Adaptive approximate Bayesian computation for complex models

Maxime Lenormand, Franck Jabot, Guillaume Deffuant

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Prototypical Policy Impacts on Multifunctional Activities in rural municipalities

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Plan

1. Motivation
2. Approximate Bayesian Computation (ABC)
3. Approximate Bayesian Computation Sequential Monte Carlo
4. Adaptive Population Monte Carlo Approximate Bayesian Computation
5. Comparison of the algorithms
6. The *SimVillages* Model
7. Conclusion

Motivation

Complex social model

- Individual-based model
- Stochastic
- High dimensional parameter space
- High computational cost by simulation

Estimate the parameter values

- Calibrate the model
- Understand the model behaviour
- Uncertainty analysis
- Validation

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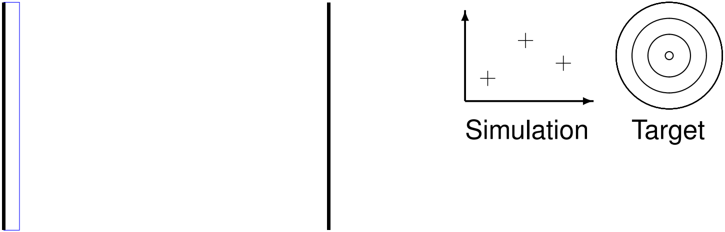
Approximate Bayesian Computation

1. Sample $\theta^* \sim \pi(\theta)$.
2. Simulate $x \sim f(x|\theta^*)$.
3. If $\rho(S(x), S(y)) \leq \epsilon$, accept θ^*, otherwise reject.
4. Repeat until a sample of the desired size is obtained



Prior distribution
$\pi(\theta)$

Posterior distribution
$\pi(\theta)P_\theta\{f(x|\theta) = y\}$

(Pritchard et al., 1999)

Derived from T. Toni 2011

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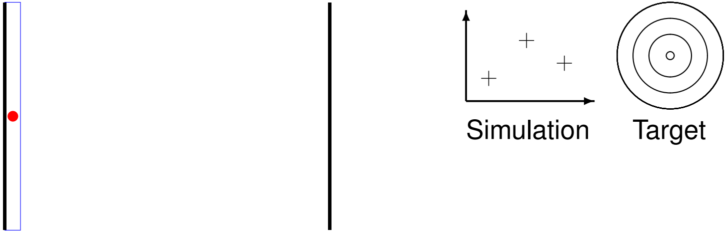

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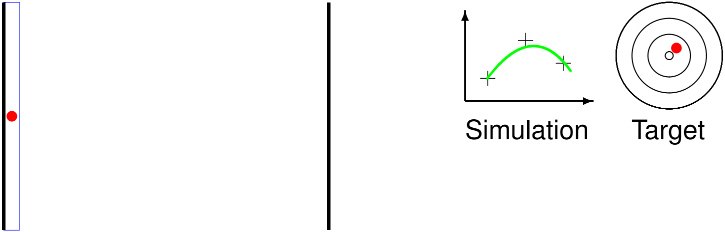

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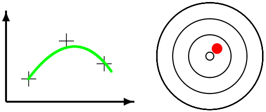

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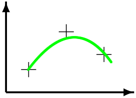
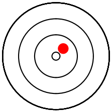


Simulation



Target

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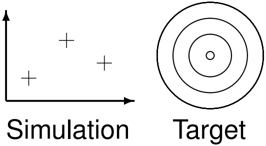

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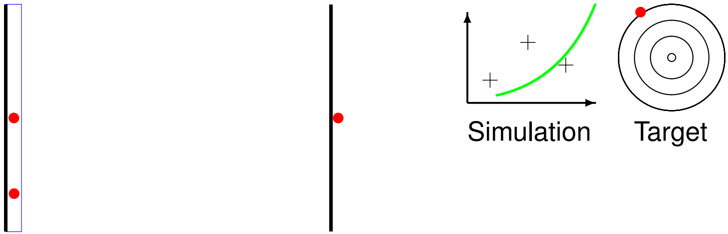

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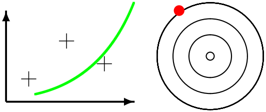

