Calibrating a complex social model

Maxime Lenormand, Guillaume Deffuant, Sylvie Huet

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Cemagref
Sciences, eaux & territoires

prima
Prototypical Policy Impacts on Multifunctional Activities in rural municipalities
A collaborative project under the EU Seventh Framework Programme

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Motivation

Complex social model

- Individual-based model
- Stochastic
- High dimensional parameter space
- High computational cost by simulation

Estimate the parameter values

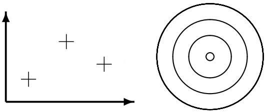
- Calibrate the model
- Understand the model behaviour
- Uncertainty analysis
- Validation

Summary

1. Approximate Bayesian Computation (ABC)
2. Adaptive approximate Bayesian computation for complex models
3. The PRIMA model

Approximate Bayesian Computation

1. Sample $\theta^* \sim \pi(\theta)$.
2. Simulate $x \sim f(x|\theta^*)$.
3. If $\rho(x, y) \leq \epsilon$, accept θ^*, otherwise reject.
4. Repeat until a sample of the desired size is obtained



Simulation Target

Prior distribution
$\pi(\theta)$

Posterior distribution
$\pi(\theta) P_\theta\{f(x|\theta) = y\}$

(Pritchard et al., 1999)

Derived from T. Toni 2011

Approximate Bayesian Computation

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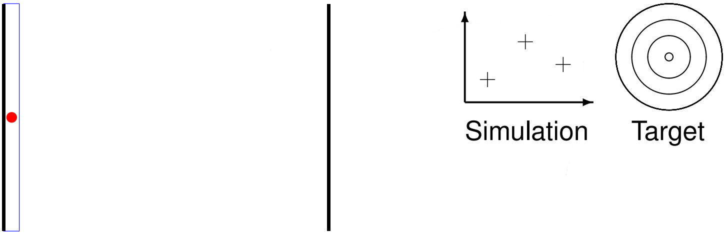

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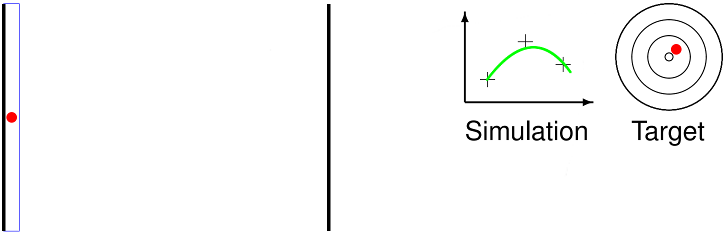

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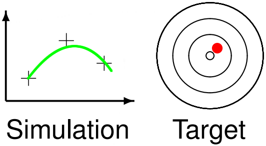

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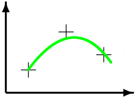
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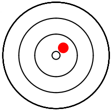
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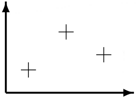

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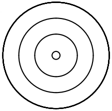

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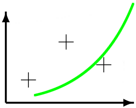

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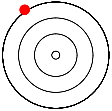

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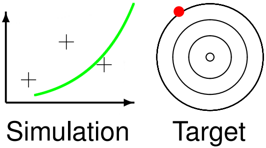

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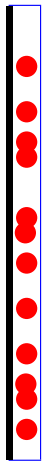

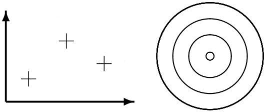

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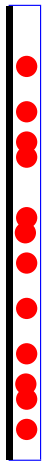

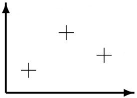

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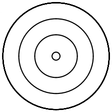

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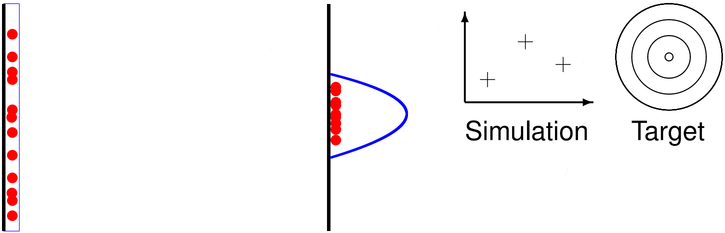

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ABC SMC (Sequential Monte-Carlo)

