



Calibrating a complex social model

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Calibrating a complex social model

Maxime Lenormand, Guillaume Deffuant & Sylvie Huet

Laboratory of Engineering for Complex Systems
Cemagref of Clermont-Ferrand

ECCS'11

September 16th 2011



Prototypical Policy Impacts on Multifunctional Activities in rural municipalities

A collaborative project under the
EU Seventh Framework Programme



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Motivation



Complex social model

- Individual-based model
- Stochastic
- High dimensional parameter space
- High computational cost by simulation

Estimate the parameter values

- Calibrate the model
- Understand the model behaviour
- Uncertainty analysis
- Validation

Summary

- 
- 1 Approximate Bayesian Computation (ABC)
 - 2 Adaptive approximate Bayesian computation for complex models
 - 3 The PRIMA model

Approximate Bayesian Computation

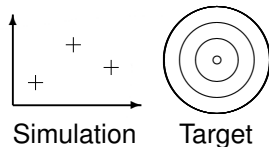
- 1 Sample $\theta^* \sim \pi(\theta)$.
- 2 Simulate $x \sim f(x|\theta^*)$.
- 3 If $\rho(x, y) \leq \epsilon$, accept θ^* , otherwise reject.
- 4 Repeat until a sample of the desired size is obtained



Prior distribution
 $\pi(\theta)$



Posterior distribution
 $\pi(\theta)P_\theta\{f(x|\theta) = y\}$



(Pritchard et al., 1999)

Derived from T. Toni 2011

Approximate Bayesian Computation

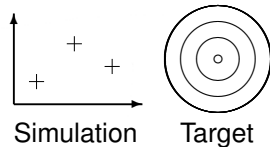
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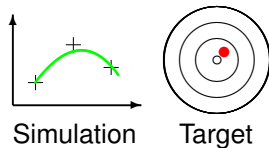
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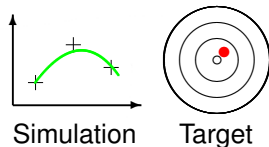
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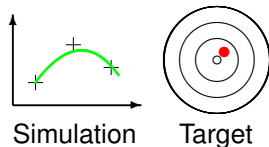
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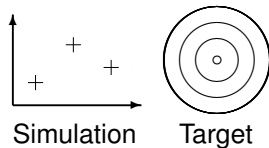
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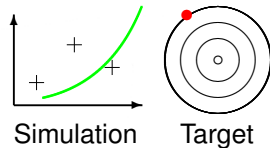
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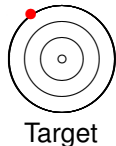
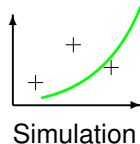
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Target



(Pritchard et al., 1999)

Derived from T. Toni 2011

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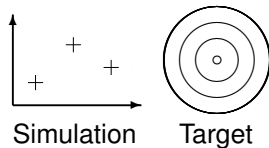
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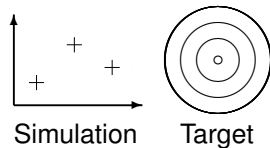
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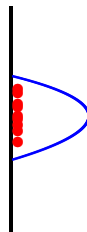
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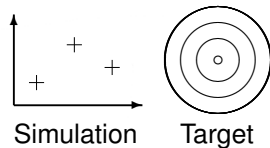
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Derived from T. Toni 2011

ABC SMC (Sequential Monte-Carlo) Algorithm



Prior

(Sisson et al., 2007)
(Beaumont et al., 2009)



