Assessment of Sub-Grid Scale Tensors in the context of Very Large Eddy Simulations

P. Chandramouli, Dominique Heitz, S. Laizet, E. Mémin

Assessment of Sub-Grid Scale Tensors in the context of Coarse Large Eddy Simulations

Pranav Chandramouli[1], Dominique Heitz[1,2], Sylvain Laizet[3], Etienne Mémin[1]

[1] INRIA, Fluminance group, Campus universitaire de Beaulieu, F-35042 Rennes Cedex, France.
[2] Irstea, UR OPAALE, F-35044 Rennes Cedex, France
[3] Department of Aeronautics, Imperial College London, London, United Kingdom
pranav.chandramouli@inria.fr

Abstract

Large Eddy Simulations (LES) are effectively used as reduced cost alternative to Direct Numerical Simulations (DNS). However, LES are still computationally expensive for complex flows. This brings into the forefront the concept of Coarse Large Eddy Simulations (cLES) involving coarser meshes and hence cheaper computations. An associated limitation with cLES is the accuracy and stability of the sub-grid scale (SGS) model to be used. This is the focus of this poster wherein several SGS models have been compared for the simple case of channel flow at coarse resolution. The development of LES SGS models has been an area of scientific research for many decades starting with Smagorinsky [6]. Recent developments in this field have produced many modern SGS models which have been shown to work better than classical models [3, 2]. In this study, the classical models namely, classic Smagorinsky as well as its dynamic version along with the Wall Adaptive Local-Eddy Viscosity (WALE) model, have been compared with the newly developed uncertainty based models of [3] and the implicit LES version of [2] in the context of cLES. The accuracy of the statistics have been analysed by comparison with the channel flow data of [4].

Introduction

Channel flow has been used as a research tool for numerous decades ever since a complete data set was provided by [4]. The simplicity of the flow and ease of simulation as well as the presence of turbulent characteristics especially close to the wall of the channel makes it an ideal flow for performing model studies. In this work, we have focused on this aspect and performed a LES-SGS model comparison at friction velocity based Reynolds number (Re_τ) of 395. The models under scrutiny include the established Smagorinsky models (dynamic and classic), WALE model, the newly developed Stochastic models by [3] and the implicit model developed by [2].

The interesting aspect of this study is not the range or type of models studied but on the simulation parameters of the flow. LES has led to a considerable reduction in computational power while maintaining statistical accuracy. Recent developments in the field of Data Assimilation (DA) (combining Experimental Fluid Dynamics (EFD) with Computational Fluid Dynamics (CFD)) have given fruitful results under low Reynolds number (Re) [1, 5]. Expansion of these studies into the realistic Re range could provide new avenues of interesting research. In order for this to be feasible, once again a reduction in computational power is required. A LES implementation in DA is still computationally unrealisable for high Reynolds number flows due to multiple orders of increase in computational power requirement. This necessitates a look into coarser resolutions which have associated problems of stability and stark decrease in statistical accuracy. Here we have focused on performing cLES of channel flow and comparing the performance of the SGS models in this framework.

Model Frameworks

Classical LES Models

The filtered Navier-Stokes (NS) equation:

$$\frac{\partial(\bar u_i)}{\partial t} + \frac{\partial(\bar u_i \bar u_j)}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \bar p}{\partial x_i} + 2\nu\frac{\partial}{\partial x_j}\bar S_{ij} - \frac{\partial \tau^r_{ij}}{\partial x_j}, \tag{1}$$

where the SGS contribution is denoted by $\tau^r_{ij} = -2\nu_r \bar S_{ij}$ and the eddy viscosity (ν_r) is modelled by:

Classic Smagorinsky (Smag):

$$\nu_r = (C_s * \Delta)^2|\bar S|, \tag{2}$$

where C_s is the Smagorinsky coefficient, Δ is the filter width and $\bar S$ is the filtered rate of strain.