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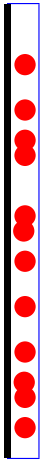

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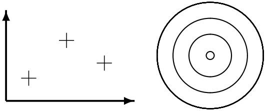

Simulation Target

(Pritchard et al., 1999)

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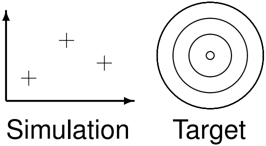

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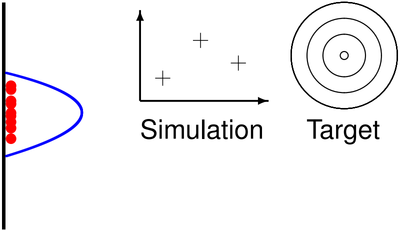

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ABC SMC
Algorithm

Prior Posterior

(Beaumont et al., 2009)
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ABC SMC
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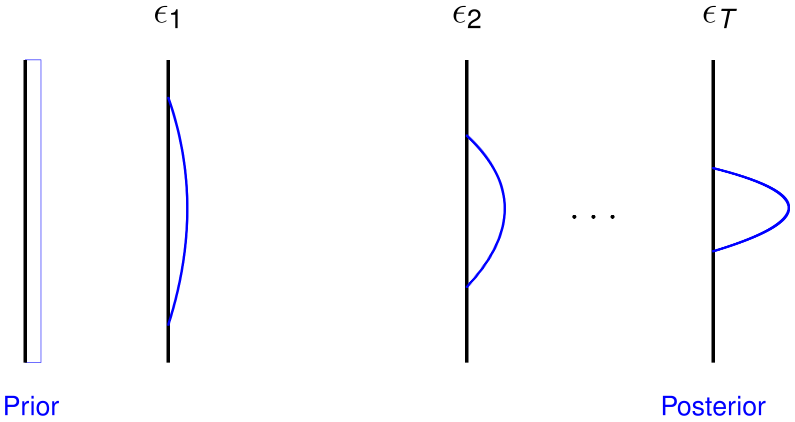

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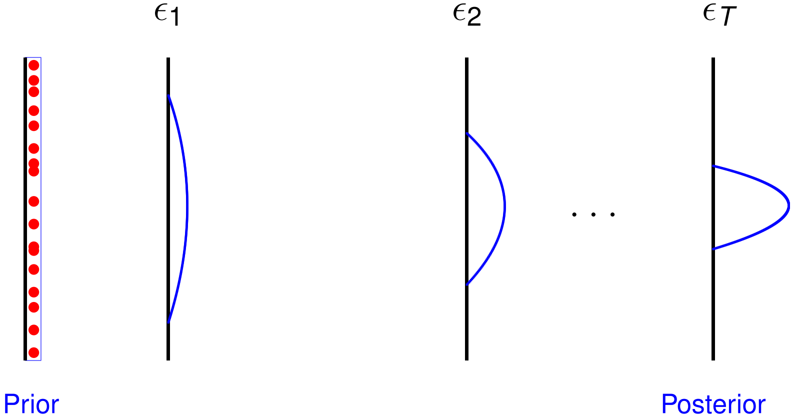

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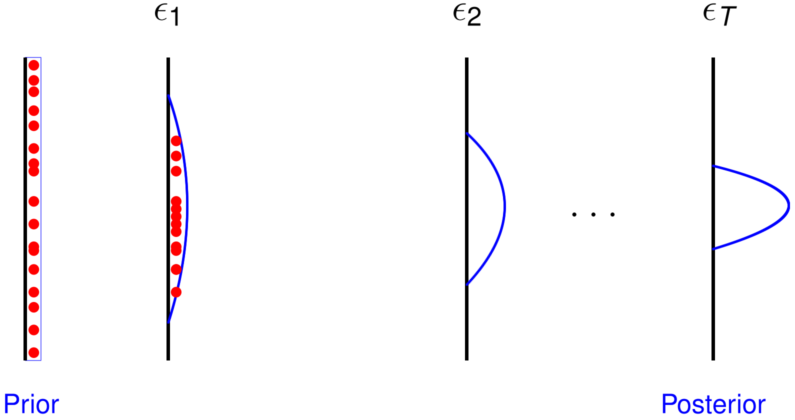

