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► **To cite this version:**

Maxime Lenormand, Guillaume Deffuant, Sylvie Huet. Calibrating a complex social model. European Conference on Complex Systems ECCS'11, Sep 2011, Vienne, Austria. hal-02596122

**HAL Id: hal-02596122**

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Submitted on 6 Jul 2020

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# Calibrating a complex social model

*Maxime Lenormand, Guillaume Deffuant & Sylvie Huet*

Laboratory of Engineering for Complex Systems  
Cemagref of Clermont-Ferrand

**ECCS'11**  
*September 16<sup>th</sup> 2011*



**Prototypical Policy Impacts on Multifunctional Activities in rural municipalities**

A collaborative project under the  
EU Seventh Framework Programme



This publication has been funded under the PRIMA (Prototypical policy impacts on multifunctional activities in rural municipalities) collaborative project, EU 7th Framework Programme (ENV 2007-1), contract no. 212345.

# Motivation




## Complex social model

- Individual-based model
- Stochastic
- High dimensional parameter space
- High computational cost by simulation

## Estimate the parameter values

- Calibrate the model
- Understand the model behaviour
- Uncertainty analysis
- Validation

# Summary

- 
- 1 Approximate Bayesian Computation (ABC)
  - 2 Adaptive approximate Bayesian computation for complex models
  - 3 The PRIMA model

# Approximate Bayesian Computation

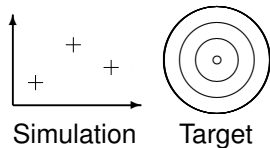
- 1 Sample  $\theta^* \sim \pi(\theta)$ .
- 2 Simulate  $x \sim f(x|\theta^*)$ .
- 3 If  $\rho(x, y) \leq \epsilon$ , accept  $\theta^*$ , otherwise reject.
- 4 Repeat until a sample of the desired size is obtained



Prior distribution  
 $\pi(\theta)$



Posterior distribution  
 $\pi(\theta)P_{\theta}\{f(x|\theta) = y\}$



*(Pritchard et al., 1999)*

*Derived from T. Toni 2011*

# Approximate Bayesian Computation

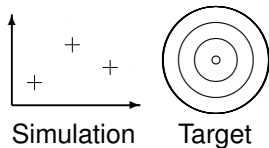
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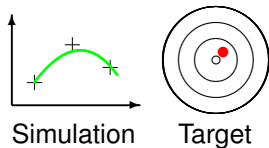
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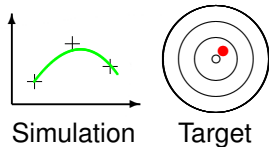
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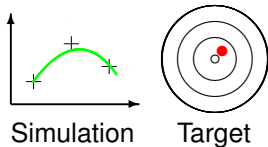
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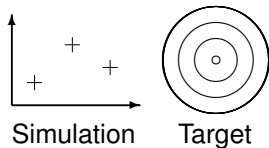
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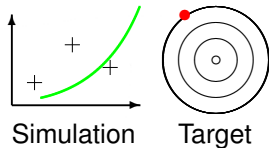
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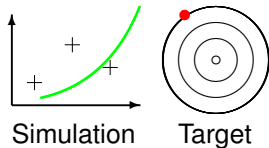
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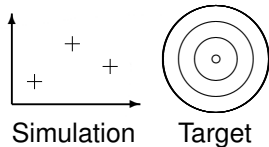
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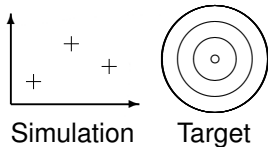
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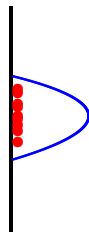
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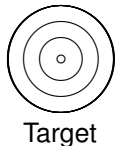
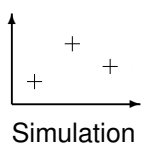
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# ABC SMC (Sequential Monte-Carlo) Algorithm



