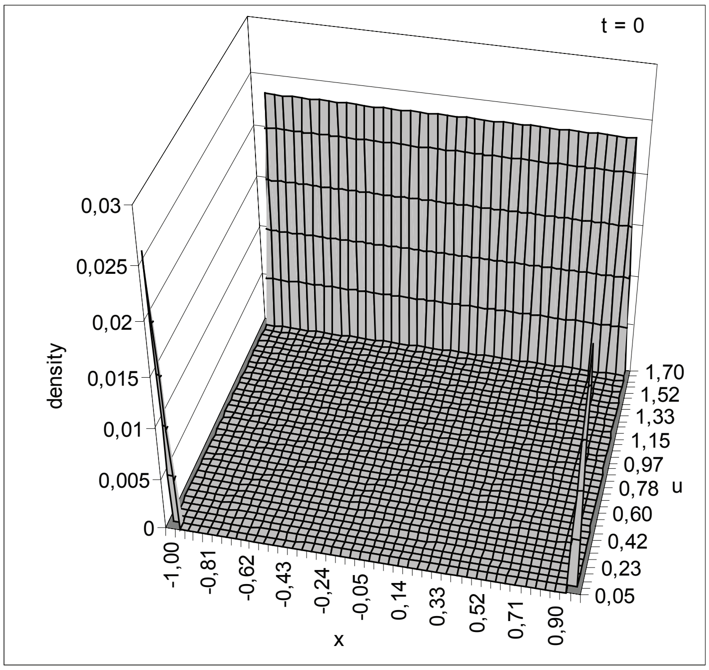
More precisely, for each site (i,j), we consider all the other sites (i',j'), and we compute the interaction between both sites. An interaction takes place only if ($\lfloor a \rfloor$ means the integer part of number a):

$$di = \lfloor \mu k_{u(j)}(x(i),x(i'),u(j')).(i'-i) \rfloor \neq 0 \quad (11)$$

$$dj = \lfloor \mu k_{u(j)}(x(i),x(i'),u(j')).(j'-j) \rfloor \neq 0 \quad (12)$$

Indeed, in this case the agents belonging to site (i',j') have an influence on agents belonging to site (i,j). This influence adds di to the opinion and dj to the uncertainty. The probability of encounter between agents of site (i',j') and agents of site (i',j') is proportional to the product $\rho(i,j)\,\rho(i',j')$. Therefore, the influence of agents of site (i',j') on agents of

Figure 2. 3D representation of the initial probability distribution on a grid in the space $x \times u$. The moderate uncertainty is 1.7 (line on the top), and the extremist uncertainty of 0.05 (the two bottom peaks), with a global density of extremists of 0.05 (0.2625 on the negative extreme, 0.2375 on the positive extreme). The discretisation includes 1,591 sites.



site (i,j) will induce an increase of $\rho(i+di, j+dj)$, and a decrease of $\rho(i,j)$, λ is the kinetic parameter of the algorithm:

$$\rho(i+di, j+dj) := \rho(i+di, j+dj) + \lambda\rho(i,j)\rho(i',j')$$
$$\rho(i,j) := \rho(i,j) - \lambda\rho(i,j)\rho(i',j') \quad (13)$$

Actually, because the change is made in parallel, we define $d\rho(i,j)$ as the distribution of the changes of $\rho(i,j)$. At each time step, we initialise $d\rho(i,j)$ with only 0 values. Then, we fill its values by computing the flow of distribution from one site (i,j) to another site (k,l). The computation is as follows:

Computation of $d\rho$
For $(i,j) \in \{0,\dots,i_m\} \times \{0,\dots,j_m\}$ do:
For $(i',j') \in \{0,\dots,i_m\} \times \{0,\dots,j_m\}$ do:
If $di = \lfloor \mu k_{u(j)}(x(i), x(i'), u(j')).(i'-i) \rfloor \neq 0$
or $dj = \lfloor \mu k_{u(j)}(x(i), x(i'), u(j')).(j'-j) \rfloor \neq 0$
$d\rho(i+di, j+dj) := d\rho(i+di, j+dj) + \lambda\rho(i,j)\rho(i',j')$
$d\rho(i,j) := d\rho(i,j) - \lambda\rho(i,j)\rho(i',j')$
end if
end for
end for

Global Algorithm

After the initialisation, we repeat the modification of ρ until changes become negligible. The stopping criterion is obtained by comparing the norm of $d\rho(i,j)$ (noted $|d\rho|$) with a threshold ε. Therefore, the global algorithm is the following:

Evolution of ρ:
Initialise ρ
Repeat
    Compute $d\rho$
    For $(i, j) \in \{0,...,i_m\} \times \{0,..., j_m\}$
$\rho(i, j) := \rho(i, j) + d\rho(i, j)$
While (ρ>εd)

In the following simulations we chose ε = 0.0001.

