Lake eutrophication and environmental change: A viability framework for resilience, vulnerability and adaptive capacity

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Context & Problem:

Environmental change in a lake can be the result of a variety of phenomena and can happen under the form of extreme events and long-term changes, interacting with each other and natural variability. Yet, these changes can have lasting ecological and economic effects.

We propose a framework that describes these changes using the mathematics of viability theory and descriptive concepts such as resilience, vulnerability and adaptation.

A viability framework for the lake eutrophication case

- **Model:** *(all quantities dimensionless)*

$$\begin{cases} P(t+1) = P(t) + \left[-b.P(t) + L(t) + r\dfrac{P(t)^8}{m^8 + P(t)^8}\right].\Delta t \\ L(t) = L^* + w(t) \textbf{ where } w(t) \sim \mathcal{N}(0,\sigma) \\ L^*(t+1) = L^*(t) + u.\Delta t \end{cases}$$

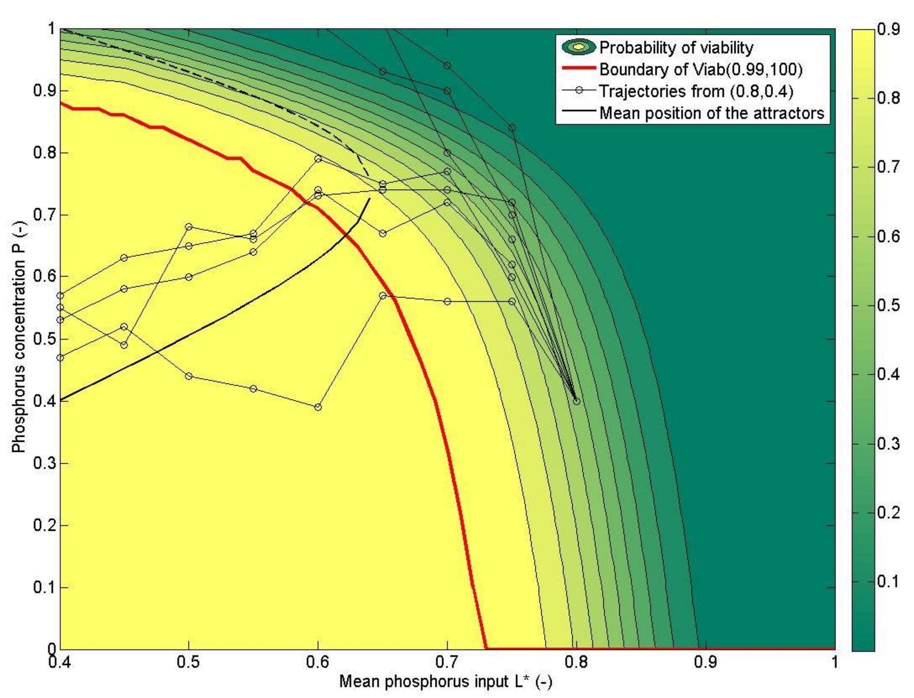
Where:

a) (L^*, P) is the state of the system; P is the phosphorus concentration, L^* is the mean input, and L is the total input
b) u is the control and represents the adaptive policies. Here we assume $|u| \leq 0.05$.
c) Parameter values are $b = 5/6$, $r = m = 1$,

- **The goal of viability is to keep the system within constraints that represent its desirable properties.** Here we have:
  1) an ecological constraint: the lake is oligotrophic for $P \leq P_{max} = 1$;
  2) an economic constraint: farming is profitable for $L^* \geq L^*_{min}$=0,4;

- **Stochastic viability kernel:** the set of states such that there is a given minimal probability β respecting the constraints for T time steps.

$$Viab(\beta, T) = \{x(0), \exists\, u(.), \boldsymbol{P}(\forall t \in [0,T], x(t) \in K) \geq \beta\}$$



Computations done through dynamic programming

(also gives the optimal control strategies)

Resilience and vulnerability to extreme events

- **Extreme event:**
  An extreme rainfall event can carry an important quantity of phosphorus from the soil into the lake, **causing an abrupt increase in P.**

- **Resilience:**
  The concept refers to the ability for a system to retain or recover its properties and functions after a perturbation.

