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► **To cite this version:**

Maxime Lenormand, Franck Jabot, Guillaume Deffuant. Adaptive approximate Bayesian computation for complex models. 8th World Congress in Probability and Statistics, Jul 2012, Istanbul, Turkey. pp.11. hal-02597181

**HAL Id: hal-02597181**

**<https://hal.inrae.fr/hal-02597181v1>**

Submitted on 6 Jul 2020

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# Adaptive approximate Bayesian computation for complex models

*Maxime Lenormand, Franck Jabot & Guillaume Deffuant*

Laboratory of Engineering for Complex Systems  
Irstea of Clermont-Ferrand

**8<sup>th</sup> World Congress in Probability and Statistics**  
*July 12<sup>th</sup> 2012*



**Prototypical Policy Impacts on Multifunctional Activities in rural municipalities**

A collaborative project under the  
EU Seventh Framework Programme



# Plan

- 1 Motivation
- 2 Approximate Bayesian Computation (ABC)
- 3 Approximate Bayesian Computation Sequential Monte Carlo
- 4 Adaptive Population Monte Carlo Approximate Bayesian Computation
- 5 Comparison of the algorithms
- 6 The *SimVillages* Model
- 7 Conclusion

# Motivation

## Complex social model

- Individual-based model
- Stochastic
- High dimensional parameter space
- High computational cost by simulation

## Estimate the parameter values

- Calibrate the model
- Understand the model behaviour
- Uncertainty analysis
- Validation

# Plan

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- 2 **Approximate Bayesian Computation (ABC)**
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# Approximate Bayesian Computation

- 1 Sample  $\theta^* \sim \pi(\theta)$ .
- 2 Simulate  $x \sim f(x|\theta^*)$ .
- 3 If  $\rho(S(x), S(y)) \leq \epsilon$ , accept  $\theta^*$ , otherwise reject.
- 4 Repeat until a sample of the desired size is obtained



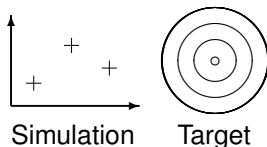
Prior distribution  
 $\pi(\theta)$

*(Pritchard et al., 1999)*



Posterior distribution  
 $\pi(\theta)P_{\theta}\{f(x|\theta) = y\}$

*Derived from T. Toni 2011*



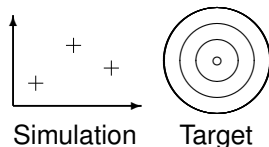
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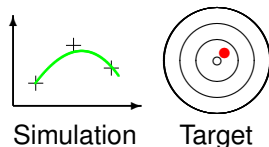
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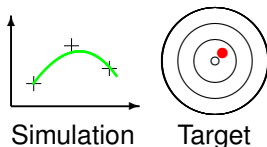
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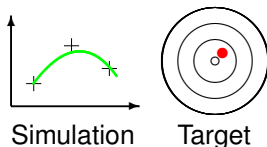
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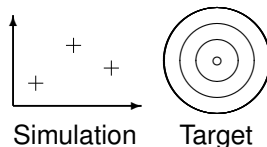
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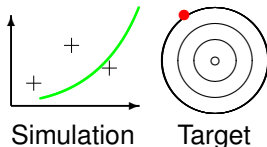
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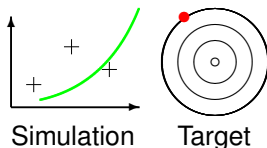
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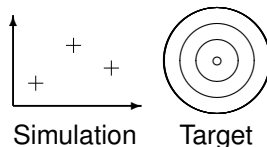
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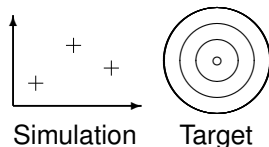
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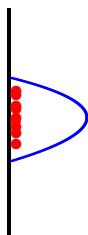
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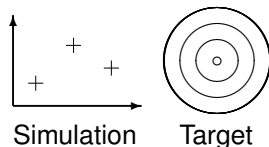
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# ABC SMC Algorithm