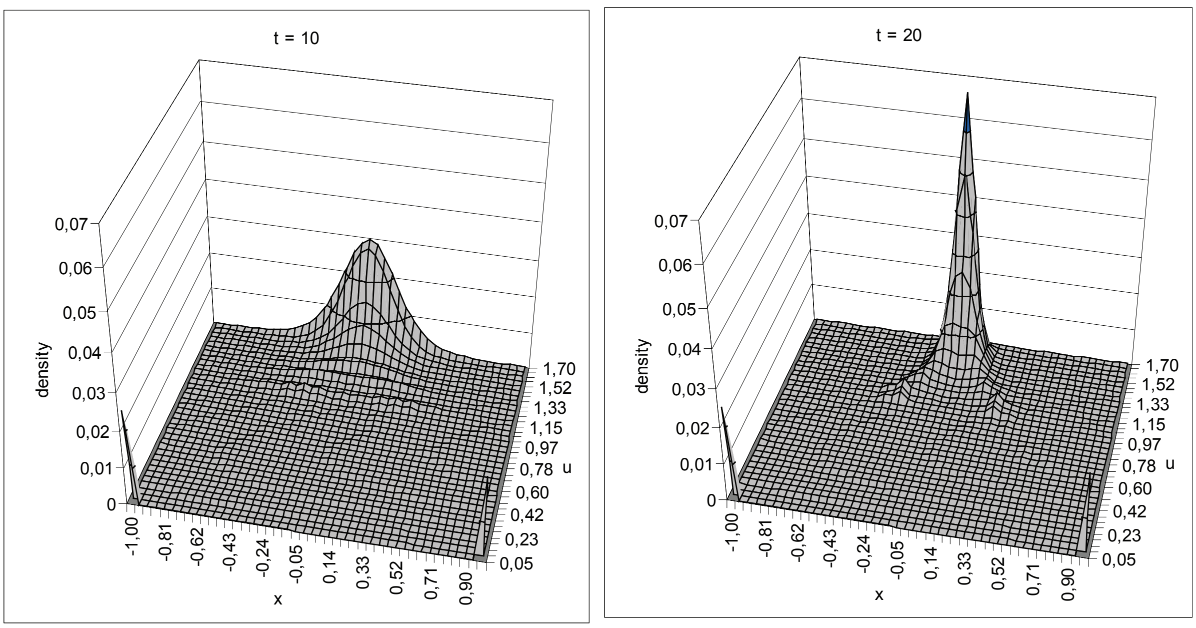
Example

Figure 3 shows a few steps of evolution of the distribution shown on Figure 2, according to the BC model.

Comparing the Distribution Dynamics with Agent-Based Simulations

We now compare attractor patterns provided by both dynamics. We focus on the variation of two parameters: *U*, the initial uncertainty of the moderates, and p_e, the initial proportion of extremists, keeping constant the other parameters, μ = 0.3 (the kinetic parameter) and u_e = 0.05 (the uncertainty of the extremists). For the agent-based model, we consider the results of Deffuant (2006): We take 51 values of *U* between 0.2 and 2, and 51 values between 0.01 and 0.21 for p_e, and *N* = 400 (number of individuals). For the distribution model, we take 21 values of *U* between 0.2 and 2, and 21 values between 0.01 and 0.21 for p_e,

Figure 3. First steps (iteration 10 and 20) of probability distribution evolution for the BC model, with a proportion of extremists p_e = 0.05, (Δ = 0.05, i.e., on the negative extreme, the initial density is 0.02625, on the positive extreme, it is 0.02375), uncertainty of extremists: u_e = 0.05, initial uncertainty of the moderates U = 1.3, μ = 0.3 (kinetic parameter of the opinion dynamics), λ = 0.5 (kinetic parameter of distribution update). We note that in this case the distribution concentrates quite rapidly.



because the simulations are longer, and there is no need to evaluate the random variations since the model is deterministic. The other parameters of the distribution model are: $\lambda = 0.5$ (kinetic parameter of distribution update), size of the uncertainty/opinion grid: 1500, $\Delta = 0.05$ (asymmetry between negative and positive extremists).

Characterisation of the Attractors

To characterise the attractor of a simulation, we consider the distribution at convergence, and we combine two indicators (the same as in Deffuant, 2006):

- The average of the absolute value of the opinions, noted X, which indicates how extreme the population is:

$$X = \sum_{i,j} \rho(i,j)\left|x(i,j)\right| \qquad (14)$$

- The generalised number of clusters, noted n, which is a smooth number of clusters obtained following the method defined in Derrida and Flyvbjerg (1986). Considering a final state of the distribution involving k clusters of average opinion x_i, of weight w_i, in the total distribution minus the weight of the initial extremists, the generalised number of clusters is defined by:

$$n = \frac{1}{\sum_{i=1}^{k} w_i} \qquad (15)$$

The weight w_i is the fraction of the distribution belonging to a cluster. The generalised number of clusters gives the exact number of clusters when they all include the same part of the distribution, and intermediate values for intermediate situations. The rationale is that small clusters count less.

In the computation of indicators defined on the AB model, we consider only the initially moderate agents. To approximate this in the distribution, we subtracted the initial values of the extremes from the final distribution. This approximation relies on the assumption that the initial extremists do not move much, which generally holds as long as the uncertainty of the extremists is low enough.

We combine the two indicators to compute the attractor type, with the following rules:

- If $X < 0.8$, then attractor = "central".
- If $X > 0.8$
    - If $n < 1.25$, then: attractor = "single extreme".
    - If $n > 1.66$, then: attractor = "double extreme".
    - If $1.25 < n < 1.66$, then: attractor = "intermediate between single and double extreme".

These attractors have a higher peak in the negative extreme, but the peak at the positive extreme is not negligible.

Patterns of Attractors in the Parameter Space

The next figures, for both the agent-based and the distribution models, represent the result of a simulation by symbols located in the U, p_e space, indicating which attractor is reached. We considered the four model versions of opinion influence: Bounded confidence (Figure 4), Relative Agreement (Figure 5), Gaussian Bounded Confidence (Figure 6), and Gaussian Bounded Confidence with Uncertainty (Figure 7).

A general observation is:

- In those regions in the parameter space when only one attractor is observed in AB dynamics whatever the sampling of initial conditions and coupled agents, the same attractor is obtained by the distribution dynamics.
- There exist regions where the attractors can be either central or single extreme, according

Figure 4. BC model. Left: AB model in total connection. Right: distribution model. Each symbol represents one simulation and the shape codes for the attractor (central, single extreme, intermediate between single and double extreme, and double extreme)