We consider that the properties are recovered when they are safe from more ordinary events, i.e. inside the stochastic viability kernel, here Viab(0.99,100).

Dynamic programming allows for the computation of the probability of entering Viab(0.99,100) within a time horizon T: this is the **probability of resilience.**

- **Vulnerability:** (IPCC definition)
  The concept refers to the degree to which a system is susceptible to, and unable to cope with, adverse effects of climate change, including climate variability and extremes.

Vulnerability is a statistic on a cost distribution found by taking into account all possible trajectories for an optimal strategy:

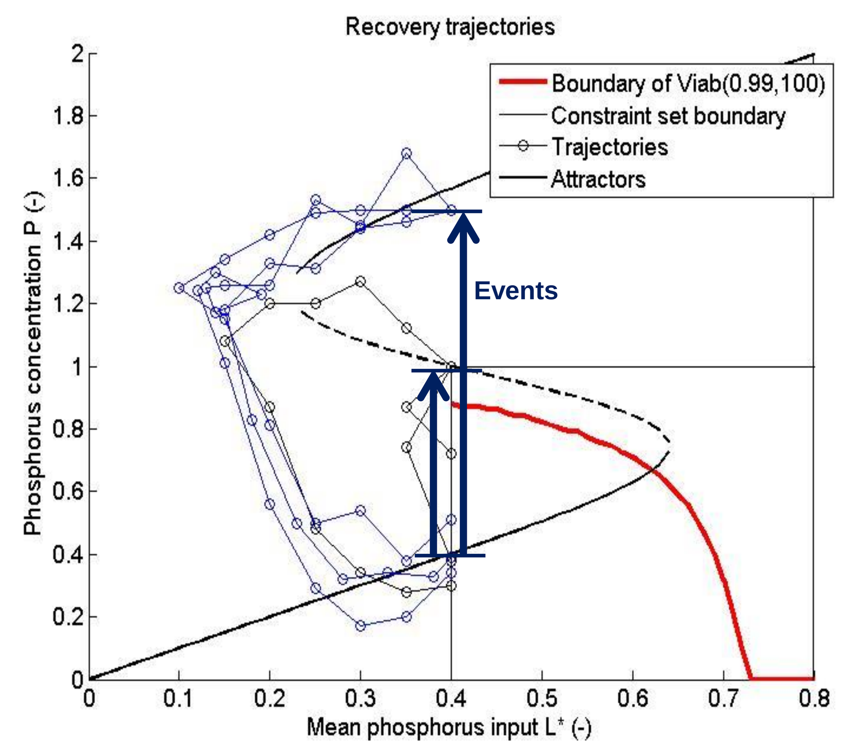
I. Recovery time
(≈ a decreasing function of resilience)