Posterior

Derived from T. Toni 2011

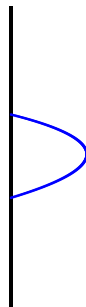
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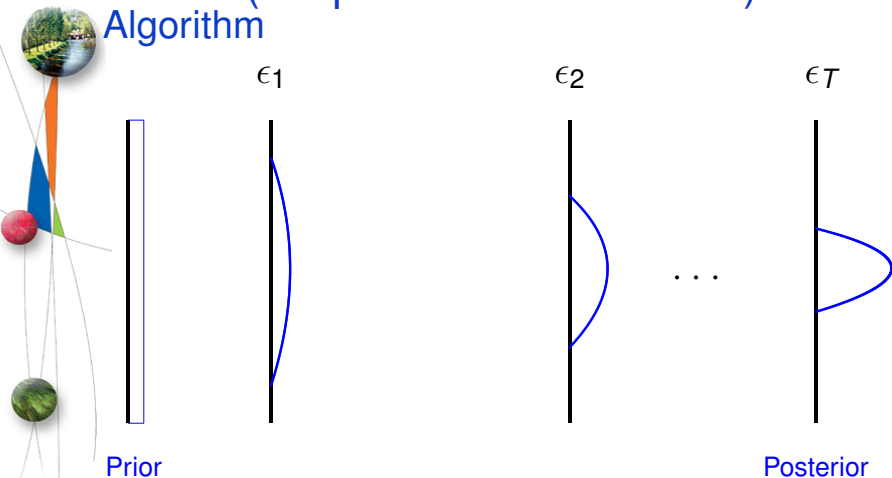
ϵ_T



