

Calibrating a complex social model

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ECCS'11
September 16th 2011



This publication has been funded under the PRIMA (Prototypical policy impacts on multifunctional activities in rural municipalities) collaborative project, EU 7th Framework Programme (ENV 2007-1), contract no. 212345.

Motivation



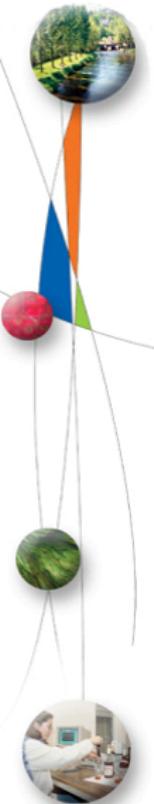
Complex social model

- Individual-based model
- Stochastic
- High dimensional parameter space
- High computational cost by simulation

Estimate the parameter values

- Calibrate the model
- Understand the model behaviour
- Uncertainty analysis
- Validation

Summary



- 1 Approximate Bayesian Computation (ABC)
- 2 Adaptive approximate Bayesian computation for complex models
- 3 The PRIMA model

Approximate Bayesian Computation

- 
- ① Sample $\theta^* \sim \pi(\theta)$.
 - ② Simulate $x \sim f(x|\theta^*)$.
 - ③ If $\rho(x, y) \leq \epsilon$, accept θ^* , otherwise reject.
 - ④ Repeat until a sample of the desired size is obtained



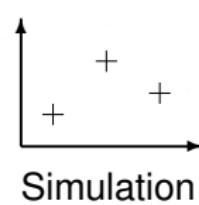
Prior distribution
 $\pi(\theta)$

(Pritchard et al., 1999)



Posterior distribution
 $\pi(\theta)P_\theta\{f(x|\theta) = y\}$

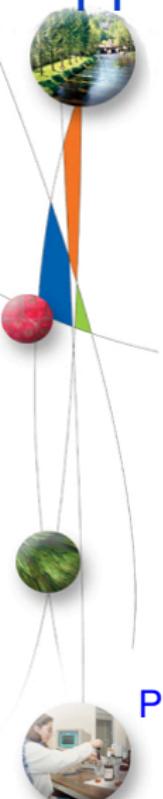
Derived from T. Toni 2011



Simulation

Target

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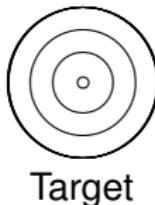
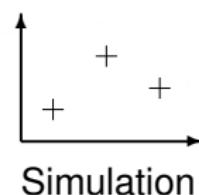
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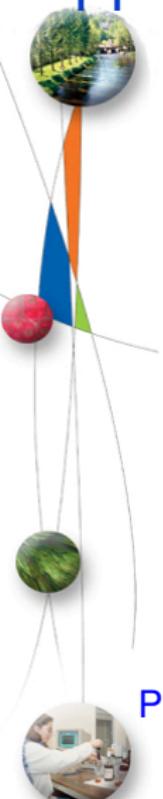


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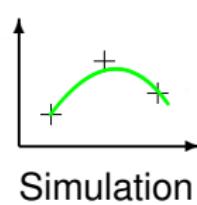
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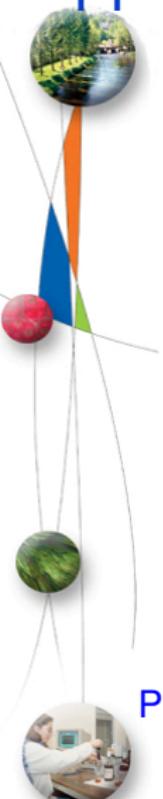


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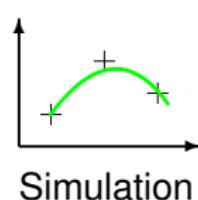
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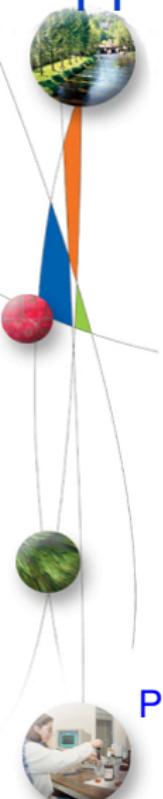


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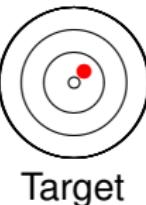
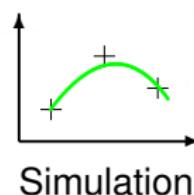


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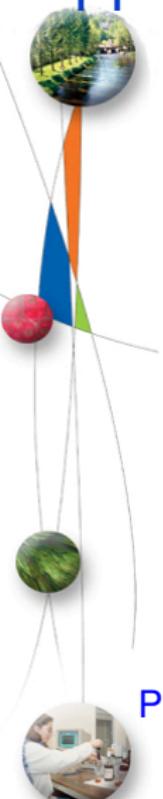
Simulation

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Derived from T. Toni 2011

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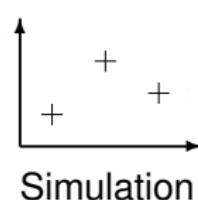
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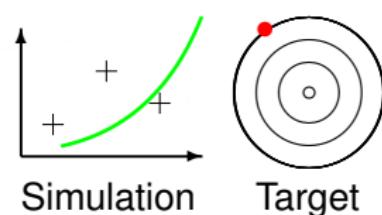
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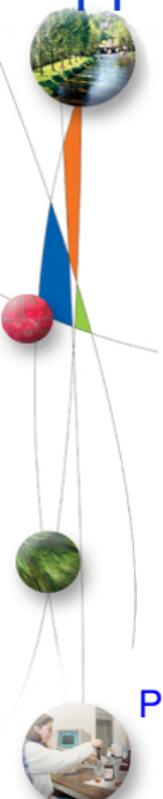


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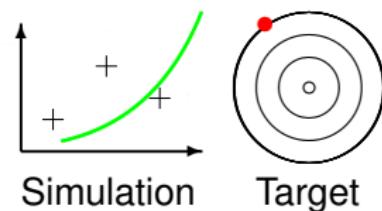