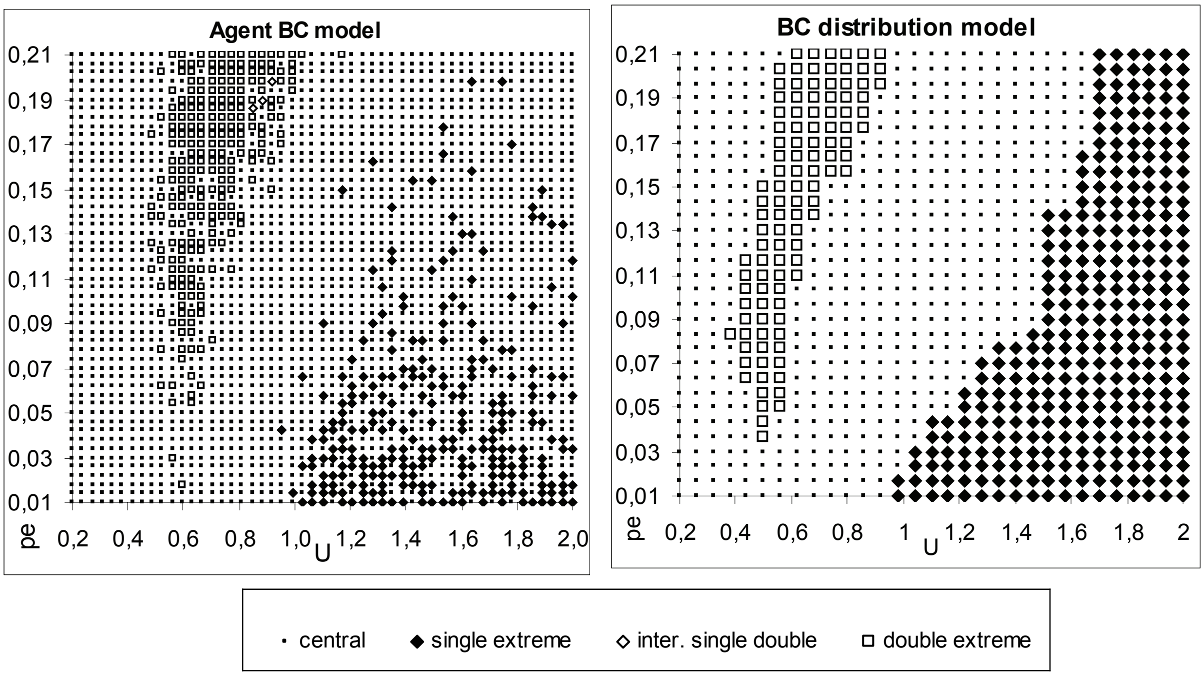


Figure 5. RA model. Left: AB model in total connection. Right: distribution model. Each symbol represents one simulation and the shape codes for the attractor (central, single extreme, intermediate between single and double extreme, and double extreme).

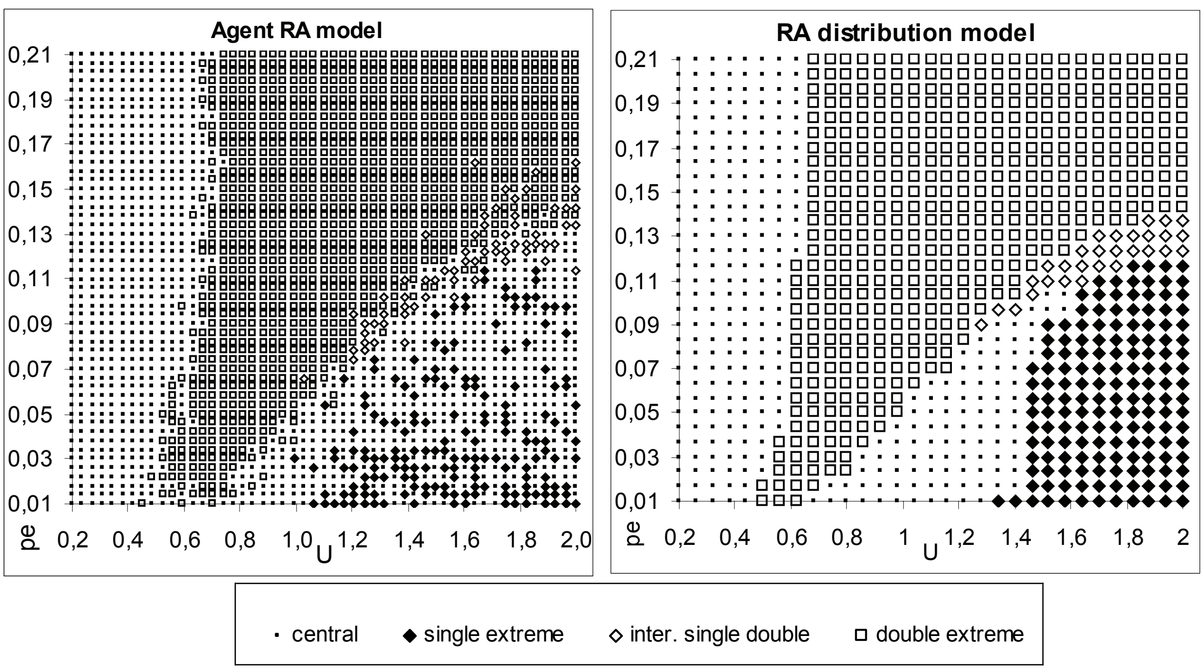


Figure 6. GBC model. Left: AB model in total connection. Right: distribution model. Each symbol represents one simulation and the shape codes for the attractor (central, single extreme, intermediate between single and double extreme, and double extreme).