Prior

*(Sisson et al., 2007)*  
*(Beaumont et al., 2009)*



Posterior

*Derived from T. Toni 2011*



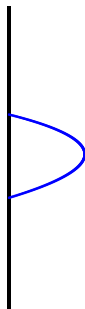
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Prior

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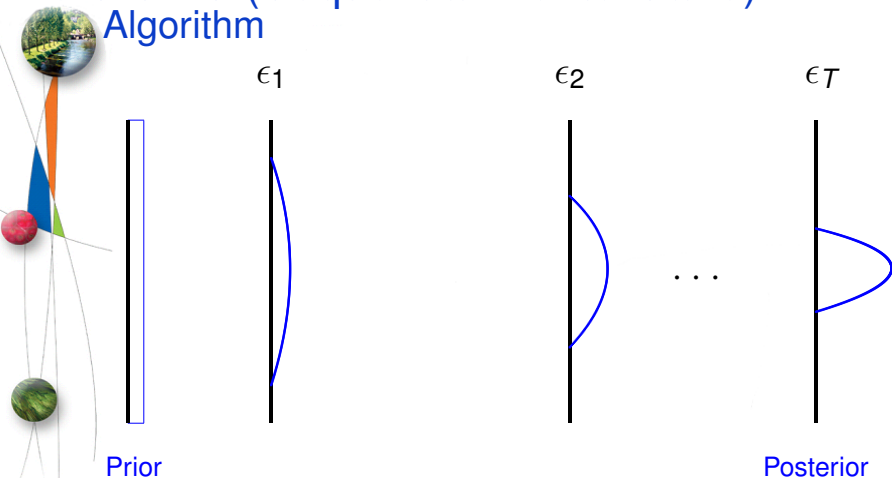
$\epsilon T$



Posterior

*Derived from T. Toni 2011*

# ABC SMC (Sequential Monte-Carlo) Algorithm



(Sisson et al., 2007)  
(Beaumont et al., 2009)

Derived from T. Toni 2011

# ABC SMC (Sequential Monte-Carlo) Algorithm

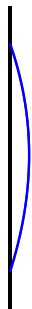


Algorithm



Prior

$\epsilon_1$

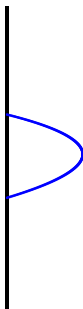


$\epsilon_2$



...

$\epsilon_T$

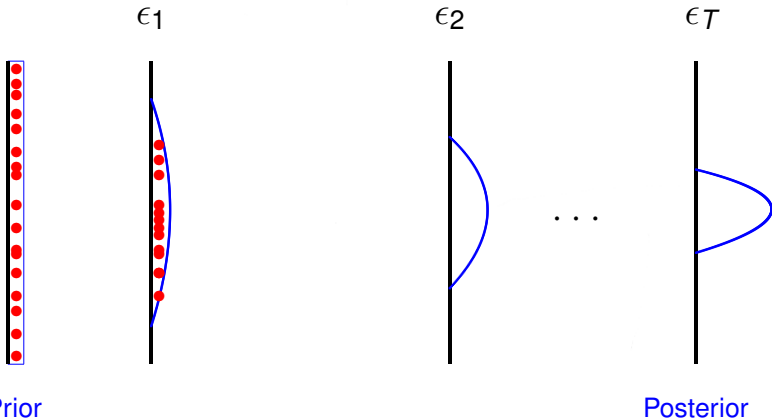


Posterior

(Sisson et al., 2007)  
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Derived from T. Toni 2011