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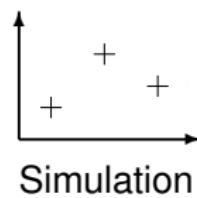
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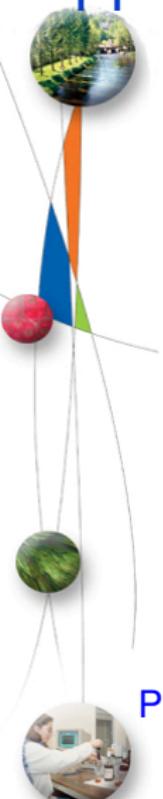
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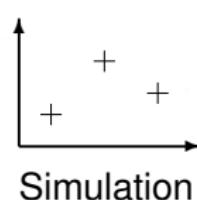
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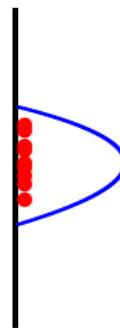
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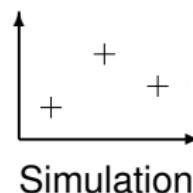
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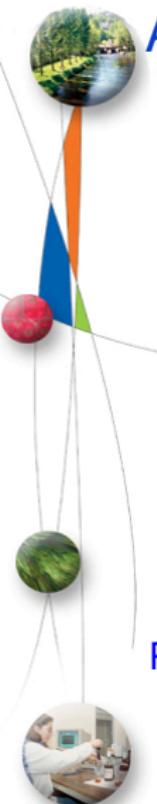


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Derived from T. Toni 2011



ABC SMC (Sequential Monte-Carlo) Algorithm



Prior

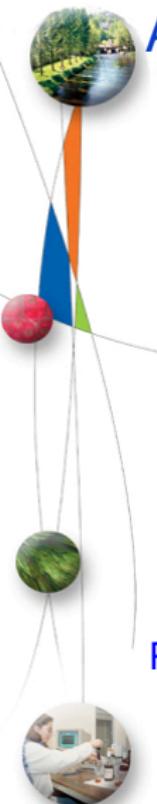
*(Sisson et al., 2007)
(Beaumont et al., 2009)*



Posterior

Derived from T. Toni 2011

ABC SMC (Sequential Monte-Carlo) Algorithm

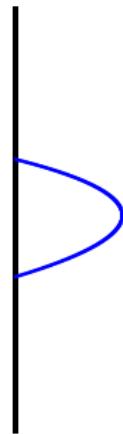


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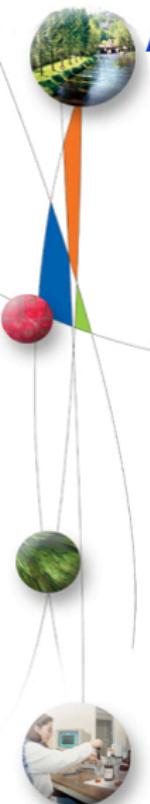
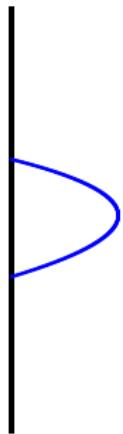
ϵ_T



Posterior

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ABC SMC (Sequential Monte-Carlo) Algorithm

 ϵ_1 ϵ_2 ϵ_T  \dots 

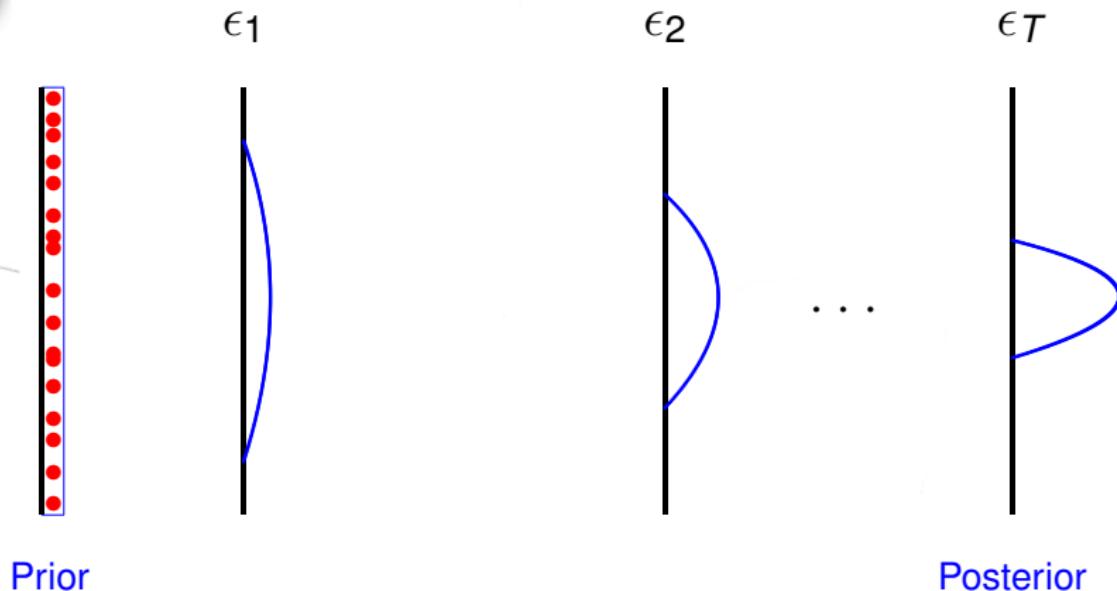
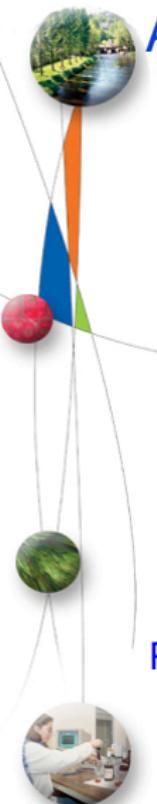
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Posterior

