

Adaptive approximate Bayesian computation for complex models

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8th World Congress in Probability and Statistics
July 12th 2012



Plan

- 1 Motivation
- 2 Approximate Bayesian Computation (ABC)
- 3 Approximate Bayesian Computation Sequential Monte Carlo
- 4 Adaptive Population Monte Carlo Approximate Bayesian Computation
- 5 Comparison of the algorithms
- 6 The *SimVillages* Model
- 7 Conclusion

Motivation

Complex social model

- Individual-based model
- Stochastic
- High dimensional parameter space
- High computational cost by simulation

Estimate the parameter values

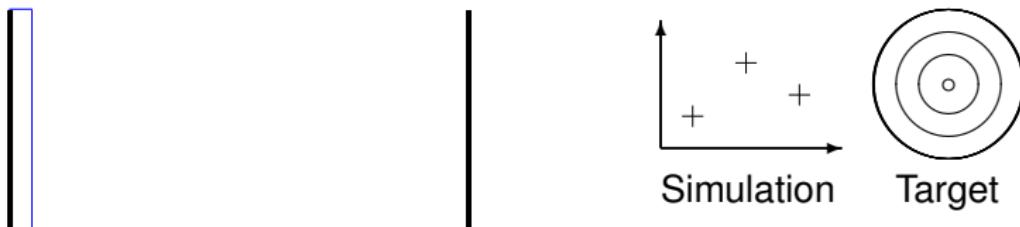
- Calibrate the model
- Understand the model behaviour
- Uncertainty analysis
- Validation

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Approximate Bayesian Computation

- ① Sample $\theta^* \sim \pi(\theta)$.
- ② Simulate $x \sim f(x|\theta^*)$.
- ③ If $\rho(S(x), S(y)) \leq \epsilon$, accept θ^* , otherwise reject.
- ④ Repeat until a sample of the desired size is obtained



Prior distribution
 $\pi(\theta)$

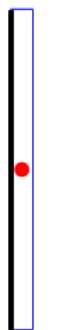
(Pritchard et al., 1999)

Posterior distribution
 $\pi(\theta) P_\theta\{f(x|\theta) = y\}$

Derived from T. Toni 2011

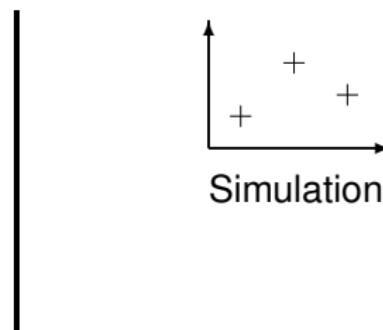
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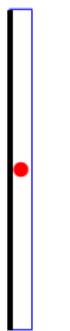


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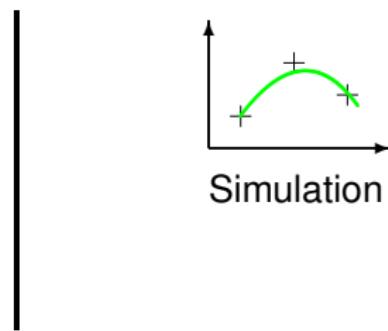
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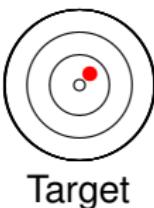
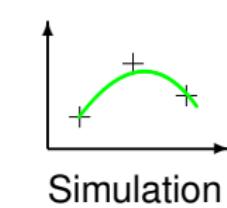


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Simulation

Target



Derived from T. Toni 2011

Approximate Bayesian Computation

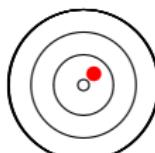
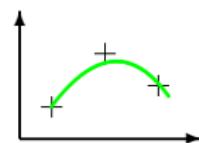
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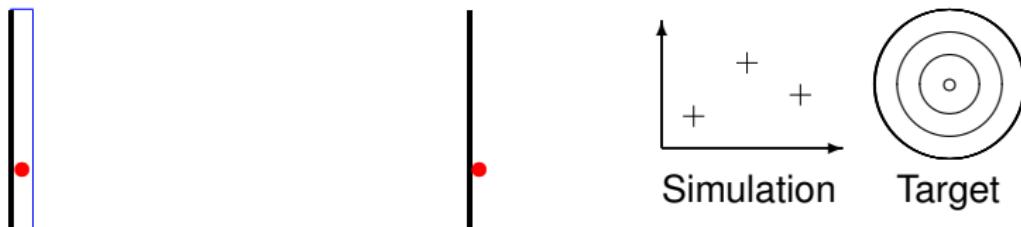


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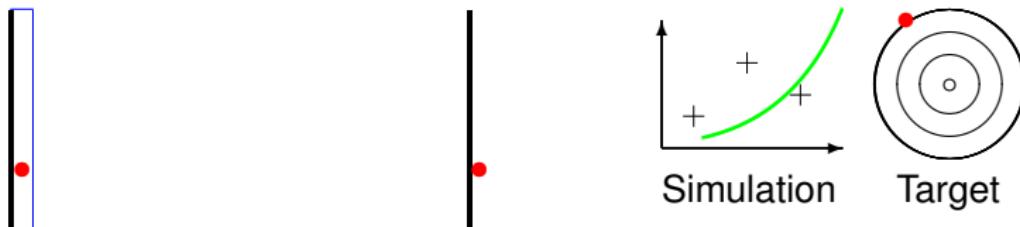
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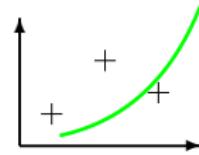
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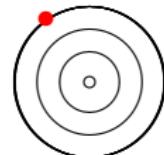
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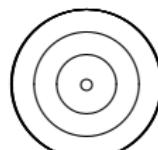


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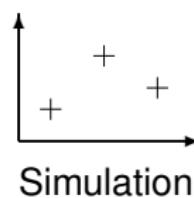


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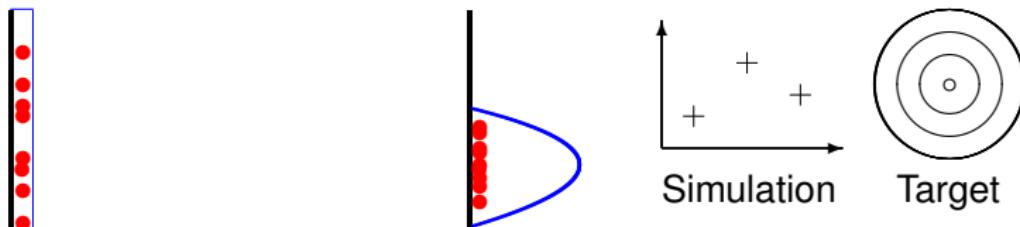
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ABC SMC Algorithm



(Beaumont *et al.*, 2009)

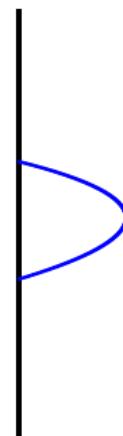
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ABC SMC Algorithm