Posterior

Derived from T. Toni 2011

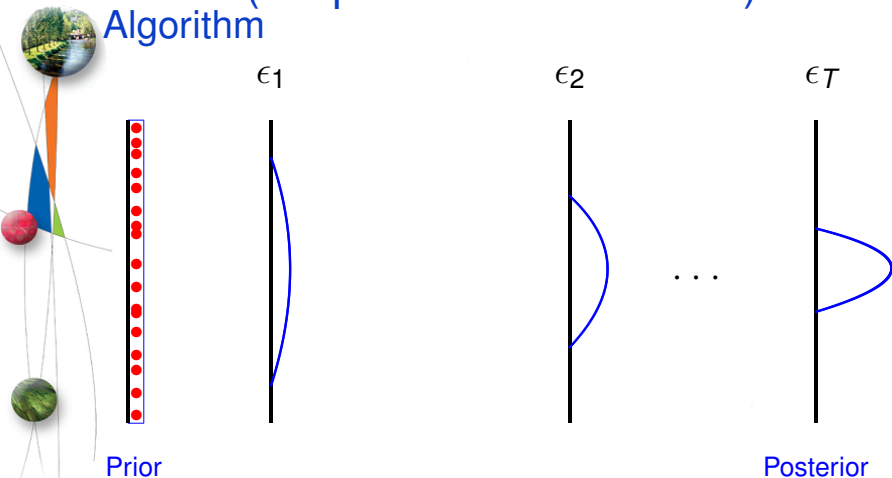
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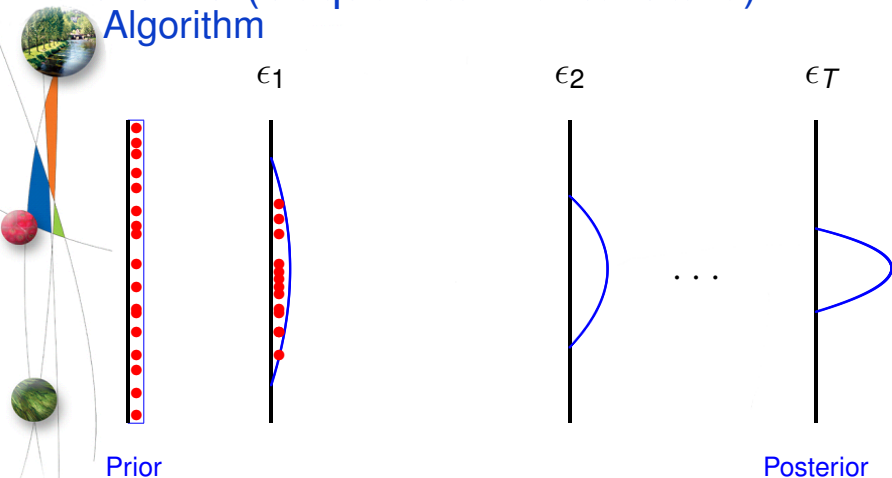
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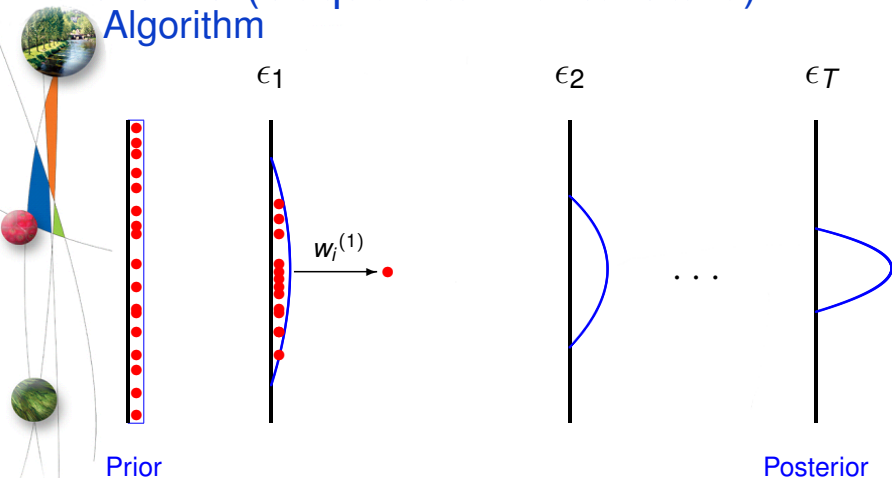
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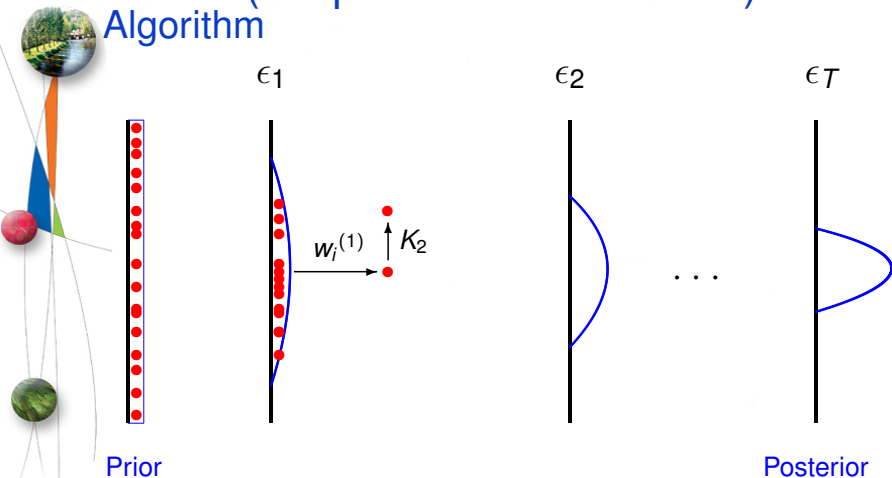
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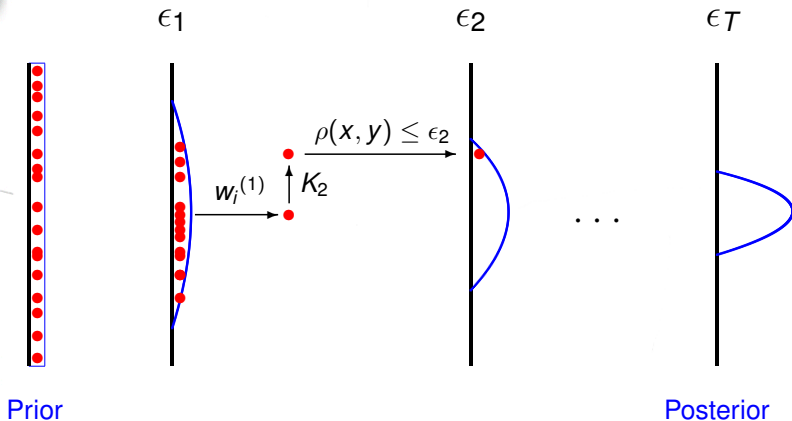
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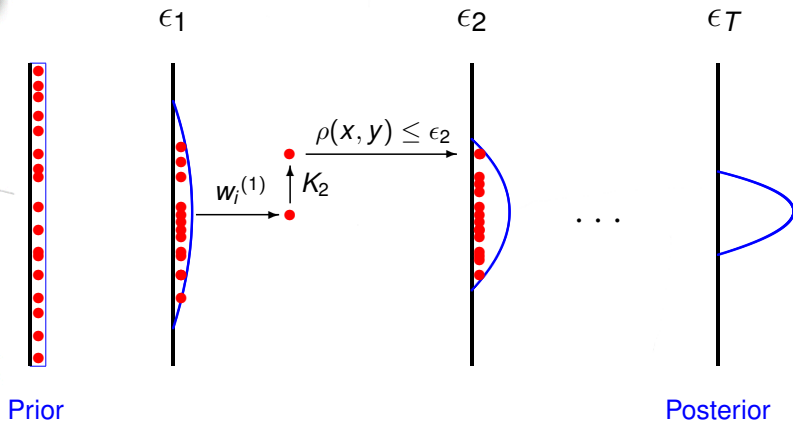
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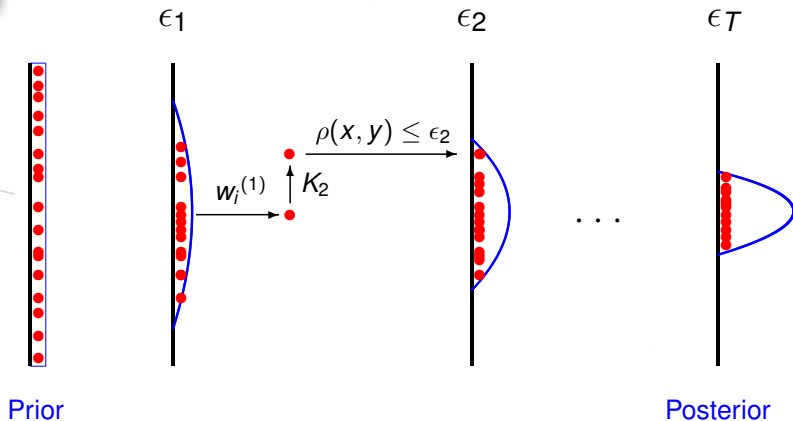
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ABC SMC (Sequential Monte-Carlo)