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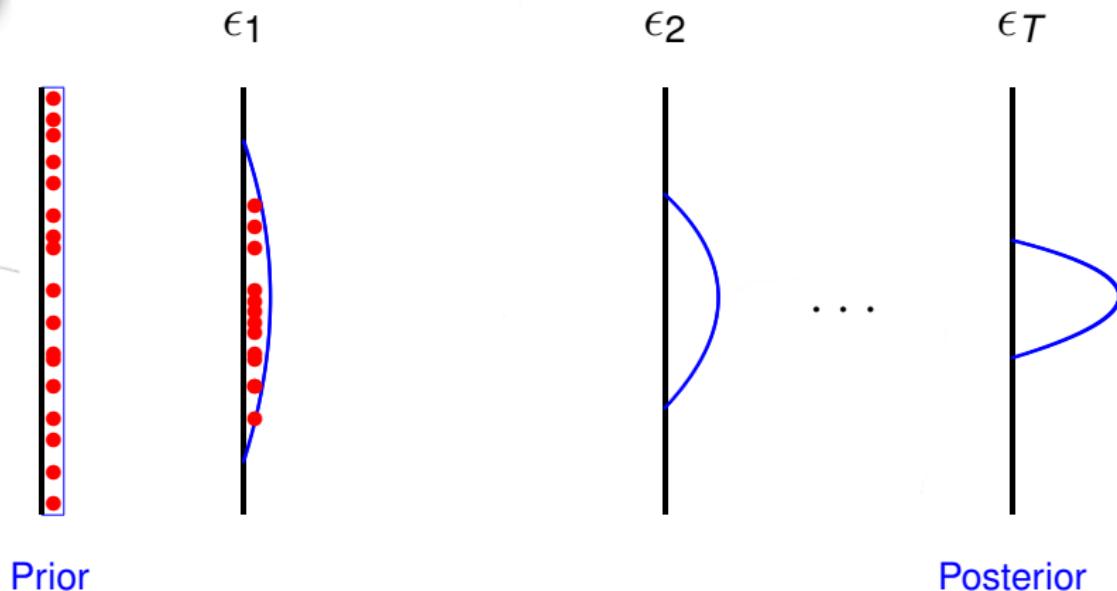
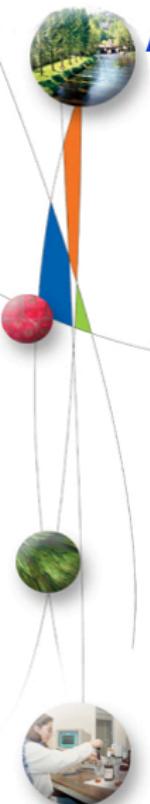
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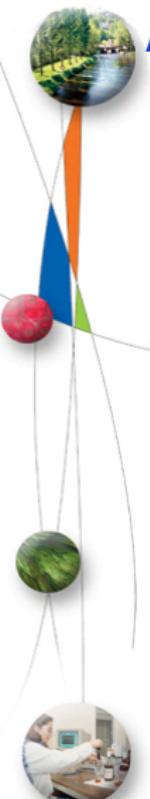
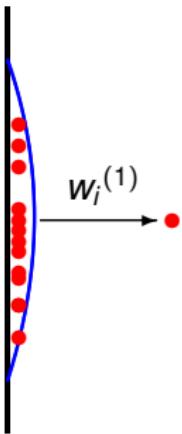
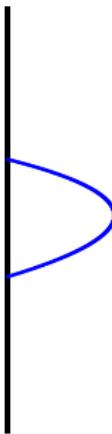
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ABC SMC (Sequential Monte-Carlo) Algorithm

