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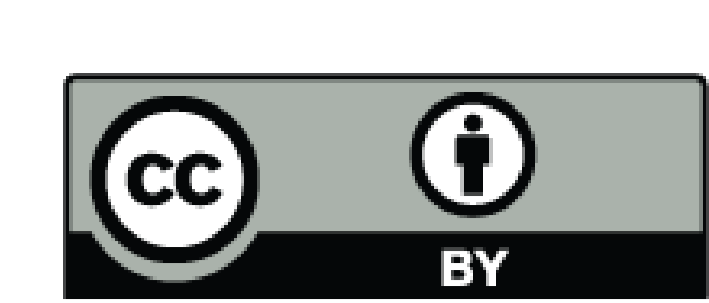
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# Lake eutrophication and environmental change: A viability framework for resilience, vulnerability and adaptive capacity



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## Context & Problem:

Environmental change in a lake can be the result of a variety of phenomena and can happen under the form of extreme events and long-term changes, interacting with each other and natural variability. Yet, these changes can have lasting ecological and economic effects.

We propose a framework that describes these changes using the mathematics of viability theory and descriptive concepts such as resilience, vulnerability and adaptation.

## A viability framework for the lake eutrophication case

**Model:** (all quantities dimensionless)

$$\begin{cases} P(t+1) = P(t) + \left[ -b \cdot P(t) + L(t) + r \frac{P(t)^8}{m^8 + P(t)^8} \right] \cdot \Delta t \\ L(t) = L^* + w(t) \text{ where } w(t) \sim \mathcal{N}(0, \sigma) \\ L^*(t+1) = L^*(t) + u \cdot \Delta t \end{cases}$$

Where:

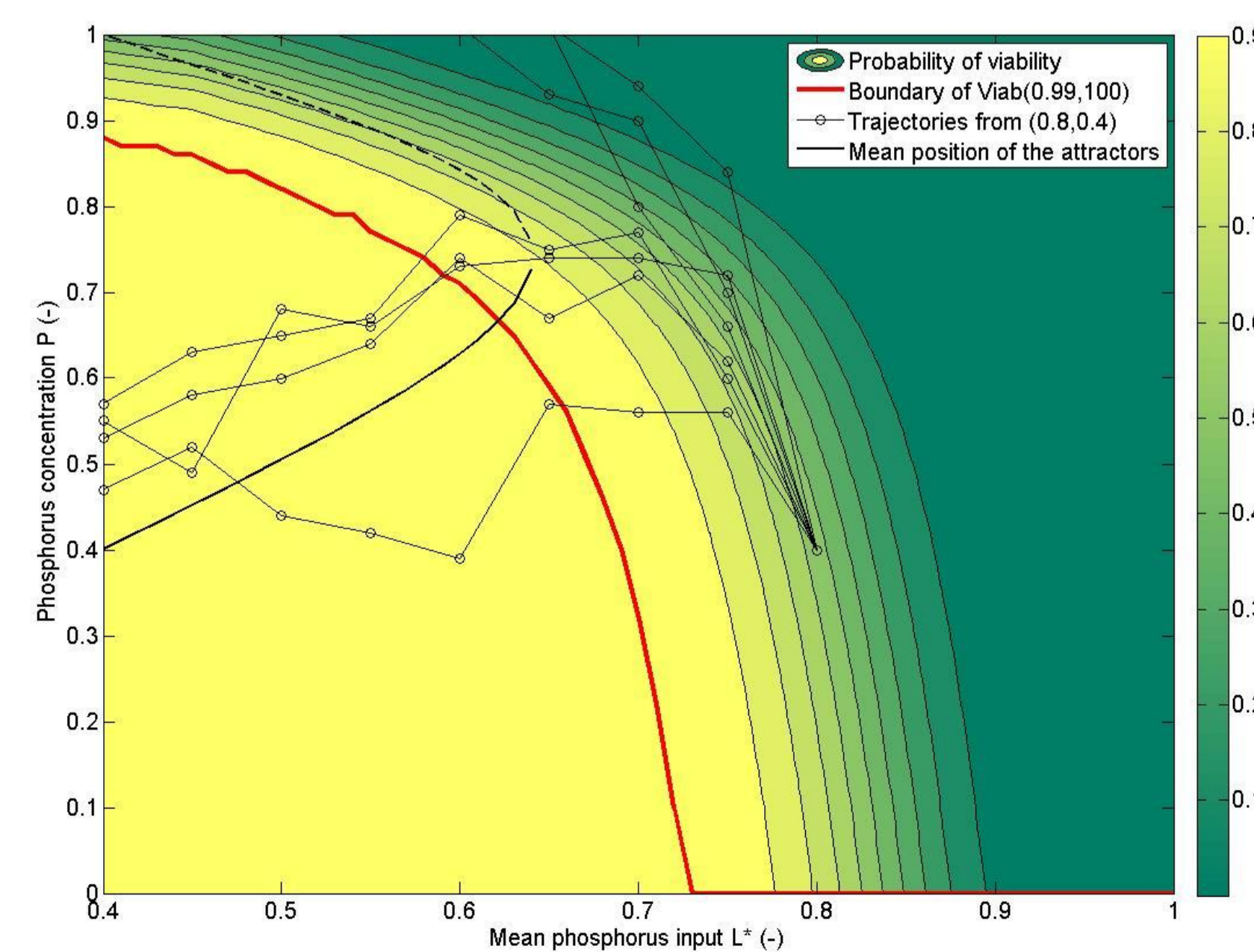
- a)  $(L^*, P)$  is the state of the system;  $P$  is the phosphorus concentration,  $L^*$  is the mean input, and  $L$  is the total input
- b)  $u$  is the control and represents the adaptive policies. Here we assume  $|u| \leq 0.05$ .
- c) Parameter values are  $b = 5/6$ ,  $r = m = 1$ ,