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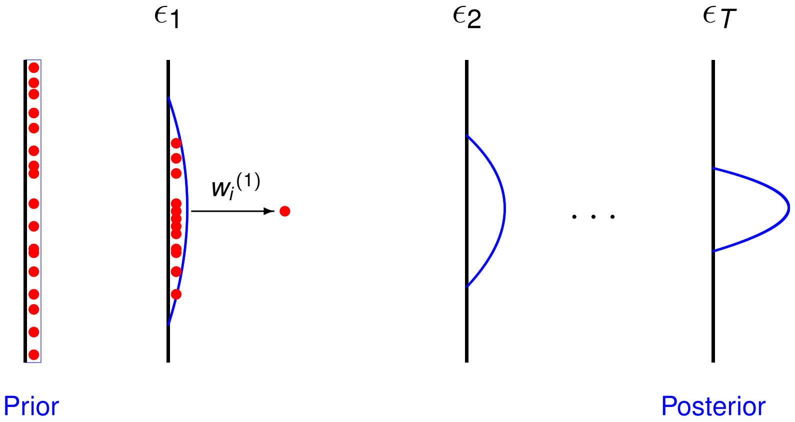

$$w_i^{(t)} = \frac{\pi(\theta_i^{(t)})}{\sum_{j=1}^{N} w_j^{(t-1)} \sigma_{t-1}^{-1} \varphi\left(\sigma_{t-1}^{-1}(\theta_i^{(t)} - \theta_j^{(t-1)})\right)}$$

(Beaumont et al., 2009)

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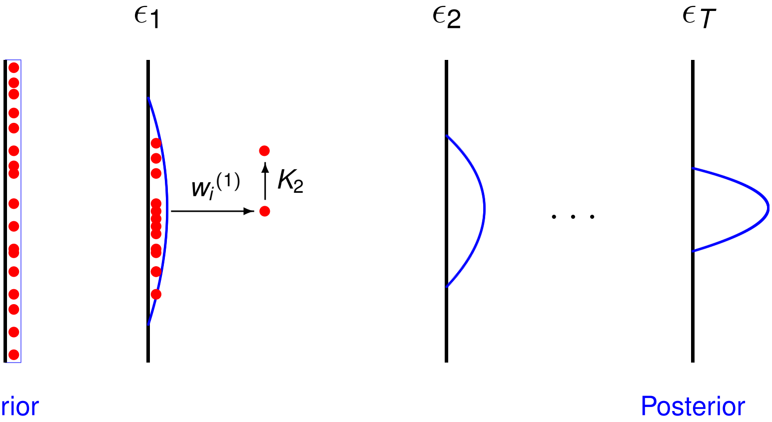

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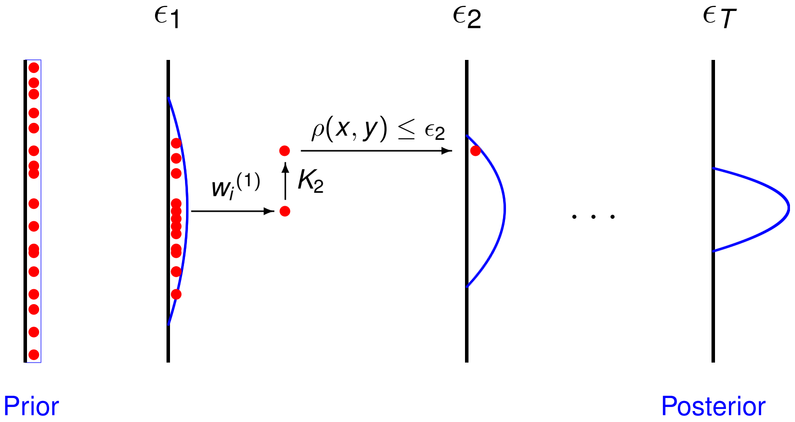