 ϵ_1 ϵ_2 ϵ_T  \dots 

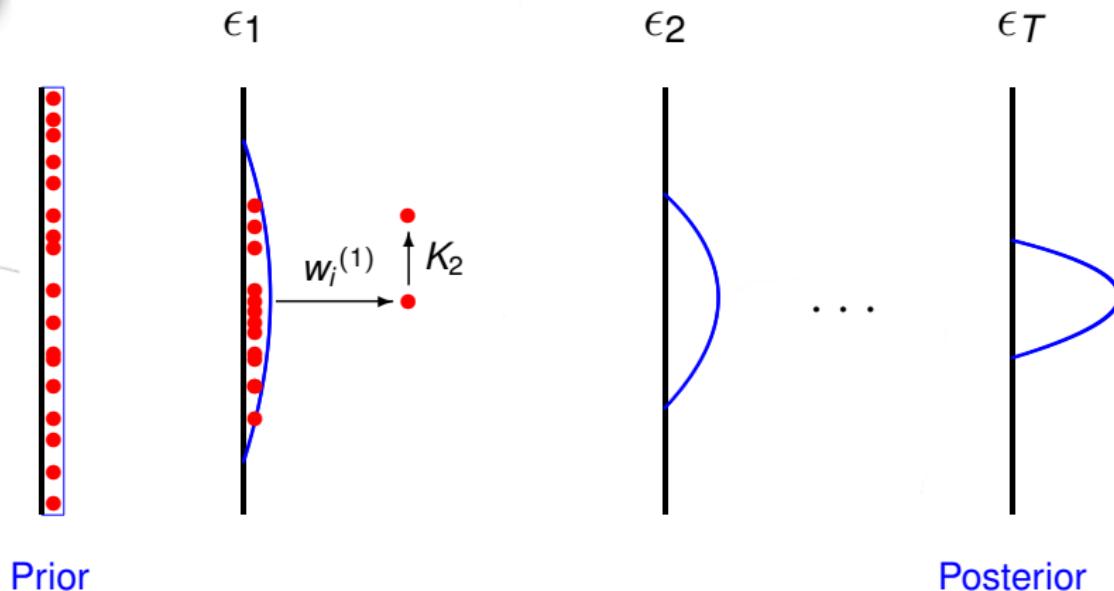
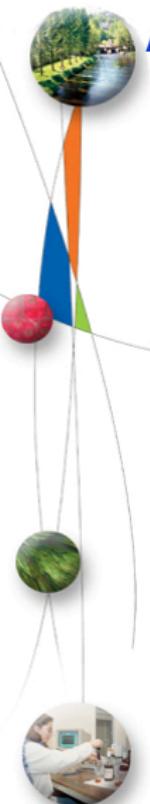
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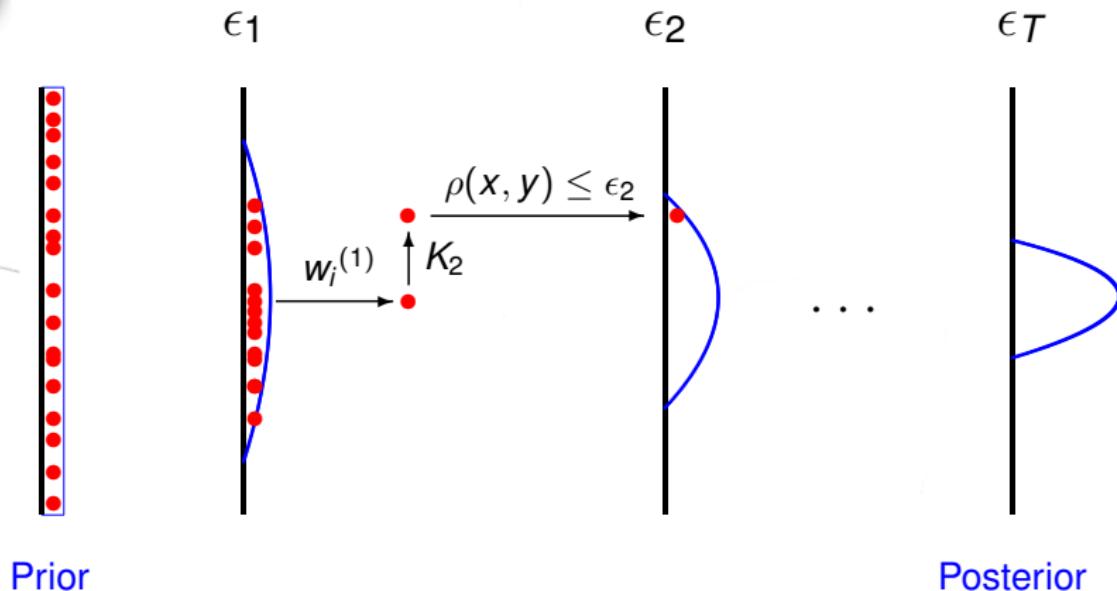
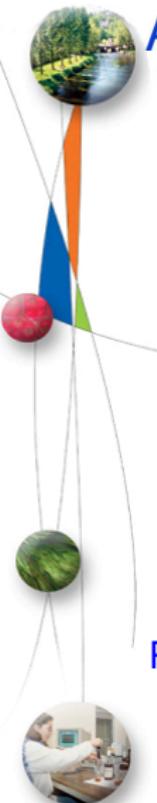
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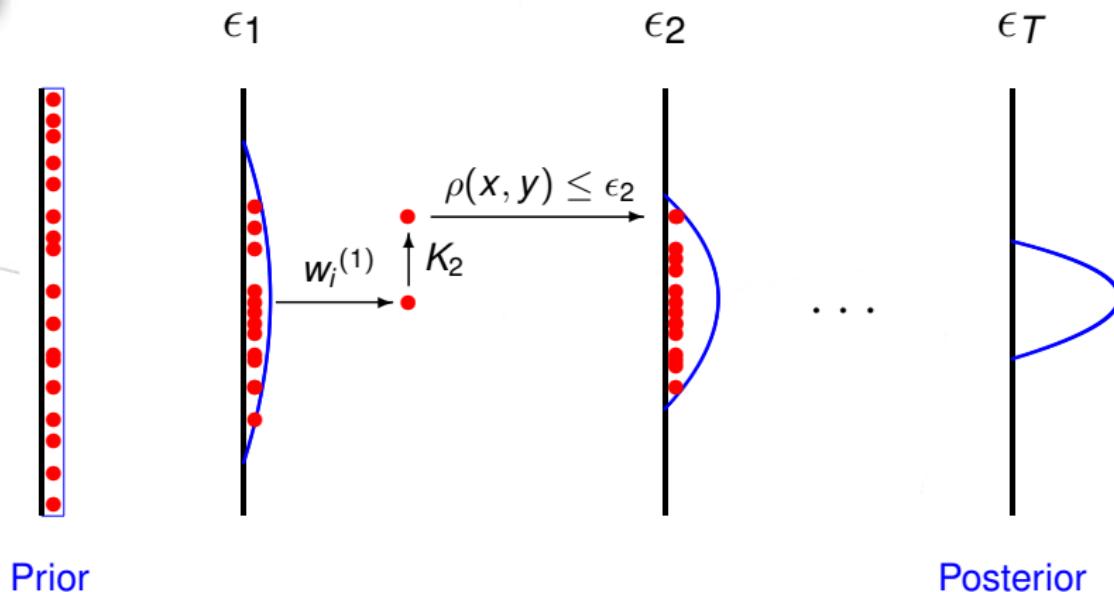
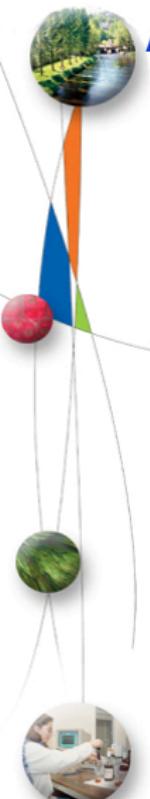
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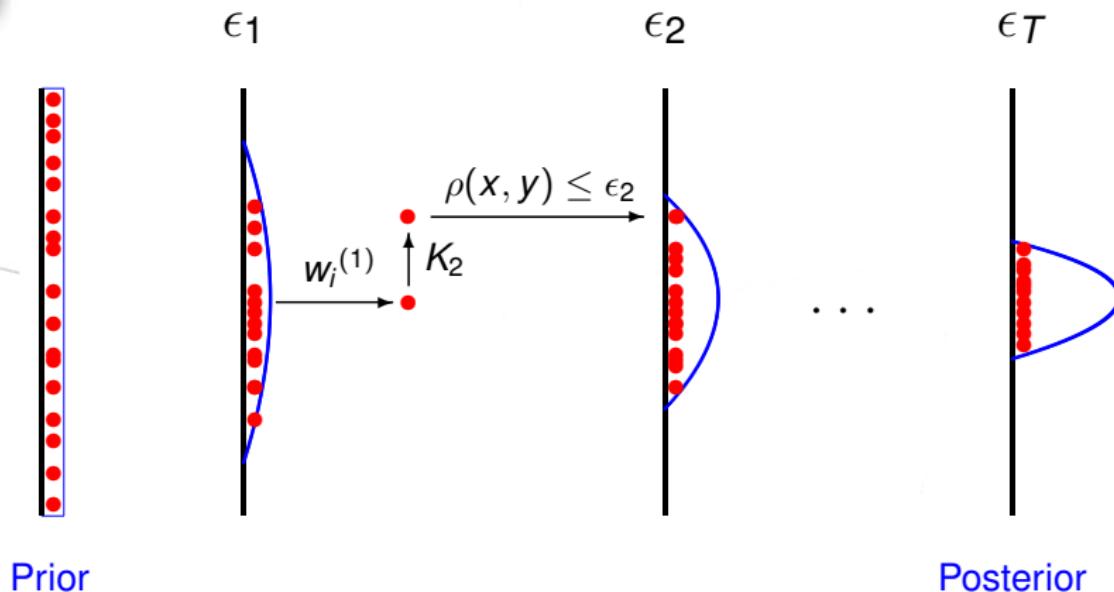
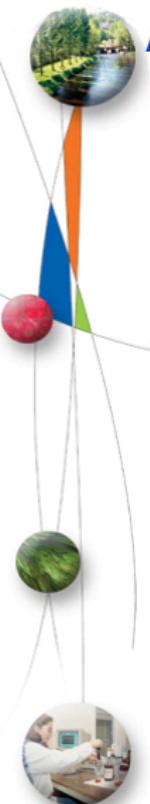
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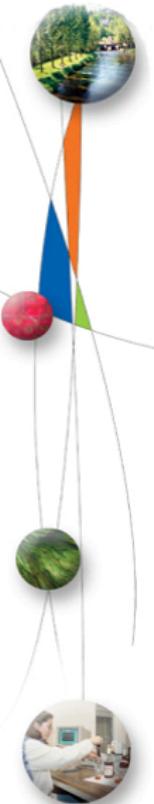
ABC SMC (Sequential Monte-Carlo)

Issues related to the model complexity

- How to control the number of simulations?
- How to determine the sequence of tolerance levels $\{\epsilon_1, \dots, \epsilon_T\}$?
- When to stop the algorithm?