**The goal of viability is to keep the system within constraints that represent its desirable properties.** Here we have:

- 1) an ecological constraint: the lake is oligotrophic for  $P \leq P_{max} = 1$ ;
- 2) an economic constraint: farming is profitable for  $L^* \geq L^*_{min} = 0.4$ ;

**Stochastic viability kernel:** the set of states such that there is a given minimal probability  $\beta$  respecting the constraints for  $T$  time steps.

$$Viab(\beta, T) = \{x(0), \exists u(\cdot), P(\forall t \in [0, T], x(t) \in K) \geq \beta\}$$



Computations done through dynamic programming (also gives the optimal control strategies)

## Resilience and vulnerability to extreme events

**Extreme event:**

An extreme rainfall event can carry an important quantity of phosphorus from the soil into the lake, causing an abrupt increase in  $P$ .

**Resilience:**

The concept refers to the ability for a system to retain or recover its properties and functions after a perturbation.

We consider that the properties are recovered when they are safe from more ordinary events, i.e. inside the stochastic viability kernel, here  $Viab(0.99, 100)$ .

Dynamic programming allows for the computation of the probability of entering  $Viab(0.99, 100)$  within a time horizon  $T$ : this is the **probability of resilience**.

**Vulnerability:** (IPCC definition)

The concept refers to the degree to which a system is susceptible to, and unable to cope with, adverse effects of climate change, including climate variability and extremes.

Vulnerability is a statistic on a cost distribution found by taking into account all possible trajectories for an optimal strategy:

**I. Recovery time**

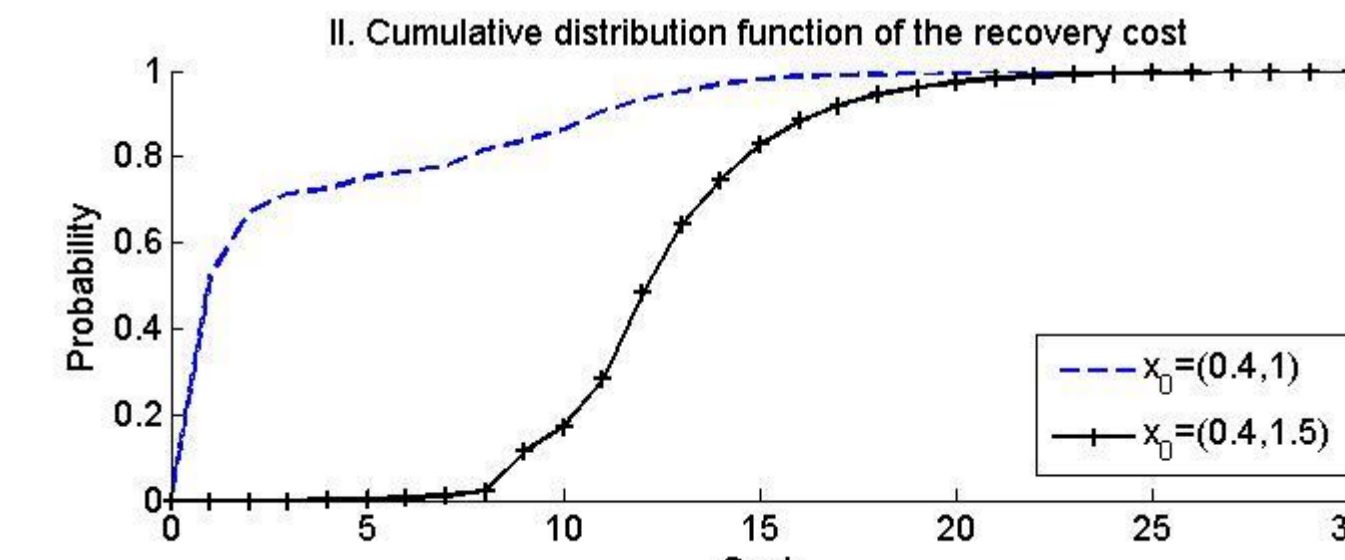
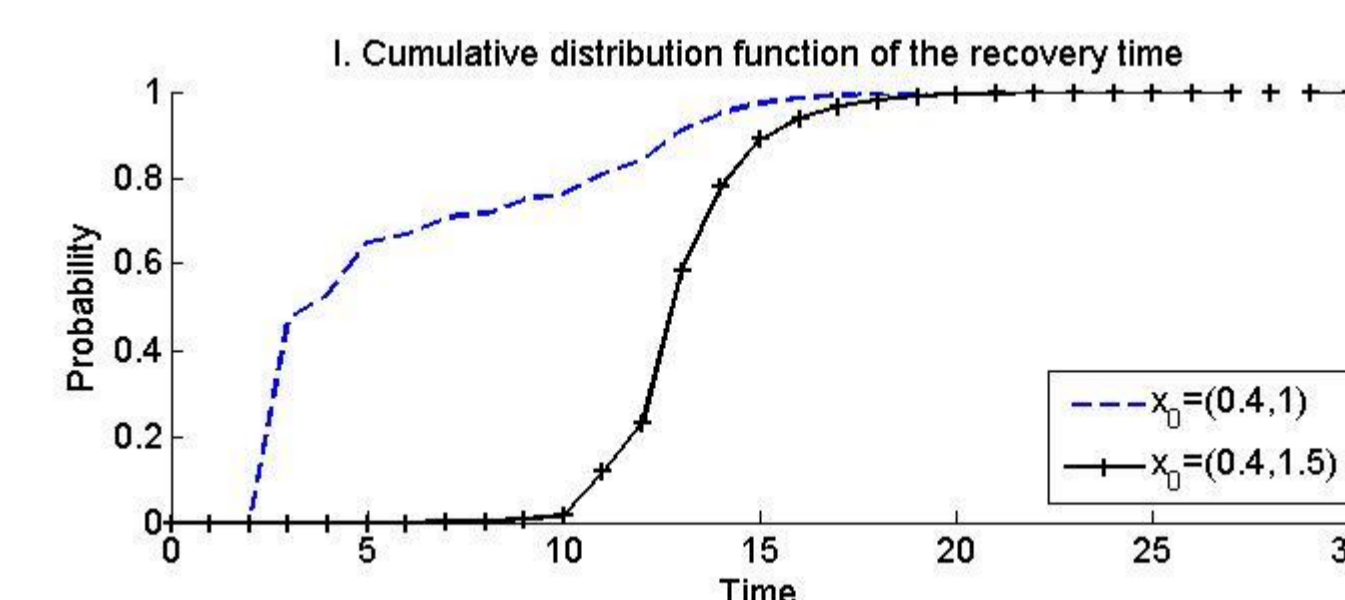