Prior



Posterior

*(Beaumont et al., 2009)*

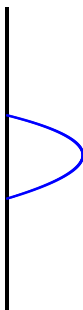
*(Toni et al., 2009)*

# ABC SMC Algorithm



Prior

$\epsilon_T$

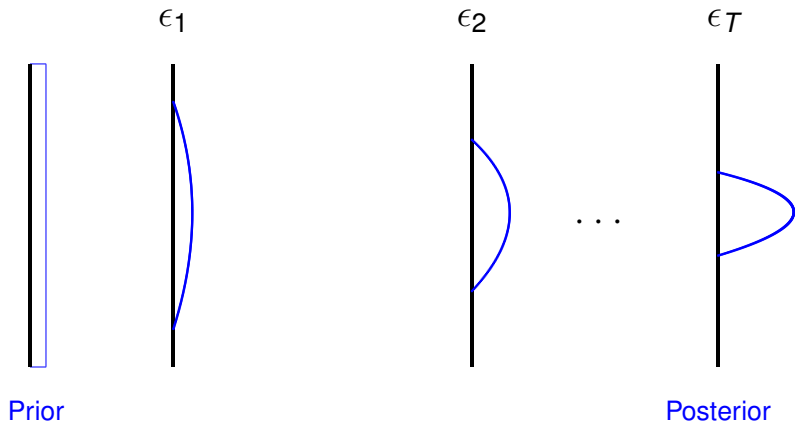


Posterior

*(Beaumont et al., 2009)*

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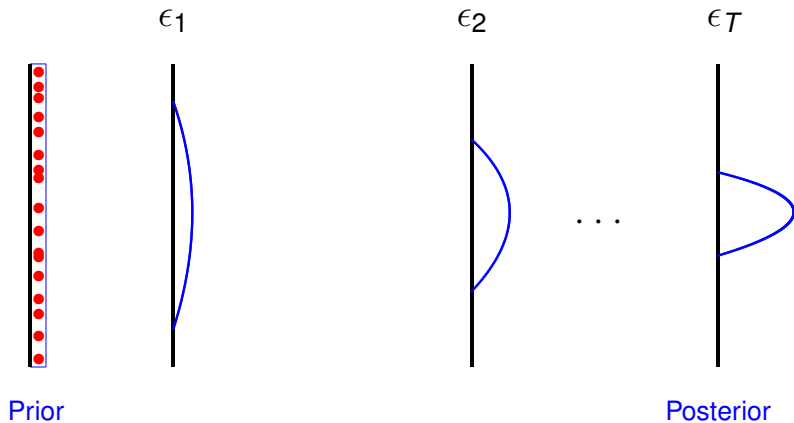
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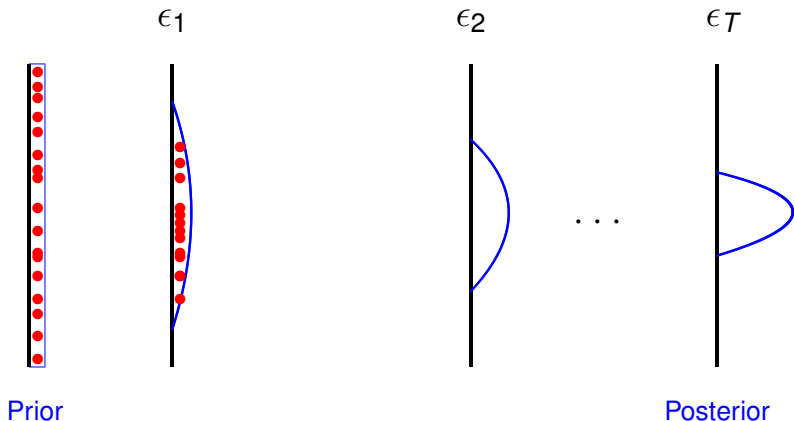
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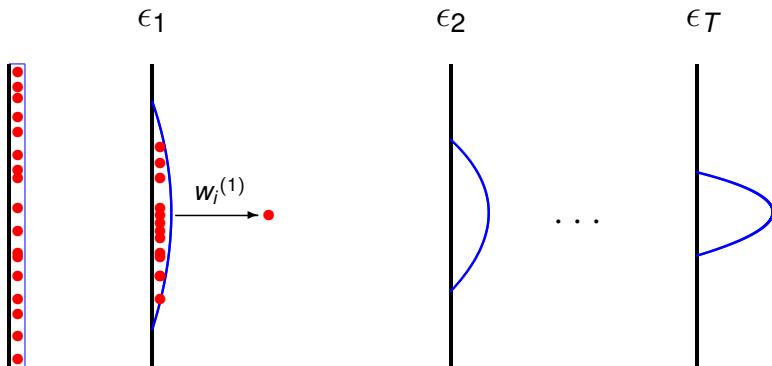
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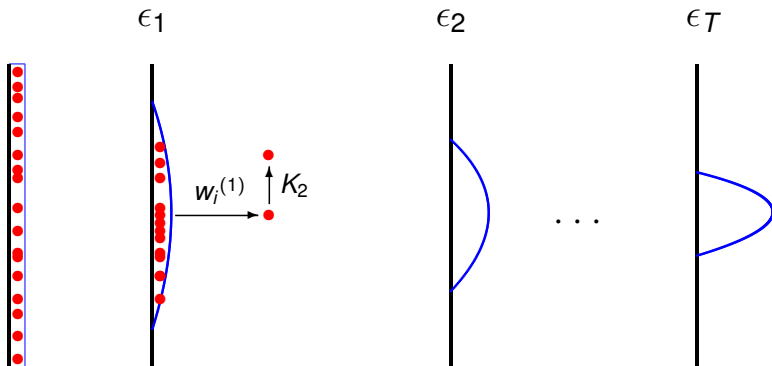
Posterior