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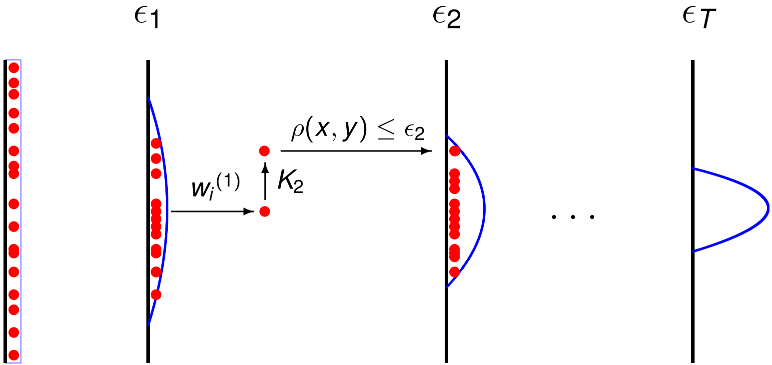

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Posterior

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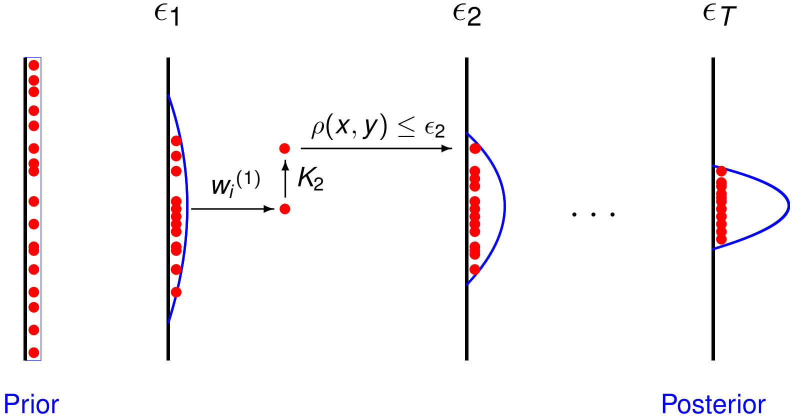

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ABC SMC
Recent Improvements

- Determination "on-line" of the decreasing sequence of tolerance levels $\{\epsilon_1, ..., \epsilon_T\}$. (DelMoral et al. (2011)) and (Drovandi and Pettitt (2011).
- From a quadratic complexity to a linear complexity for the computation of the weights thanks to a MCMC kernel. (DelMoral et al. (2011)) and (Drovandi and Pettitt (2011).
- Realisation of *M* pseudo-observations (DelMoral et al. (2011))

$\Longrightarrow$ The MCMC kernel leads to the problem of particles duplication!!!

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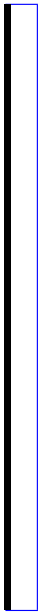

Prior

(Lenormand et al.)

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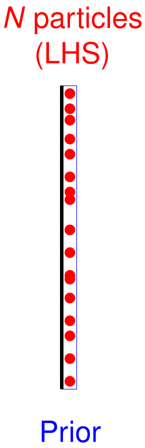

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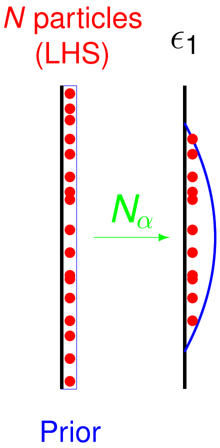

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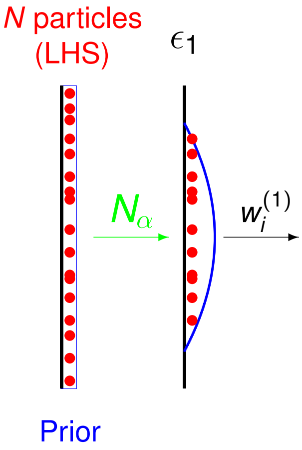

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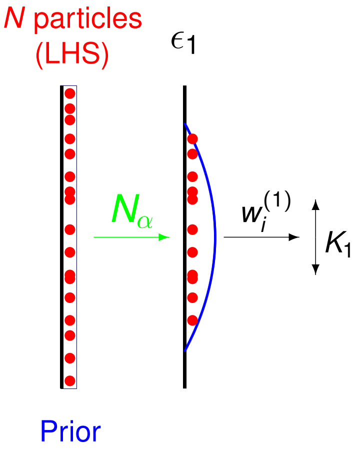