Issues related to the model complexity

- How to control the number of simulations?
- How to determine the sequence of tolerance levels $\{\epsilon_1, \dots, \epsilon_T\}$?
- When to stop the algorithm?

Summary

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- 1 Approximate Bayesian Computation (ABC)
 - 2 Adaptive approximate Bayesian computation for complex models
 - 3 The PRIMA model

Adaptive ABC SMC for complex models

Algorithm



Prior

(Lenormand et al.)

Adaptive ABC SMC for complex models

Algorithm

N particles
(LHS)



Prior

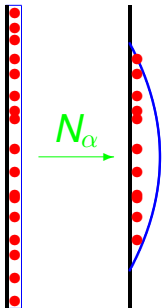
(Lenormand et al.)

Adaptive ABC SMC for complex models

Algorithm

N particles
(LHS)

ϵ_1

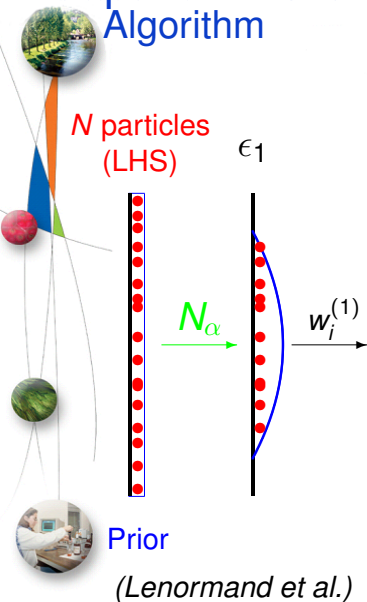


Prior

(Lenormand et al.)