# ABC SMC (Sequential Monte-Carlo) Algorithm



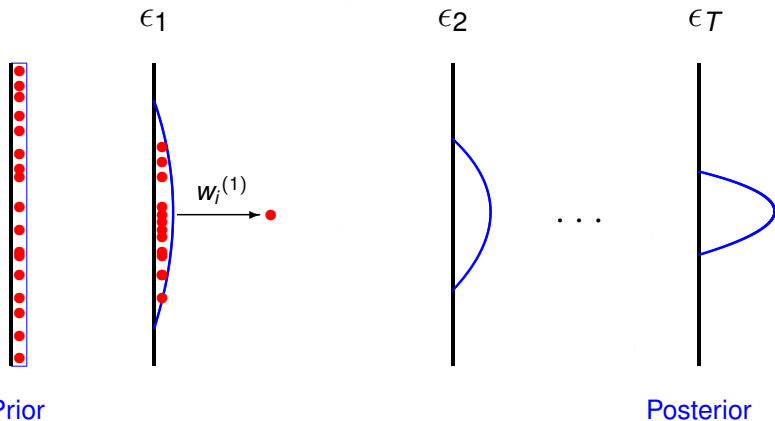
Prior

Posterior

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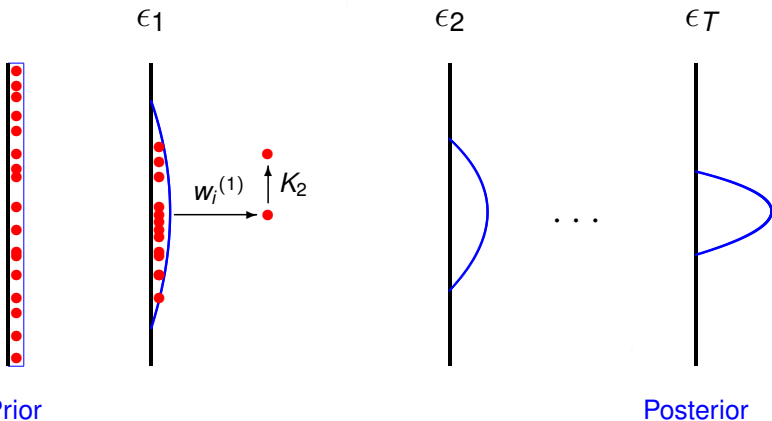
Prior

Posterior

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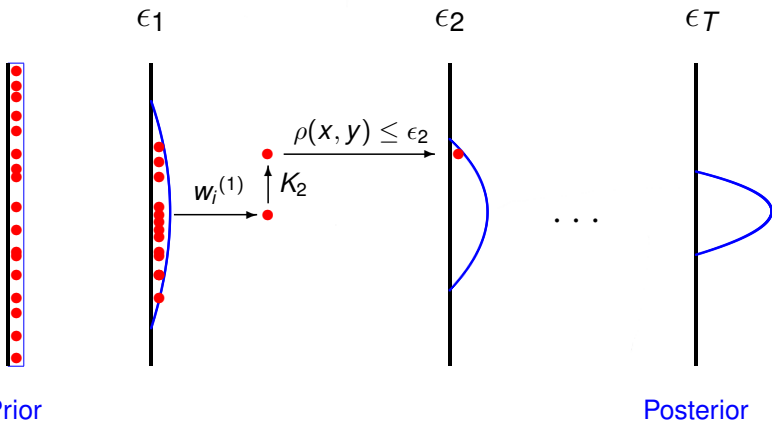
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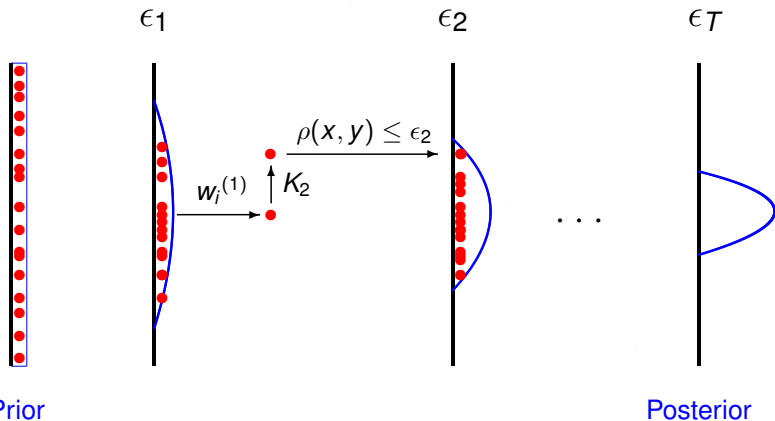
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# ABC SMC (Sequential Monte-Carlo) Algorithm



