Calibrating a complex social model

Maxime Lenormand, Guillaume Deffuant, Sylvie Huet

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Laboratory of Engineering for Complex Systems
Cemagref of Clermont-Ferrand

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Cemagref
Sciences, eaux & territoires

Prototypical Policy Impacts on Multifunctional Activities in rural municipalities
A collaborative project under the EU Seventh Framework Programme

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Motivation

Complex social model

- Individual-based model
- Stochastic
- High dimensional parameter space
- High computational cost by simulation

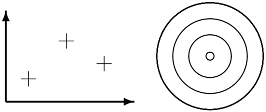
Estimate the parameter values

- Calibrate the model
- Understand the model behaviour
- Uncertainty analysis
- Validation

Summary

Approximate Bayesian Computation

1. Sample $\theta^* \sim \pi(\theta)$.
2. Simulate $x \sim f(x|\theta^*)$.
3. If $\rho(x, y) \leq \epsilon$, accept θ^*, otherwise reject.
4. Repeat until a sample of the desired size is obtained



Prior distribution
$\pi(\theta)$

Posterior distribution
$\pi(\theta) P_\theta\{f(x|\theta) = y\}$

(Pritchard et al., 1999)

Derived from T. Toni 2011

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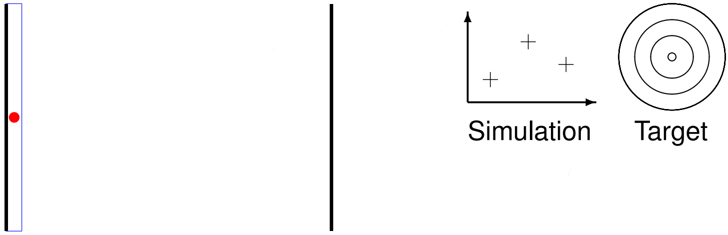

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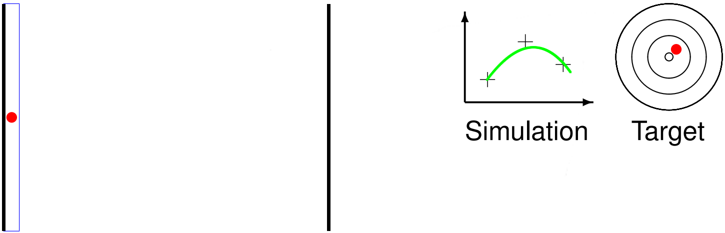

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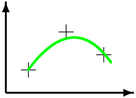
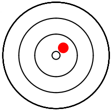


Simulation



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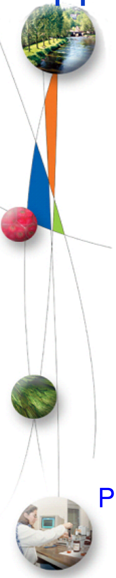

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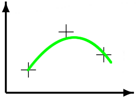

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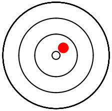

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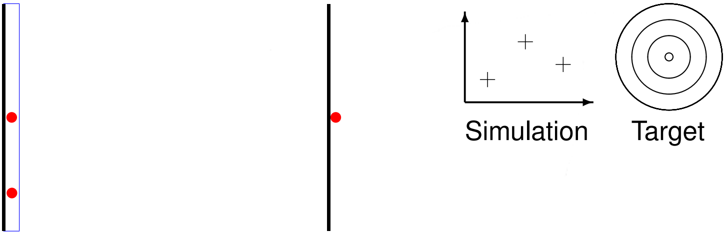

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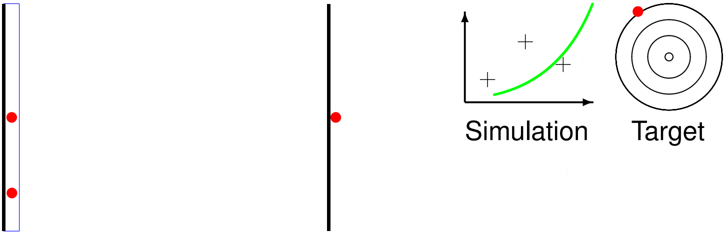