ϵ_T



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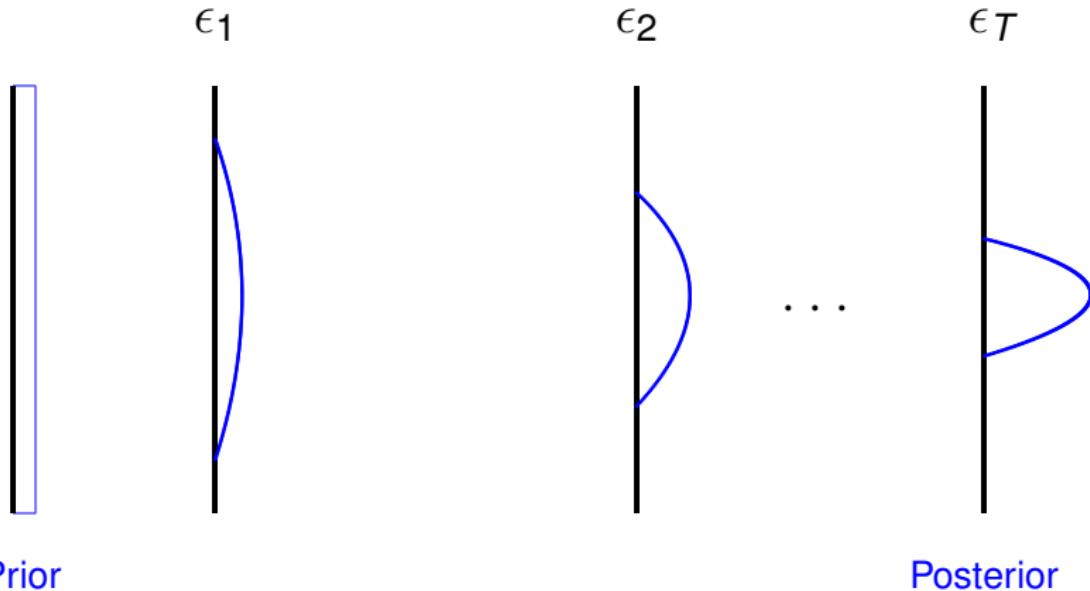


