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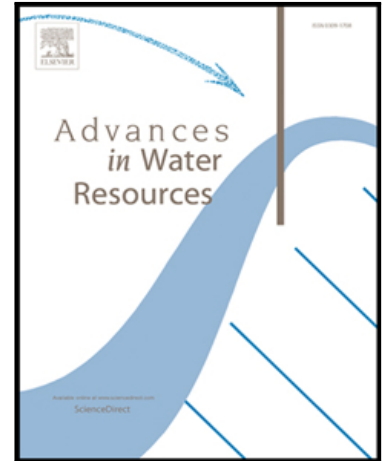
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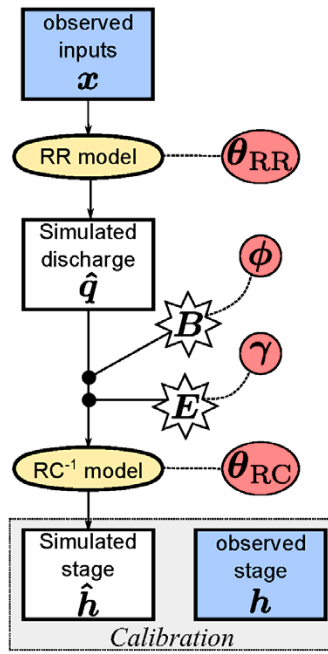
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Highlights

- Method for quantifying rating curve uncertainties in discharge prediction is proposed
- A rainfall-stage model is developed and calibrated in stage space
- Such a rainfall-stage model couples a hydrological model with an inverse rating curve
- We consider both structural and parametric uncertainties of the rating curve
- Shares of these errors in the total uncertainty of stages and discharges are assessed
- Structural uncertainties of hydrological model dominates other uncertainty sources
- Ignoring rating curve errors affects the estimation of hydrological model parameters

Graphical Abstract



Calibrating a hydrological model in stage space to account for rating curve uncertainties: general framework and key challenges

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Abstract

Hydrological models are typically calibrated with discharge time series derived from a rating curve, which is subject to parametric and structural uncertainties that are usually neglected. In this work, we develop a Bayesian approach to probabilistically represent parametric and structural rating curve errors in the calibration of hydrological models. To achieve this, we couple the hydrological model with the inverse rating curve yielding the rainfall-stage model that is calibrated in stage space. Acknowledging uncertainties of the hydrological and the rating curve models allows assessing their contribution to total uncertainties of stages and discharges. Our results from a case study in France indicate that a) ignoring rating curve uncertainty leads to changes in hydrological parameters, and b) structural uncertainty of hydrological model dominates other uncertainty sources. The paper ends with discussing key challenges that remain to be addressed to achieve a meaningful quantification of various uncertainty sources that affect hydrological model, as including input errors.

Keywords: rating curve, rainfall-stage model, structural uncertainty, parametric uncertainty, Bayesian inference, hydrological modeling

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1. Introduction

1.1. The importance of rating curve uncertainty in hydrological modeling

Flood risk analysis relies on estimates of hydrological models and associated uncertainty [1, 2, 3]. This uncertainty results mainly from four components: (i) parametric uncertainty of the hydrological model, (ii) its limited approximation of the catchment hydrological processes (model structural error), (iii) uncertainty in external model inputs (typically rainfall, temperature or evapotranspiration), and in (iv) output calibration data (typically discharge series) [4, 5, 6, 7, 3, 8, 9].

Among these four uncertainty contributors, input errors are considered to be one of the major uncertainty sources in hydrological models [10, 11, 12, 3] and thus more research has been devoted to investigate their effect on hydrological predictions than the effect of output uncertainty. Hence, different techniques have been proposed to represent input uncertainty which include a rainfall multiplier approach [10, 11, 13, 3], an addition to the bias [14, 8], or a more advanced stochastic description [15]. All of these studies, however, indicated that the inclusion of input errors raises several challenges. First, the computational cost is much higher than with traditional calibration. But even more importantly, substantial difficulties arise from the interaction between input errors and other uncertainty components. For instance, Renard et al. [6] discussed the challenge of identifying both input and structural errors; similarly, Del Giudice et al. [15] reported difficulties in distinguishing between different observational errors (input and output) if they have similar properties, i.e., are systematic. Hence, in this study we do not describe input errors explicitly, to be able to focus entirely on the effect of output uncertainty (due to the rating curve) on calibration and prediction of a hydrological model. Input errors will be implicitly encompassed in the structural error of the hydrological model.