Prior

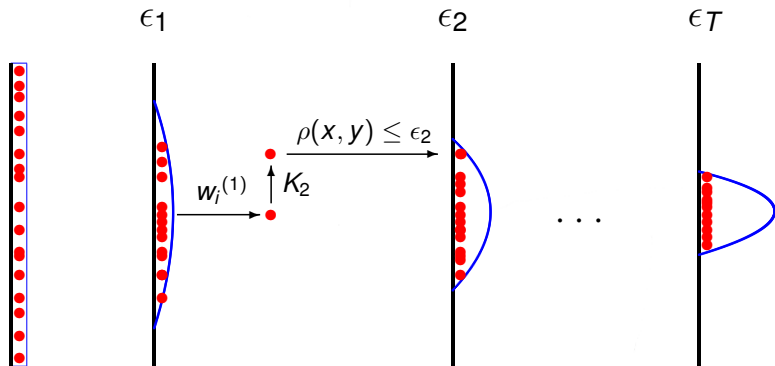
Posterior

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Derived from T. Toni 2011



# ABC SMC (Sequential Monte-Carlo) Algorithm



Prior

Posterior

(Sisson et al., 2007)  
(Beaumont et al., 2009)


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# ABC SMC (Sequential Monte-Carlo)

## Issues related to the model complexity

- How to control the number of simulations?
- How to determine the sequence of tolerance levels  $\{\epsilon_1, \dots, \epsilon_T\}$ ?
- When to stop the algorithm?

# Summary

- 
- 1 Approximate Bayesian Computation (ABC)
  - 2 Adaptive approximate Bayesian computation for complex models
  - 3 The PRIMA model

# Adaptive ABC SMC for complex models

## Algorithm



Prior

*(Lenormand et al.)*

# Adaptive ABC SMC for complex models

## Algorithm



$N$  particles  
(LHS)



Prior

*(Lenormand et al.)*

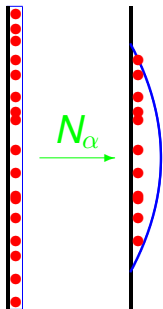
# Adaptive ABC SMC for complex models

## Algorithm



$N$  particles  
(LHS)

$\epsilon_1$



Prior

*(Lenormand et al.)*

# Adaptive ABC SMC for complex models

## Algorithm



$N$  particles  
(LHS)

$\epsilon_1$



$N_\alpha$



$w_i^{(1)}$

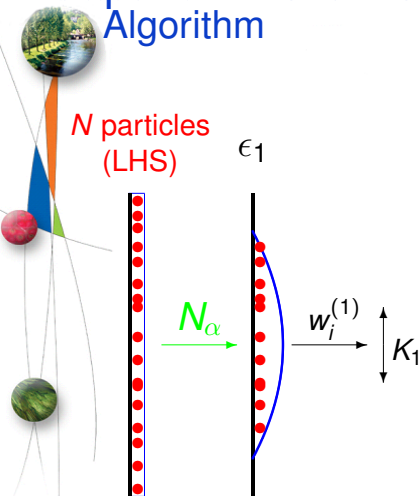


Prior

*(Lenormand et al.)*

# Adaptive ABC SMC for complex models

## Algorithm



Prior

(Lenormand et al.)



# Adaptive ABC SMC for complex models

## Algorithm



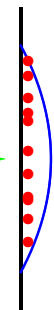
$N$  particles  
(LHS)

$\epsilon_1$

$N - N_\alpha$   
particles



$N_\alpha$



$w_i^{(1)}$

$K_1$



Prior

(Lenormand et al.)

$$p_{acc} = \frac{\sum_{k=N_\alpha+1}^N \mathbb{1}_{\rho(x,y) \leq \epsilon_1}}{N - N_\alpha}$$

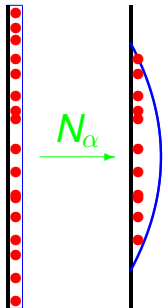
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## Algorithm



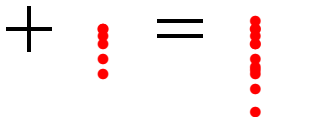
$N$  particles  
(LHS)

$\epsilon_1$



$N - N_\alpha$   
particles

$N$   
particles



Prior

(Lenormand et al.)