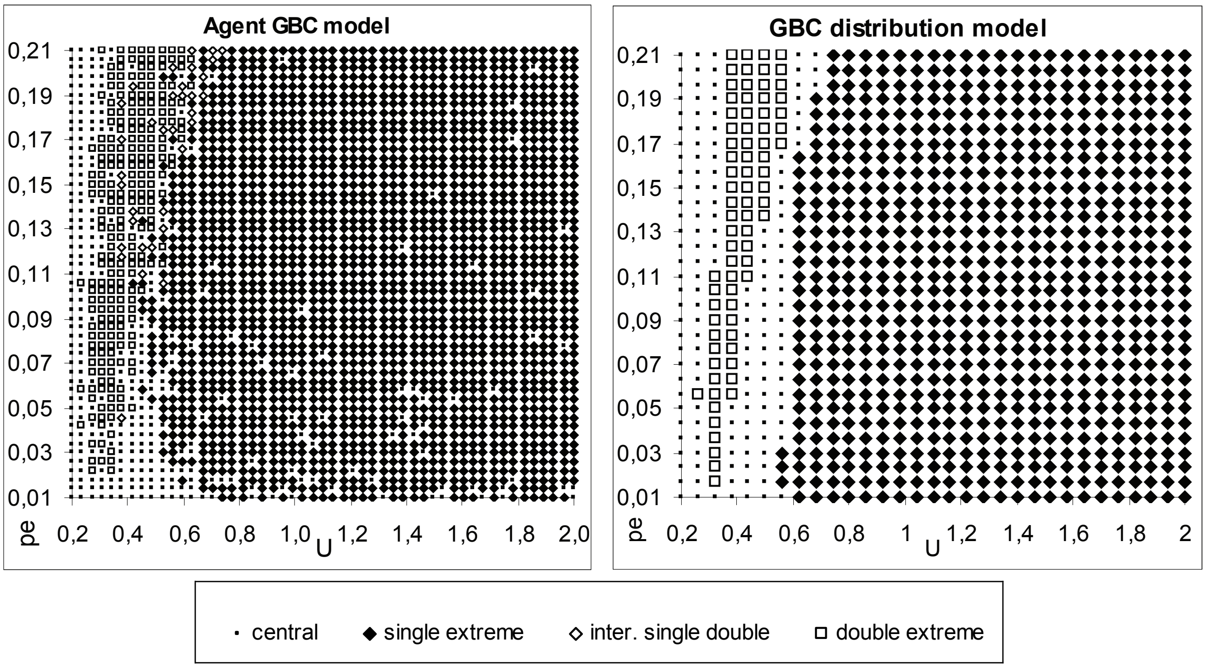
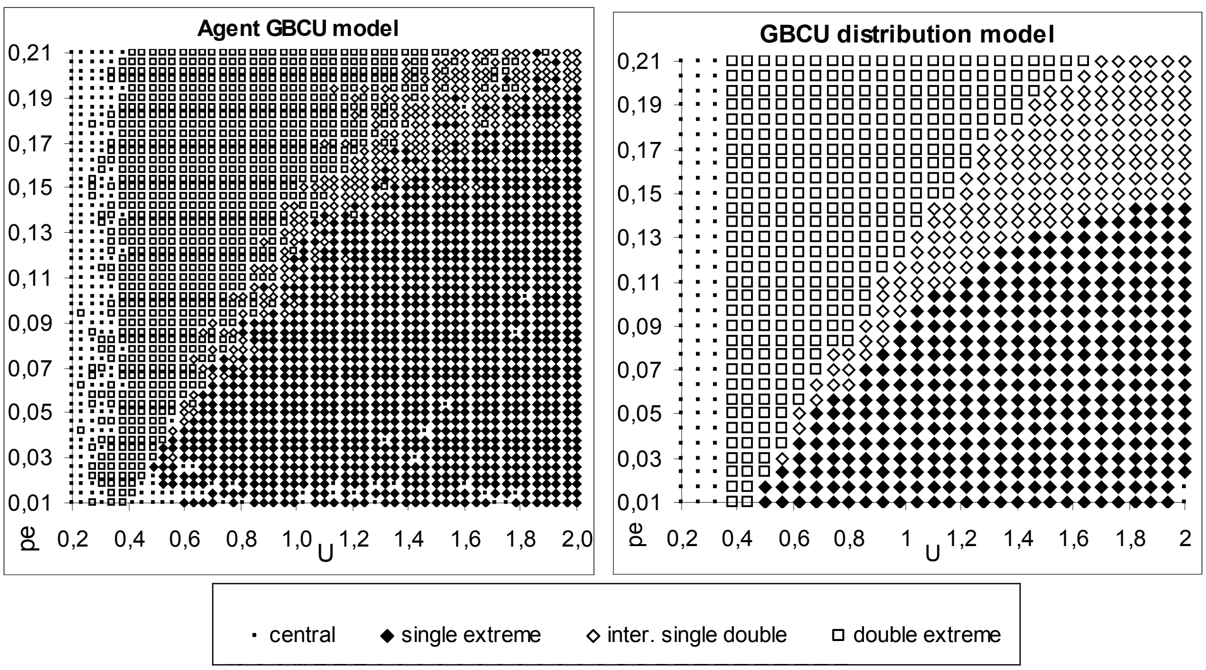


Figure 7. GBCU model. Left: AB model in total connection. Right: distribution model. Each symbol represents one simulation and the shape codes for the attractor (central, single extreme, intermediate between single and double extreme, and double extreme).



to the sampling of initial conditions and of coupled agents in AB dynamics, for the same values of the parameters. In these regions, the distribution dynamics always yield the same attractor for the same parameters because it is deterministic. The boundaries between the attractor regions depend upon the magnitude of the asymmetry parameter.

STUDY OF THE EXTREME ATTRACTOR IN THE DISTRIBUTION MODEL

In order to better understand the process of convergence, a particularly useful tool is to visualise the influence zones of the extremists on the grid. We distinguish four zones which are represented in Figure 8. By having an influence, we mean that the extremes will induce a flow from the sites located in the zone. From the equations of the models, one can derive for both the RA and BC models:

- The condition for site (x, u) to be influenced by the negative extreme: $x - u < -1$.
- The condition for site (x, u) to be influenced by the positive extreme: $x + u > +1$.

We focus first on the BC and RA models, because they use sharp boundaries of the influence zone.

Figure 8 visualises these conditions.

These zones will be particularly useful to understand the evolution of the distribution when double or single extreme attractors take place.

Single Extreme Attractor

Figure 9 shows some pictures of the evolution of the distribution in a case of convergence to a single extreme attractor for the BC model, and Figure 10 shows a convergence to a central attractor. The comparison between these cases helps to understand the convergence to a single extreme attractor.

Figure 8. The influence of the extremes on the grid opinion/uncertainty. The black lines show the limit of the influence of positive and negative extremes, and define four zones. In the bottom triangle the extremes have no influence, in the left triangle only the negative extreme has an influence, in the right triangle only the positive extreme has an influence, and in the top triangle both extremes have an influence.