As opposed to input errors, less attention has been given to the output uncertainty which is often assumed to be relatively small in comparison to the other three parts and thus has been evenly neglected in uncertainty analysis frameworks [16, 17]. Such a strong assumption might be justified for a direct measurement of discharge, for which measurement errors of 5% on average could be assumed [18, 19]. For practical applications, however, measuring

35 discharge continuously becomes impossible [20, 21]. Instead, a measure of dis-
36 charge is obtained from an observed stage using a stage - discharge relation-
37 ship (called rating curve) [22, 23]. This relationship needs to be established
38 at a hydrometric station with few direct (discrete) measurements of gaug-
39 ing pairs (stage and discharge) [18]. Using pre-established rating curves to
40 compute discharges therefore allows deriving continuous quasi-observed dis-
41 charge series [22, 23], which next serve for calibration of hydrological models
42 [24, 25].

43 Awkwardly, these computed discharge series are often communicated to
44 modellers or practitioners without any uncertainty statement [22, 26]. It is
45 however clear that such estimated discharge series contain several errors. It
46 has been reported in literature that although these errors are on average
47 about 3 – 6% of an estimated value, they may increase to about 20% under
48 poor measurement conditions [27], and to more than 25% outside the range
49 of measured stage-discharge pairs [16, 28]. However, the level of these errors
50 is case-specific [29] and results from many sources: measurement errors of
51 gauging pairs (instrumental errors, measurement technique), temporal shifts
52 in the rating curve (unstable stream channel due to vegetation, bank erosion,
53 sediment deposition, ice jams, etc.), transient hydrological conditions during
54 measurement of gauging pairs, hysteresis effect, and rating curve parametric
55 and structural uncertainties [12, 30, 31, 26, 32].

56 All these errors affect calibration of the hydrological model and have
57 serious implications for discharge simulations [12, 26, 23], flood frequency
58 analysis [33, 34, 35], and for regionalization of model parameters [36]. As
59 these errors are often not explicitly considered in uncertainty estimation,
60 their effect on discharge uncertainty cannot be quantified. Moreover, when
61 fully neglected, the uncertainty caused by rating curve errors may be wrongly
62 attributed to other uncertainty source(s), leading to biased estimates that
63 might be misunderstood by practitioners [35]. Given the above considerations
64 and the number of studies dealing with calibration of hydrological models
65 based on such quasi-observed discharge series, an accurate assessment of
66 the rating curve uncertainties and their impact on the hydrological model
67 becomes essential for flood risk assessment and management.

68 *1.2. Existing approaches to describe rating curve uncertainty*

69 Although a number of recent studies have investigated different aspects
70 of rating curve uncertainties [20, 22, 37, 24, 38, 30, 39], the contribution
71 of the rating curve to the uncertainty in hydrological simulations has not

72 been assessed systematically so far. In many uncertainty frameworks, rating
73 curve errors are either not explicitly represented or are combined with other
74 error sources. For instance, a common practice in uncertainty analysis is
75 to pool all uncertainties (apart from parametric uncertainty but including
76 rating curve uncertainty) into a lumped error term, which properties need
77 to be mathematically described [40]. We call the latter approach when only
78 parametric and structural errors of hydrological model are represented and
79 the model is calibrated against discharges computed from rating curves as
80 *a standard uncertainty estimation approach*. Another possible solution is
81 mapping all uncertainty sources (including rating curve errors) to parameter
82 uncertainty as in the original GLUE (generalized likelihood uncertainty esti-
83 mation) methodology [41]. Further developments allowed to relate "limits of
84 acceptability" with the rating curve uncertainty, although the need to extend
85 these limits to account for other error sources (input errors in particular) was
86 recognized [42]. Other approaches allow distinguishing input and structural
87 errors [43, 11]. However, they don't explicitly represent rating curve errors,
88 which are hence implicitly merged with structural errors. Finally, a recently
89 introduced bias addition approach [14] gives the possibility to distinguish,
90 aside from the parametric uncertainty, two different structural error types
91 of the hydrological model, i.e., systematic and random errors. These er-
92 rors are interpreted as structural and observational errors respectively. The
93 bias approach pools however all observational errors (i.e., input and output)
94 together and thus the uncertainty linked to the rating curve cannot be as-
95 sessed. Hence, the major drawback of all these different approaches available
96 to assess uncertainty of hydrological models is their inability to quantify the
97 uncertainty contribution of the rating curve in total uncertainty estimates of
98 hydrological models.