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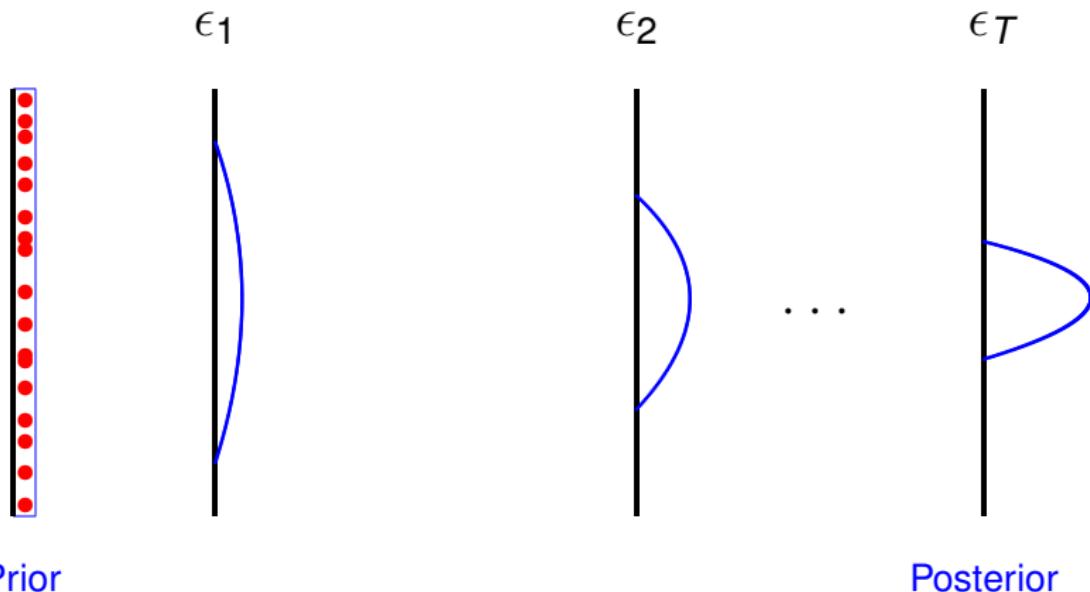
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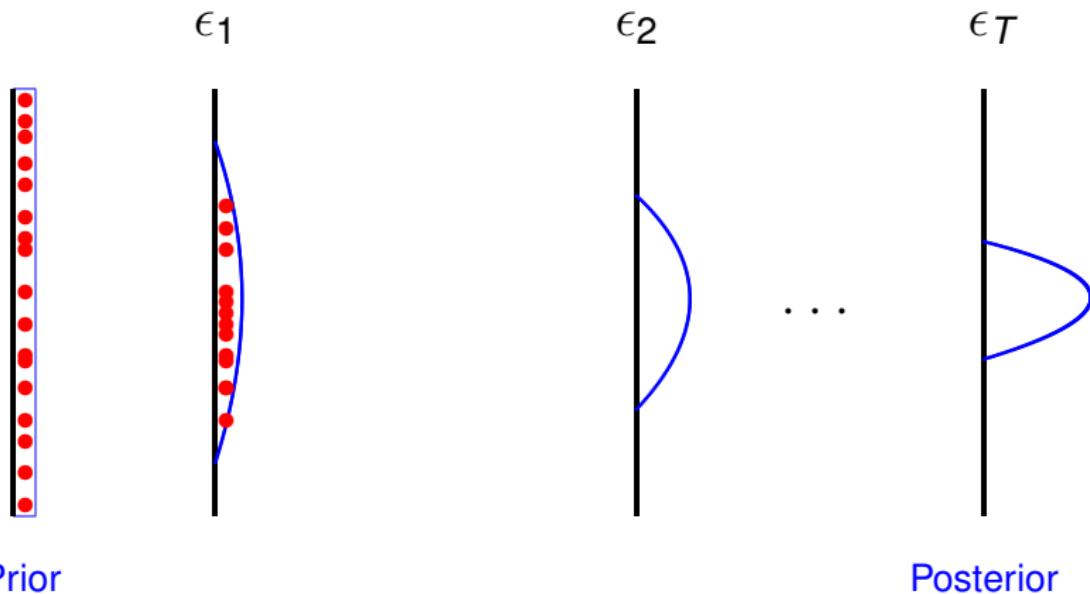
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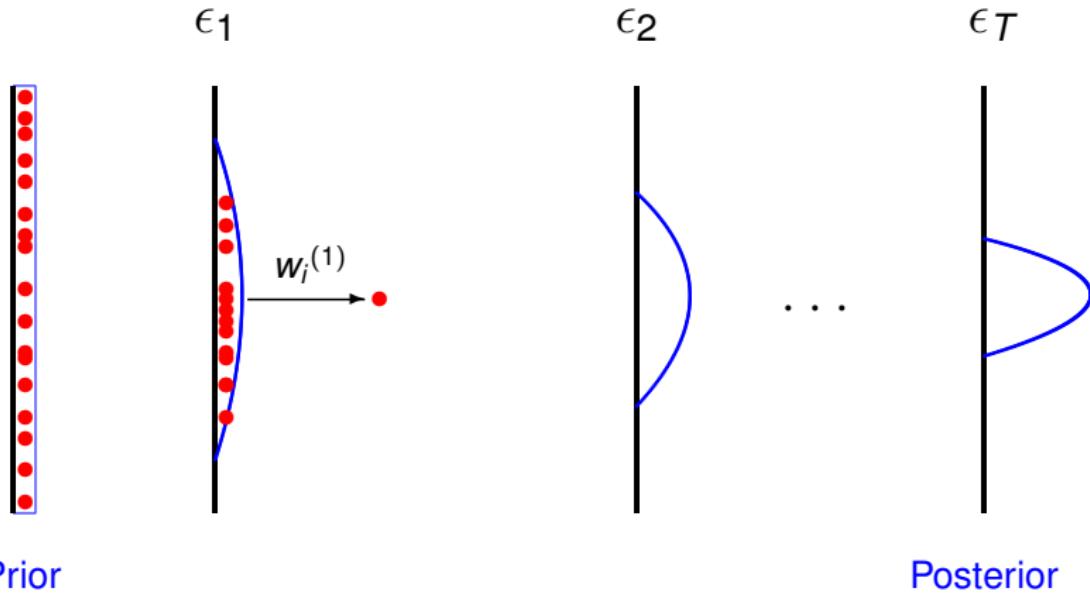
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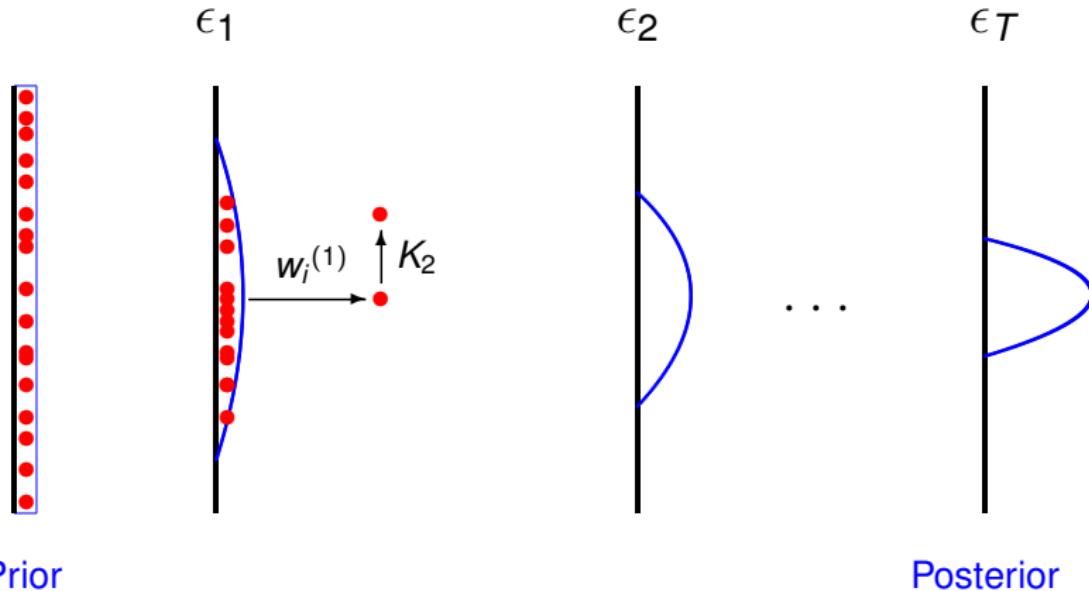
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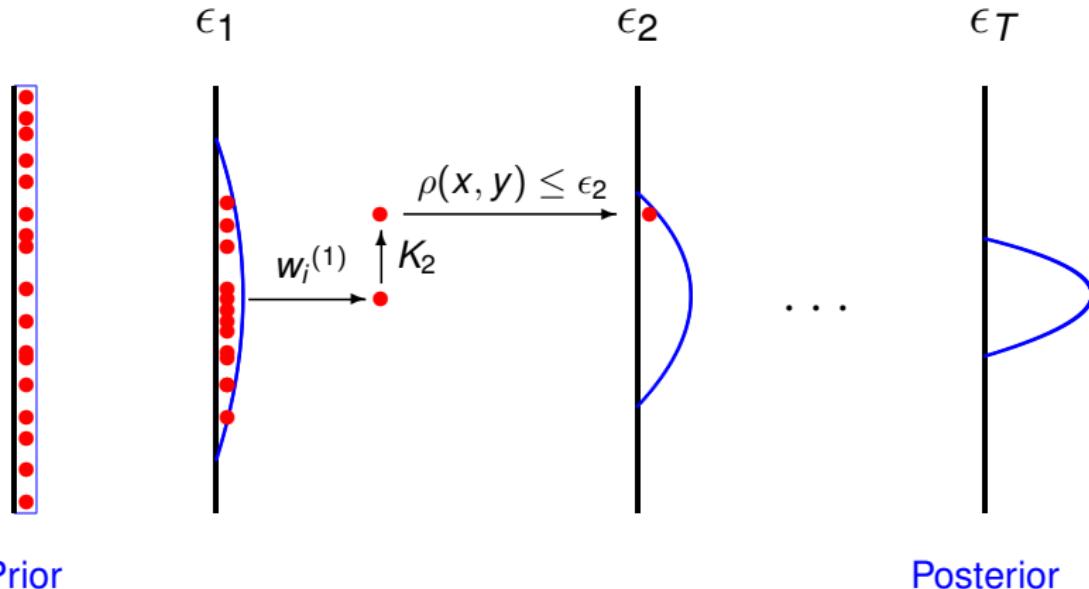
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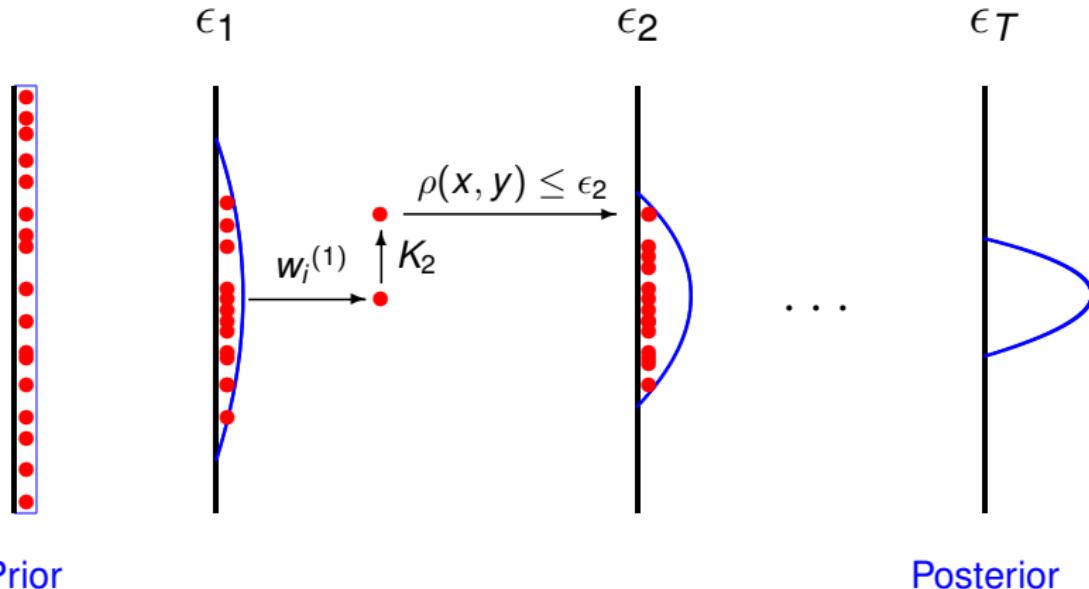
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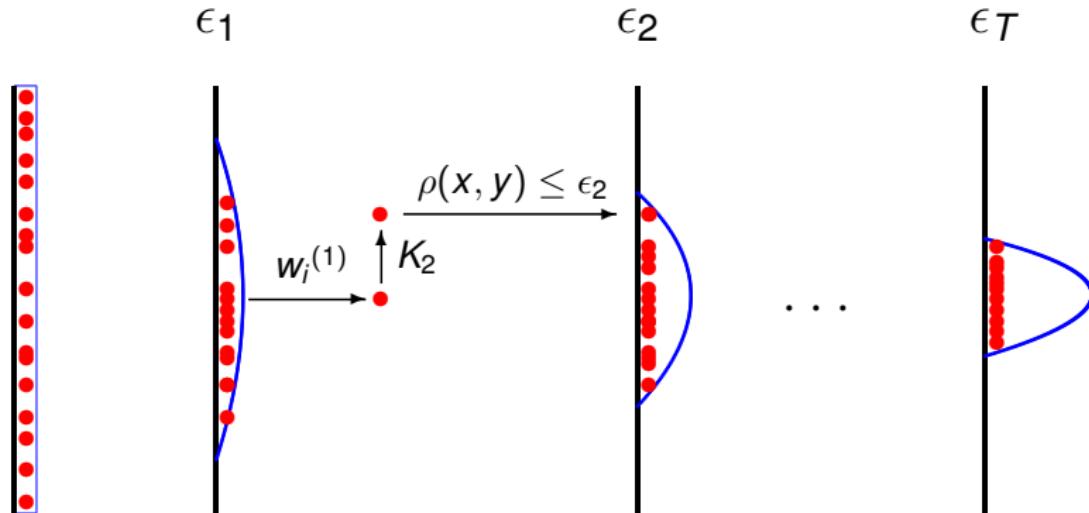
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ABC SMC