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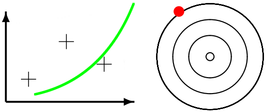

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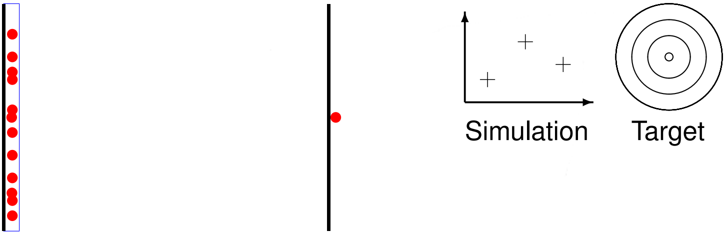

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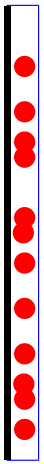

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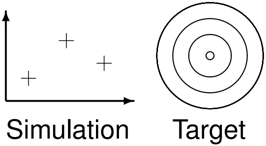

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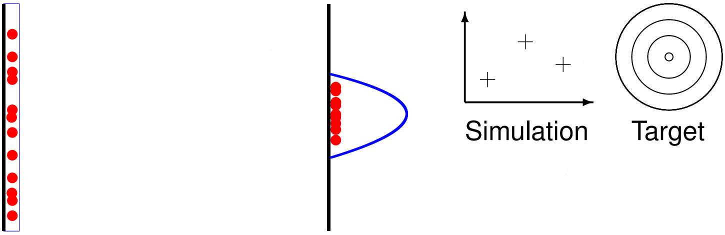

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ABC SMC (Sequential Monte-Carlo)

Algorithm

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Posterior

(Sisson et al., 2007)
(Beaumont et al., 2009)

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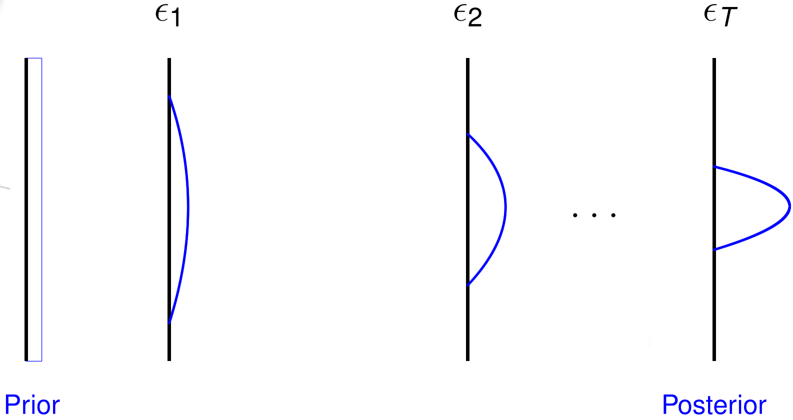

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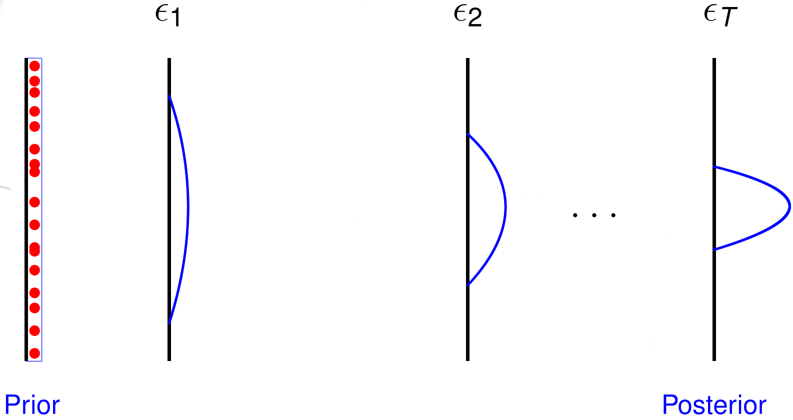