Algorithm

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Posterior

(Sisson et al., 2007)
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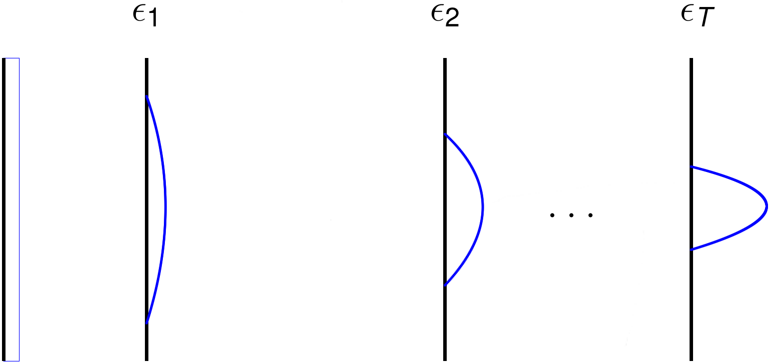

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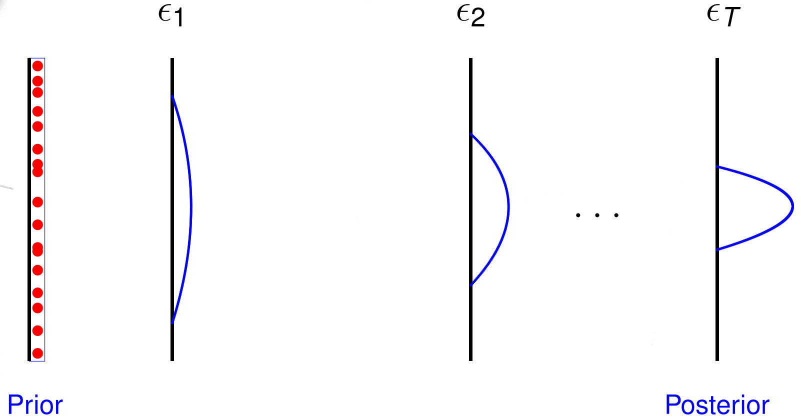

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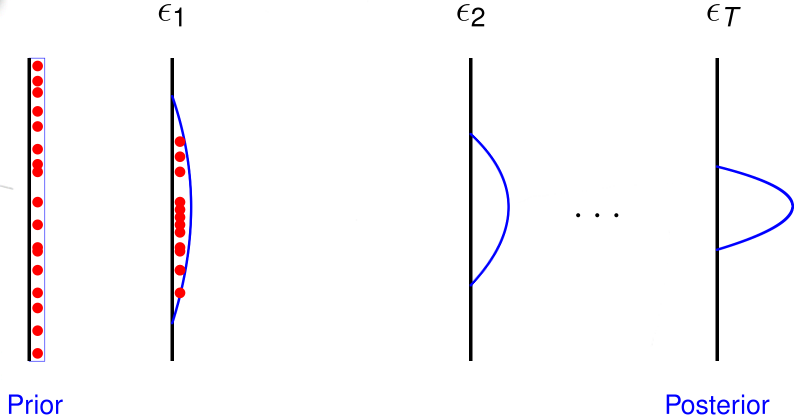

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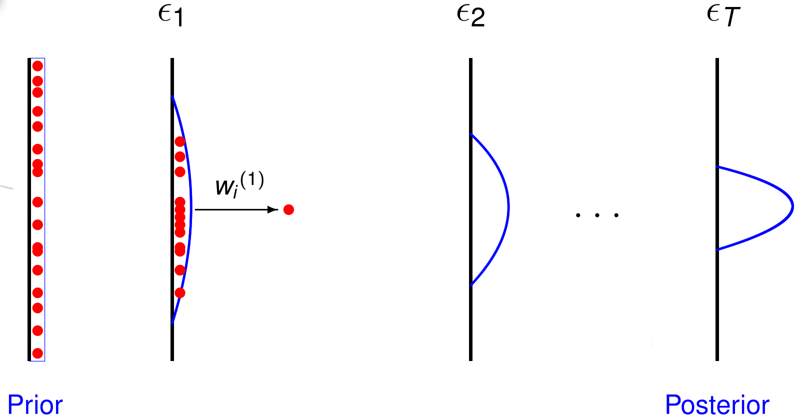