99 One possibility to indirectly tackle rating curve uncertainty is to prop-
100 agate rating curve errors to discharge series which are then represented as
101 spaghetti lines or uncertainty bands [44]. Such multiple realizations of dis-
102 charge series yield however a practical question of how to calibrate a hydro-
103 logical model with hundreds of "observed" discharges.

104 As an alternative, Sikorska et al. [26] and Thyer et al. [45] have recently
105 proposed to avoid the issue of multiple "observed" discharges by simulating
106 directly stages instead of discharges. Thus, they proposed to couple the
107 hydrological model with the inverse rating curve yielding a so-called *rainfall-*
108 *stage* model, for which uncertainty was evaluated in the stage space. In this
109 way, rating curve uncertainty could be directly incorporated into simulations

110 of the hydrological model and the contribution of the rating curve uncertainty
111 could be assessed. Yet, this method was mostly suitable to estimate stages
112 while it was lacking the possibility to provide discharge predictions along with
113 their uncertainty estimates (as discharge was only an intermediate step and
114 was not directly modelled). Moreover, only the assessment of the parametric
115 rating curve uncertainty was possible, while the structural errors of the rating
116 curve could not be separated from those of the hydrological model.

117 Finally, other authors proposed specific error models to describe rating
118 curve errors, based on an analysis of the rating curve itself [24]. Thyer et
119 al. [37] and Renard et al. [12] proposed a specific error model within the
120 Bayesian total error analysis methodology (BATEA) of Kavetski et al. [43,
121 10], to represent structural errors of rating curves in discharge data along
122 other uncertainty components (input and structural errors of hydrological
123 model). In this way, contributions of those three main uncertainty compo-
124 nents could be evaluated. Yet, they did not make an explicit distinction
125 between parametric and structural uncertainties of rating curves, pooling all
126 rating curve errors into a lumped structural error.

127 *1.3. Objectives*

128 Therefore, within this work, we further advance uncertainty quantifica-
129 tion of rating curves by developing a Bayesian approach to probabilistically
130 represent rating curve errors in the estimation of the hydrological model.
131 In contrast to previous works, for the first time, we explicitly represent the
132 parametric and the structural uncertainties of both the hydrological and the
133 rating curve models. To achieve this, we couple the hydrological model with
134 the inverse rating curve yielding the rainfall-stage model that can be cali-
135 brated in stage space, as previously proposed by Sikorska et al. [26]. Specifi-
136 cally, we describe structural errors of the hydrological model as an Ornstein-
137 Uhlenbeck process [46] in the form implemented by Sikorska et al. [47], and
138 the structural errors of the rating curve as Gaussian errors with a zero mean
139 and a standard deviation proportional to the discharge value following the
140 BaRatin method [23]. Because of such an explicit consideration of different
141 uncertainty components of the rating curve and the hydrological model, the
142 coupled total error can be decomposed into its constitutive sources. Hence,
143 the approach is suitable for providing both stage and discharge simulations
144 along with their associated uncertainties.

145 Specifically, we formulate the following objectives for this study:

- 146 1. Propose a generic framework for quantifying parametric and structural
 147 uncertainties of rating curves in hydrological models, and derive the
 148 corresponding inference equations;
- 149 2. Examine the effects of ignoring a specific source of rating curve uncer-
 150 tainty (parametric or structural) in the inference of model parameters
 151 and in model simulations;
- 152 3. Discuss pros and contras of using an advanced calibration approach
 153 (representing both structural and parametric rating curve errors ex-
 154 plicitly) over a “standard” uncertainty estimation approach (when un-
 155 certainty is attributed only to parametric and structural errors of the
 156 hydrological model and uncertainties of rating curve are neglected).

157 Our approach is developed and tested on a medium-size study catchment in
 158 France. This study restricts its attention solely to investigate uncertainties
 159 in output (discharge) of hydrological models, while uncertainty in input data
 160 (typically rainfall), although non negligible, is not explicitly acknowledged
 161 and is implicitly represented in structural errors of the hydrological model.
 162 We debate possible consequences of this assumption in the discussion part.
 163 Moreover, we recognize that an explicit and reliable treatment of all error
 164 sources remains a key challenge for hydrologic modeling: while not the ob-
 165 jective of this paper, we also discuss this long-term objective in section 5.5.