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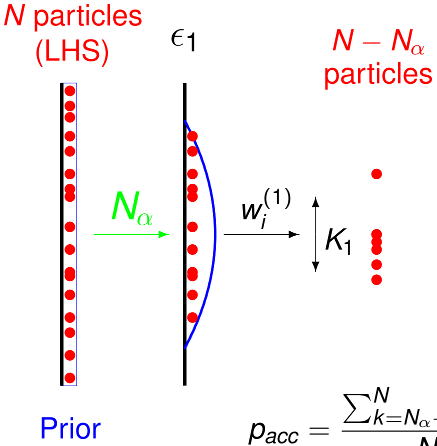

$$p_{acc} = \frac{\sum_{k=N_\alpha+1}^{N} \mathbb{1}_{\rho(x,y)\leq\epsilon_1}}{N - N_\alpha}$$

(Lenormand et al.)

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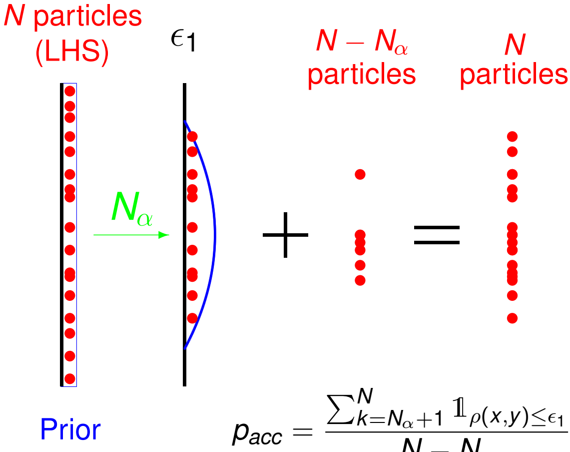

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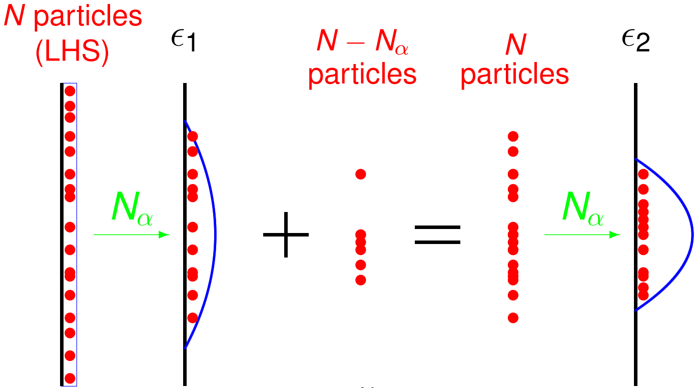

Prior

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Adaptive PMC ABC
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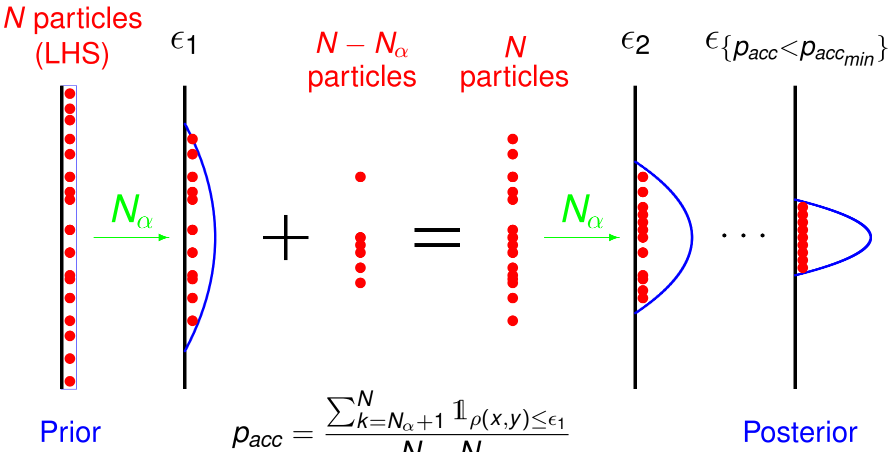

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Adaptive PMC ABC
Pros and Cons

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- Control the number of simulations at each iteration ($N - N_\alpha$ simulations).