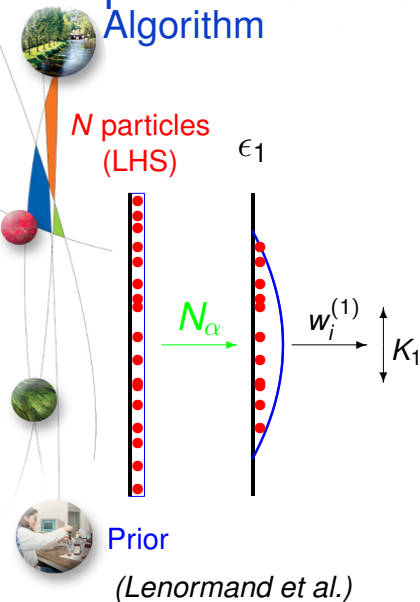
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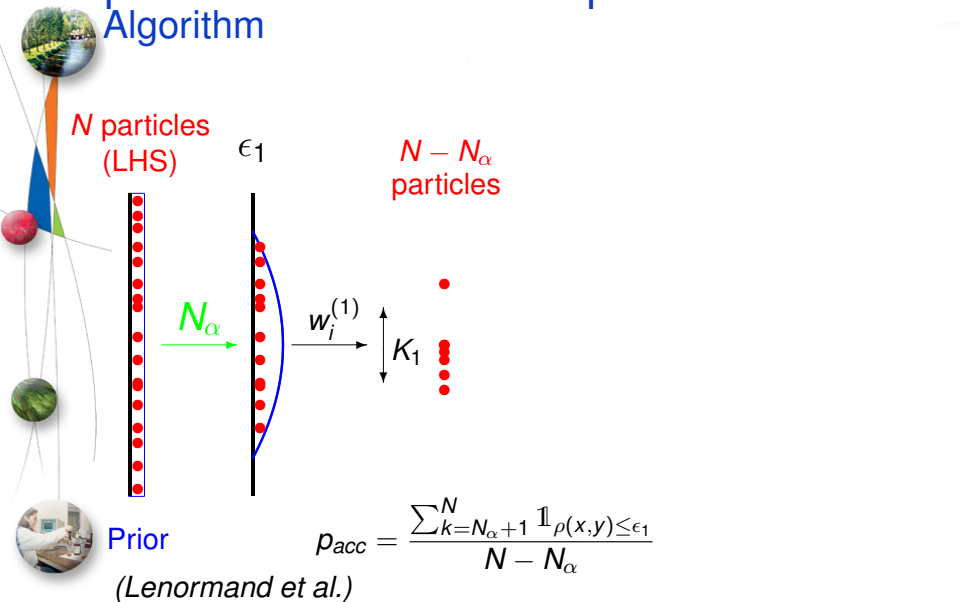
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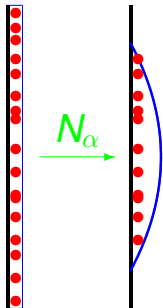
Adaptive ABC SMC for complex models

Algorithm



N particles
(LHS)