166 2. Uncertainty representation

167 2.1. Rating curve

168 2.1.1. Rating curve model

169 We describe an instantaneous discharge at time t predicted with the rating
 170 curve (RC), \check{q}_t , as

$$\check{q}_t = f_{\text{RC}}(h_t, \boldsymbol{\theta}_{\text{RC}}) \quad (1)$$

171 where $f_{\text{RC}}(h_t, \boldsymbol{\theta}_{\text{RC}})$ is the deterministic RC equation, h_t is the instantaneous
 172 stage at time t and $\boldsymbol{\theta}_{\text{RC}} = (\theta_{\text{RC}_1}, \dots, \theta_{\text{RC}_w})$ are parameters of the RC. Because
 173 parameters of the RC are unknown, they must be calibrated and thus they
 174 will introduce parametric uncertainty to the RC (see Sect. 3.1 for description
 175 of model calibration).

176 2.1.2. Structural error

177 The rating curve equation is a simplified mathematical representation of
 178 the true stage-discharge relationship prevailing at the gauging station. We

179 therefore introduce a structural error E_t to describe the difference between
 180 the RC-predicted discharge \check{q}_t and the (unknown) true discharge q_t :

$$\check{q}_t = q_t + E_t(\gamma) \quad (2)$$

181 The structural error E_t is assumed to be a realization from a Gaussian distri-
 182 bution with mean zero and standard deviation varying with the RC-predicted
 183 discharge as parameterized below:

$$E_t \stackrel{indep}{\sim} N(0, g(\check{q}_t, \gamma)^2); \quad g(\check{q}_t, \gamma) = \gamma_1 + \gamma_2 \cdot \check{q}_t \quad (3)$$

184 where $\gamma = (\gamma_1, \gamma_2)$ are the unknown parameters of the RC structural error
 185 model. This equation calls for the following comments:

- 186 1. The assumption that the standard deviation of structural errors is an
 187 affine function of the RC-predicted discharge is made to account for
 188 heteroscedasticity, which is often observed in practice (see e.g. [22,
 189 23]). A homoscedastic model can easily be obtained by fixing $\gamma_2 = 0$.
 190 Conversely, more complex heteroscedasticity models can in principle
 191 be derived by replacing the affine function g by another function (e.g.
 192 an higher-order polynomial), at the cost of introducing more unknown
 193 parameters;
- 194 2. Since the true discharge q_t is unknown, we assume that the standard
 195 deviation of structural errors is a function of the RC-predicted discharge
 196 \check{q}_t ;
- 197 3. Eq. 3 also makes the strong assumption that structural errors are in-
 198 dependent in time. This will be further discussed in section 5.2.

199 2.1.3. Gauging measurement error

200 The RC is typically calibrated using gaugings, i.e., pairs of stage-discharge
 201 values measured at different stage levels and flow conditions [48, 49, 50].
 202 The measurement error on stage is assumed to be negligible. Conversely, the
 203 measurement error on the gauged discharge can be considerable. Hence, we
 204 represent the gauged discharge observed at time t , \tilde{q}_t , as the sum of the true
 205 discharge q_t and a measurement error W_t :

$$\tilde{q}_t = q_t + W_t \quad (4)$$

206 The measurement error W_t is further assumed to be a realization from a
 207 Gaussian distribution with mean zero and known standard deviation δ_t :

$$W_t \stackrel{indep}{\sim} N(0, \delta_t^2), \quad (5)$$

208 This equation calls for the following comments:

- 209 1. We assume that δ_t is known because the uncertainty of the gauged discharge can be quantified before RC estimation by analyzing the measurement process (see e.g. [51, 52, 23]). Note that each gauging has its
210 specific uncertainty;
211
- 212 2. As for structural errors, eq. 5 also makes the assumption that measurement errors are independent in time. However this assumption is
213 probably much more realistic here.
214
215

216 2.2. Hydrological model

217 2.2.1. Rainfall-runoff model

218 For simplicity sake, we prefer to substitute the hydrological model with a
219 rainfall-runoff model which abbreviates to RR since h notation is restricted
220 for stage and thus could be confused with the abbreviation of a hydrological
221 model. We represent a RR-predicted discharge at time t , \hat{q}_t , as:

$$\hat{q}_t = f_{\text{RR}}(\mathbf{x}_{1:t}, \boldsymbol{\theta}_{\text{RR}}) \quad (6)$$

222 where $f_{\text{RR}}(\mathbf{x}_{1:t}, \boldsymbol{\theta}_{\text{RR}})$ represent the deterministic RR equations, $\mathbf{x}_{1:t}$ are in-
223 puts time series up to time t and $\boldsymbol{\theta}_{\text{RR}} = (\theta_{\text{RR}_1}, \dots, \theta_{\text{RR}_z})$ are the parameters.
224 Note that for simplicity this notation makes initial conditions implicit. Sim-
225 ilarly to the parameters of RC, parameters of the RR are unknown and they
226 must be estimated from observations. Hence they will introduce parametric
227 uncertainty to the RR model (see further Sect. 3.2 describing model calibra-
228 tion).