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$$w_i^{(t)} = \frac{\pi(\theta_i^{(t)})}{\sum_{j=1}^N w_j^{(t-1)} \sigma_{t-1}^{-1} \varphi\left(\sigma_{t-1}^{-1}(\theta_i^{(t)} - \theta_j^{(t-1)})\right)}$$

# ABC SMC Algorithm



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Posterior

(Beaumont et al., 2009)

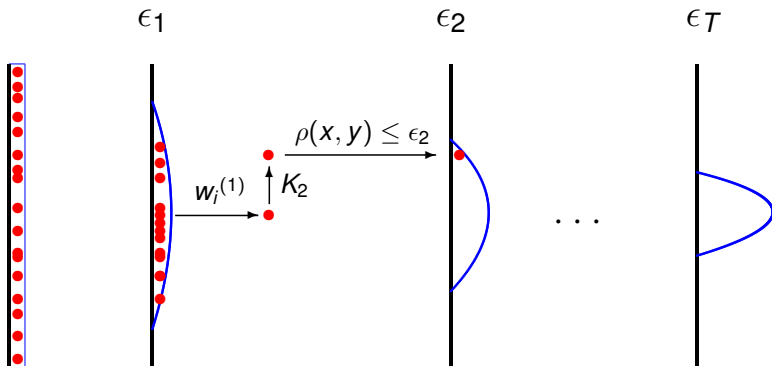
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Posterior