Recent Improvements

- Determination "on-line" of the decreasing sequence of tolerance levels $\{\epsilon_1, \dots, \epsilon_T\}$. (DelMoral et al. (2011)) and (Drovandi and Pettitt (2011)).
- From a quadratic complexity to a linear complexity for the computation of the weights thanks to a MCMC kernel. (DelMoral et al. (2011)) and (Drovandi and Pettitt (2011)).
- Realisation of M pseudo-observations (DelMoral et al. (2011))

⇒ The MCMC kernel leads to the problem of particles duplication!!!

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Adaptive PMC ABC Algorithm



Prior

(Lenormand et al.)

Adaptive PMC ABC Algorithm

N particles
(LHS)



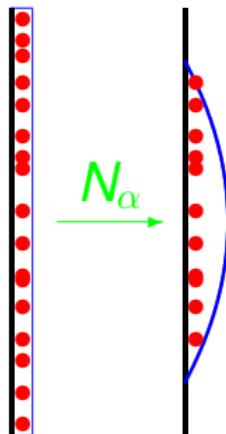
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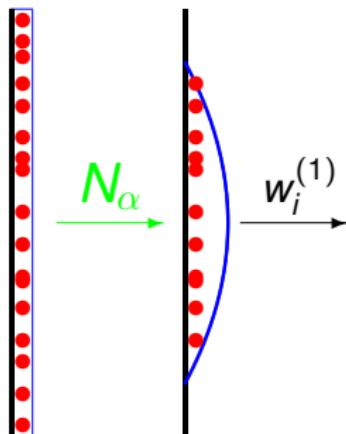
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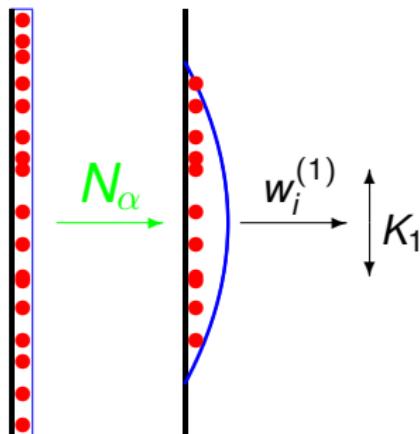
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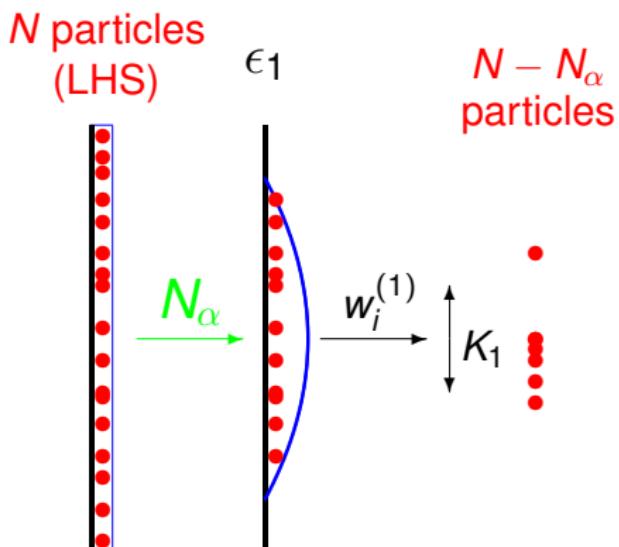
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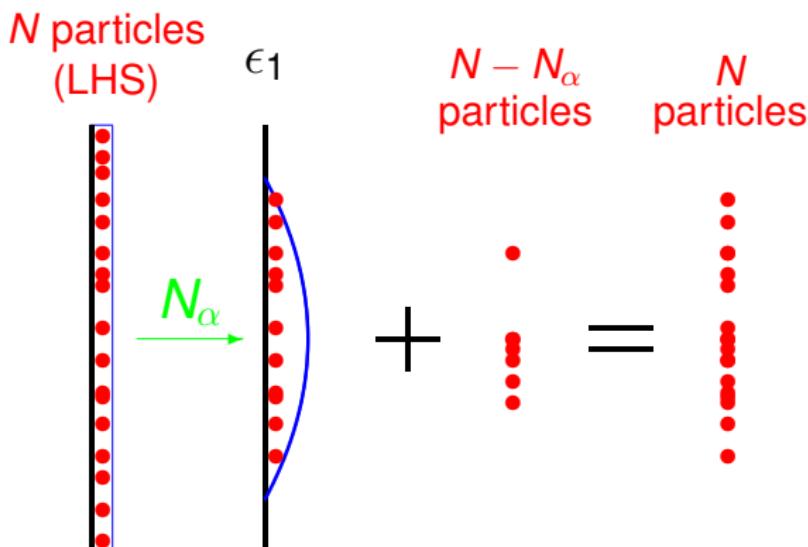
Adaptive PMC ABC Algorithm



$$p_{acc} = \frac{\sum_{k=N_\alpha+1}^N \mathbb{1}_{\rho(x,y) \leq \epsilon_1}}{N - N_\alpha}$$

(Lenormand et al.)