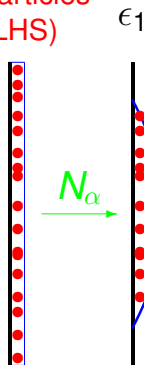
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# Adaptive ABC SMC for complex models

## Algorithm



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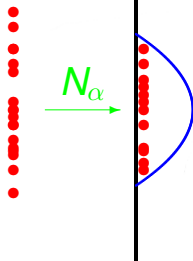
$N - N_\alpha$   
particles

$N$   
particles

+

=

$\epsilon_2$



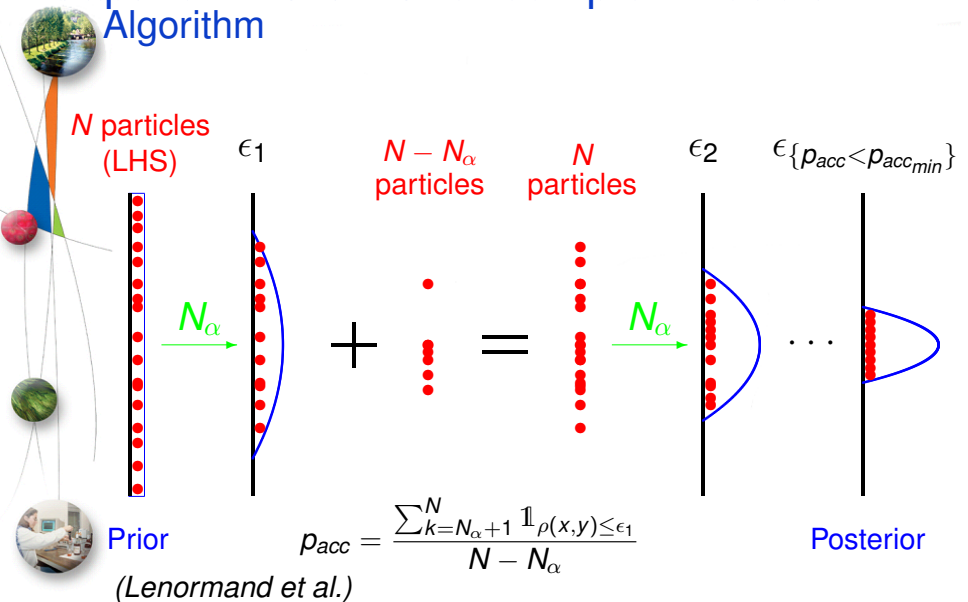
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# Adaptive ABC SMC for complex models

## Algorithm



# Adaptive ABC SMC for complex models

## Issues related to the model complexity

- How to control the number of simulations?

# Adaptive ABC SMC for complex models

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- How to control the number of simulations?  
⇒  $N - N_\alpha$  simulations at each iteration

# Adaptive ABC SMC for complex models

## Issues related to the model complexity

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 $\implies N - N_\alpha$  simulations at each iteration
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 $\implies \epsilon_t = \alpha$ -quantile of the  $N$  distances to the data



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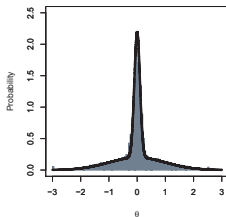
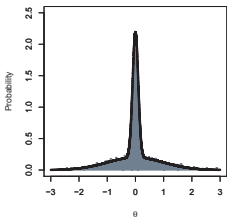
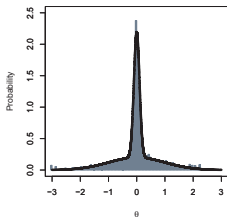
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- How to control the number of simulations?  
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- How to determine the sequence of tolerance levels  $\{\epsilon_1, \dots, \epsilon_T\}$ ?  
 $\implies \epsilon_t = \alpha$ -quantile of the  $N$  distances to the data
- When to stop the algorithm?  
 $\implies \rho_{acc} < \rho_{acc_{min}}$