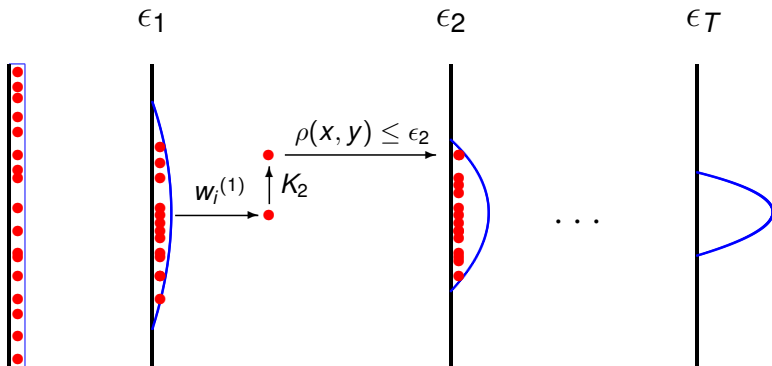
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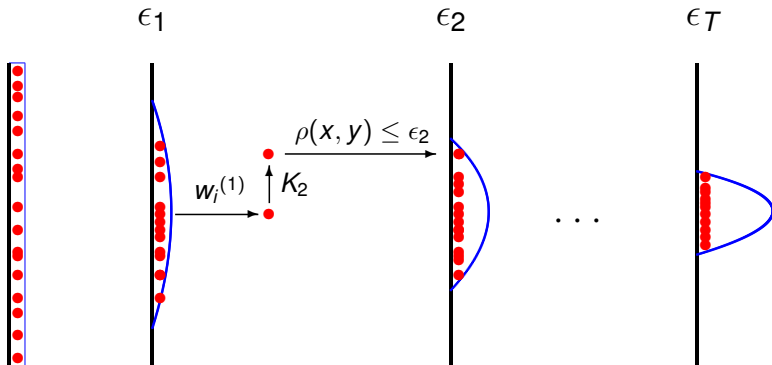
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# ABC SMC Algorithm



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$$K_t(\theta^* | \theta) = \sigma_{t-1}^{-1} \varphi(\sigma_{t-1}^{-1}(\theta^* - \theta))$$

# ABC SMC

## Recent Improvements

- Determination "on-line" of the decreasing sequence of tolerance levels  $\{\epsilon_1, \dots, \epsilon_T\}$ . (DelMoral et al. (2011)) and (Drovandi and Pettitt (2011)).
- From a quadratic complexity to a linear complexity for the computation of the weights thanks to a MCMC kernel. (DelMoral et al. (2011)) and (Drovandi and Pettitt (2011)).
- Realisation of  $M$  pseudo-observations (DelMoral et al. (2011))

⇒ The MCMC kernel leads to the problem of particles duplication!!!

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# Adaptive PMC ABC Algorithm



Prior

*(Lenormand et al.)*

# Adaptive PMC ABC Algorithm

$N$  particles  
(LHS)



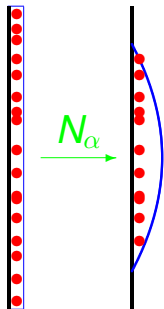
Prior

*(Lenormand et al.)*

# Adaptive PMC ABC Algorithm

$N$  particles  
(LHS)

$\epsilon_1$



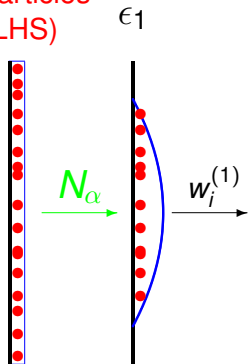
Prior

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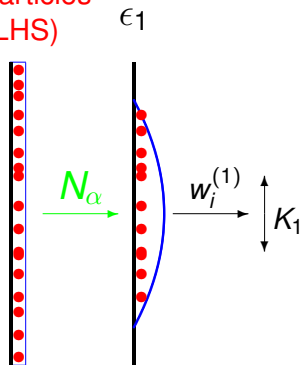


Prior

*(Lenormand et al.)*

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$N$  particles  
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$N$  particles  
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Prior

(Lenormand et al.)

$\epsilon_1$

$N_\alpha$

$w_i^{(1)}$

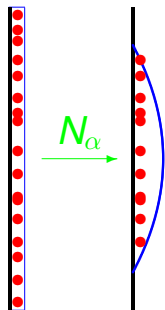
$N - N_\alpha$   
particles

$K_1$

$$\rho_{acc} = \frac{\sum_{k=N_\alpha+1}^N \mathbb{1}_{\rho(x,y) \leq \epsilon_1}}{N - N_\alpha}$$

# Adaptive PMC ABC Algorithm

$N$  particles  
(LHS)



$\epsilon_1$

$N - N_\alpha$   
particles

$N$   
particles



Prior

(Lenormand et al.)