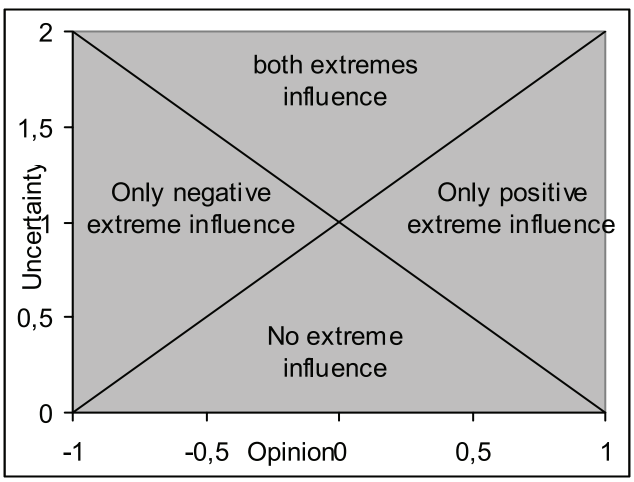
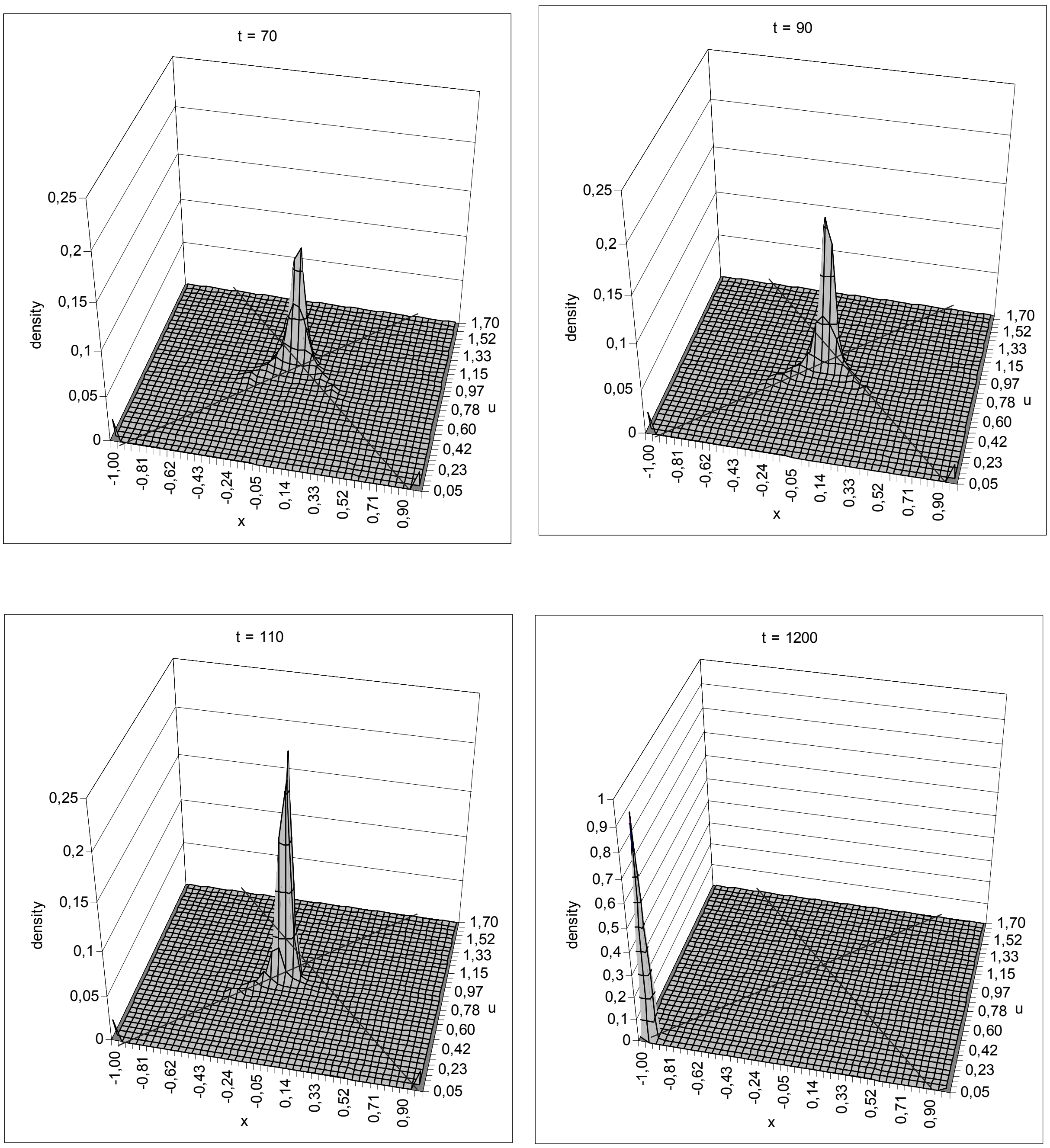


Figure 9. Pictures of distribution evolution for a single extreme attractor with the BC model. U = 1.7, ue= 0.05 size of the grid: 1500, μ = 0.3, l = 0.5. The initial density of extremists is 0.05, on the negative extreme: 0.02625, on the positive extreme: 0.02375.



The value t on top of each graph is the number of iterations. When the distribution goes down through the intersection of the extreme influence zone limits (black lines) it is already concentrated, and the maximum of the distribution is located in the negative extreme influence zone (see at $t = 90$). This enhances the dissymmetry of the distribution (t = 110), which keeps an important part in the negative extreme influence zone. However, the density maximum lies in the zone where the extremes have no influence, which explains why the convergence is generally quite slow. The process continues until the single extreme convergence: More than 90 percent of the initial distribution is finally at the negative extreme (after around 1,200 time steps).

The importance of these zones relies on the fact that the convergence occurs in two time scales:

- First the moderates' cloud converges in the neighbourhood of the centre of gravity of the initial distribution, with a fast decrease of uncertainty (the vertical axis). This process takes place because the cloud is located in the influence zone of both extremes (above the intersection of the lines).
- The crucial moment which determines if the attractor will be single extreme or central is when the distribution crosses the intersection of the lines separating the influence zones:
    - If the distribution is very concentrated (as in Figure 9), a major part of the distribution tends to go into the zone where only the negative extreme is influent because this extreme is slightly more influent from the beginning, and only a small deviation tends to have the maximum of the distribution inside the negative extreme influence zone (see Figure 9, $t = 90$). The asymmetry of the distribution tends then to keep a significant part inside the negative extreme influence zone. This part of the distribution pulls the rest (which remains in the zone of no extreme influence) slowly toward the negative extreme. The whole process leads to the convergence to a single extreme.
    - If the distribution is not concentrated (as in Figure 10), the difference between the part of the distribution which goes into the positive influence zone and into the negative influence zone is much lower. Indeed, at $t = 30$, for instance, there is a significant part of the density in the positive extreme influence zone (which is never the case in Figure 9). Therefore, the asymmetry is not enhanced as in Figure 9. The distribution lies mainly in the triangle where the extremes have no influence, which leads to the convergence to a central attractor.

It appears that the critical moment which decides between moderate and single extreme convergence is when the distribution goes through the intersection of the limits of the extreme influence zones, and the concentration or the dispersion of the distribution at this moment is particularly important.