ϵ_1



$N - N_\alpha$
particles

N
particles

+

=



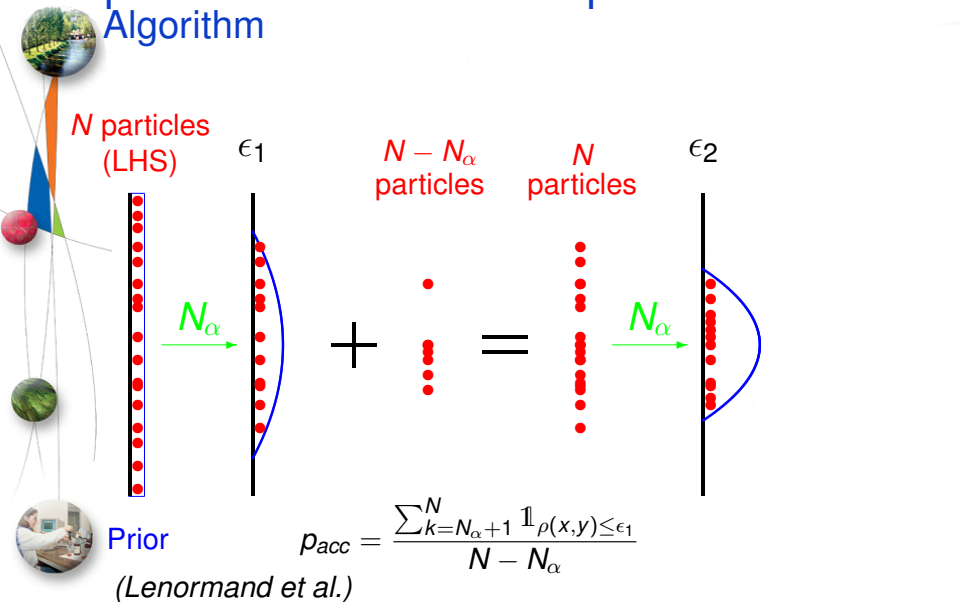
Prior

(Lenormand et al.)

$$p_{acc} = \frac{\sum_{k=N_\alpha+1}^N \mathbb{1}_{\rho(x,y) \leq \epsilon_1}}{N - N_\alpha}$$

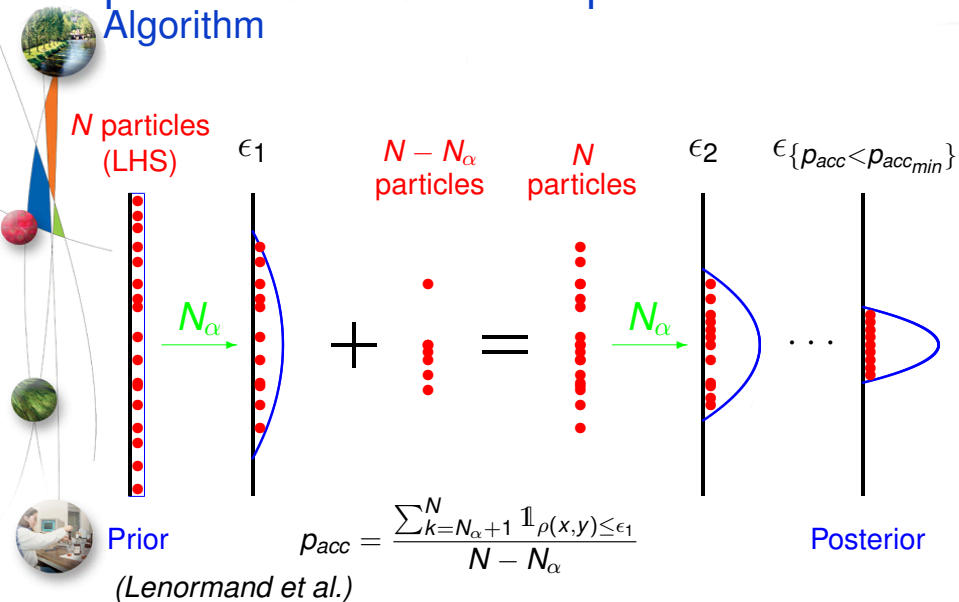
Adaptive ABC SMC for complex models

Algorithm



Adaptive ABC SMC for complex models

Algorithm



Adaptive ABC SMC for complex models

Issues related to the model complexity

- How to control the number of simulations?

Adaptive ABC SMC for complex models

Issues related to the model complexity

- How to control the number of simulations?
⇒ $N - N_\alpha$ simulations at each iteration

Adaptive ABC SMC for complex models

Issues related to the model complexity

- How to control the number of simulations?
 $\implies N - N_\alpha$ simulations at each iteration
- How to determine the sequence of tolerance levels $\{\epsilon_1, \dots, \epsilon_T\}$?