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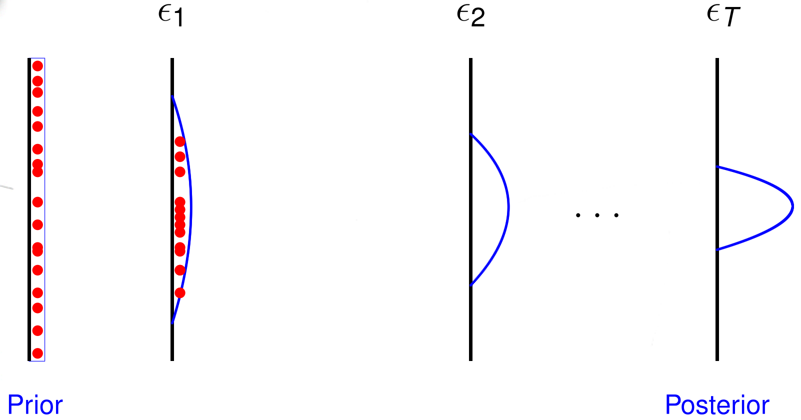

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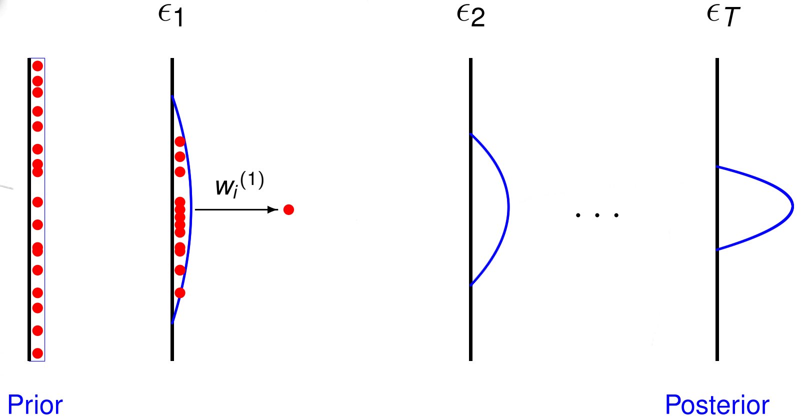

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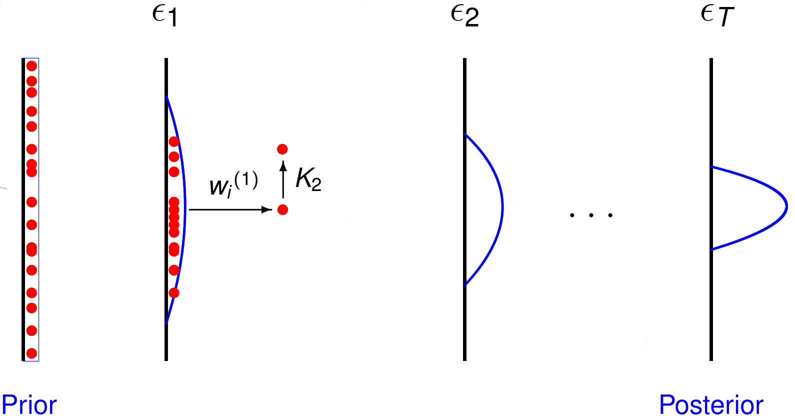

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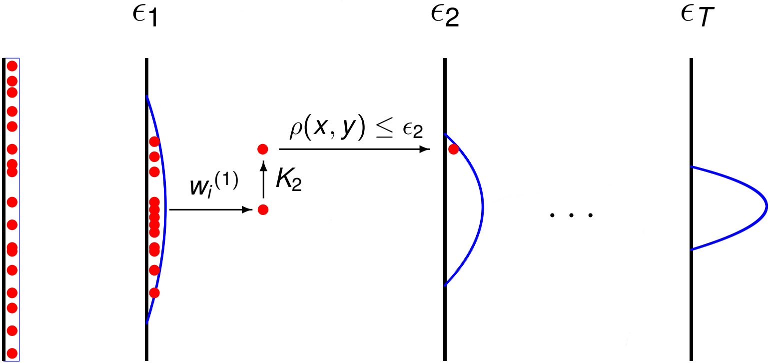