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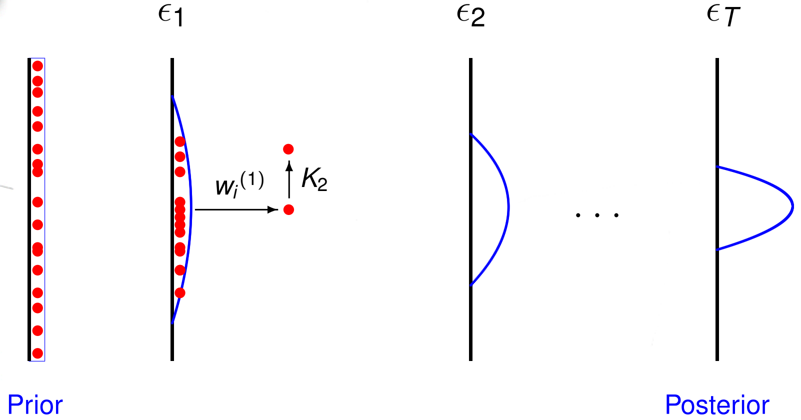

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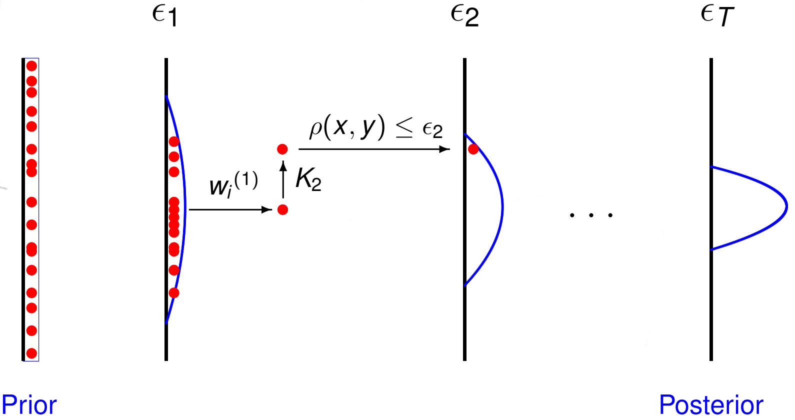

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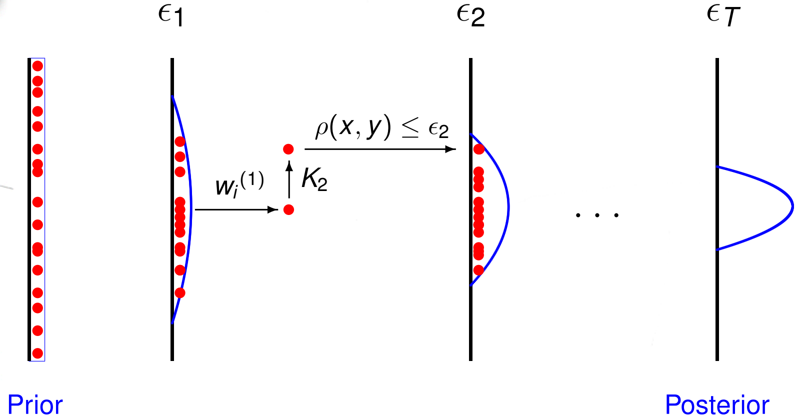

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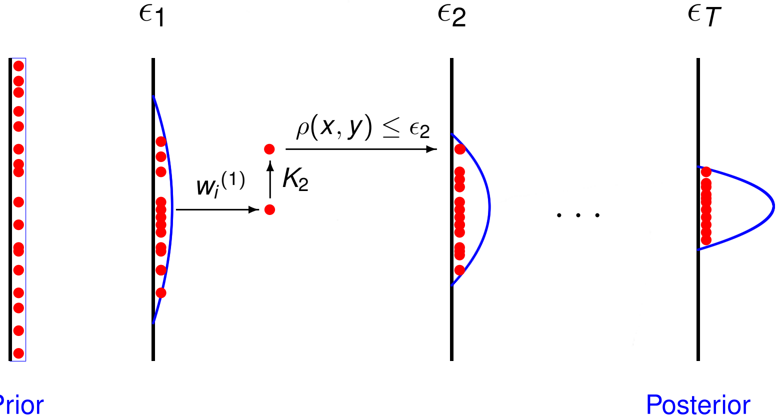

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ABC SMC (Sequential Monte-Carlo)
Issues related to the model complexity

- How to control the number of simulations?
- How to determine the sequence of tolerance levels $\{\epsilon_1, ..., \epsilon_T\}$?
- When to stop the algorithm?

Summary

1. Approximate Bayesian Computation (ABC)
2. Adaptive approximate Bayesian computation for complex models
3. The PRIMA model

Adaptive ABC SMC for complex models

Algorithm

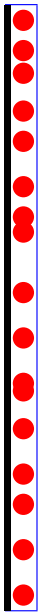
Prior

(Lenormand et al.)