Adaptive ABC SMC for complex models

Issues related to the model complexity

- How to control the number of simulations?
 $\Rightarrow N - N_\alpha$ simulations at each iteration
- How to determine the sequence of tolerance levels $\{\epsilon_1, \dots, \epsilon_T\}$?
 $\Rightarrow \epsilon_t = \alpha$ -quantile of the N distances to the data

Adaptive ABC SMC for complex models

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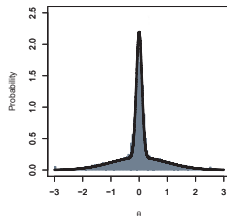
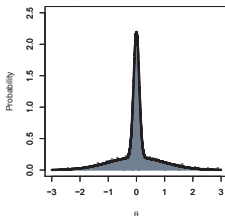
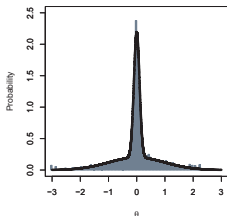
Issues related to the model complexity

- How to control the number of simulations?
 $\Rightarrow N - N_\alpha$ simulations at each iteration
- How to determine the sequence of tolerance levels $\{\epsilon_1, \dots, \epsilon_T\}$?
 $\Rightarrow \epsilon_t = \alpha$ -quantile of the N distances to the data
- When to stop the algorithm?
 $\Rightarrow p_{acc} < p_{acc_{min}}$

Adaptive ABC SMC for complex models

Toy example: Presentation

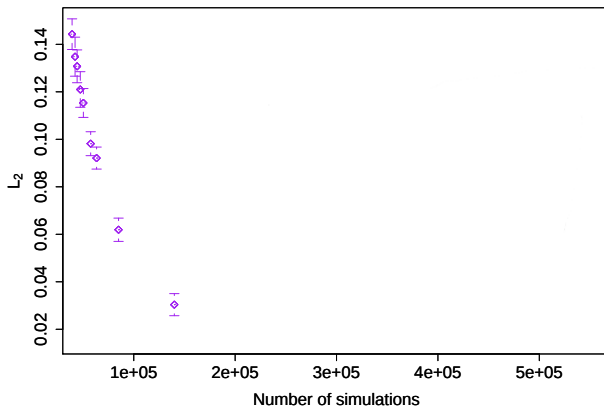
$$f(x|\theta) \sim \frac{1}{2}\phi\left(\theta, \frac{1}{100}\right) + \frac{1}{2}\phi(\theta, 1) \text{ and } \theta \sim \mathcal{U}_{[-10,10]}$$



Adaptive ABC SMC for complex models

Toy example: Parameters Study

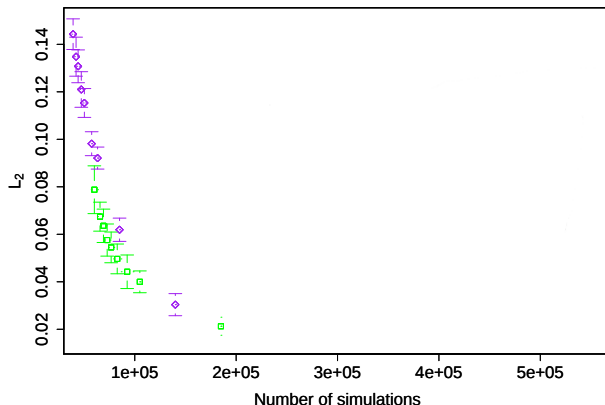
$N_\alpha = 5000$; α from 0.9 to 0.1 corresponding to $N = 5555$ to 50000