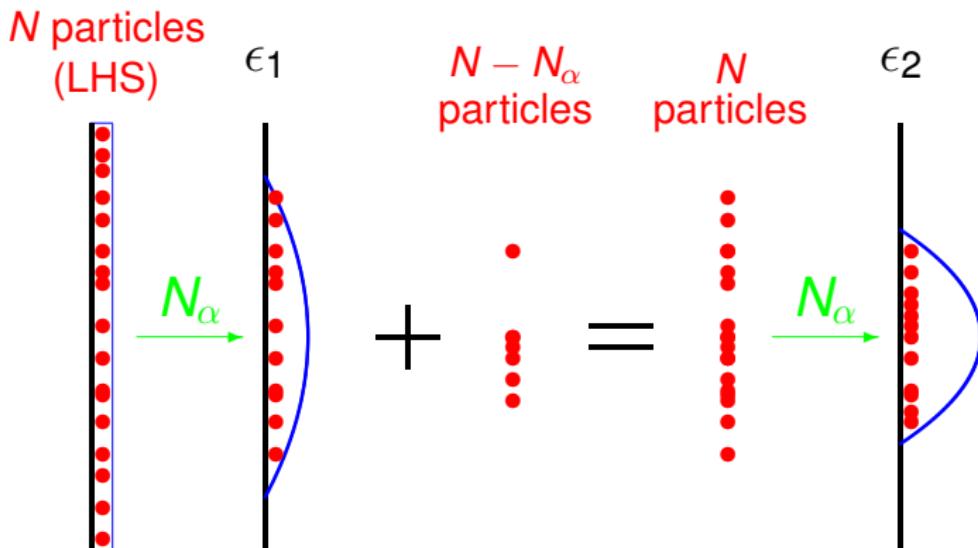
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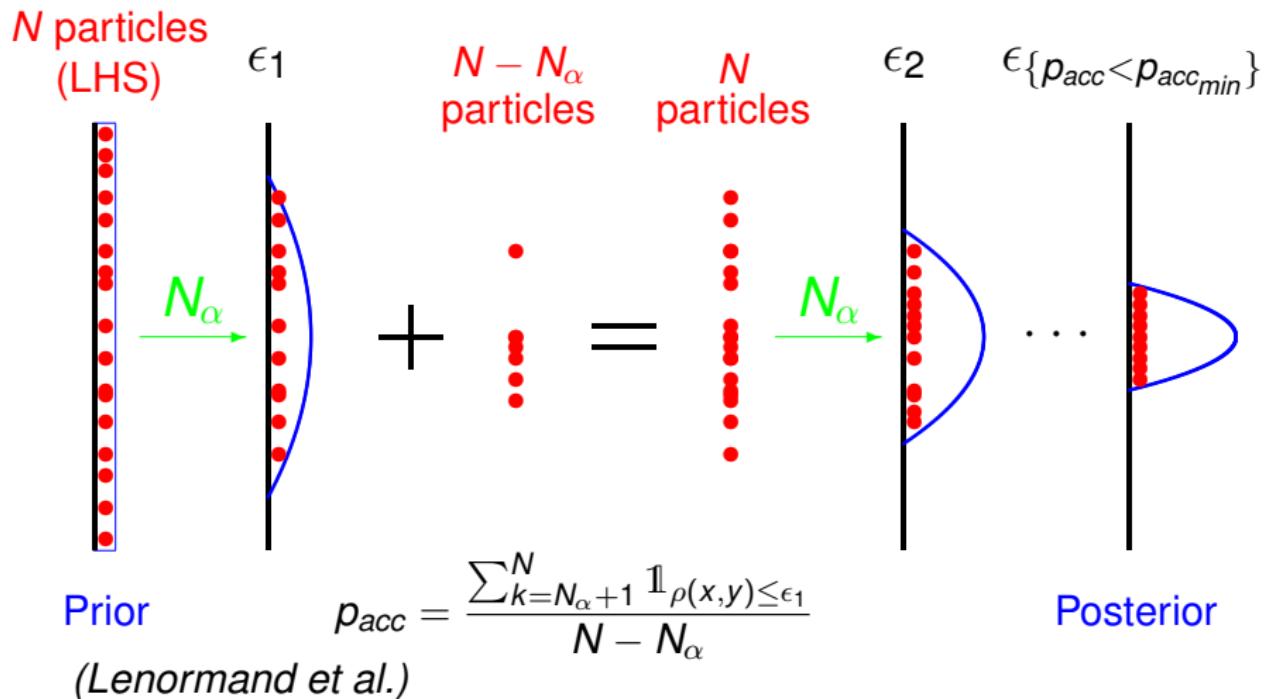
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Adaptive PMC ABC

Pros and Cons

Pros

- Control the number of simulations at each iteration ($N - N_\alpha$ simulations).