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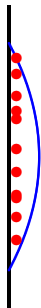
# Adaptive PMC ABC Algorithm

$N$  particles  
(LHS)

$\epsilon_1$



$N_\alpha$



+

$N - N_\alpha$   
particles



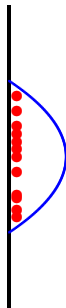
=

$N$   
particles



$N_\alpha$

$\epsilon_2$



Prior

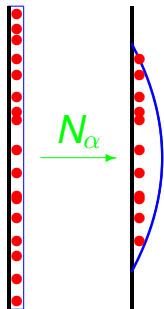
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(LHS)

$\epsilon_1$



Prior

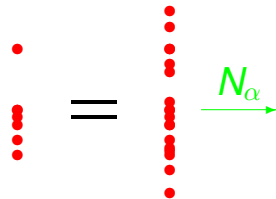
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$N - N_\alpha$   
particles

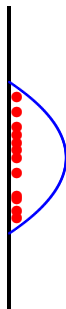
$N$   
particles

+

=

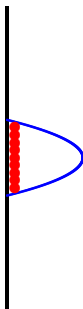


$\epsilon_2$



$\epsilon_{\{p_{acc} < p_{acc_{min}}\}}$

...



Posterior

$$p_{acc} = \frac{\sum_{k=N_\alpha+1}^N \mathbb{1}_{\rho(x,y) \leq \epsilon_1}}{N - N_\alpha}$$

# Adaptive PMC ABC

## Pros and Cons

### Pros

- Control the number of simulations at each iteration ( $N - N_\alpha$  simulations).

# Adaptive PMC ABC

## Pros and Cons

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- Control the number of simulations at each iteration ( $N - N_\alpha$  simulations).
- Determination "on-line" of the decreasing sequence of tolerance levels.



# Adaptive PMC ABC

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# Adaptive PMC ABC

## Pros and Cons

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- Determination "on-line" of the decreasing sequence of tolerance levels.
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- Stopping criterion

# Adaptive PMC ABC

## Pros and Cons

### Pros

- Control the number of simulations at each iteration ( $N - N_\alpha$  simulations).
- Determination "on-line" of the decreasing sequence of tolerance levels.
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- Stopping criterion

### Cons

- Complexity  $O(N_\alpha^2)$  for the computation of the weights

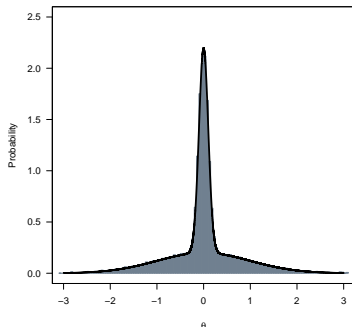
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# Comparison of the algorithms

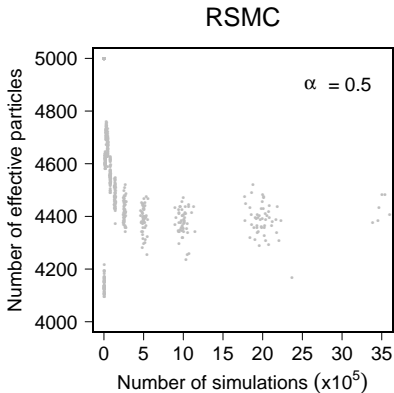
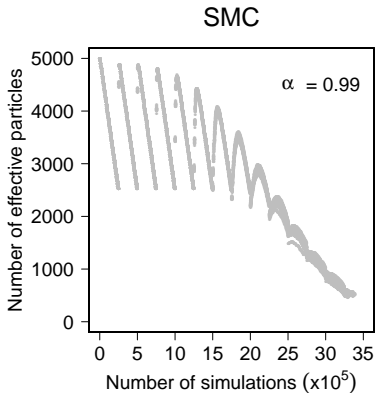
## Toy Example : Presentation

$$f(x|\theta) \sim \frac{1}{2}\phi\left(\theta, \frac{1}{100}\right) + \frac{1}{2}\phi(\theta, 1) \text{ and } \theta \sim \mathcal{U}_{[-10,10]}$$