Adaptive ABC SMC for complex models

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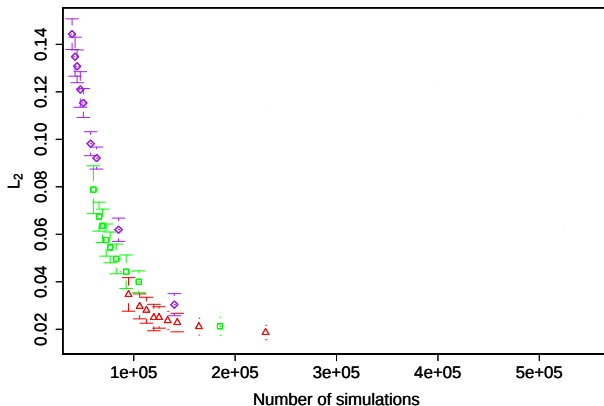
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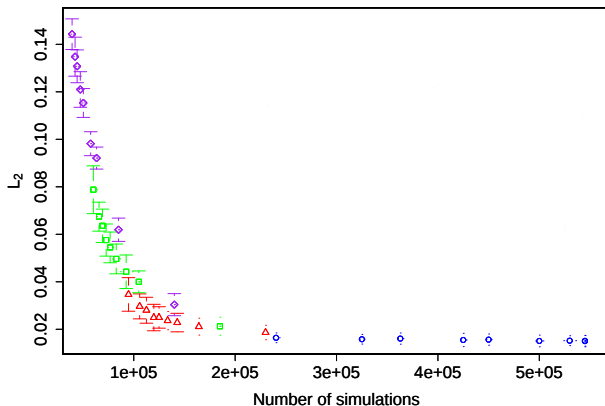
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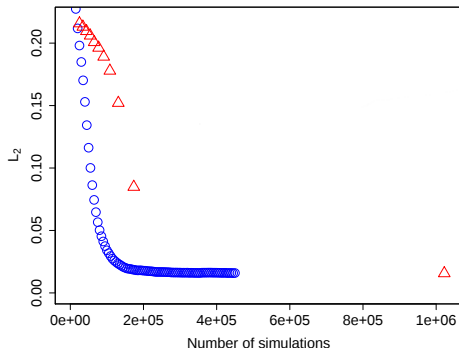
Toy example: Parameters Study

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Adaptive ABC SMC for complex models

Toy example: Model comparison



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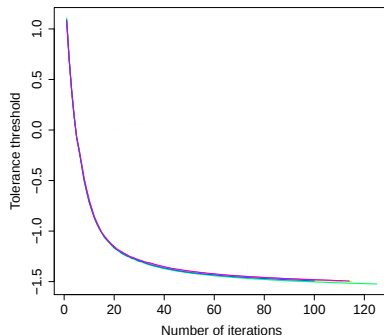
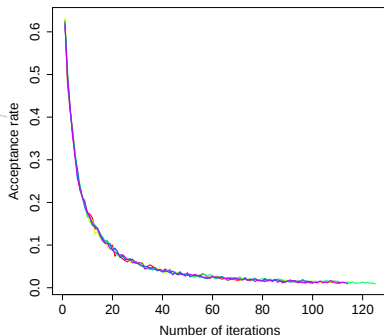
The PRIMA model

Parameters and summary statistics

- 4 parameters
- 8 summary statistics
- $\|(\rho_m(S_m, S'_m))_{1 \leq m \leq M}\|_{\infty} = \sup_{1 \leq m \leq M} |\rho_m(S_m, S'_m)|$

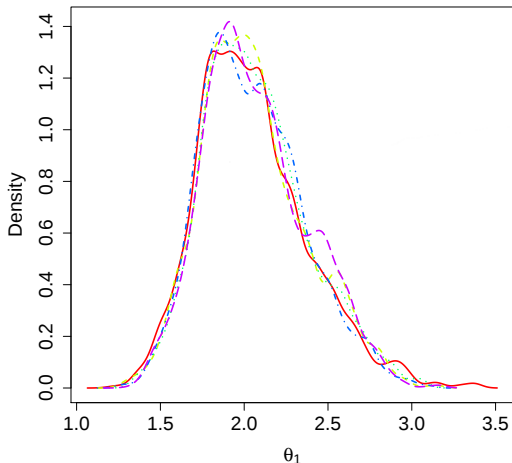
The PRIMA model

Acceptance rate and threshold evolution



The PRIMA model

Posterior density of a parameter

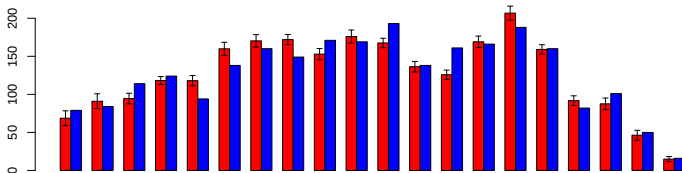


The PRIMA model

Concrete results

- 1.4 second by simulations
- 400,000 simulations
- 6 days

Age pyramid



Conclusion

- 
- We have answered the three research questions
 - Comparison with a well-known method
 - Calibration of a complex social model