229 2.2.2. Structural error

230 To account for the imperfect nature of the RR model, a structural error B_t
231 is introduced to describe the mismatch between the RR-predicted discharge
232 and the (unknown) true discharge q_t :

$$\psi(\hat{q}_t) = \psi(q_t) + B_t(\boldsymbol{\phi}) \quad (7)$$

233 where $\psi(\cdot)$ is a transformation function applied to the true and the RR-
234 predicted discharges (typically, a Box-Cox transformation, see appendix sec-
235 tion Appendix A). The aim of this transformation is to make the proba-
236 bilistic model used to describe B_t (described next) more realistic.

237 In order to explicitly describe the autocorrelated nature of structural errors,

238 B_t is represented as an Ornstein-Uhlenbeck (OU) process [46] with param-
 239 ters $\phi = (\phi_1, \phi_2)$.

$$B_t \sim OU(\phi_1, \phi_2) \quad (8)$$

240 The OU process is a continuous-time equivalent of more standard time se-
 241 ries models such as the autoregressive (AR) error model, which are only
 242 defined for data sampled at regular discrete times. Such a continuous-time
 243 model allows dealing with unequally spaced data, which are commonly used
 244 for routine monitoring of instantaneous water stage or discharge (typically,
 245 more frequent records during floods than during low flows). We choose the
 246 correlation structure of B_t in such a way that it becomes similar to the AR(1)
 247 model [14, 47] with the variance at time t_i conditioned on a previous time
 248 step t_j being equal to:

$$\text{Var}(B_{t_i|j}) = \phi_1^2 \cdot \left(1 - \exp\left(-\frac{2 \cdot |t_i - t_j|}{\phi_2}\right) \right) \quad (9)$$

249 ϕ_1 can be interpreted as the asymptotic standard deviation (for infinitely-
 250 spaced time points) and ϕ_2 is a characteristic correlation time.

251 2.3. Rainfall-stage model

252 The basic idea behind the construction of the rainfall-stage (RS) model is
 253 to apply the inverse of the RC to the discharge simulated by the RR model
 254 [26]. The advantage of such a RS model is that its parameters encompass
 255 both the RR and the RC parameters, which allows explicitly accounting for
 256 RC uncertainty in the calibration of the RR parameters. However, the struc-
 257 tural errors affecting both the RR and the RC models propagate to the RS
 258 model and therefore need to be accounted for, as described next.

259 2.3.1. Structural error

260 Let h_t denote the true stage value at time t . From the RC model eqs. 1
 261 and 2 we get:

$$f_{\text{RC}}(h_t, \boldsymbol{\theta}_{\text{RC}}) = q_t + E_t(\boldsymbol{\gamma}) \quad (10)$$

262 Inverting the RC therefore yields the following relation:

$$h_t = f_{\text{RC}}^{-1}(q_t + E_t(\boldsymbol{\gamma}), \boldsymbol{\theta}_{\text{RC}}) \quad (11)$$

263 Moreover from the RR structural error model eq. 7 we get:

$$q_t = \psi^{-1}(\psi(\hat{q}_t) - B_t(\phi)) \quad (12)$$

264 where $\psi(\cdot)$ and $\psi^{-1}(\cdot)$ are the forward and the backward transformation.

265 Combining eqs. 11 and 12, the true instantaneous stage at time t can be
266 written as:

$$h_t = f_{\text{RC}}^{-1} \left(\psi^{-1} \left[\underbrace{\psi \left(f_{\text{RR}}(\mathbf{x}_{1:t}, \boldsymbol{\theta}_{\text{RR}}) \right)}_{\text{RR model}} - \underbrace{B_t(\phi)}_{\text{RR structural error}} \right] + \underbrace{E_t(\gamma)}_{\text{RC structural error}}, \boldsymbol{\theta}_{\text{RC}} \right) \quad (13)$$

267 We stress that the structural error model described in eq. 13 is a pure con-
268 sequence of the individual error models used for the RR and the RC models:
269 no new assumption has been made to derive eq. 13.

270 2.3.2. Input/output measurement errors

271 The RS model needs to be calibrated using observations of its input/output
272 variables. The input variables typically comprise precipitation and potential
273 evapotranspiration, while the output variable is stage.

