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Lake eutrophication and environmental change: A viability framework for resilience, vulnerability and adaptive capacity

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Context & Problem:

Environmental change in a lake can be the result of a variety of phenomena and can happen under the form of extreme events and long-term changes, interacting with each other and natural variability. Yet, these changes can have lasting ecological and economic effects.

We propose a framework that describes these changes using the mathematics of viability theory and descriptive concepts such as resilience, vulnerability and adaptive capacity.

A viability framework for the lake eutrophication case

- Model: (all quantities dimensionless)

\[
\begin{align*}
&\dot{P}(t) = b_P(t) + \frac{1}{\alpha} (P(t) - L(t)) + r_P(t) + P(t(0)) - P(t) \\
&L(t) = \frac{1}{\beta} (L(t)) + w(t) \quad \text{where } w(t) \sim \mathcal{N}(0, \sigma) \\
&\dot{L}(t) = \frac{1}{\gamma} (L(t)) + u(t)
\end{align*}
\]

where:

a) \((L', P')\) is the state of the system; \(P\) is the phosphorus concentration, \(L\) the mean input, and \(L\) is the lost input
b) \(u\) is the control and represents the adaptive policies. Here we assume \(|u| \leq 0.5\).  

- The goal of viability is to keep the system within constraints that represent its desirable properties. Here we have:

1) an ecological constraint: the lake is oligotrophic for \(P \leq P_{\text{min}} = 1\);  
2) an economic constraint: farming is profitable for \(L \geq 2.7\)  

- Stochastic viability kernel: the set of states such that there is a given minimal probability of respecting the constraints for \(T\) time steps.  

\[\mathcal{V}_T(\beta, t) = \{x(t) : 3 \leq u < x, \mathbb{P}(\tau \in [0,T], x(t) \in \mathbb{E}) \geq \beta\}\]

Resilience and vulnerability to extreme events

- Extreme event: An extreme rainfall event can carry an important quantity of phosphorus from the soil into the lake, causing an abrupt increase in \(P\).

- Resilience: The concept refers to the ability of a system to retain or recover its properties and functions after a perturbation. We consider that the properties are recovered when they are safe from more ordinary events (i.e., inside the stochastic viability kernel, here \(\mathcal{V}(0.99,100)\)).

- Dynamic programming allows for the computation of the probability of entering \(\mathcal{V}(0.99,100)\) within a time horizon \(T\): this is the probability of resilience.

- Vulnerability: (PCC definition) The concept refers to the degrees to which a system is susceptible to and unable to cope with, adverse effects of climate change, including climate variability and extremes.

Vulnerability is a statistic on a cost distribution found by taking into account all possible trajectories for an optimal strategy:

1. Recovery time (a decreasing function of resilience)

\[\tau_{\text{fin}}(t) = \frac{t}{\alpha} \quad \text{and} \quad x(t) = \left\{ \begin{array}{ll}
0 & \text{if } t < t_{\text{fin}} \\
1 & \text{otherwise}
\end{array} \right.\]

2. Recovery cost, the distance from the desirable properties:

\[\text{Cost} = \sum_{t=1}^{T} \text{Cost}(t)\]

Here, \(\alpha = 0.2\).

Extension to environmental changes (change in model parameters)

- Example: reduction of the outflow by 25%

Assuming that the phosphorus sink \(-b_P\) is solely due to outflow, the lake becomes irreversible: the oligotrophic property \((P < 1)\) cannot be recovered after it is lost. The value of \(b\) decreases to 5/8.

Then vulnerability to this change is the difference in vulnerability before and after the change. Here for vulnerability as the time spent outside of \(K\), this is also a resilience loss.

Adaptive capacity

Adaptive capacity can be defined as the vulnerability reduction due to the introduction of new controls

Example: new technological developments or management practices lower the minimum economically acceptable phosphorus input to \(L^*<0.35\).

Before change, \(b=5/8\):

After change, \(b=5/8\):