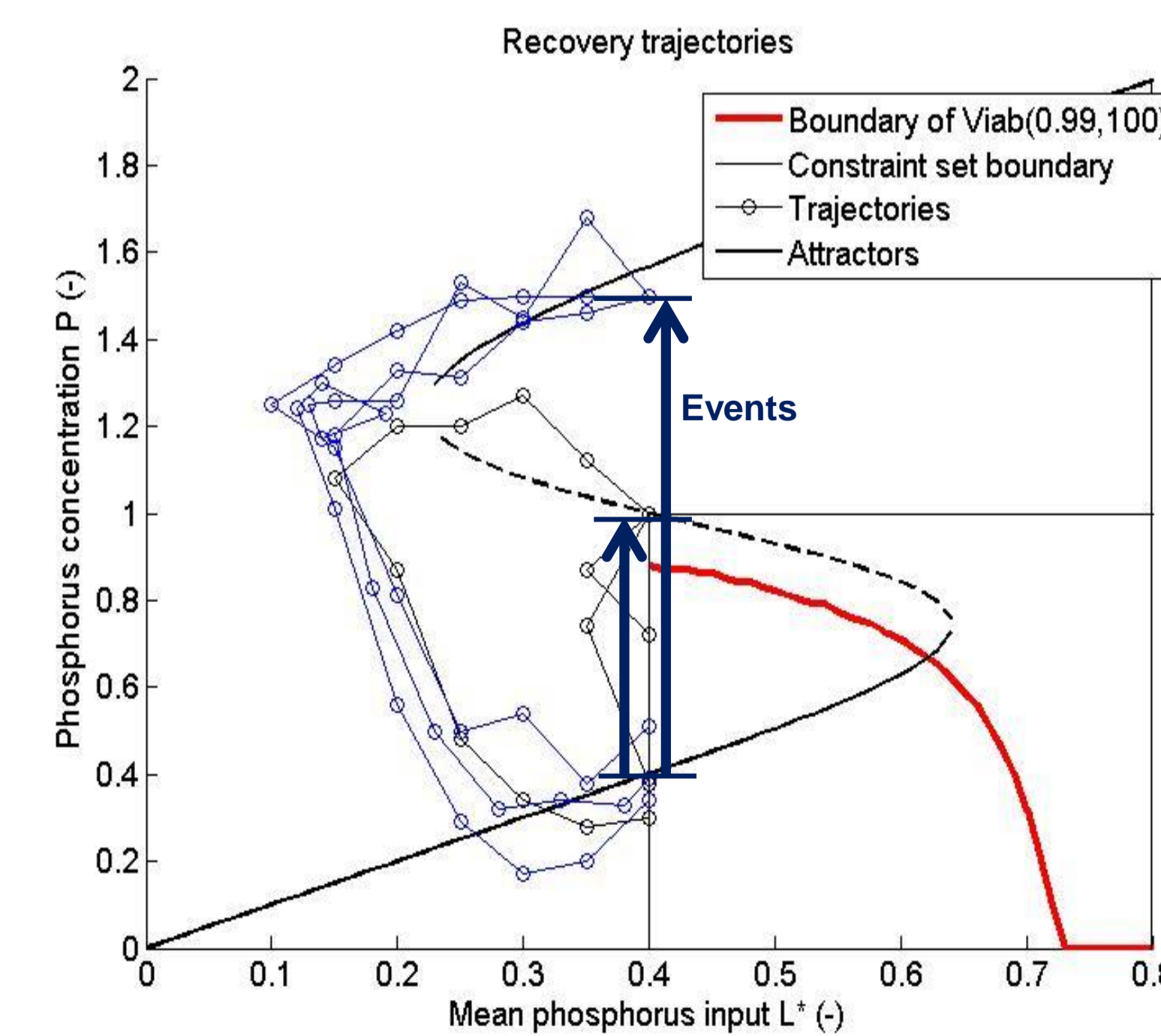
( $\approx$  a decreasing function of resilience)