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Cons

- Complexity $O({N_\alpha}^2)$ for the computation of the weights

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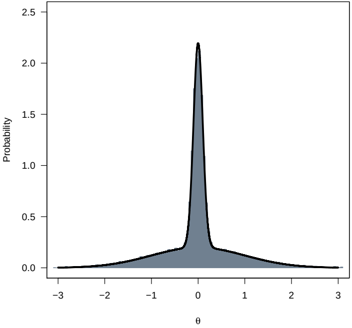
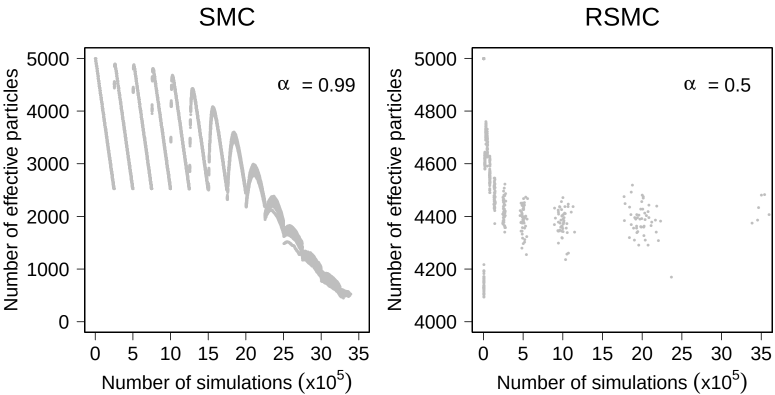
Comparison of the algorithms
Toy Example : Presentation

$$f(x|\theta) \sim \frac{1}{2}\phi\left(\theta, \frac{1}{100}\right) + \frac{1}{2}\phi(\theta, 1) \text{ and } \theta \sim \mathcal{U}_{[-10,10]}$$

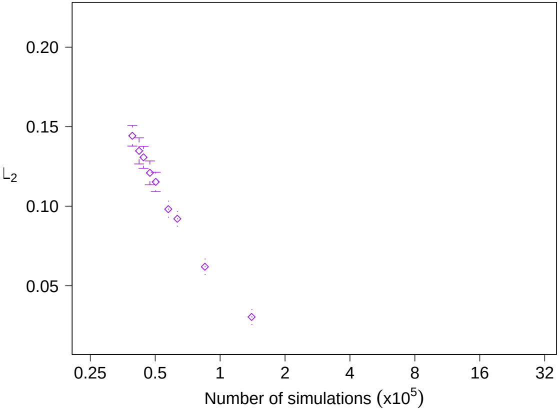


Comparison of the algorithms
Toy Example : Particles Duplication



Comparison of the algorithms
Toy Example : Comparison

$N_\alpha = 5000$; α from 0.9 to 0.1 corresponding to $N = 5555$ to 50000



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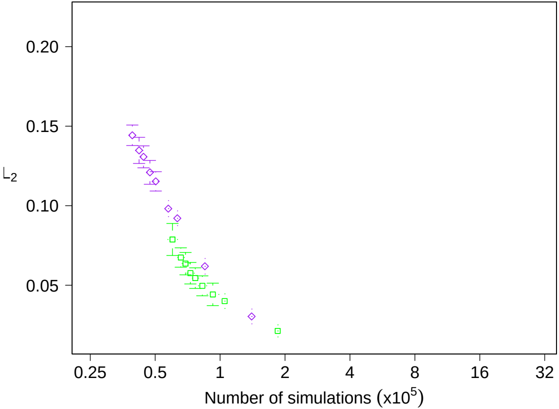