Adaptive ABC SMC for complex models

Algorithm

N particles
(LHS)



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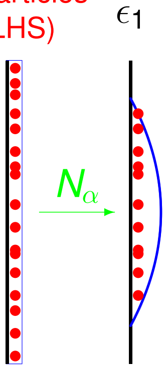

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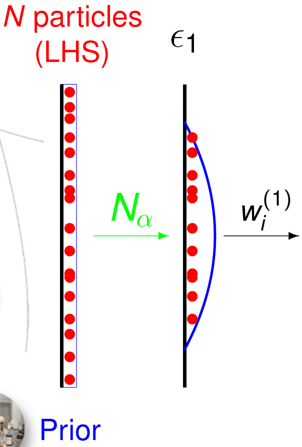

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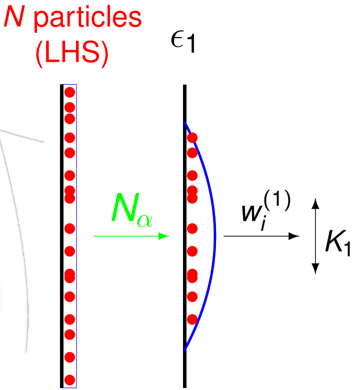

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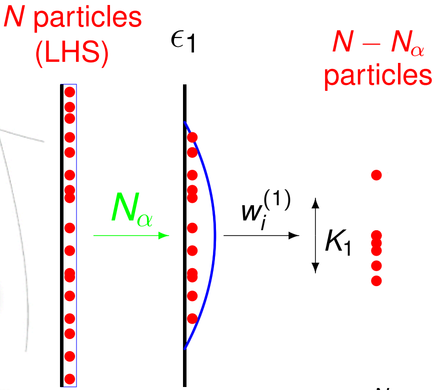

Prior

$$p_{acc} = \frac{\sum_{k=N_\alpha+1}^{N} \mathbb{1}_{\rho(x,y)\leq\epsilon_1}}{N - N_\alpha}$$

(Lenormand et al.)

Adaptive ABC SMC for complex models

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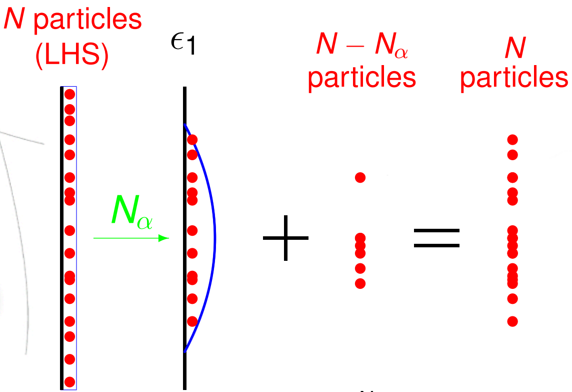

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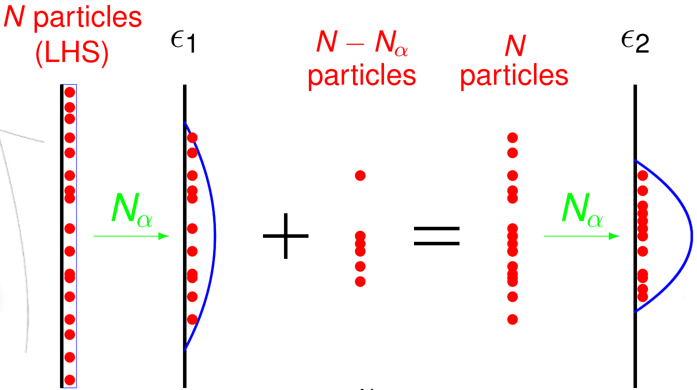

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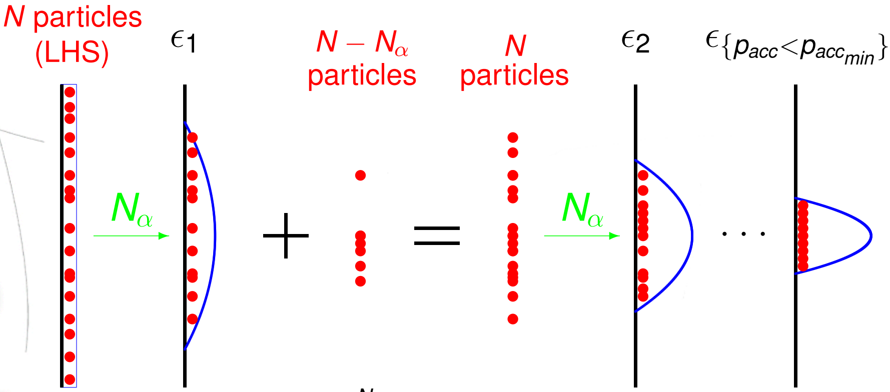

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Posterior

(Lenormand et al.)