Double Extreme Attractor

The same type of study can be done in order to better understand the convergence to a double extreme attractor. In the next figures, we compare the evolution of the distribution, with the same parameters, for the BC and RA models. The BC model yields a central attractor, whereas the RA model yields a double extreme attractor.

On Figure 11 (RA model), at $t = 50$, there is a larger part of the distribution located in the extreme influence zones (above the black lines), which leads to a reinforcement of the attraction to the extremes (visible at $t = 70$, 100). On the contrary, on Figure 12 (BC model), at $t = 40$, there

Figure 10. Pictures of distribution evolution for central attractor with the BC model. U = 1.2, u_e= 0.05, size of the grid: 1500, μ = 0.3, λ = 0.5. The initial density of extremists is 0.1, on the negative extreme: 0.0525, on the positive extreme: 0.0475. The value t at the top of each graph is the number of iterations. When the distribution goes down through the intersection of the extreme influence zones, limits (t = 30), it is much less concentrated than in Figure 9. Therefore, the dissymmetry is not so much enhanced (t = 40). The distribution is globally attracted down into the zone where extremes have no (direct) influence, which leads to a concentration of the distribution at an opinion close to 0 (t = 50), which will continue until the distribution is completely in the zone where extremists have no influence (for t = 460).

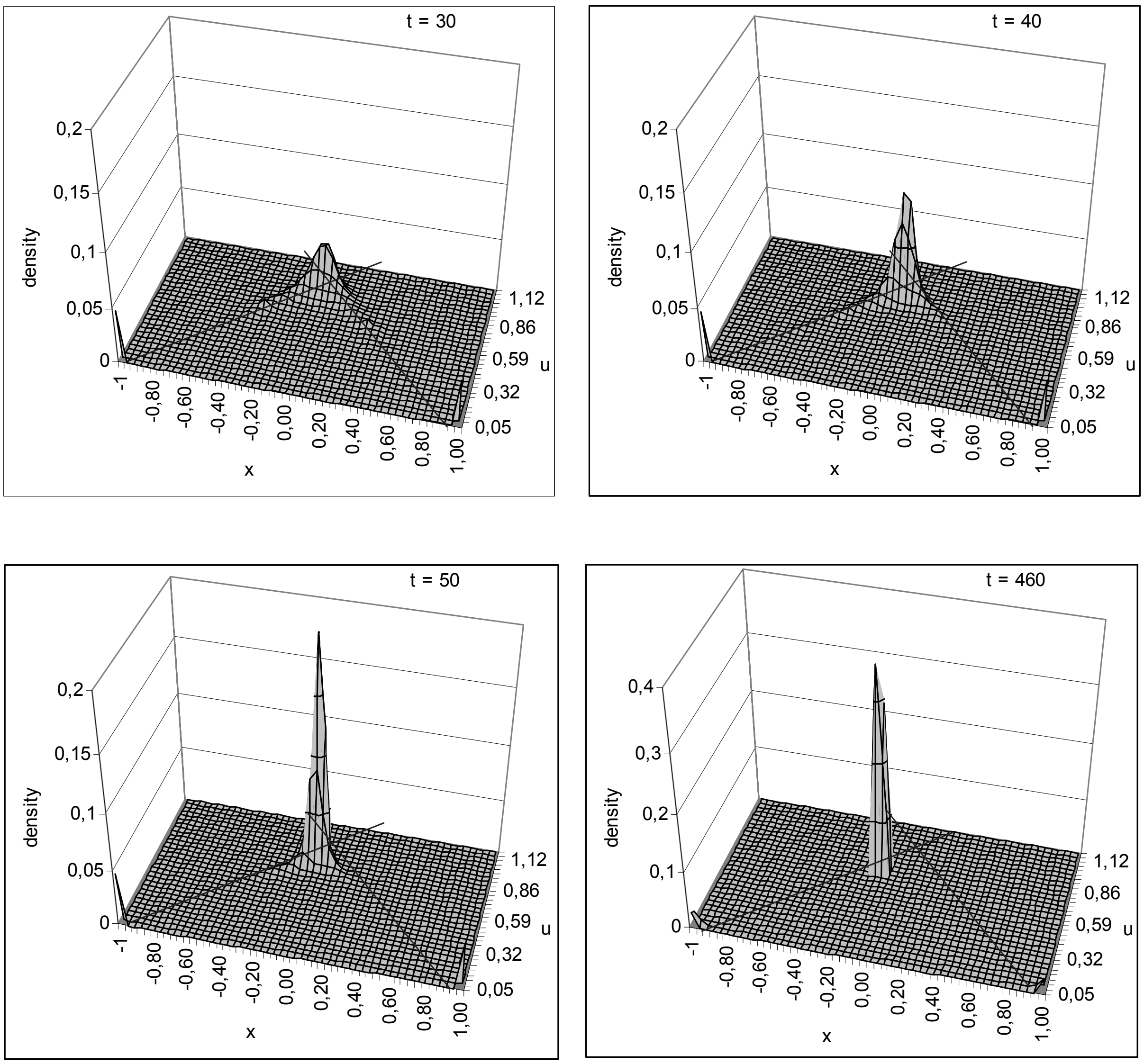
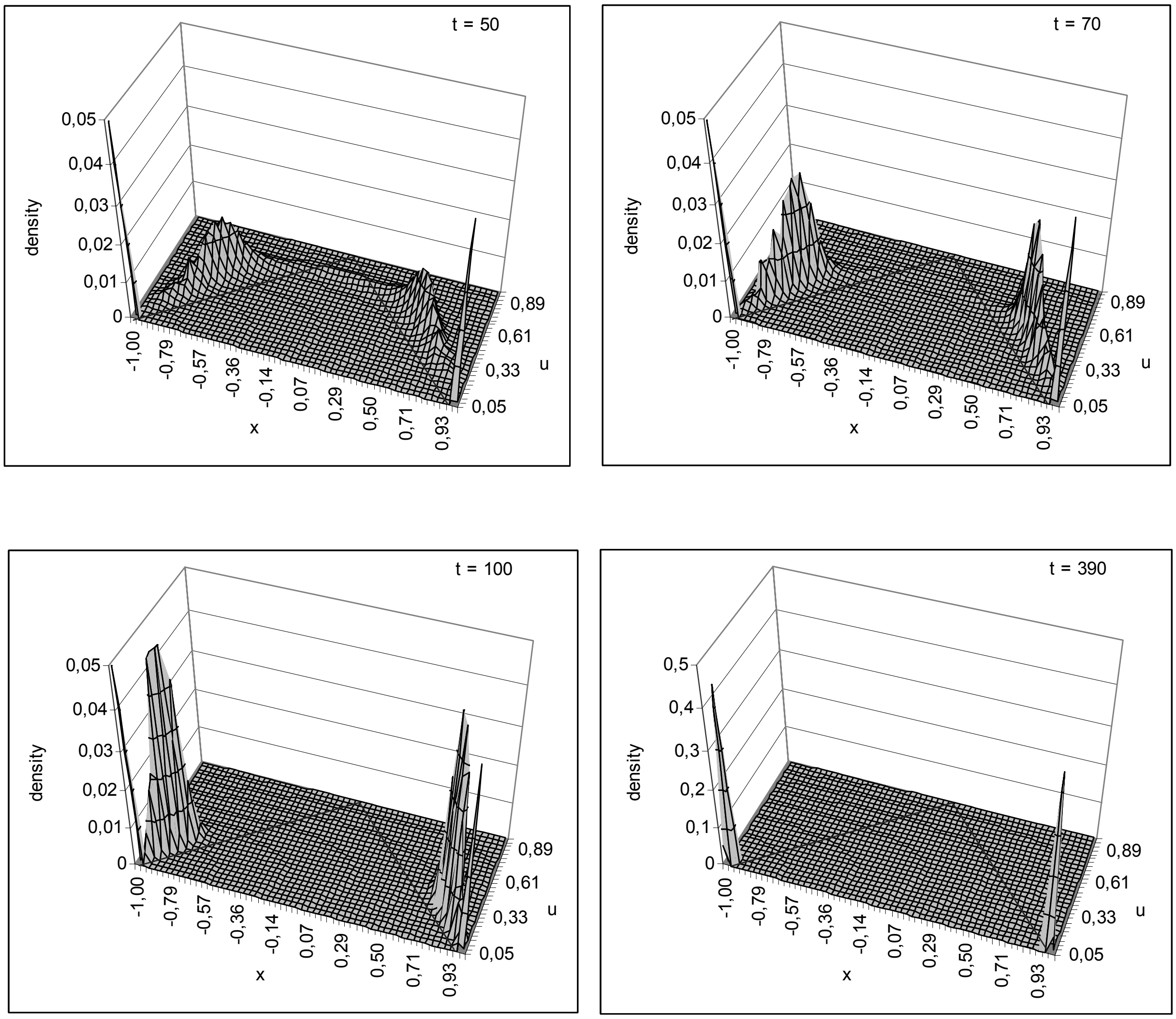