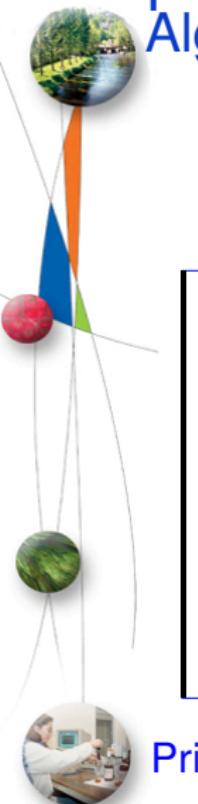


Summary



- 1 Approximate Bayesian Computation (ABC)
- 2 Adaptive approximate Bayesian computation for complex models
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Adaptive ABC SMC for complex models Algorithm



Prior

(Lenormand et al.)

Adaptive ABC SMC for complex models

Algorithm



N particles
(LHS)



Prior

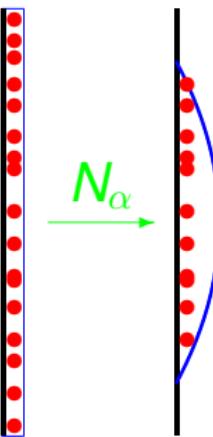
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Adaptive ABC SMC for complex models Algorithm



N particles
(LHS)

ϵ_1



Prior

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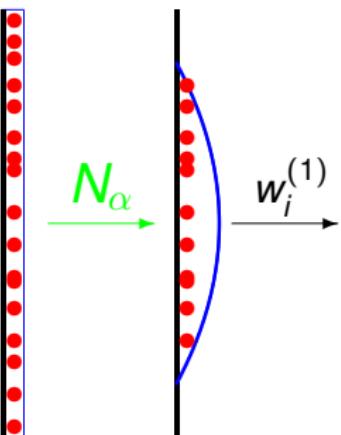
Adaptive ABC SMC for complex models

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N particles
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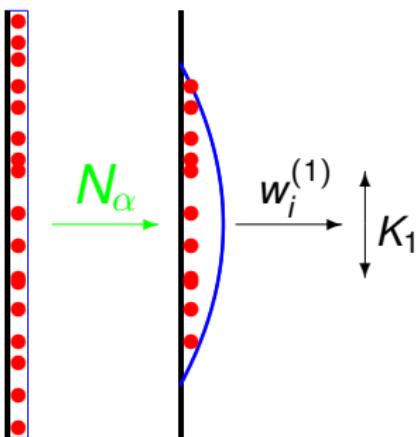
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Adaptive ABC SMC for complex models

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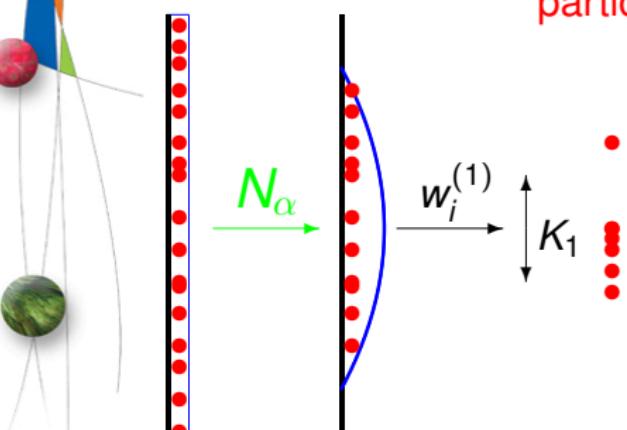


N particles
(LHS)

N_α

ϵ_1

$N - N_\alpha$
particles



Prior

(Lenormand et al.)

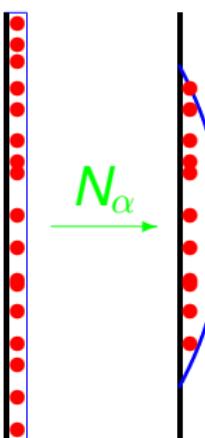
$$p_{acc} = \frac{\sum_{k=N_\alpha+1}^N \mathbb{1}_{\rho(x,y) \leq \epsilon_1}}{N - N_\alpha}$$

Adaptive ABC SMC for complex models

Algorithm



N particles
(LHS)



$N - N_\alpha$ particles N particles

+

•

=

•

•

•

•

•

Prior

(Lenormand et al.)

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Adaptive ABC SMC for complex models

Algorithm



N particles
(LHS)