Adaptive ABC SMC for complex models

Issues related to the model complexity

- How to control the number of simulations?

Adaptive ABC SMC for complex models
Issues related to the model complexity

- How to control the number of simulations?
  $\Longrightarrow N - N_\alpha$ simulations at each iteration

Adaptive ABC SMC for complex models

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Adaptive ABC SMC for complex models

Issues related to the model complexity

- How to control the number of simulations?
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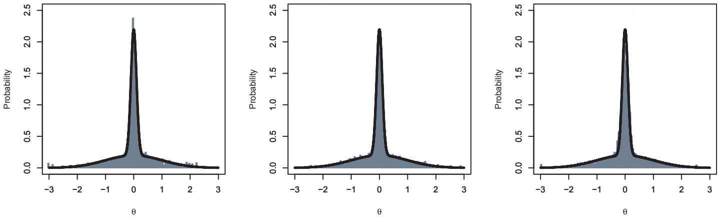
- How to determine the sequence of tolerance levels $\{\epsilon_1, \dots, \epsilon_T\}$?
  $\Longrightarrow \epsilon_t = \alpha$-quantile of the N distances to the data
- When to stop the algorithm?
  $\Longrightarrow p_{acc} < p_{acc_{min}}$

Adaptive ABC SMC for complex models

Toy example: Presentation

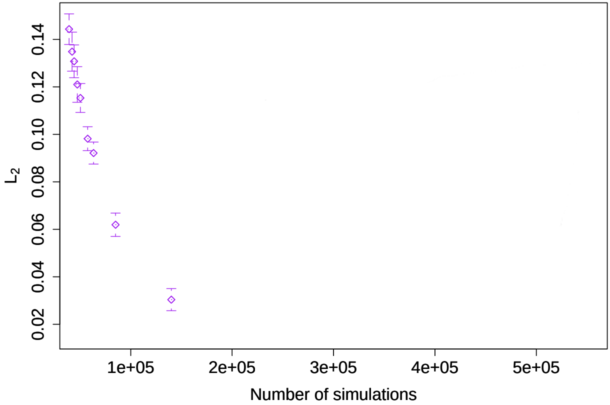
$$f(x|\theta) \sim \frac{1}{2}\phi\left(\theta, \frac{1}{100}\right) + \frac{1}{2}\phi(\theta, 1) \text{ and } \theta \sim \mathcal{U}_{[-10,10]}$$



Adaptive ABC SMC for complex models

Toy example: Parameters Study

$N_\alpha = 5000$; α from 0.9 to 0.1 corresponding to $N = 5555$ to 50000



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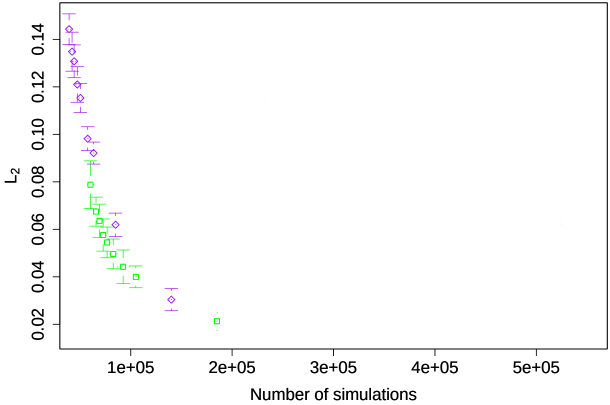

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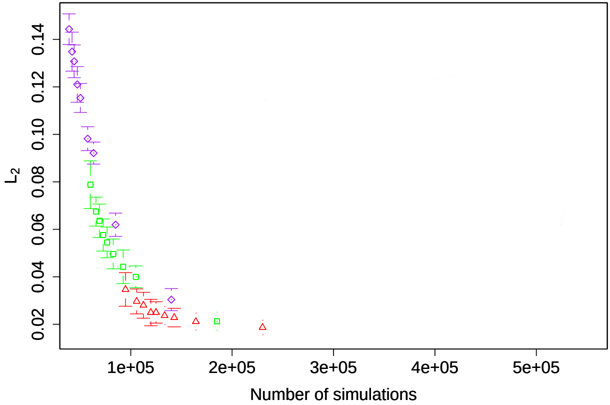