$$\text{So } C(t, x) = \begin{cases} 0 & \text{if } x \in K \\ 1 & \text{otherwise} \end{cases} \text{ and } v = \sum_t C(t)$$

**II. Recovery cost, the distance from the desirable properties:**

1. Economic cost  $C_1(t)$ : distance to  $L=0.4$
2. Ecological cost  $C_2(t)$ : distance to  $P=1$

Here  $k = 0.2$ .

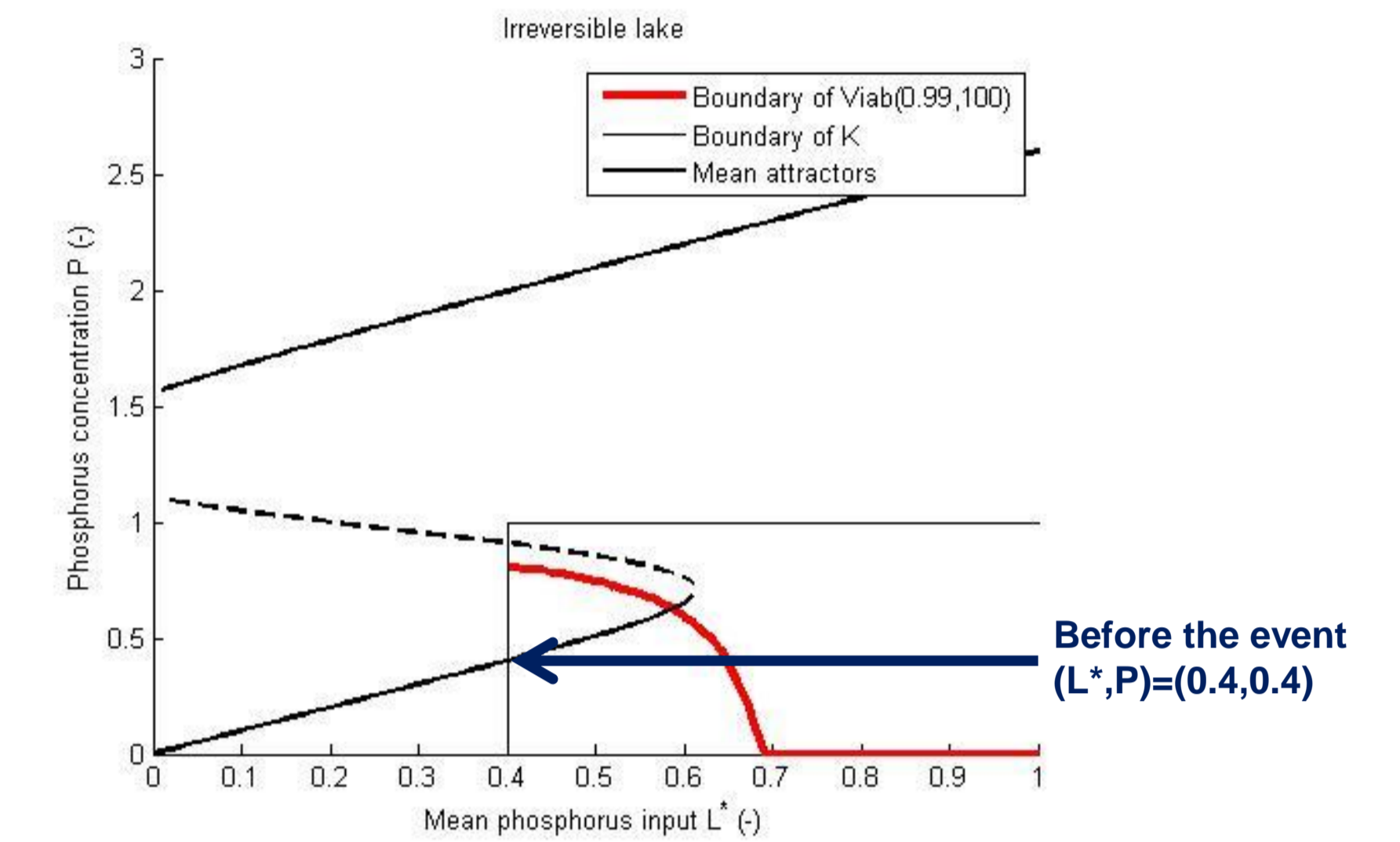


## Extension to environmental changes (change in model parameters)

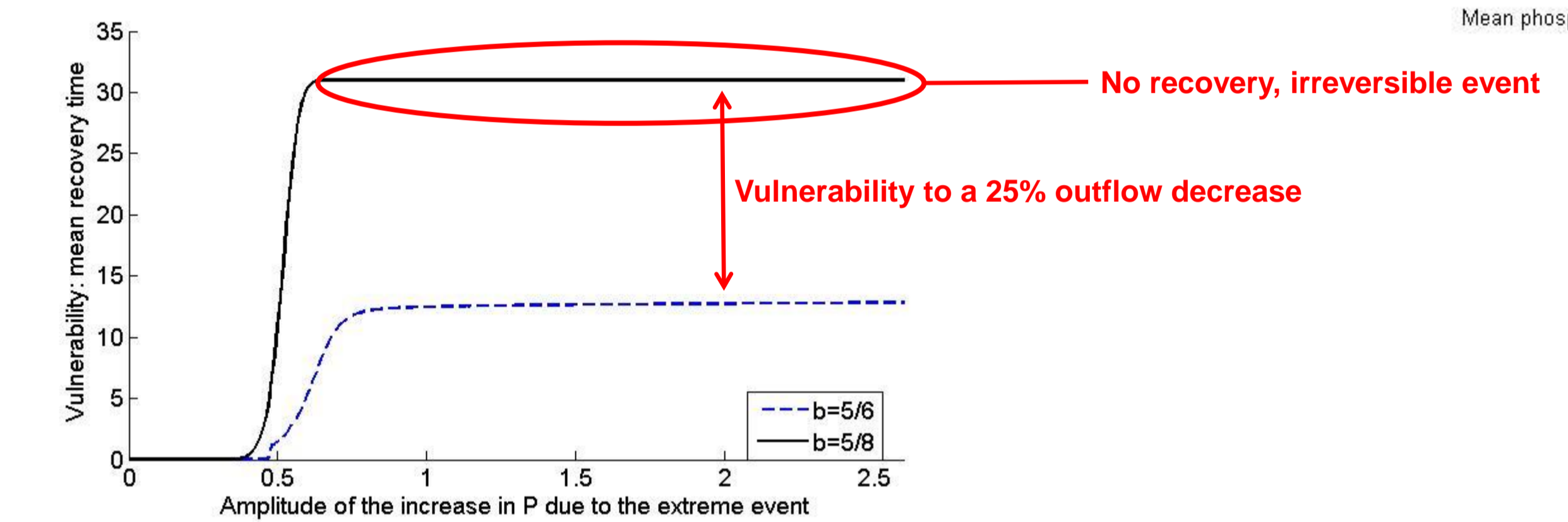
**Example: reduction of the outflow by 25%**

Assuming that the phosphorus sink term  $-b \cdot P$  is solely due to outflow, the lake becomes irreversible: the oligotrophic property ( $P < 1$ ) cannot be recovered after it is lost. The value of  $b$  decreases to  $5/8$ .

Then **vulnerability to this change** is the difference in vulnerability before and after the change. Here for vulnerability as the time spent outside of  $K$ , this is also a **resilience loss**.