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Adaptive PMC ABC

Pros and Cons

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Cons

- Complexity $O(N_\alpha^2)$ for the computation of the weights

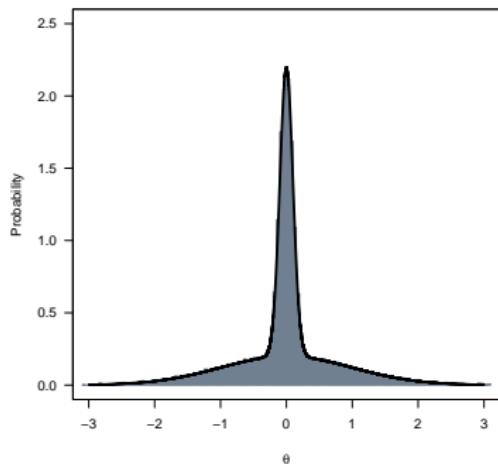
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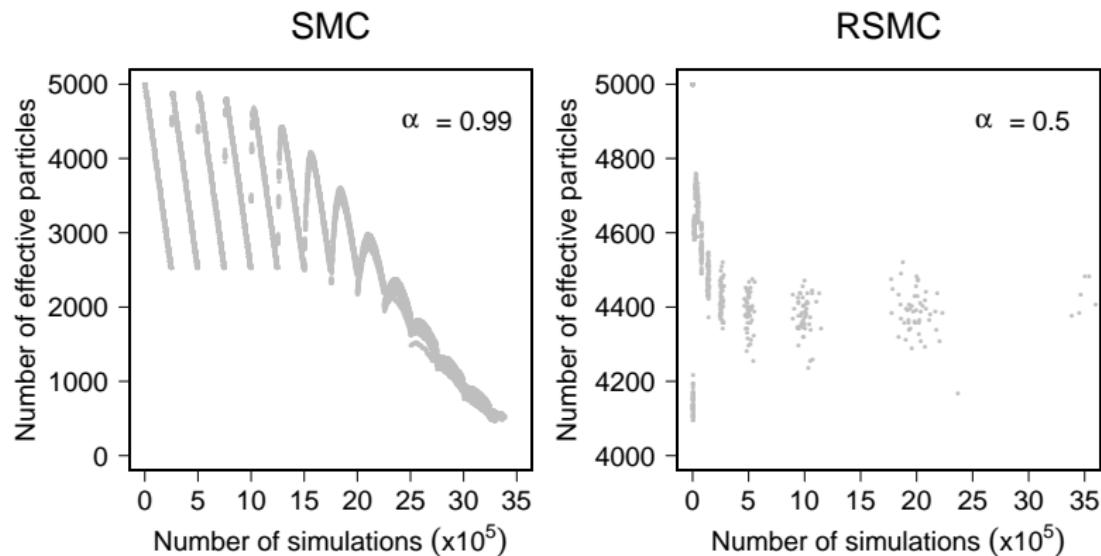
Toy Example : Presentation

$$f(x|\theta) \sim \frac{1}{2}\phi\left(\theta, \frac{1}{100}\right) + \frac{1}{2}\phi(\theta, 1) \text{ and } \theta \sim \mathcal{U}_{[-10, 10]}$$



Comparison of the algorithms

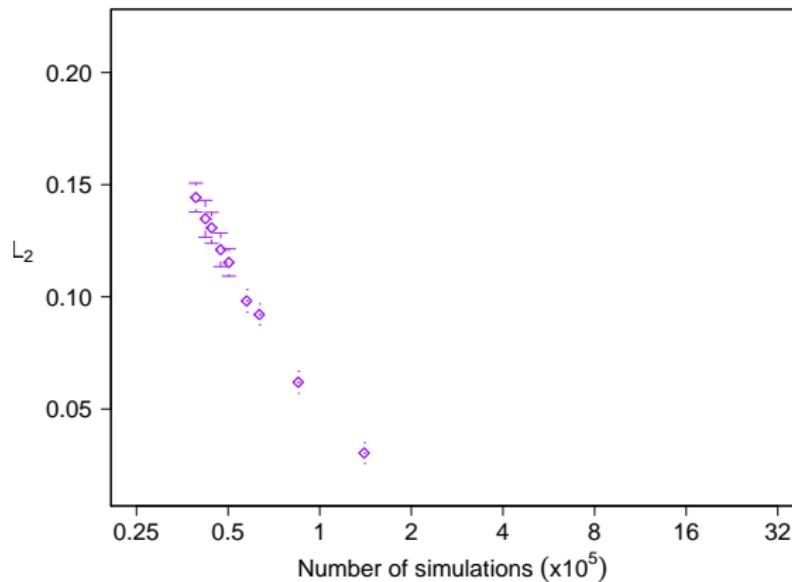
Toy Example : Particles Duplication



Comparison of the algorithms

Toy Example : Comparison

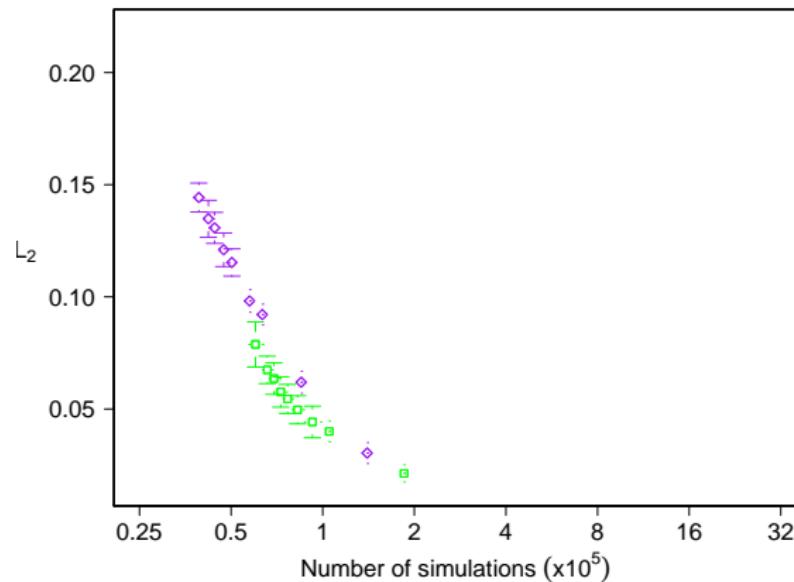
$N_\alpha = 5000$; α from 0.9 to 0.1 corresponding to N = 5555 to 50000