# Comparison of the algorithms

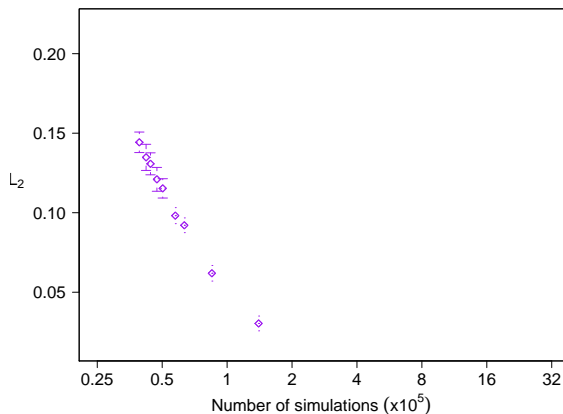
## Toy Example : Particles Duplication



# Comparison of the algorithms

## Toy Example : Comparison

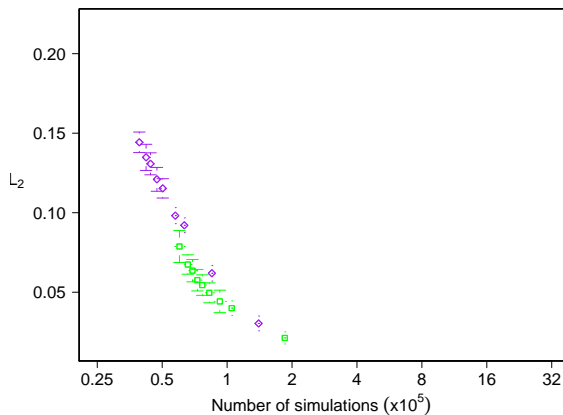
$N_\alpha = 5000$ ;  $\alpha$  from 0.9 to 0.1 corresponding to  $N = 5555$  to 50000



# Comparison of the algorithms

## Toy Example : Comparison

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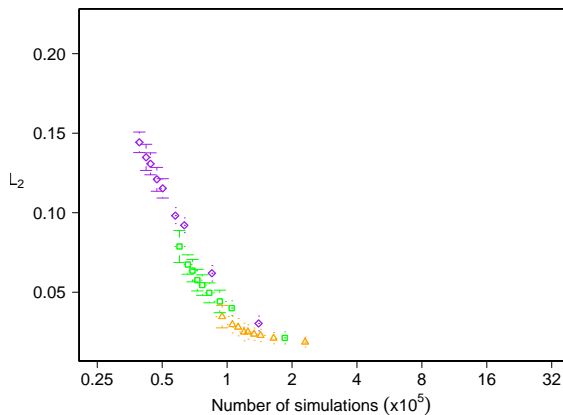




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## Toy Example : Comparison

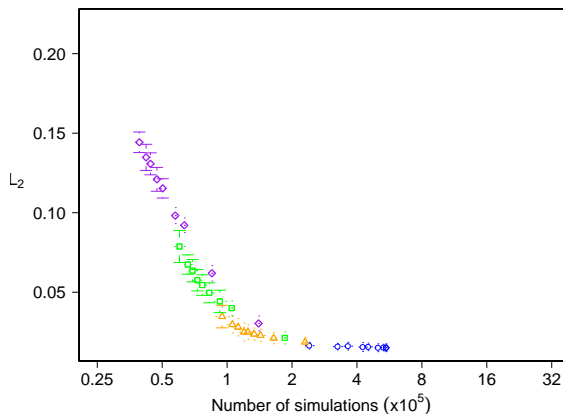
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## Toy Example : Comparison