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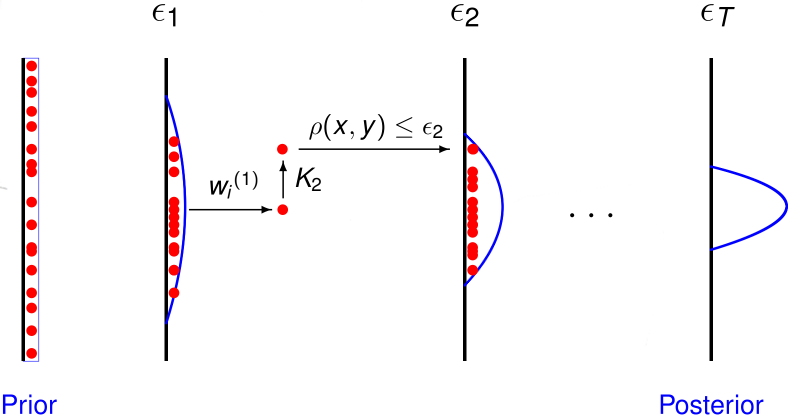

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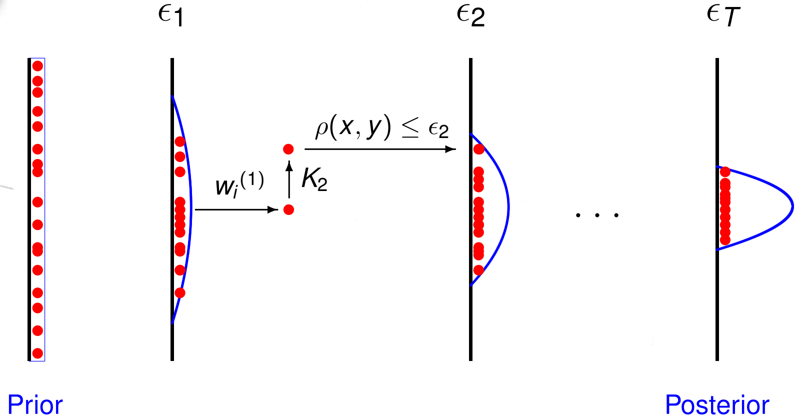

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ABC SMC (Sequential Monte-Carlo)

Issues related to the model complexity

- How to control the number of simulations?
- How to determine the sequence of tolerance levels $\{\epsilon_1, ..., \epsilon_T\}$?
- When to stop the algorithm?

Summary

Adaptive ABC SMC for complex models

Algorithm

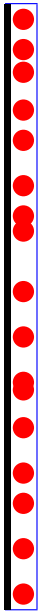
Prior

(Lenormand et al.)

Adaptive ABC SMC for complex models

Algorithm

N particles
(LHS)



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Adaptive ABC SMC for complex models

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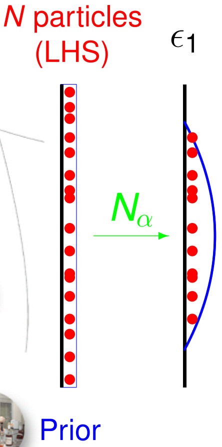

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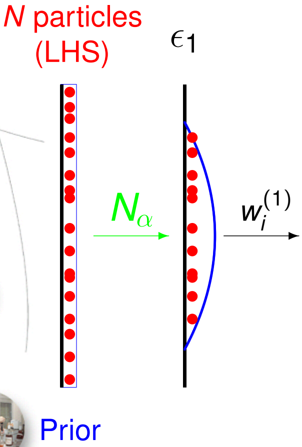

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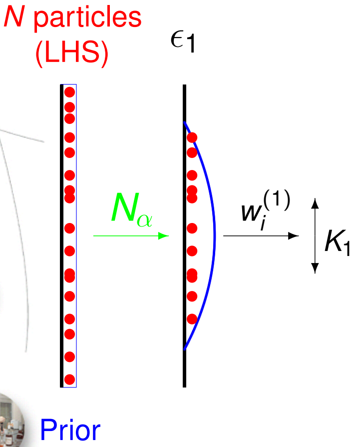