Comparison of the algorithms

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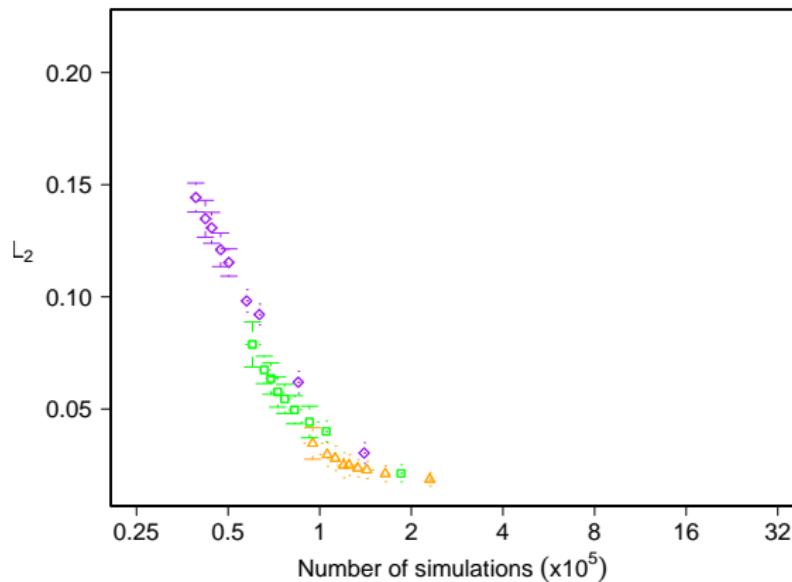
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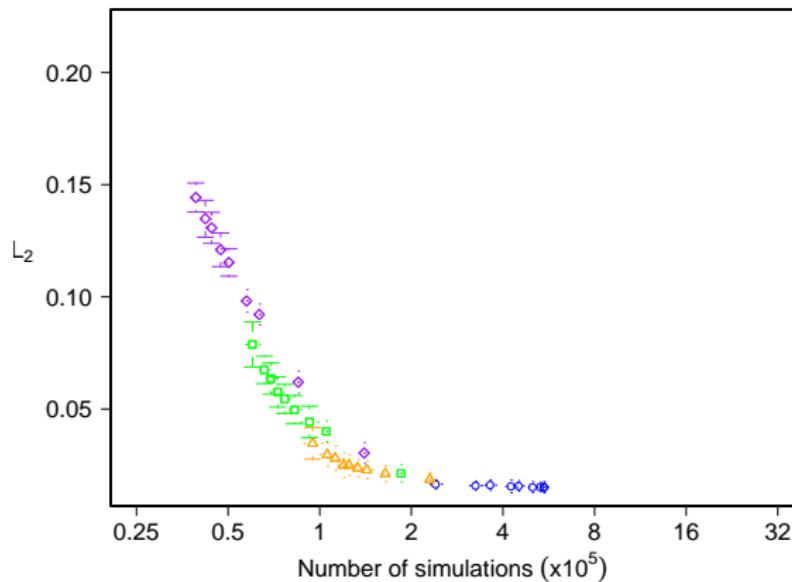
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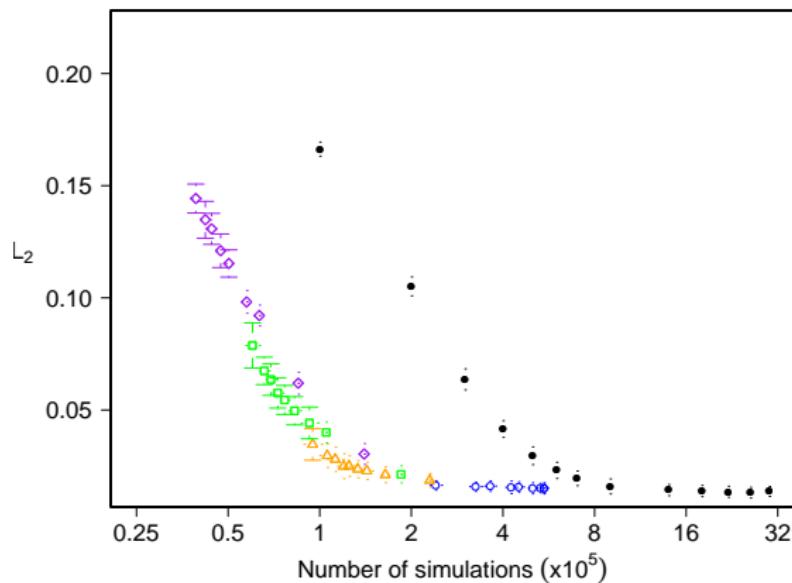
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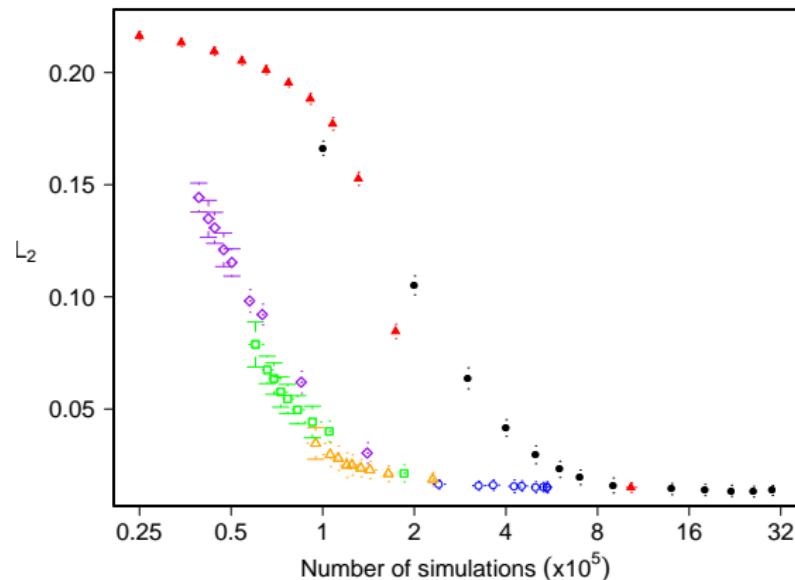
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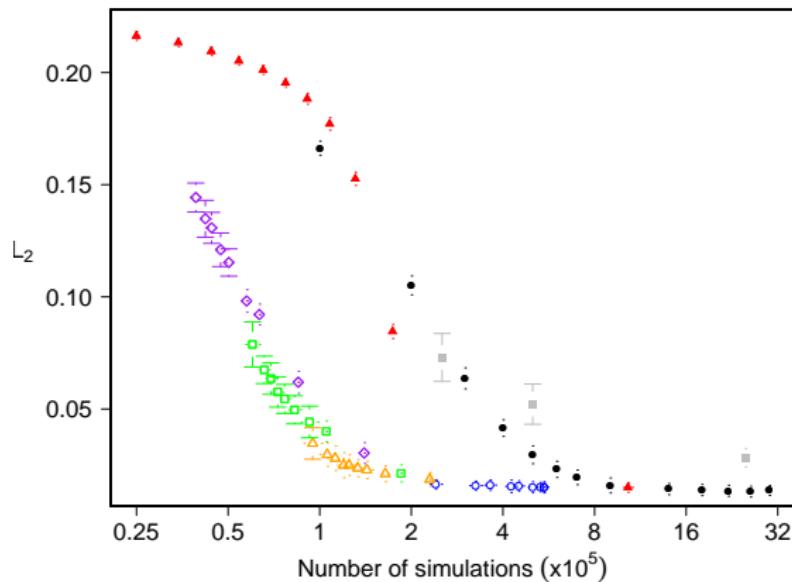
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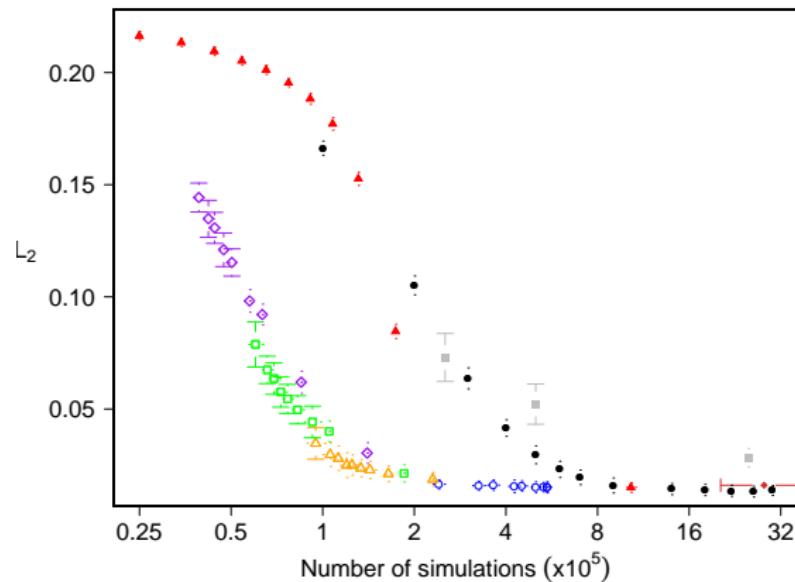
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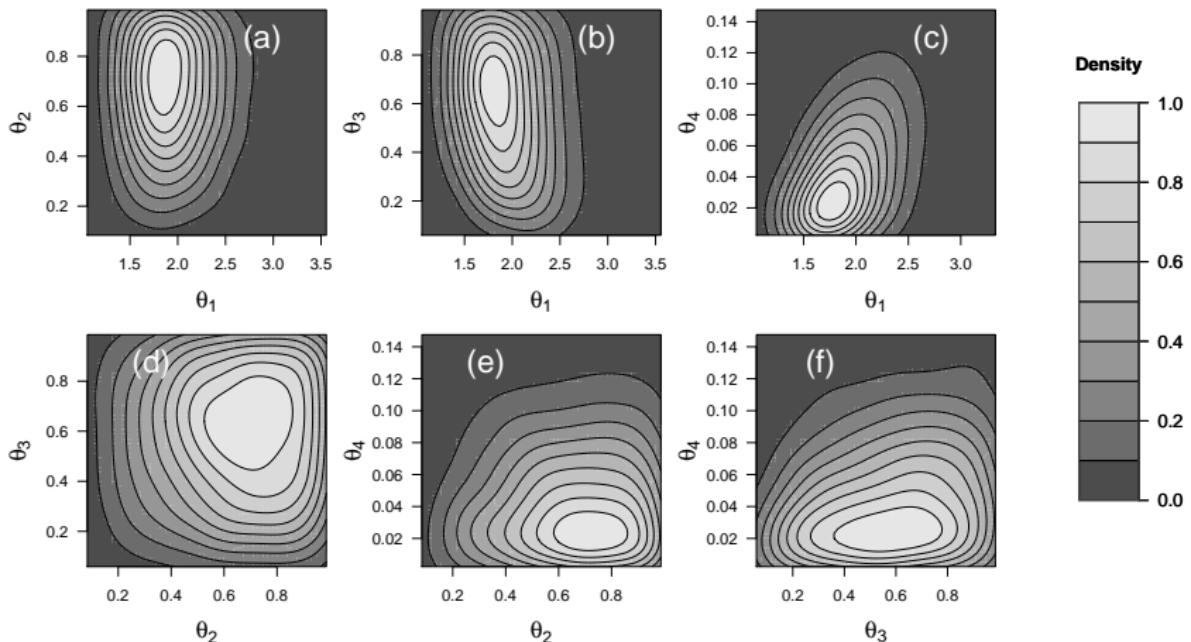
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The *SimVillages* Model