274 In this paper, we make the strong assumption that measurement errors
275 in all input/output variables are negligible. We acknowledge that this as-
276 sumption is unrealistic in most studies. For instance, errors in estimating
277 areal precipitation may be large when the raingauge density is small (see e.g.
278 [53, 12]). Similarly, continuously-measured stage values may be affected by
279 non-negligible errors, of both random and systematic nature. Typically, the
280 inherent uncertainty of the stage sensor corresponds to a random error, while
281 the periodic recalibration of the stage sensor with respect to the staff gauge
282 produces an unknown systematic error between two successive recalibrations
283 (for more details, see e.g. [32]).

284 Making this restrictive assumption allows focusing entirely on the un-
285 certainty induced by the rating curve while minimizing possible interactions
286 between input and output errors. In practice, unaccounted input/output er-
287 rors will be implicitly absorbed by the structural error terms (B_t and E_t).
288 One should therefore keep in mind that while these terms are intended to
289 represent structural errors, they may also encompass the effect of ignored
290 input/output errors.

291 3. Calibration

292 In this paper, we apply Bayesian estimation to estimate all unknown
 293 parameters. The posterior distributions are explored by means of an adaptive
 294 Markov Chain Monte Carlo sampler described in Haario et al. [54]. The
 295 convergence of the chains is assessed visually by plotting the simulated chains
 296 and verifying their stationarity.

297 The general calibration strategy is made of two successive steps. We
 298 first estimate the RC using available gauging pairs (these gaugings are not
 299 used afterwards). In a second stage, we estimate the RS model combining
 300 the RC and the RR submodels (thus the RC model is re-calibrated). Since
 301 the RS model comprises parameters related to the RC (namely, $\boldsymbol{\theta}_{\text{RC}}$ and
 302 $\boldsymbol{\gamma}$, see section 2.1), the posterior distribution of these parameters obtained
 303 after stage 1 becomes their prior distribution in stage 2. Note that this
 304 informative prior for the RC model, based on an analysis of rating curve data,
 305 strongly constrains the inference. This allows avoiding non-identifiability and
 306 equifinality problems in the estimation of all parameters during stage 2.

307 3.1. Stage 1: rating curve calibration

308 From the assumptions described in section 2.1, the gauged discharge at
 309 time t can be written as follows (combining equations 2 and 4):

$$\tilde{q}_t = f_{\text{RC}}(\tilde{h}_t, \boldsymbol{\theta}_{\text{RC}}) - E_t(\boldsymbol{\gamma}) + W_t \quad (14)$$

310 Conditional on unknown parameters, the gauged discharge \tilde{q}_t is therefore a
 311 realization from a Gaussian distribution with mean $\check{q}_t = f_{\text{RC}}(\tilde{h}_t, \boldsymbol{\theta}_{\text{RC}})$ and
 312 variance $(\gamma_1 + \gamma_2 \cdot \check{q}_t)^2 + \delta_t^2$. The likelihood function can therefore be written:

$$p(\tilde{\boldsymbol{q}} | \boldsymbol{\theta}_{\text{RC}}, \boldsymbol{\gamma}, \tilde{\boldsymbol{h}}) = \prod_{k=1}^{N_{\text{gauging}}} f_G(\tilde{q}_{t_k}; \check{q}_{t_k}, (\gamma_1 + \gamma_2 \cdot \check{q}_{t_k})^2 + \delta_{t_k}^2) \quad (15)$$

313 where $f_G(u; m, v)$ is the Gaussian pdf with mean m and variance v , evalu-
 314 ated at u .

315 The posterior distribution is then computed up to a constant of proportion-
 316 ality using Bayes' theorem:

$$p(\boldsymbol{\theta}_{\text{RC}}, \boldsymbol{\gamma} | \tilde{\boldsymbol{q}}, \tilde{\boldsymbol{h}}) \propto p(\tilde{\boldsymbol{q}} | \boldsymbol{\theta}_{\text{RC}}, \boldsymbol{\gamma}, \tilde{\boldsymbol{h}}) \cdot p(\boldsymbol{\theta}_{\text{RC}}, \boldsymbol{\gamma}) \quad (16)$$

317 The prior distribution for RC parameters $\boldsymbol{\theta}_{\text{RC}}$ is derived from an analysis of
 318 the hydraulic configuration of the gauging station, as will be described in
 319 the case study (for more general considerations, see Le Coz et al. [23]). For
 320 the parameters γ governing the standard deviation of structural errors, wide
 321 non-informative priors are used.