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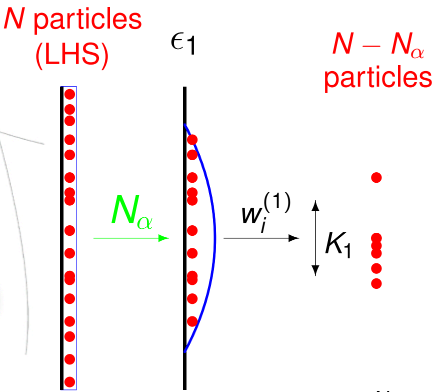

Prior

$$p_{acc} = \frac{\sum_{k=N_\alpha+1}^{N} \mathbb{1}_{\rho(x,y)\leq\epsilon_1}}{N - N_\alpha}$$

(Lenormand et al.)

Adaptive ABC SMC for complex models

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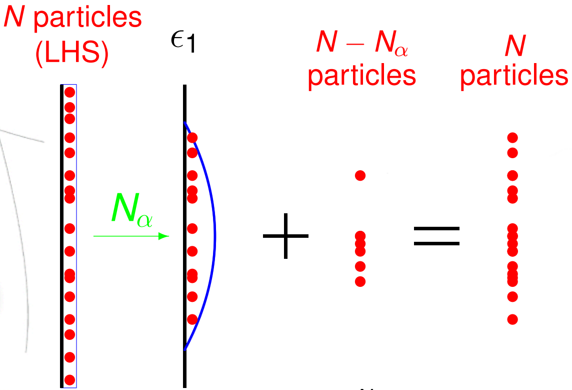

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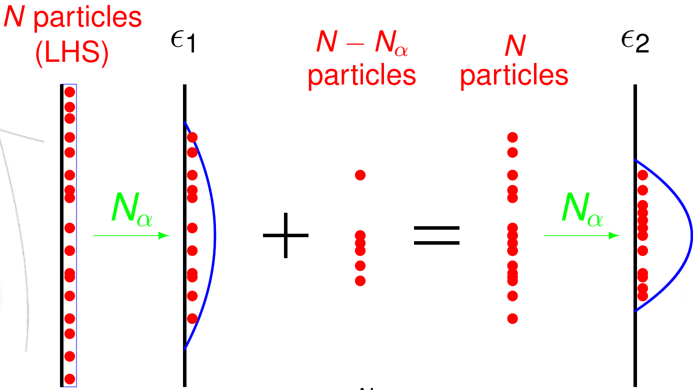

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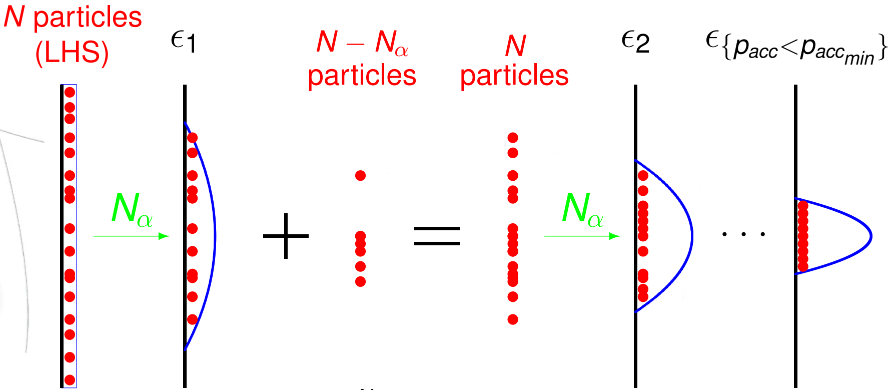

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Posterior

(Lenormand et al.)

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Issues related to the model complexity

- How to control the number of simulations?