ϵ_1

$N - N_\alpha$
particles

N particles

ϵ_2

+

=

N_α

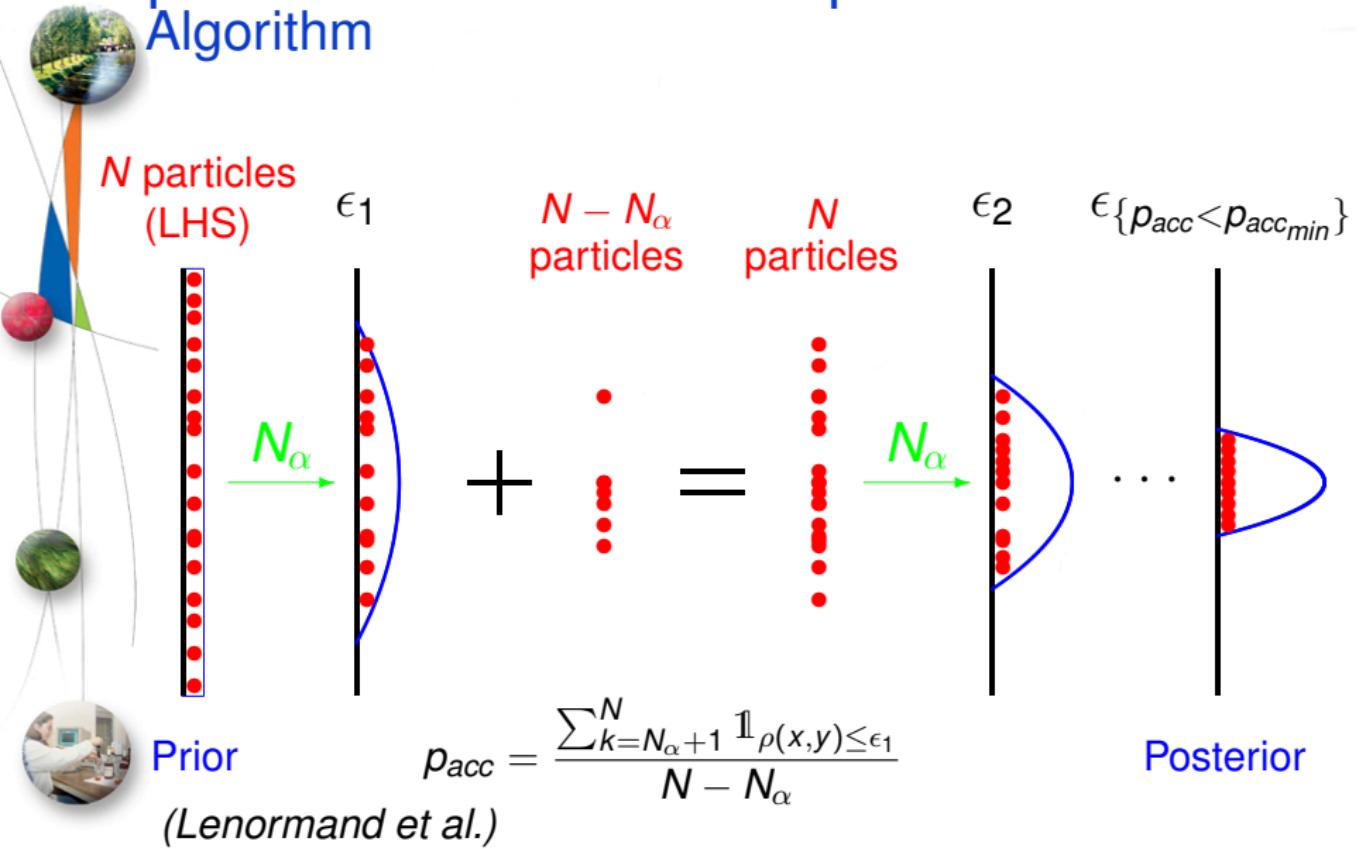
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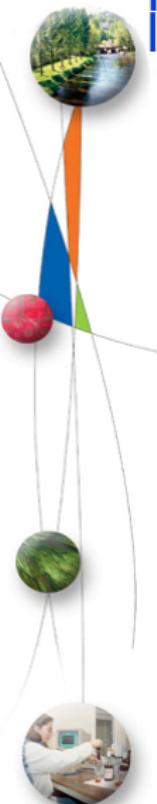
Adaptive ABC SMC for complex models

Algorithm



Adaptive ABC SMC for complex models

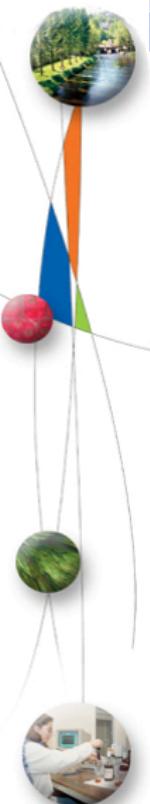
Issues related to the model complexity



- How to control the number of simulations?

Adaptive ABC SMC for complex models

Issues related to the model complexity



- How to control the number of simulations?
 $\Rightarrow N - N_\alpha$ simulations at each iteration

Adaptive ABC SMC for complex models

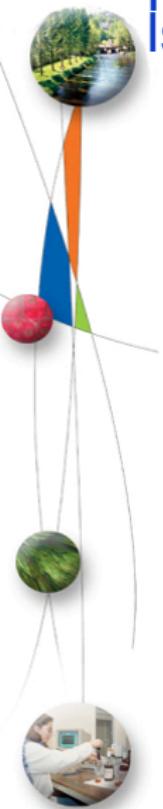
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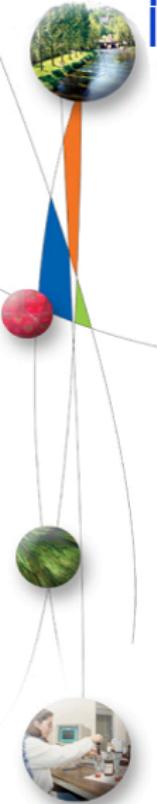
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 $\implies \epsilon_t = \alpha$ -quantile of the N distances to the data

Adaptive ABC SMC for complex models

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 - When to stop the algorithm?

Adaptive ABC SMC for complex models

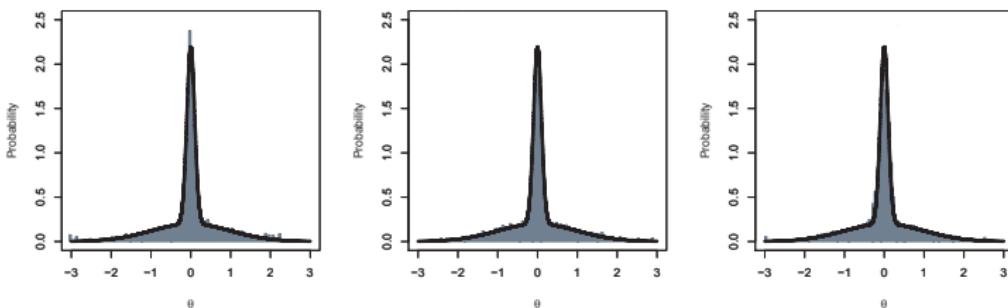
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 $\implies \epsilon_t = \alpha\text{-quantile of the } N \text{ distances to the data}$
- When to stop the algorithm?
 $\implies p_{acc} < p_{acc_{min}}$

Adaptive ABC SMC for complex models

Toy example: Presentation

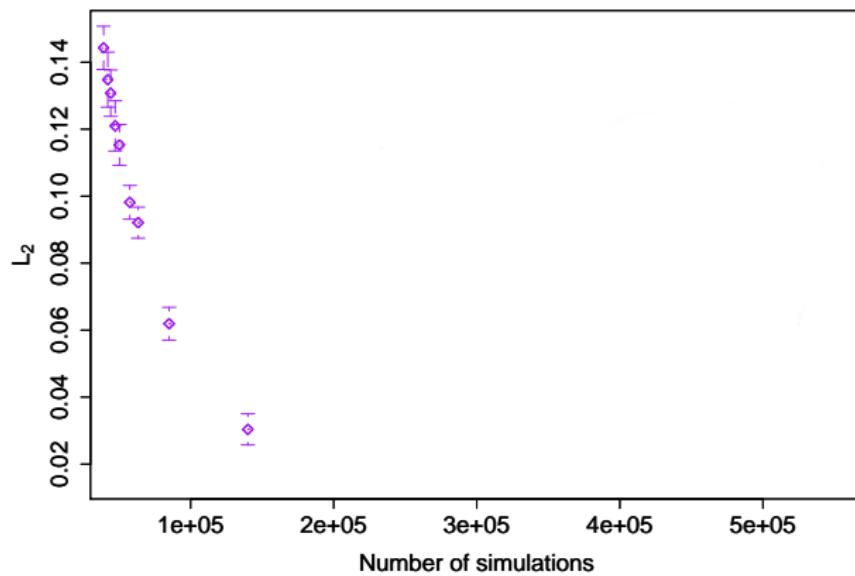
$$f(x|\theta) \sim \frac{1}{2}\phi\left(\theta, \frac{1}{100}\right) + \frac{1}{2}\phi(\theta, 1) \text{ and } \theta \sim \mathcal{U}_{[-10, 10]}$$