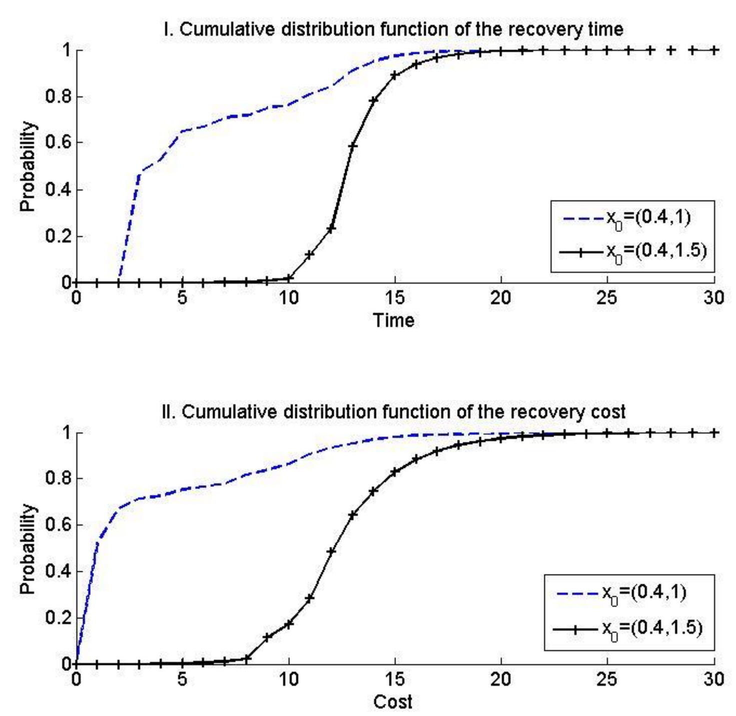
So $C(t,x) = \begin{cases} 0 \text{ if } x \in K \\ 1 \text{ otherwise} \end{cases}$ and $V = \sum_t C(t)$

II. Recovery cost, the distance from the desirable properties:
1. Economic cost $C_1(t)$: distance to L=0.4
2. Ecological cost $C_2(t)$: distance to P=1

$V = \sum_t C_1(t) + kC_2(t)$

Here $k = 0.2$.