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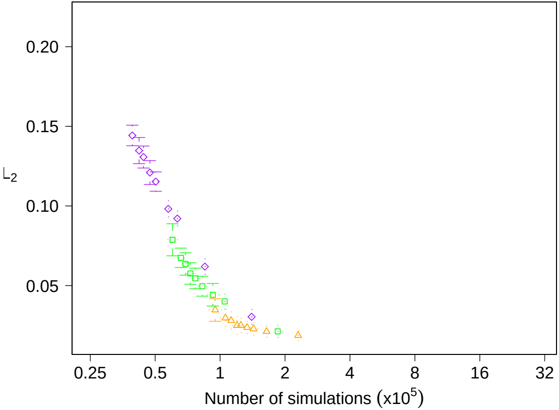

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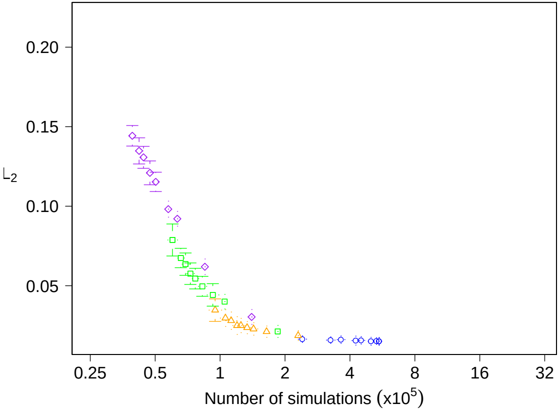

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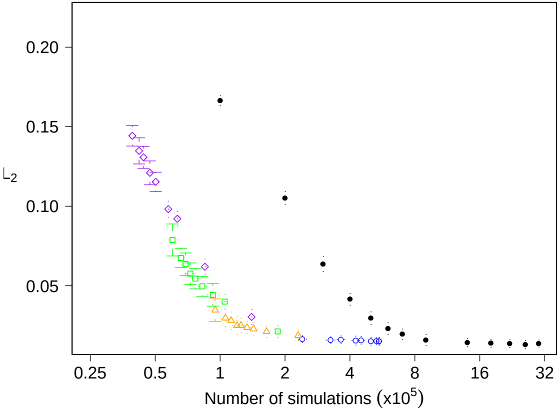

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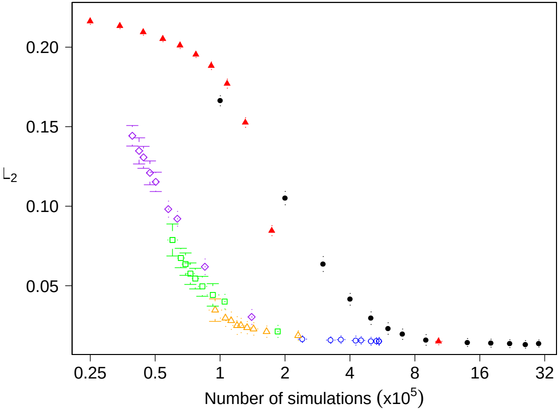

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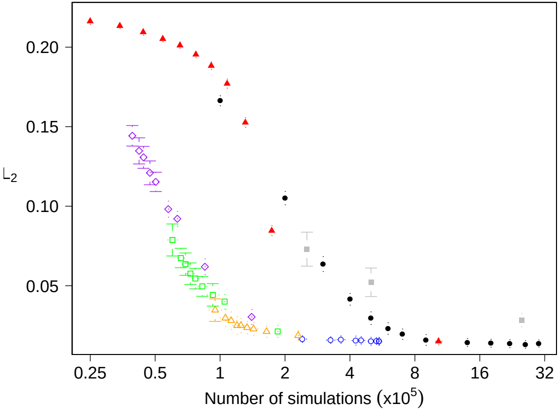
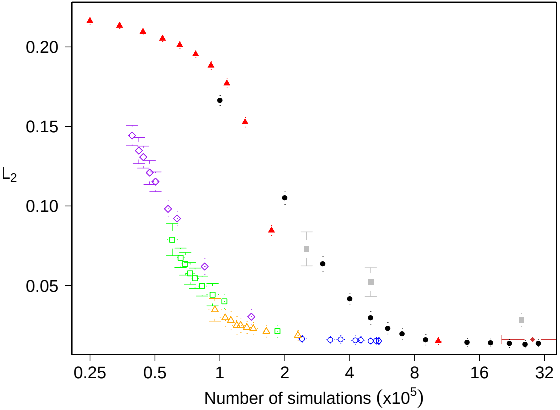