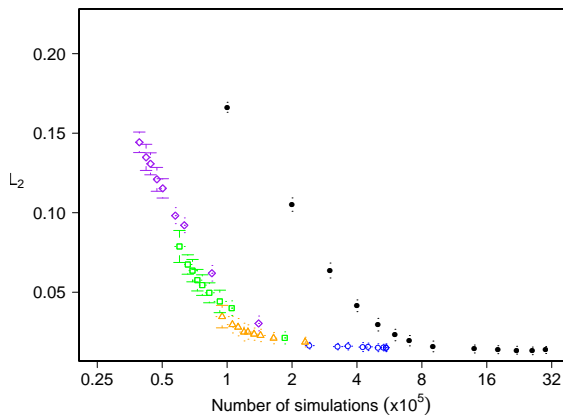
$N_\alpha = 5000$ ;  $\alpha$  from 0.9 to 0.1 corresponding to  $N = 5555$  to 50000



# Comparison of the algorithms

## Toy Example : Comparison

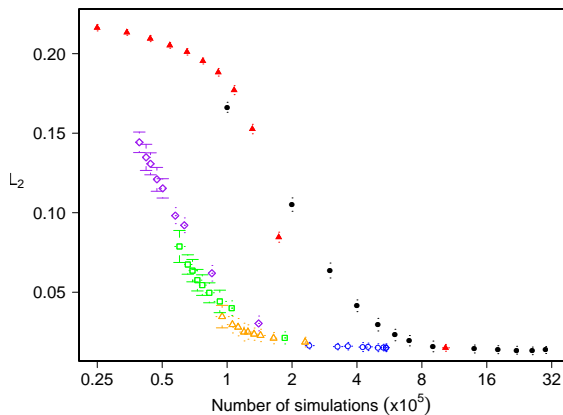
$N_\alpha = 5000$ ;  $\alpha$  from 0.9 to 0.1 corresponding to  $N = 5555$  to 50000



# Comparison of the algorithms

## Toy Example : Comparison

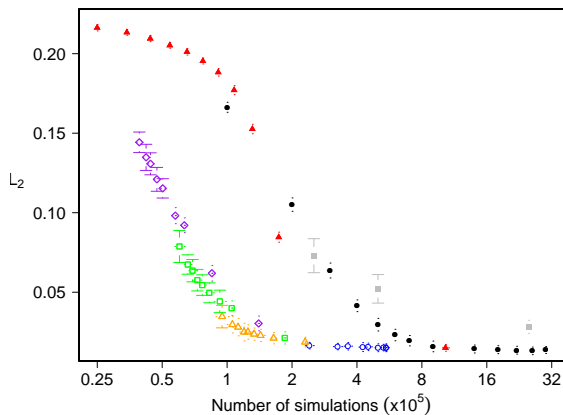
$N_\alpha = 5000$ ;  $\alpha$  from 0.9 to 0.1 corresponding to  $N = 5555$  to 50000



# Comparison of the algorithms

## Toy Example : Comparison

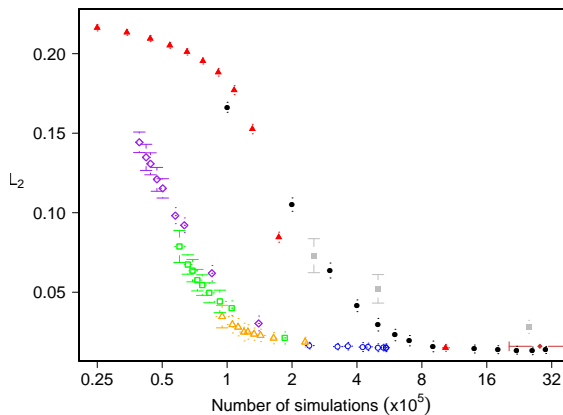
$N_\alpha = 5000$ ;  $\alpha$  from 0.9 to 0.1 corresponding to  $N = 5555$  to 50000



# Comparison of the algorithms

## Toy Example : Comparison

$N_\alpha = 5000$ ;  $\alpha$  from 0.9 to 0.1 corresponding to  $N = 5555$  to 50000