# Adaptive ABC SMC for complex models

## Toy example: Presentation

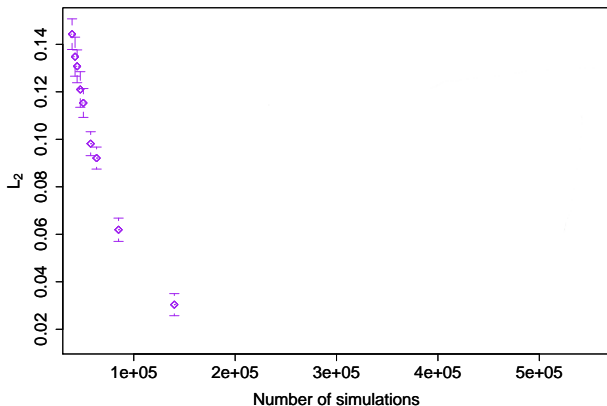
$$f(x|\theta) \sim \frac{1}{2}\phi\left(\theta, \frac{1}{100}\right) + \frac{1}{2}\phi(\theta, 1) \text{ and } \theta \sim \mathcal{U}_{[-10,10]}$$



# Adaptive ABC SMC for complex models

## Toy example: Parameters Study

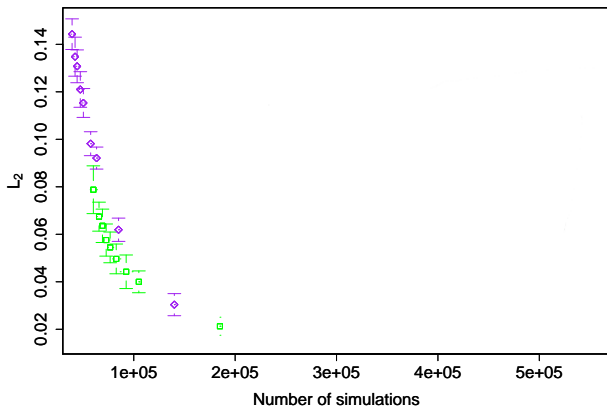
$N_\alpha = 5000$ ;  $\alpha$  from 0.9 to 0.1 corresponding to  $N = 5555$  to 50000



# Adaptive ABC SMC for complex models

## Toy example: Parameters Study

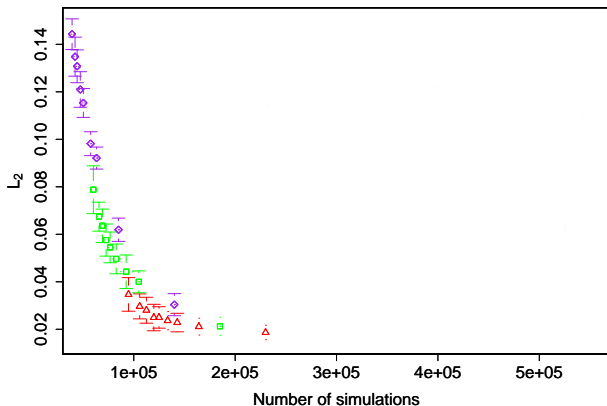
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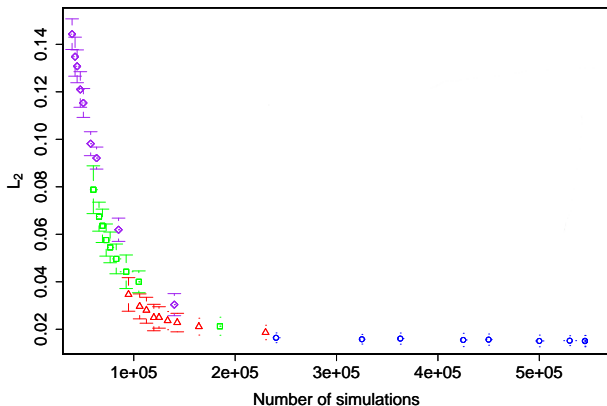
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# Adaptive ABC SMC for complex models