Comparison of the algorithms
Toy Example : Comparison

$N_\alpha = 5000$; α from 0.9 to 0.1 corresponding to $N = 5555$ to 50000



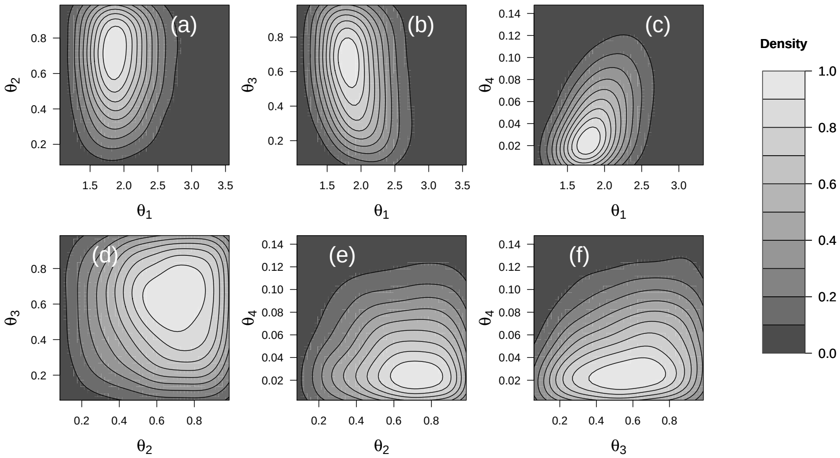
Plan

1. Motivation
2. Approximate Bayesian Computation (ABC)
3. Approximate Bayesian Computation Sequential Monte Carlo
4. Adaptive Population Monte Carlo Approximate Bayesian Computation
5. Comparison of the algorithms
6. The *SimVillages* Model
7. Conclusion

The *SimVillages* Model
Parameter and Summary Statistics

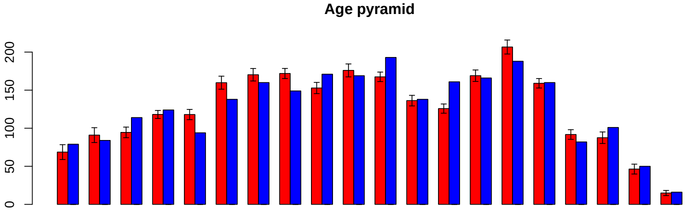
- 4 parameters
- 8 statistics
- $\|(\rho_m(S_m, S_m^{'}))_{1 \leq m \leq M}\|_\infty = \sup_{1 \leq m \leq M} |\rho_m(S_m, S_m^{'})|$

The *SimVillages* Model
Posterior Density

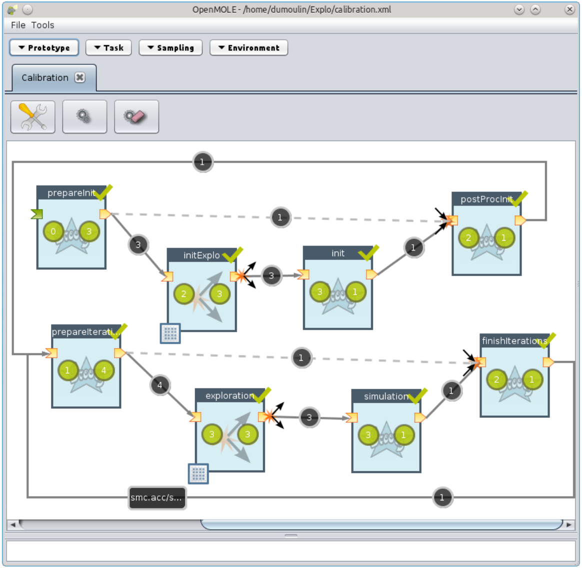


The *SimVillages* Model
Concrete Results

- 1.4 second by simulation
- 400 000 simulations
- 6 days



The *SimVillages* Model
Open Mole



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Conclusion

- We have developed a new algorithm to reduce the number of simulations in SMC ABC.
- Comparison with three methods.
- Calibration of a complex social model.

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