Adaptive ABC SMC for complex models

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  $\Longrightarrow N - N_\alpha$ simulations at each iteration

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  $\Longrightarrow \epsilon_t = \alpha$-quantile of the N distances to the data

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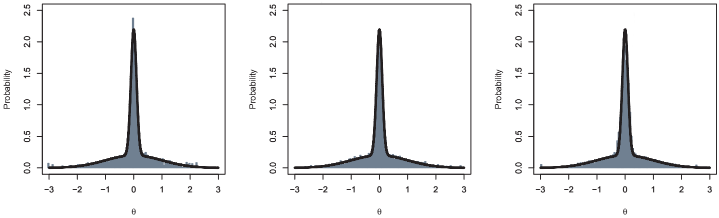
- How to determine the sequence of tolerance levels $\{\epsilon_1, ..., \epsilon_T\}$?
  $\Longrightarrow \epsilon_t = \alpha$-quantile of the N distances to the data
- When to stop the algorithm?
  $\Longrightarrow p_{acc} < p_{acc_{min}}$

Adaptive ABC SMC for complex models

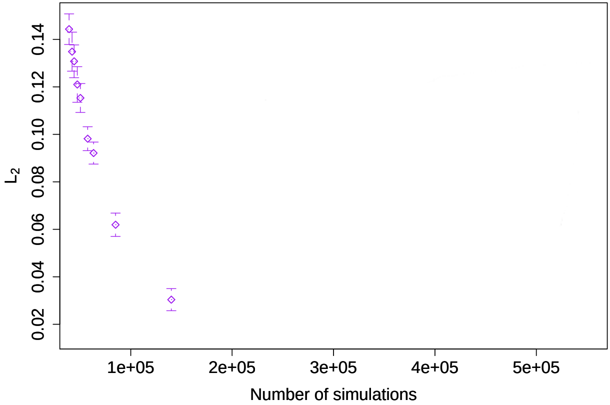
Toy example: Presentation

$$f(x|\theta) \sim \frac{1}{2}\phi\left(\theta, \frac{1}{100}\right) + \frac{1}{2}\phi(\theta, 1) \text{ and } \theta \sim \mathcal{U}_{[-10,10]}$$



Adaptive ABC SMC for complex models
Toy example: Parameters Study

$N_\alpha = 5000$; α from 0.9 to 0.1 corresponding to $N = 5555$ to 50000



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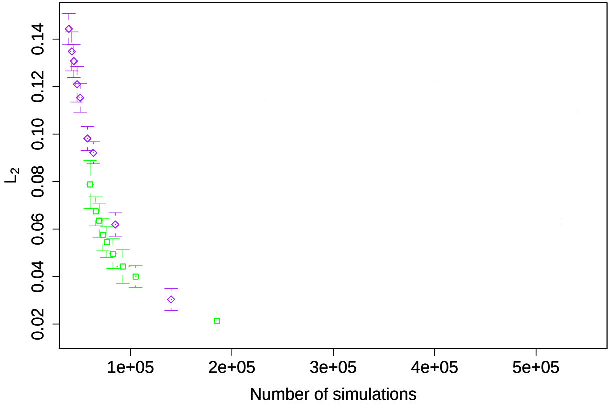

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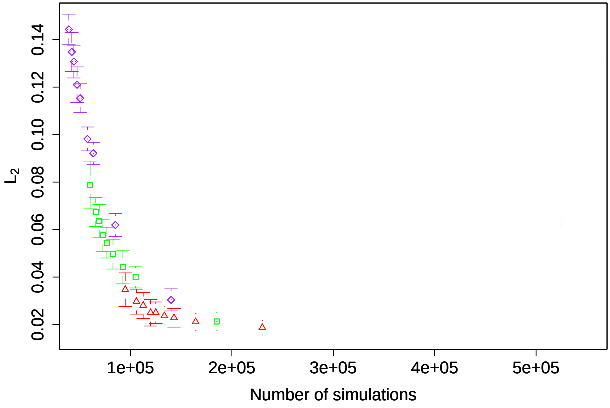