Recovery time before and after change (horizon  $T=30$ )

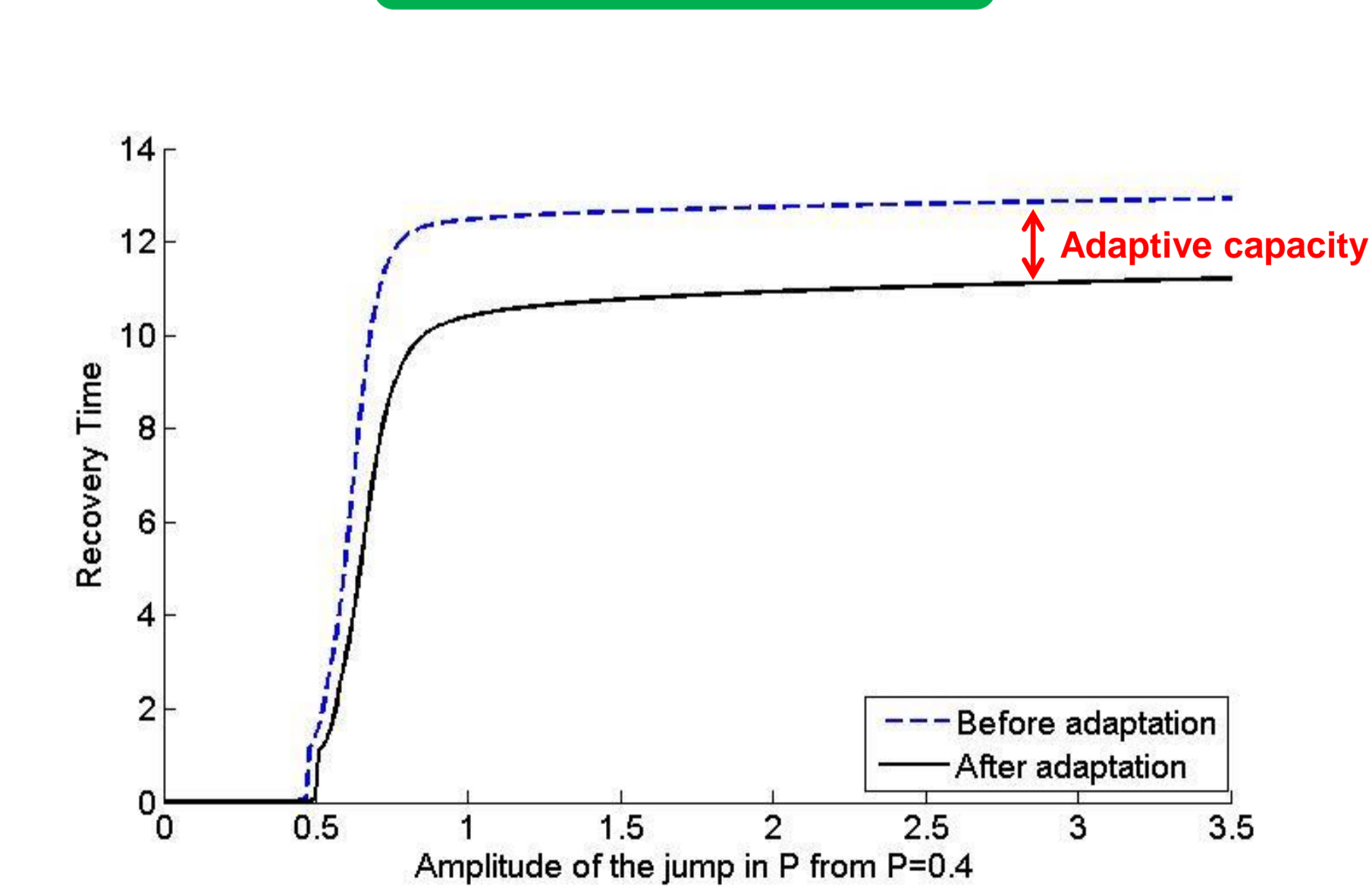


## Adaptive capacity

Adaptive capacity can be defined as the vulnerability reduction due to the introduction of new controls

**Example:** new technological developments or management practices lower the minimum economically acceptable phosphorus input to  $L^*=0.35$ .

Before change,  $b=5/6$



After change,  $b=5/8$