Dynamic Smagorinsky (DSmag):

$$\nu_r = (C_s*\Delta)^2|\bar S|, \quad L_{ij} = \widehat{\bar u_i \bar u_j} - \hat{\bar u}_i\hat{\bar u}_j, \quad M_{ij} = -2\Delta^2(\alpha'^2|\hat{\bar S}|\hat{\bar S}_{ij} - \widehat{|\bar S|\bar S_{ij}}), \quad C_s^2 = \frac{\langle L_{ij}M_{ij}\rangle}{\langle M_{ij}M_{ij}\rangle}, \tag{3}$$

where α' stands for the scale ratio between test ($\hat{}$) and LES ($\bar{}$) filters.

Wall Adaptive Local-Eddy Viscosity (WALE):

$$\nu_t = (C_w\Delta)^2\frac{(\varsigma^d_{ij}\varsigma^d_{ij})^{3/2}}{(\bar S_{ij}\bar S_{ij})^{5/2} + (\varsigma^d_{ij}\varsigma^d_{ij})^{5/4}}, \quad \varsigma^d_{ij}\varsigma^d_{ij} = \frac{1}{6}(S^2S^2 + \Omega^2\Omega^2) + \frac{2}{3}S^2\Omega^2 + 2IV_{S\Omega}, \tag{4}$$

where $S^2 = \bar S_{ij}\bar S_{ij}, \quad \Omega^2 = \bar\Omega_{ij}\bar\Omega_{ij}, \quad IV_{S\Omega} = \bar S_{ik}\bar S_{kj}\bar\Omega_{jl}\bar\Omega_{li}$.

Stochastic LES Models

The stochastic version of the NS equation [3]:

$$\left(\partial_t \boldsymbol{w} + \boldsymbol{w}\boldsymbol{\nabla}^T(\boldsymbol{w} - \frac{1}{2}\boldsymbol{\nabla}\cdot\boldsymbol{a}) - \frac{1}{2}\sum_{ij}\partial_{x_i}(a_{ij}\partial_{x_j}\boldsymbol{w})\right)\rho = \rho\boldsymbol{g} - \boldsymbol{\nabla}p + \mu\Delta\boldsymbol{w}, \tag{5}$$

where the SGS contributions (highlighted) are calculated from the variance tensor $\boldsymbol{a}$ which is modelled by:

Stochastic Smagorinsky (StSm):

$$\boldsymbol{a}(\boldsymbol{x}, t) = C||\boldsymbol{S}||\mathbb{I}_3, \tag{6}$$

where C is a constant, $||\boldsymbol{S}|| = \frac{1}{2}\left[\sum_i j(\partial_{x_i}w_j + \partial_{x_j}w_i)^2\right]^{\frac{1}{2}}$, and $\mathbb{I}_3$ is the 3×3 identity.

Stochastic Spatial Variance (StSp):

$$\boldsymbol{a}(\boldsymbol{x}, n\delta t) = \frac{1}{|\Gamma|-1}\sum_{x_i\in\eta(x)}(\boldsymbol{w}(x_i, n\delta t) - \bar w(x, n\delta t))(\boldsymbol{w}(x_i, n\delta t) - \bar w(x, n\delta t))^T C_{sp}, \quad C_{sp} = \left(\frac{L}{\eta}\right)^{\frac{5}{3}}\tau_L, \tag{7}$$

where $\bar w(x, n\delta t)$ is the empirical mean around the arbitrarily selected local spatial neighbourhood Γ.

Stochastic Temporal Variance (StTe):

$$\boldsymbol{a}(\boldsymbol{x}, t) = \frac{1}{|\Gamma|-1}\sum_{t_i\in\eta(t)}(\boldsymbol{w}(x, t_i) - \bar w(x, t_i))(\boldsymbol{w}(x, t_i) - \bar w(x, t_i))^T C_{st}, \quad C_{st} = \left(\frac{\tau_L}{\tau_\eta}\right)^{\frac{5}{3}}\tau_L, \tag{8}$$

where $\bar w(x, n\delta t)$ is the empirical mean computed over the arbitrarily defined local temporal neighbourhood Γ.