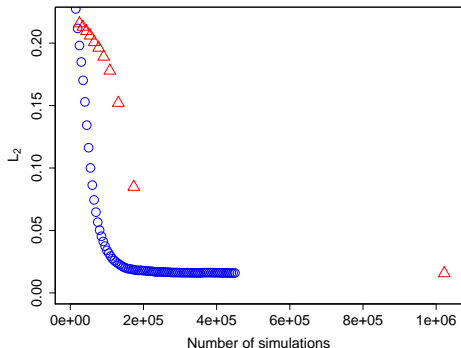
## Toy example: Parameters Study

$N_\alpha = 5000$ ;  $\alpha$  from 0.9 to 0.1 corresponding to  $N = 5555$  to 50000




# Adaptive ABC SMC for complex models

## Toy example: Model comparison





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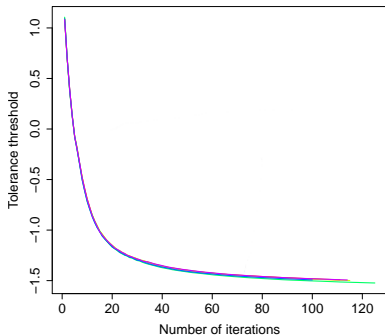
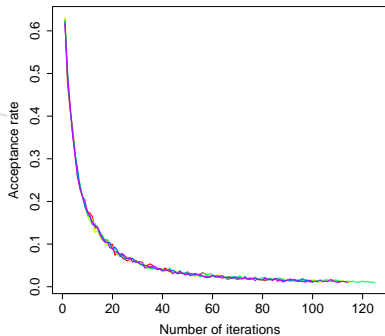
# The PRIMA model

## Parameters and summary statistics

- 4 parameters
- 8 summary statistics
- $\|(\rho_m(\mathcal{S}_m, \mathcal{S}'_m))_{1 \leq m \leq M}\|_{\infty} = \sup_{1 \leq m \leq M} |\rho_m(\mathcal{S}_m, \mathcal{S}'_m)|$

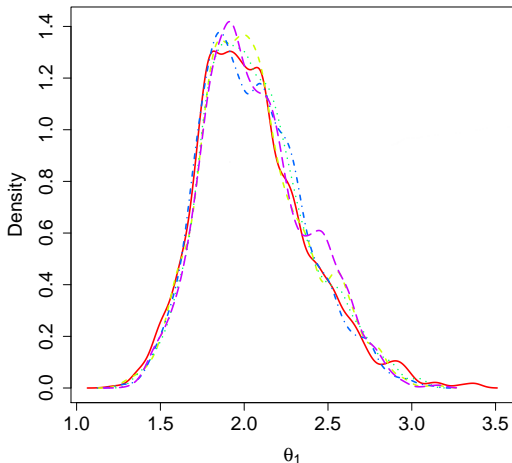
# The PRIMA model

## Acceptance rate and threshold evolution



# The PRIMA model

## Posterior density of a parameter

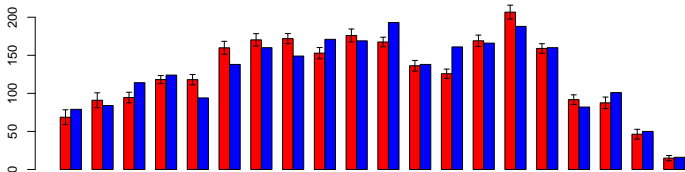


# The PRIMA model


## Concrete results

- 1.4 second by simulations
- 400,000 simulations
- 6 days

Age pyramid



# Conclusion

- 
- We have answered the three research questions
  - Comparison with a well-known method
  - Calibration of a complex social model