# Plan

- 1 Motivation
- 2 Approximate Bayesian Computation (ABC)
- 3 Approximate Bayesian Computation Sequential Monte Carlo
- 4 Adaptive Population Monte Carlo Approximate Bayesian Computation
- 5 Comparison of the algorithms
- 6 The *SimVillages* Model**
- 7 Conclusion

# The *SimVillages* Model

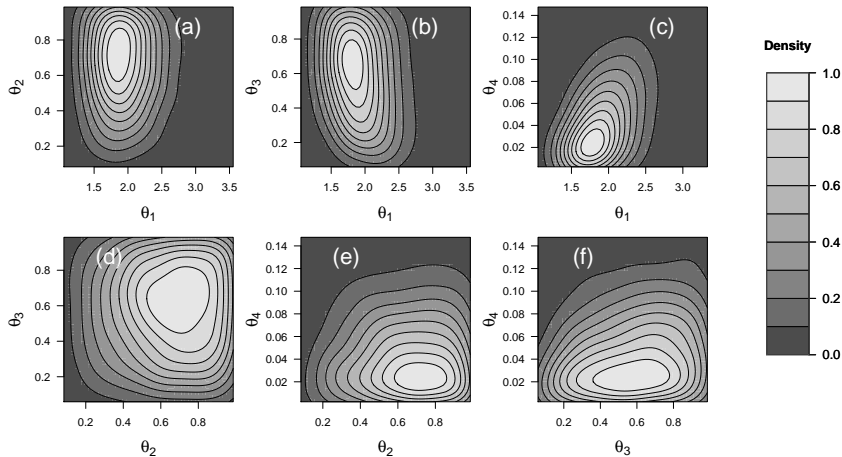
## Parameter and Summary Statistics

- 4 parameters
- 8 statistics
- $\|(\rho_m(\mathcal{S}_m, \mathcal{S}'_m))_{1 \leq m \leq M}\|_{\infty} = \sup_{1 \leq m \leq M} |\rho_m(\mathcal{S}_m, \mathcal{S}'_m)|$



# The *SimVillages* Model

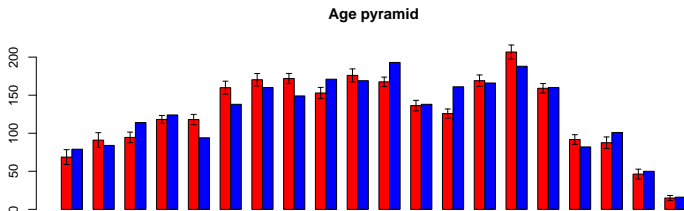
## Posterior Density



# The *SimVillages* Model

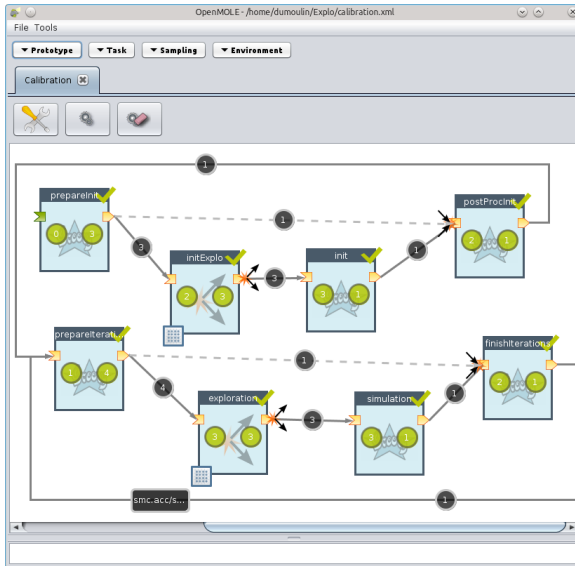
## Concrete Results

- 1.4 second by simulation
- 400 000 simulations
- 6 days



# The *SimVillages* Model

## Open Mole







# Plan

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# Conclusion

- We have developed a new algorithm to reduce the number of simulations in SMC ABC.
- Comparison with three methods.
- Calibration of a complex social model.

# Bibliography

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