Adaptive ABC SMC for complex models

Toy example: Parameters Study

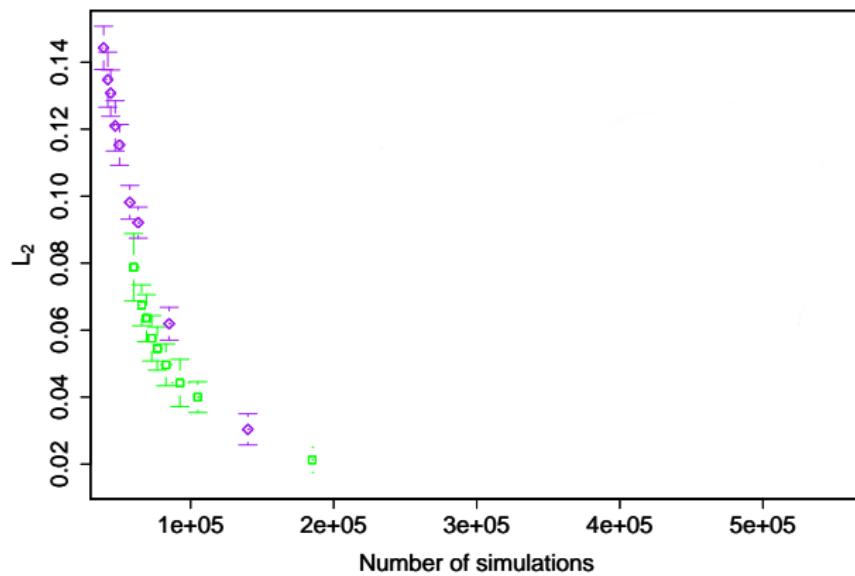
$N_\alpha = 5000$; α from 0.9 to 0.1 corresponding to $N = 5555$ to 50000



Adaptive ABC SMC for complex models

Toy example: Parameters Study

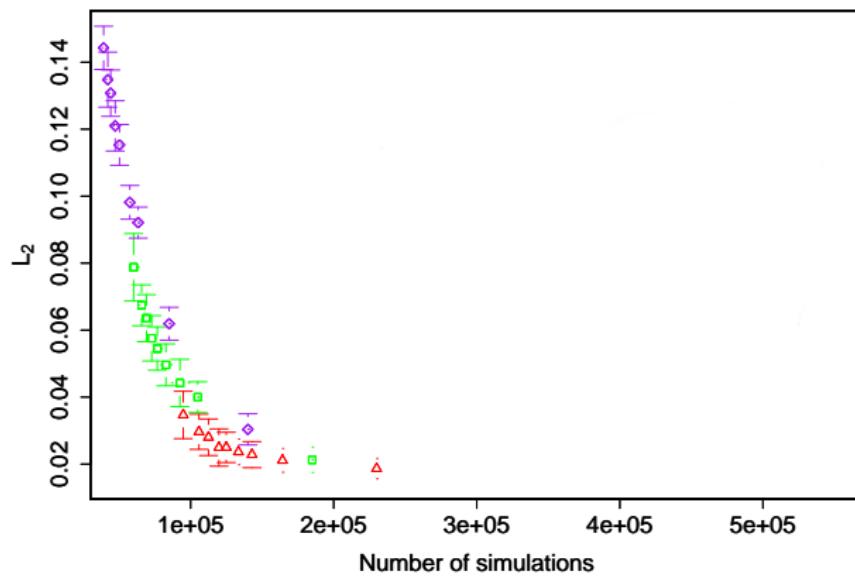
$N_\alpha = 5000$; α from 0.9 to 0.1 corresponding to N = 5555 to 50000



Adaptive ABC SMC for complex models

Toy example: Parameters Study

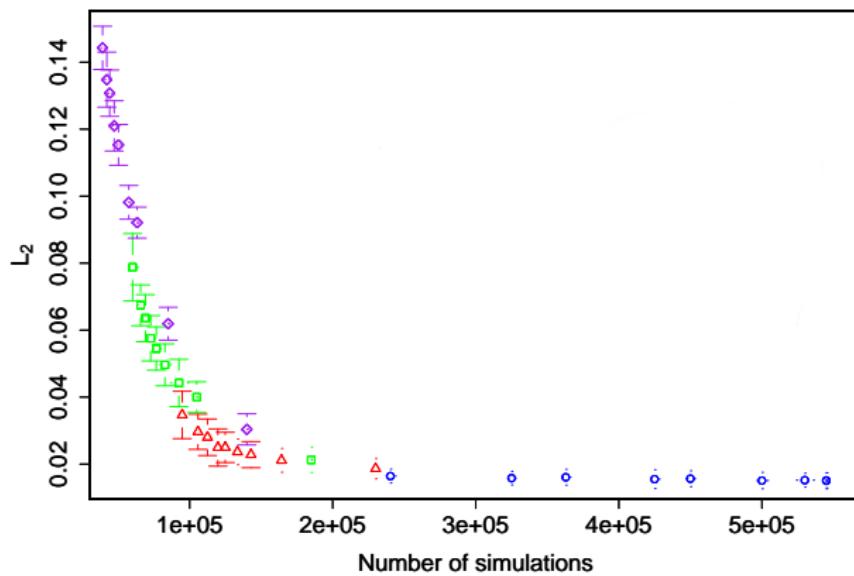
$N_\alpha = 5000$; α from 0.9 to 0.1 corresponding to N = 5555 to 50000