Implicit Large Eddy Simulation (ILES) [2]:

Dissipation added via the viscous term with a customized modified square wavenumber with no additional term in the NS equation.

$$k''\Delta x^2 = \frac{2a[1-cos(k\Delta x)] + \frac{b}{2}[1-cos(2k\Delta x)] + \frac{2c}{9}[1-cos(3k\Delta x)]}{1+2\alpha cos(k\Delta x)}, \quad \nu_s(k) = \nu\frac{k''(k)-k^2}{k^2} \tag{9}$$

Results

	$n_x\times n_y\times n_z$	$l_x\times l_y\times l_z$	Δx	Δy	Δz	Δt
cLES	48×81×48	6.28×2×3.14	0.13	0.005-0.12	0.065	0.002
Ref	256×257×256	6.28×2×3.14	0.024	0.0077	0.012	-

Table 1: Simulation Parameters

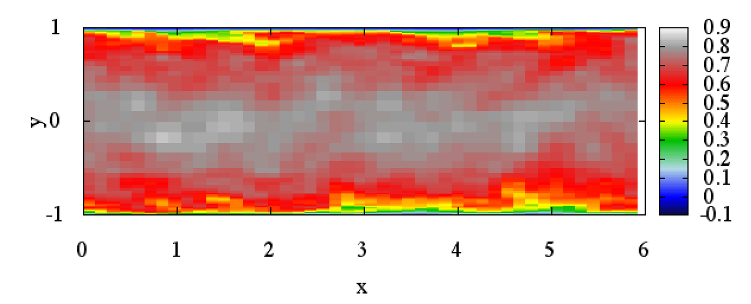

Figure 1: Instantaneous streamwise velocity contour at the z axis middle plane (StSp)

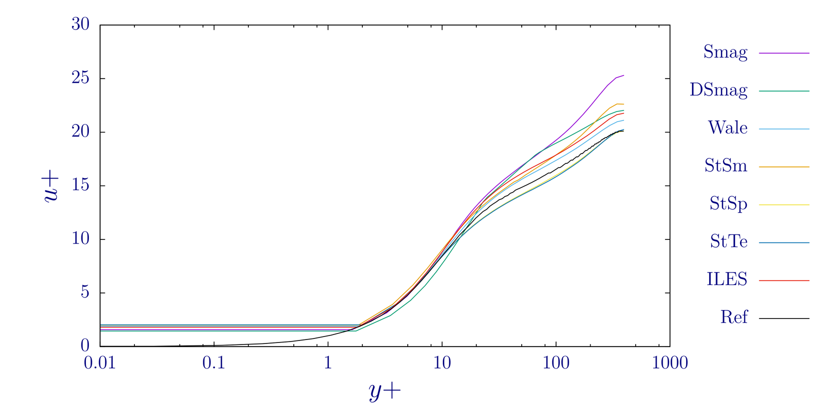

Figure 2: Mean velocity profile for turbulent channel flow at $Re_\tau = 395$

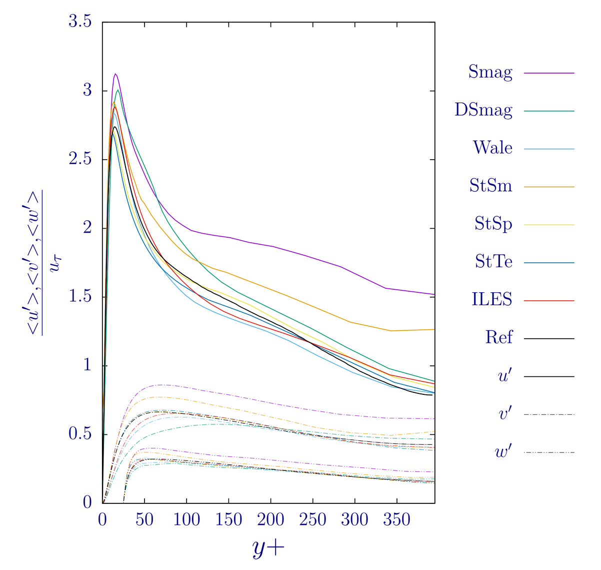

Figure 3: Velocity fluctuation profiles for turbulent channel flow at $Re_\tau = 395$