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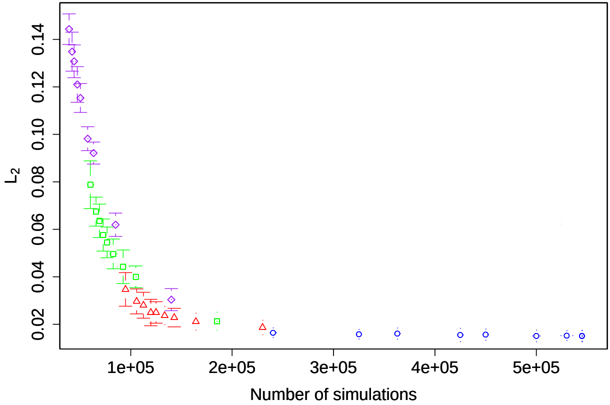
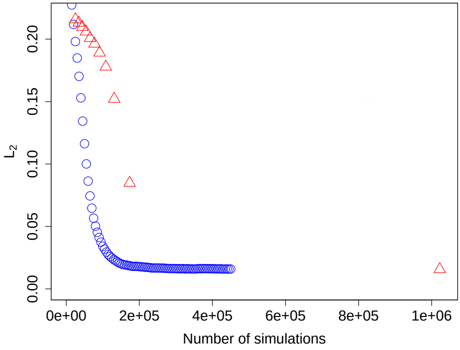


Adaptive ABC SMC for complex models

Toy example: Model comparison



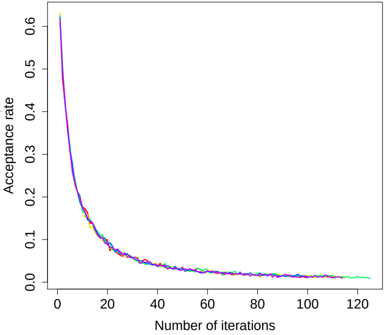
Summary

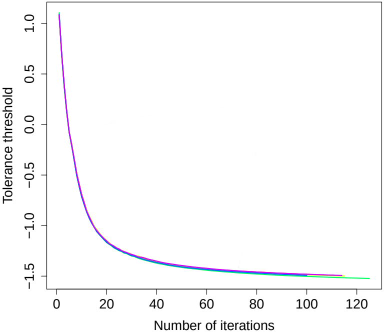
The PRIMA model

Parameters and summary statistics

- 4 parameters
- 8 summary statistics
- $\left\|(\rho_m(S_m, S_m'))_{1 \le m \le M}\right\|_\infty = \sup_{1 \le m \le M} |\rho_m(S_m, S_m')|$

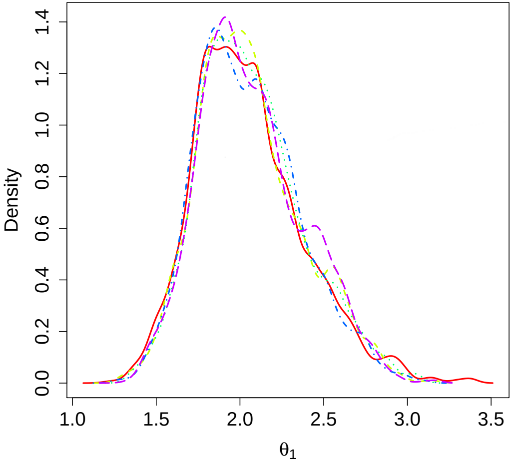
The PRIMA model
Acceptance rate and threshold evolution





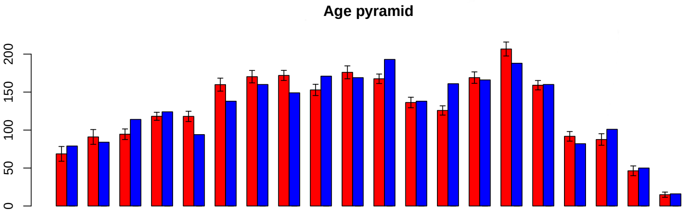
The PRIMA model

Posterior density of a parameter



The PRIMA model
Concrete results

- 1.4 second by simulations
- 400,000 simulations
- 6 days



Conclusion

- We have answered the three research questions
- Comparison with a well-known method
- Calibration of a complex social model