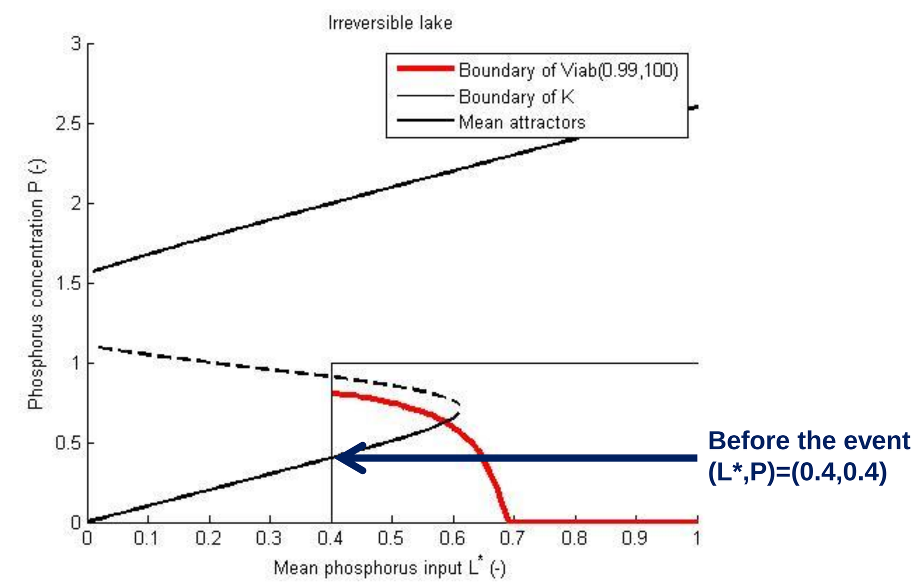


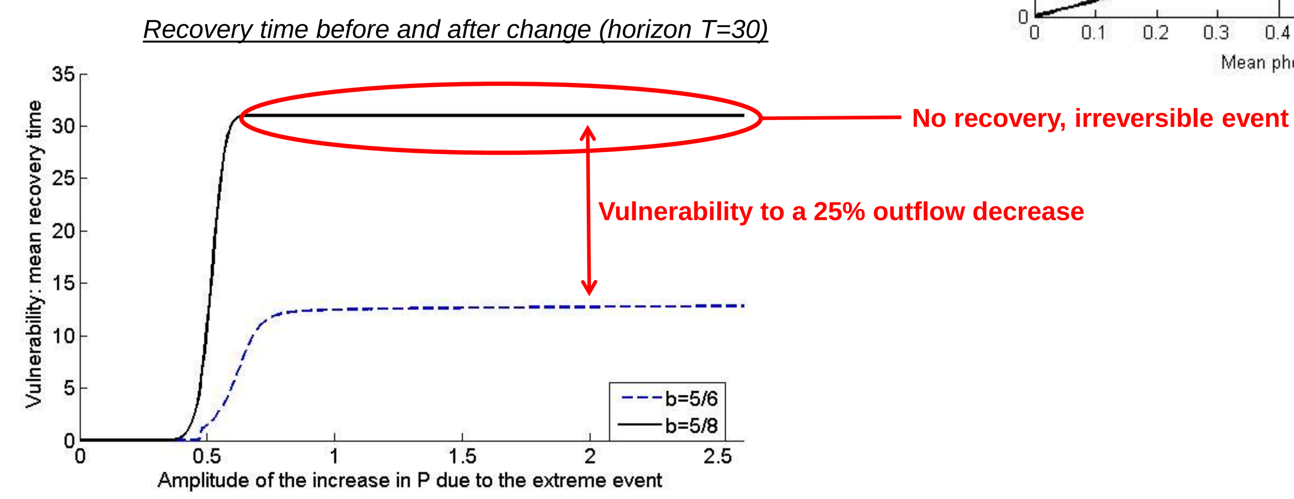
Extension to environmental changes (change in model parameters)

- **Example: reduction of the outflow by 25%**

Assuming that the phosphorus sink term −b.P is solely due to outflow, the lake becomes irreversible: the oligotrophic property (P<1) cannot be recovered after it is lost. The value of b decreases to 5/8.

Then **vulnerability to this change** is the difference in vulnerability before and after the change. Here for vulnerability as the time spent outside of K, this is also a **resilience loss.**



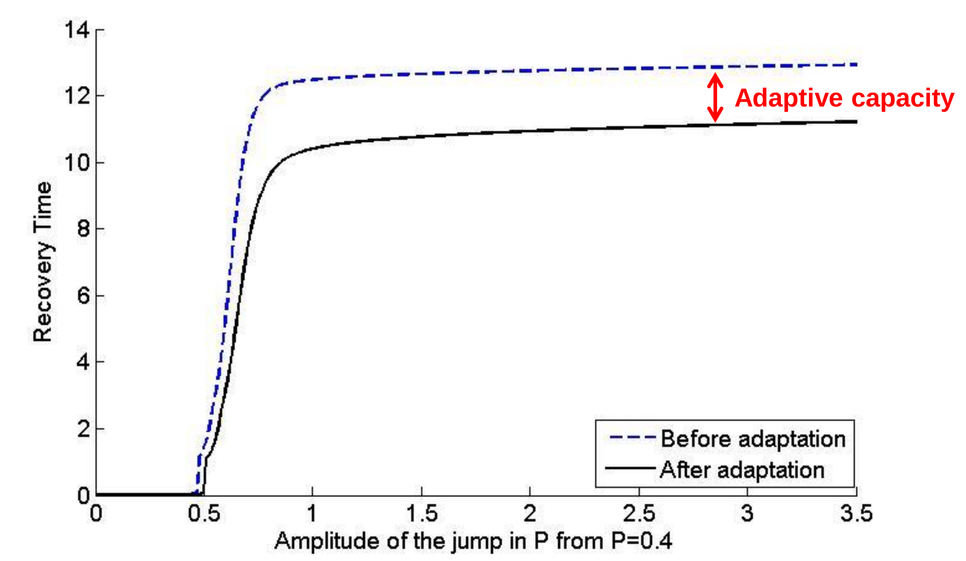


Adaptive capacity

Adaptive capacity can be defined as the vulnerability reduction due to the introduction of new controls

Example: new technological developments or management practices lower the minimum economically acceptable phosphorus input to L*=0.35.

Before change, b=5/6



After change, b=5/8