Adaptive ABC SMC for complex models

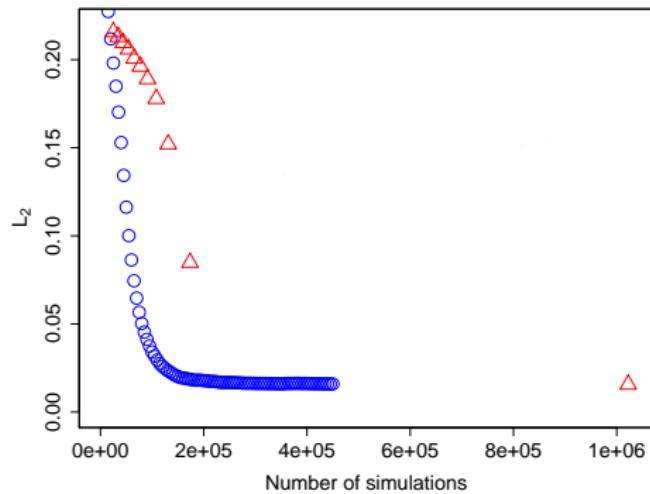
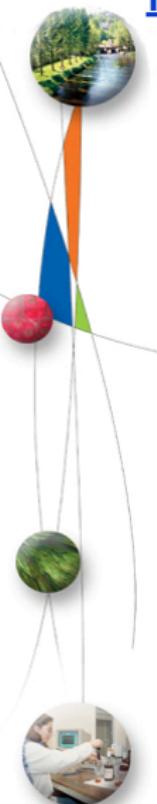
Toy example: Parameters Study

$N_\alpha = 5000$; α from 0.9 to 0.1 corresponding to N = 5555 to 50000



Adaptive ABC SMC for complex models

Toy example: Model comparison



Summary

- 
- 1 Approximate Bayesian Computation (ABC)
 - 2 Adaptive approximate Bayesian computation for complex models
 - 3 The PRIMA model

The PRIMA model

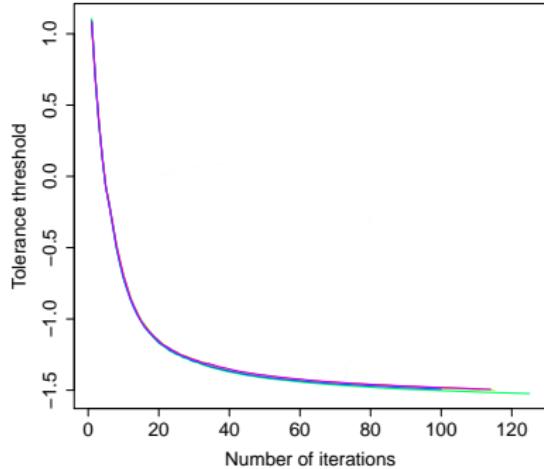
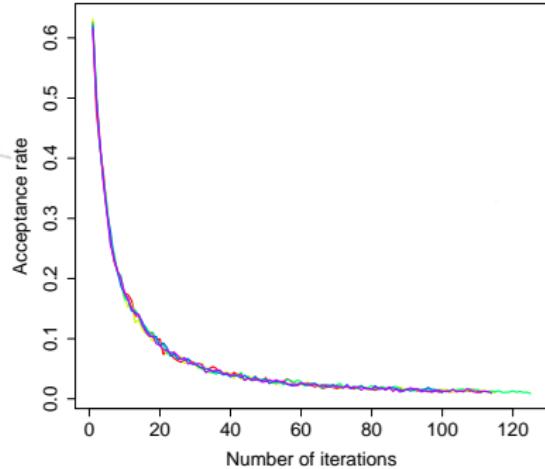
Parameters and summary statistics

- 4 parameters
- 8 summary statistics
- $\|(\rho_m(S_m, S'_m))_{1 \leq m \leq M}\|_\infty = \sup_{1 \leq m \leq M} |\rho_m(S_m, S'_m)|$



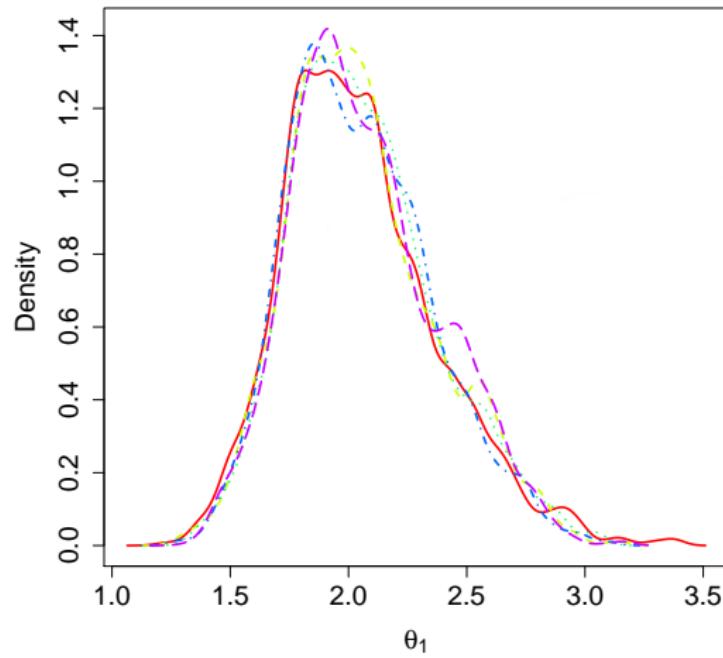
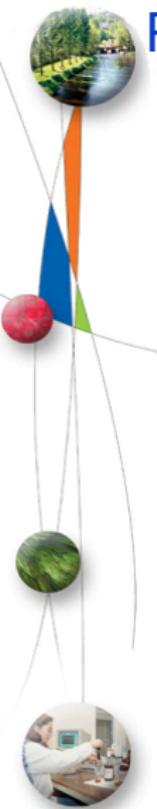
The PRIMA model

Acceptance rate and threshold evolution



The PRIMA model

Posterior density of a parameter

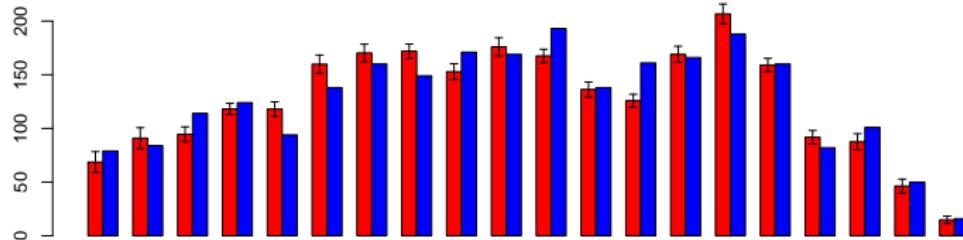


The PRIMA model

Concrete results

- 1.4 second by simulations
- 400,000 simulations
- 6 days

Age pyramid



Conclusion

- We have answered the three research questions
- Comparison with a well-known method
- Calibration of a complex social model