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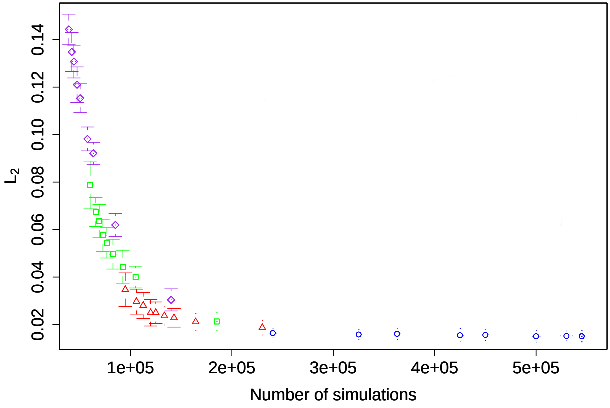
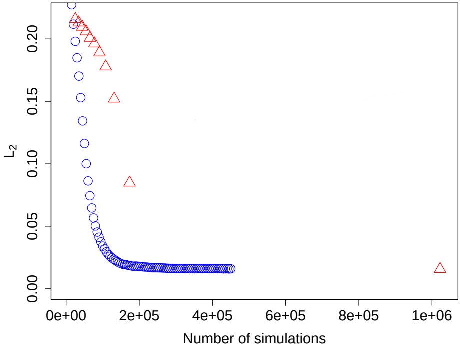


Adaptive ABC SMC for complex models

Toy example: Model comparison



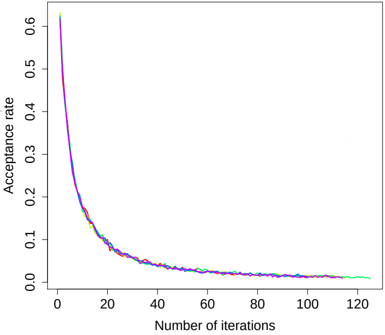
Summary

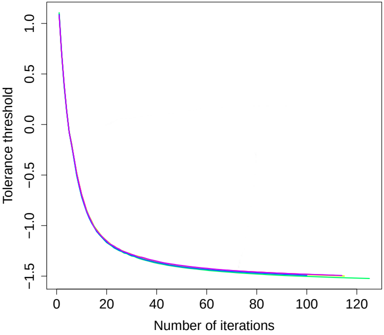
The PRIMA model
Parameters and summary statistics

- 4 parameters
- 8 summary statistics
- $\left\|(\rho_m(S_m, S_m'))_{1 \leq m \leq M}\right\|_\infty = \sup_{1 \leq m \leq M} |\rho_m(S_m, S_m')|$

The PRIMA model

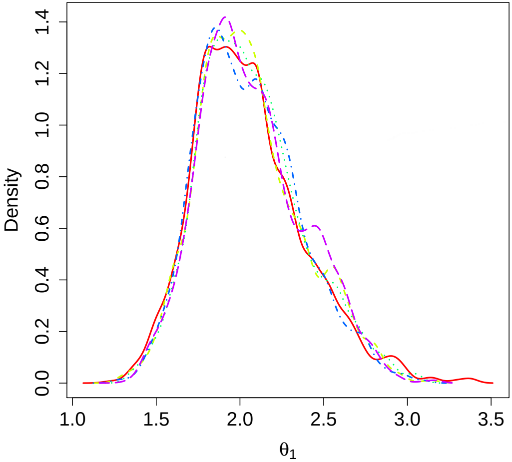
Acceptance rate and threshold evolution





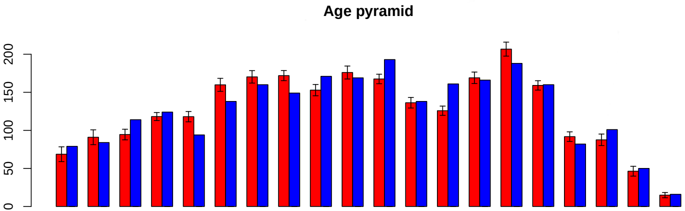
The PRIMA model

Posterior density of a parameter



The PRIMA model
Concrete results

- 1.4 second by simulations
- 400,000 simulations
- 6 days



Conclusion

- We have answered the three research questions
- Comparison with a well-known method
- Calibration of a complex social model