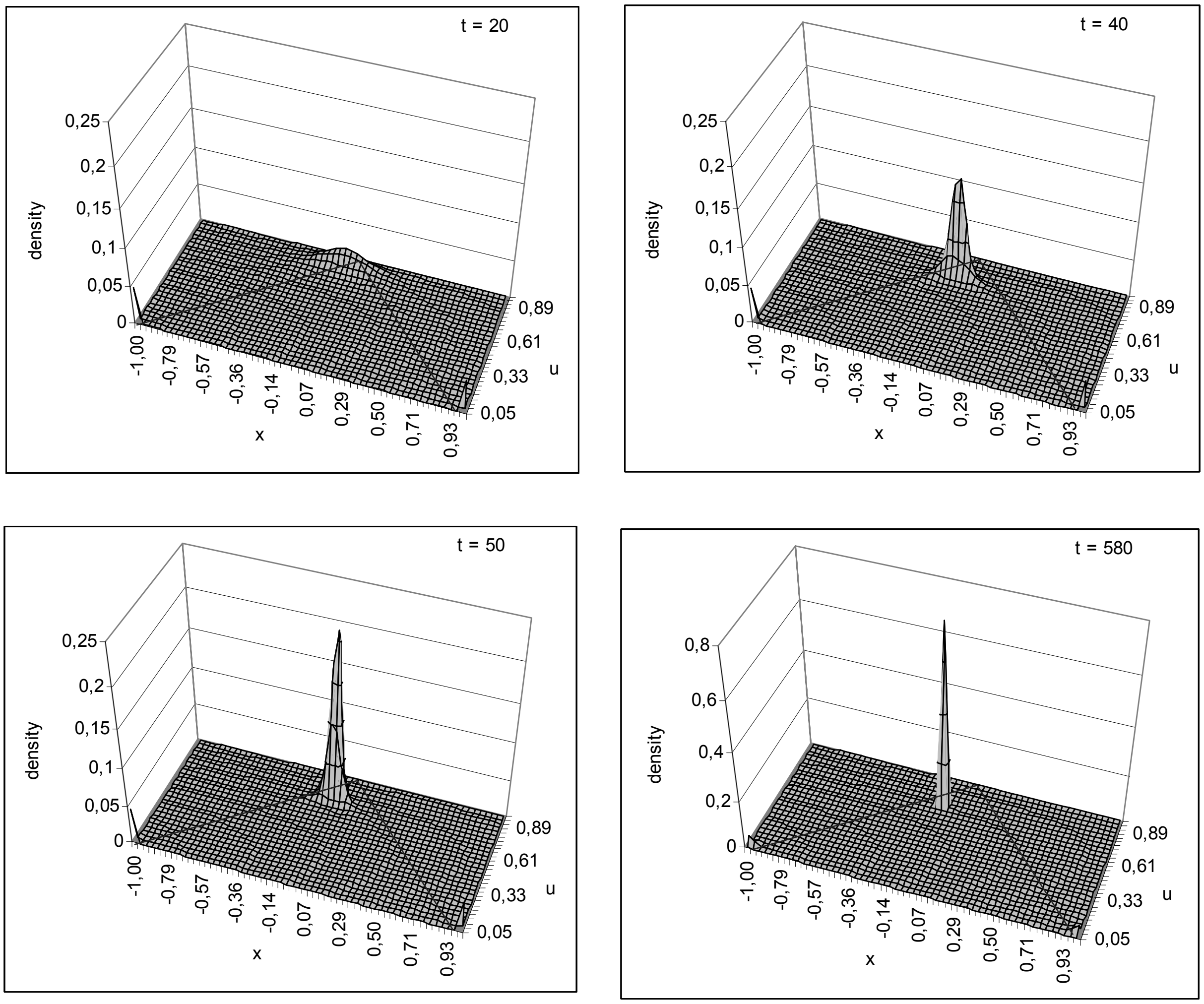
Figure 11. Views of convergence to double extreme attractor with the RA model. U = 1.0, ue= 0.05 size of the grid: 1500, μ = 0.3, l = 0.5. The initial density of extremists is 0.1, on the negative extreme: 0.0525, on the positive extreme: 0.0475. We note that at t = 50, the concentration tends to be higher in the extreme influence zones (above the black lines). This leads to the formation of two peaks, one at each extreme (double extreme convergence). The final attractor (double extreme) is reached at t = 390.