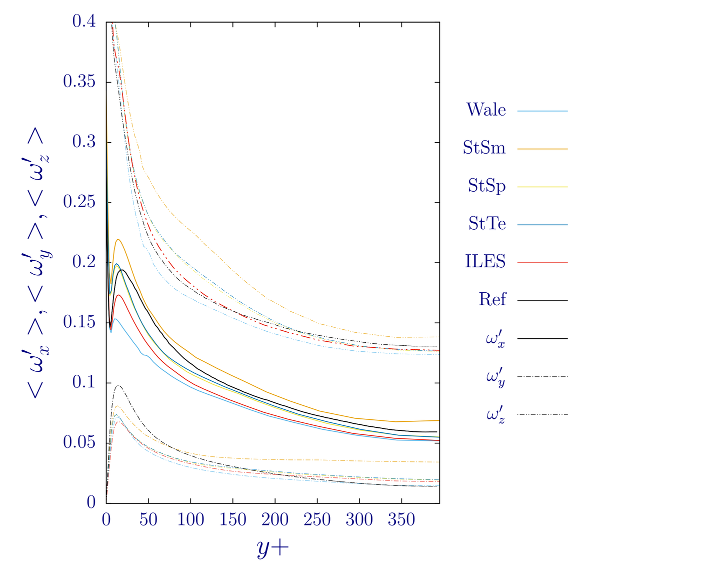

Figure 4: Vorticity fluctuation profiles for turbulent channel flow at $Re_\tau = 395$

Model	Advantages	Drawbacks
Smag	Cost effective, Easy to implement	Statistically Inaccurate, Wall inaccuracies
DSmag	-	Statistically Inaccurate, Wall inaccuracies, Computationally expensive, Stability Issues
WALE	Accurate velocity statistics	Inaccurate vorticity statistics
StSm	Cost effective, Reduced wall inaccuracies	Statistically inaccurate
StSp	Statistically accurate, Easy to implement	Computationally expensive
StTe	Statistically accurate, Easy to implement	Memory limitations
ILES	Statistically accurate, Cost effective	Arbitrary inputs

Table 2: Model advantages and drawbacks

Conclusions

- Classical models are not accurate or stable at coarser resolutions.
- Stochastic variance models perform admirably at coarser resolutions maintaining stability and accuracy.
- Implicit LES model also performs accurately however the associated arbitrariness is a cause for concern.
- Stochastic variance models and ILES are also capable of capturing the vorticity fluctuation statistics accurately along with velocity fluctuation statistics.

Forthcoming Research

Identification of a suitable SGS model for LES under coarse resolution can be utilised to perform Data Assimilation studies in 3D, a fact currently restricted by the increased computational cost of Data Assimilation. Using PIV and tomo-PIV data combined with coarse LES, we hope to perform Data Assimilation studies in 3D at nominal costs.

References

[1] A. Gronskis, D. Heitz, and E. Mémin. Inflow and initial conditions for direct numerical simulation based on adjoint data assimilation. *Journal of Computational Physics*, 242:480–497, June 2013.

[2] E. Lamballais, V. Fortuné, and S. Laizet. Straightforward high-order numerical dissipation via the viscous term for direct and large eddy simulation. *Journal of Computational Physics*, 230(9):3270–3275, May 2011.

[3] E. Mémin. Fluid flow dynamics under location uncertainty. *Geophysical & Astrophysical Fluid Dynamics*, 108(2):119–146, March 2014.

[4] R.D. Moser, J. Kim, and N.N. Mansour. Direct numerical simulation of turbulent channel flow up to Re_τ=590. *Physics of Fluids*, 11(4):943–945, April 1999.

[5] C. Robinson. *Image assimilation techniques for Large Eddy Scale models : Application to 3D reconstruction*. Scientific, Université de Rennes 1, Rennes, 2015.

[6] J. Smagorinsky. General circulation experiments with the primitive equations. *Monthly Weather Review*, 91(3):99–164, 1963.