322 3.2. Stage 2: rainfall-stage model calibration

323 Let $\mathbf{h} = (h_{t_k})_{k=1:N}$ denote the observed time series of stage values used
 324 to calibrate the RS model. Computing the likelihood requires deriving the
 325 distribution of \mathbf{h} conditional on all inferred quantities. Unfortunately, this
 326 cannot be done directly on the basis of eq. 13. Indeed, this conditional
 327 distribution is not Gaussian, because the Gaussian error terms E_t and B_t
 328 transit through nonlinear models (the backward transformation ψ^{-1} and the
 329 inverse rating curve f_{RC}^{-1}). Moreover, this non-Gaussian pdf cannot be derived
 330 analytically. Indeed, eq. 13 involves the sum of two independent random
 331 variables. The pdf of this sum can be obtained by convolution, but this
 332 convolution has no analytical solution because one of the random variables
 333 is not Gaussian.

334 In order to circumvent this issue, we partly linearize eq. 13 as described next.
 335 We introduce the following shorthand notation for this section:

$$\begin{aligned} \hat{q}_t(\boldsymbol{\theta}_{\text{RR}}) &= f_{\text{RR}}(\mathbf{x}_{1:t}, \boldsymbol{\theta}_{\text{RR}}) \\ d_t^{(\psi)}(\boldsymbol{\theta}_{\text{RR}}) &= \psi'(\hat{q}_t) \end{aligned} \quad (17)$$

336 Using this notation and linearizing the backward transformation ψ^{-1} , eq. 13
 337 can be approximated as follows (see Appendix B for details):

$$h_t \approx f_{\text{RC}}^{-1} \left(\underbrace{\hat{q}_t(\boldsymbol{\theta}_{\text{RR}}) - \frac{B_t(\phi)}{d_t^{(\psi)}(\boldsymbol{\theta}_{\text{RR}})} + E_t(\gamma)}_{Z_t}, \boldsymbol{\theta}_{\text{RC}} \right) \quad (18)$$

338 The term Z_t is now the sum of a constant plus two Gaussian terms, and
 339 is therefore itself Gaussian. More precisely, the vector $\mathbf{Z} = (Z_{t_1}, \dots, Z_{t_N})$
 340 follows a multivariate Gaussian distribution, with mean vector $\boldsymbol{\mu}$ (size N)
 341 and covariance matrix $\boldsymbol{\Sigma}$ (size $N \times N$) defined as follows:

$$\boldsymbol{\mu}(\boldsymbol{\theta}_{\text{RR}}) = (\hat{q}_{t_1}(\boldsymbol{\theta}_{\text{RR}}), \dots, \hat{q}_{t_N}(\boldsymbol{\theta}_{\text{RR}})) \quad (19)$$

$$\Sigma(\boldsymbol{\theta}_{\text{RR}}, \boldsymbol{\phi}, \boldsymbol{\gamma}) = \mathbf{D}^{(\psi)} \Sigma^{(\text{RR})} \mathbf{D}^{(\psi)} + \Sigma^{(\text{RC})} \quad (20)$$

342 In the latter equation, $\mathbf{D}^{(\psi)}$ denotes the square $N \times N$ diagonal matrix whose
 343 diagonal terms are equal to $1/d_t^{(\psi)}$, while $\Sigma^{(\text{RR})}$ and $\Sigma^{(\text{RC})}$ are the $N \times N$
 344 covariance matrices of RR and RC structural errors:

$$\mathbf{D}^{(\psi)}(i, i) = \frac{1}{d_{t_i}^{(\psi)}(\boldsymbol{\theta}_{\text{RR}})}; \quad \mathbf{D}^{(\psi)}(i, j) = 0 \text{ if } i \neq j \quad (21)$$

345

$$\Sigma^{(\text{RR})}(i, j) = \phi_1^2 \cdot \exp\left(-\frac{|t_i - t_j|}{\phi_2}\right) \quad (22)$$

346

$$\Sigma^{(\text{RC})}(i, i) = (\gamma_1 + \gamma_2 \cdot \hat{q}_{t_i})^2; \quad \Sigma^{(\text{RC})}(i, j) = 0 \text{ if } i \neq j \quad (23)$$

347 Having derived the pdf of \mathbf{Z} , the pdf of $\mathbf{h} \approx f_{\text{RC}}^{-1}(\mathbf{Z})$ (eq. 18) can be ob-
 348 tained by applying the change-of-variables formula. After some computation
 349 (see Appendix B for details), this yields the following likelihood:

$$p(\mathbf{h} | \boldsymbol{\theta}_{\text{RR}}, \boldsymbol{\theta}_{\text{RC}}, \boldsymbol{\phi}, \boldsymbol{\gamma}, \mathbf{x}) = f_{MG}(f_{\text{RC}}(\mathbf{h}, \boldsymbol{\theta}_{\text{RC}}); \boldsymbol{\mu}(\boldsymbol{\theta}_{\text{RR}}), \Sigma(\boldsymbol{\theta}_{\text{RR}}, \boldsymbol{\phi}, \boldsymbol{\gamma})) \times \prod_{k=1}^N |f'_{\text{RC}}(h_{t_k}, \boldsymbol{\theta}_{\text{RC}})| \quad (24)$$