Parameter and Summary Statistics

- 4 parameters
- 8 statistics
- $\|(\rho_m(S_m, S'_m))_{1 \leq m \leq M}\|_\infty = \sup_{1 \leq m \leq M} |\rho_m(S_m, S'_m)|$

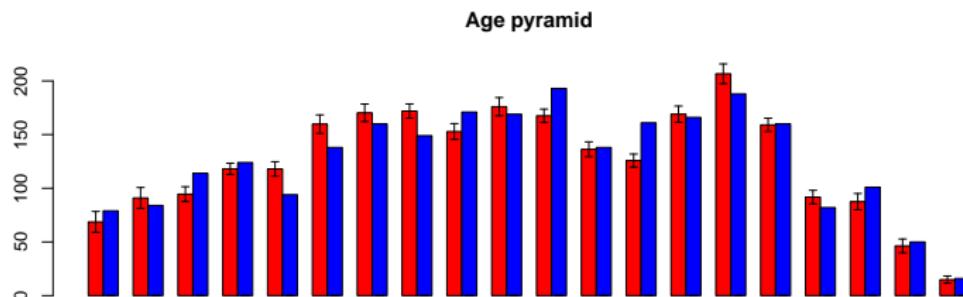
The *SimVillages* Model Posterior Density



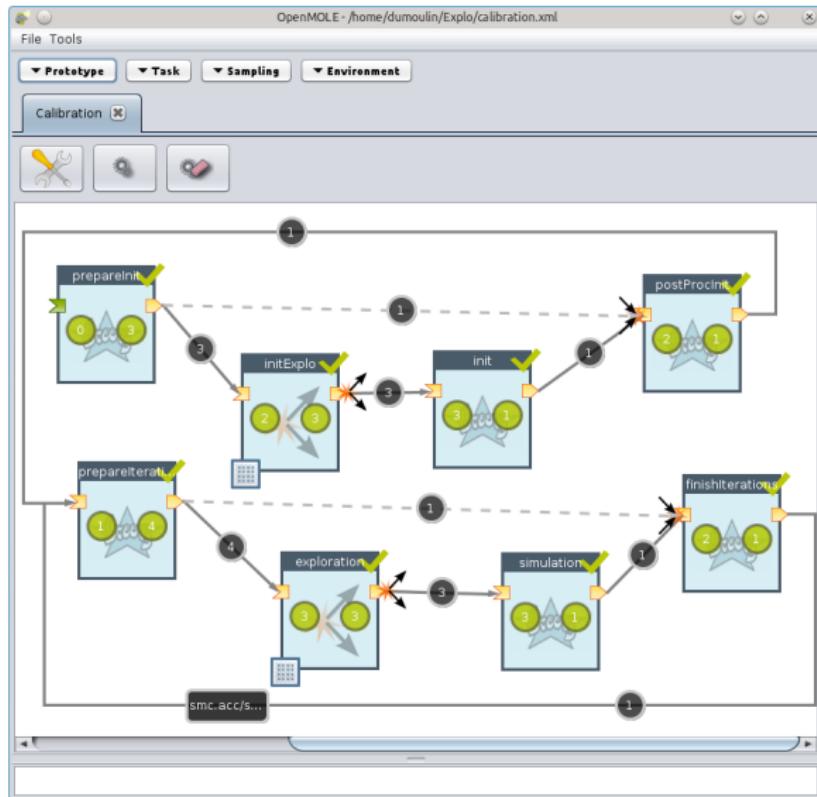
The *SimVillages* Model

Concrete Results

- 1.4 second by simulation
- 400 000 simulations
- 6 days



The *SimVillages* Model Open Mole



Plan

- 1 Motivation
- 2 Approximate Bayesian Computation (ABC)
- 3 Approximate Bayesian Computation Sequential Monte Carlo
- 4 Adaptive Population Monte Carlo Approximate Bayesian Computation
- 5 Comparison of the algorithms
- 6 The *SimVillages* Model
- 7 Conclusion

Conclusion

- We have developed a new algorithm to reduce the number of simulations in SMC ABC.
- Comparison with three methods.
- Calibration of a complex social model.

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