is a larger part of the distribution which is located in the zone where the extremes have no (direct) influence, which leads to a concentration of the distribution at an opinion close to 0 and a high uncertainty (visible at $t = 50$ and at convergence at $t = 580$).

The difference is due to the fact that the RA model gives relatively more influence to the ex-

Figure 12. Views of moderate convergence with the BC model, with the same parameters as in figure 11. U = 1.0, ue= 0.05, size of the grid: 1500, μ = 0.3, l = 0.5. The initial density of extremists is 0.1, on the negative extreme: 0.0525, on the positive extreme: 0.0475. We note that at t = 20, there is already a higher concentration in the centre, in the zone where the extremes have no influence (under the black lines). This leads to the formation of a peak located at an opinion which is close to 0, and with an uncertainty remaining high, as shown at t = 40 and t = 50. The final distribution (after around 600 time steps) is classified as central by our rules.



tremists, which attract the initial distribution to the extremes more quickly than the BC model. The double-extreme convergence takes place when the distribution splits into two almost equal parts, each located in the influence zone of one extreme.

DISCUSSION–CONCLUSION

We have shown that distribution dynamics, similar to that proposed in Ben Naim et al. (2003), yields attractor patterns in the parameter space which are similar to the ones obtained with the agent-based model for different variants of the influence model

(BC, RA, GBC, and GBCU). This result opened the possibility to study the distribution model in order to understand the process of convergence taking place in AB simulations.

We then studied the single and double extreme convergence, for BC and RA models. We observed the distribution in the extreme zones of influence. The observation is particularly relevant for the single-extreme convergence because it reveals that the shape of the distribution when going through the intersection of the extreme influence zones limits is crucial. Indeed, if the distribution is concentrated at this moment, it has very likely to enhance any small asymmetry very strongly. The same type of observations can be done for the other variants of the social influence models (GBC and GBCU), although the limits of the influence zones are not strict.

The interpretation of this observation in terms of collective social behaviour would be that the convergence to a single extreme is much facilitated in groups with a strong tendency to uniformity. There is a moment when the group uncertainty decreases to a threshold which makes it vulnerable to one extreme because it became indifferent to the other extreme. The process requires that the uncertainty of agents decreases when it interacts with both extremes (otherwise the central cloud would remain above the uncertainty threshold) which is certainly a questionable assumption.

Beyond the discussion about the realism of the model, we would like to stress the interest of deriving an aggregated model from an agent-based model. Of course, a similar study could have been done with AB models. However, the interpretation of the observed process is easier with the distribution model; the shape of this distribution appears crucial in the process. Therefore, in this example, the aggregated model helped to explain why the AB model converges to one or the other attractor.

REFERENCES

Amblard F., & Deffuant, G. (2004). The role of network topology on extremism propagation with relative agreement opinion dynamics. *Physica A.*, *343*, 725-738.

Ben-Naim, E., Krapivsky, P.L.,Vazquez, F., & Redner, S. (2003). Unity and discord in opinion dynamics. *Physica A.*, *330*(1-2), 99-106

Deffuant, G., Neau, D., Amblard, F., & Weisbuch, G. (2001). Mixing beliefs among interacting agents. *Advances in Complex Systems*, 3, 87-98.

Deffuant, G., Amblard, F., Weisbuch, G., & Faure, T. (2002). How can extremism prevail? A study based on the relative agreement interaction model. *Journal of Artificial Societies and Social Simulation, 5,4.* http://jasss.soc.surrey.ac.uk/5/4/1.html

Deffuant, G. (2006). Comparing extremism propagation patterns in continuous opinion models. *Journal of Artificial Societies and Social Simulation*, *9*(3). http://jasss.soc.surrey.ac.uk/9/3/8.html

Deffuant G., & Huet, S. (2006). Collective reinforcement of first impression bias. *Proceedings of the 1st World Conference on Social Simulation*. Kyoto.

Derrida, B., & Flyvbjerg, H. (1986). Multivalley structure in Kauffman's model: Analogy with spin glasses. *J. Phys. A*19, L1003-L1008.

Edwards M., Huet, S., Goreaud F., &Deffuant, G. (2003). Comparing an individual-based model of behaviour diffusion with its mean field aggregate approximation. *Journal of Artificial Societies and Social Simulation*, *6*(40).

Hegselmann, R., & Krause, U. (2002). Opinion dynamics and bounded confidence models, analysis and simulation. *Journal of Artificial Societies* *and Social Simulation*, *5*(3). http://jasss.soc.surrey.ac.uk/5/3/2.html

Stauffer, D., Sousa, A., & Schulze, C. (2004). Discretized opinion dynamics of the Deffuant model on scale-free networks. *Journal of Artificial Societies and Social Simulation*, *7*(3). http://jasss.soc.surrey.ac.uk/7/3/7.html

Urbig, D. (2003). Attitude dynamics with limited verbalisation capabilities. *Journal of Artificial Societies and Social Simulation*, *6*(1). http://jasss.soc.surrey.ac.uk/6/1/2.html

Urbig, D., & Lorenz J. (2004, September 16-19). Communication regimes in opinion dynamics: Changing the number of communicating agents. In *Proceedings of the Second Conference of the European Social Simulation Association (ESSA)*, Valladolid, Spain.

Weidlich, W., & Haag, G.(1999). *An integrated model of transport and urban evolution*. Springer.

Weisbuch, G., Deffuant, G., Amblard, F., & Nadal, J.P. (2002). Meet, discuss and segregate! *Complexity*, *7*(3), 55-63.