350 where $f_{MG}(\mathbf{u}; \mathbf{m}, \mathbf{v})$ is the multivariate Gaussian pdf with mean vector \mathbf{m}
 351 (size N) and covariance matrix \mathbf{v} (size $N \times N$), evaluated at vector \mathbf{u} (size
 352 N).

353 The posterior distribution is then computed up to a constant of proportion-
 354 ality using Bayes' theorem:

$$p(\boldsymbol{\theta}_{\text{RR}}, \boldsymbol{\theta}_{\text{RC}}, \boldsymbol{\phi}, \boldsymbol{\gamma} | \mathbf{h}, \mathbf{x}) \propto p(\mathbf{h} | \boldsymbol{\theta}_{\text{RR}}, \boldsymbol{\theta}_{\text{RC}}, \boldsymbol{\phi}, \boldsymbol{\gamma}, \mathbf{x}) \cdot p(\boldsymbol{\theta}_{\text{RR}}, \boldsymbol{\theta}_{\text{RC}}, \boldsymbol{\phi}, \boldsymbol{\gamma}) \quad (25)$$

355 The prior distribution for RC-related parameters $\boldsymbol{\theta}_{\text{RC}}$ and $\boldsymbol{\gamma}$ is set to the
 356 posterior distribution obtained after calibration of the RC using gaugings
 357 at stage 1 (eq. 16). For the parameters of the RR model ($\boldsymbol{\theta}_{\text{RR}}$), priors are
 358 case-specific and related to the RR model and available information. For the
 359 parameters $\boldsymbol{\phi}$ governing the properties of RR structural errors, wide non-
 360 informative priors are used.

361 Note that the RS model is calibrated against time series with observed
 362 stages. However, during the evaluation both the output of the RS model,
 363 stage, and the output of the RR model, discharge, will be examined. This is
 364 possible thanks to the explicit treatment of RC and RR errors.

365 *3.3. Calibration strategies*

366 The posterior distribution in eq. 25 corresponds to a full calibration strat-
 367 egy, schematized in Figure 1: the parameters related to both the RR model
 368 and the RC are estimated together, thus enabling interactions between them
 369 and hence assessing how RC uncertainties impact the estimation of RR pa-
 370 rameters. In particular, both parametric (θ_{RC}) and structural (\mathbf{E}) uncer-
 371 tainties of the RC are accounted for. In order to understand in more depth
 372 the impact of these two types of uncertainty, we also implement incomplete
 373 calibration strategies, where some uncertainty sources are ignored. As shown
 374 in Table 1, these strategies are the following:

- 375 1. Strategy NoS ignores RC structural uncertainty. This corresponds to
 376 assuming that $\mathbf{E} = 0$, which is achieved by using $\Sigma^{(RC)} = 0$ in eq. 20.
 377 A similar representation of RC uncertainty has been used by Steinbakk
 378 [55] in the context of flood frequency analysis, and by Sikorska et al. [26]
 379 in the context of model calibration.
- 380 2. Strategy NoP ignores RC parametric uncertainty. This is achieved by
 381 removing θ_{RC} from the list of inferred parameters. The RC is therefore
 382 used with a fixed parameter vector $\hat{\theta}_{RC}$, taken as the maxpost esti-
 383 mate (i.e. the vector maximizing the stage-1 posterior of eq. 16). This
 384 strategy is similar to the representation of RC uncertainty used by e.g.
 385 Thyer et al. [37] or Renard et al. [6].
- 386 3. Strategy NoPNoS ignores both RC parametric and structural uncer-
 387 tainty, hence using both a fixed parameter vector $\hat{\theta}_{RC}$ and setting
 388 $\Sigma^{(RC)} = 0$. In this strategy, there is no explicit representation of RC
 389 uncertainty, which corresponds to the most widely-used approach in
 390 hydrological modeling (*standard uncertainty estimation approach*).
- 391 4. Strategy FULL* is similar to the full strategy, except that the prior for
 392 RC parameters θ_{RC} is truncated. More precisely, we set the prior pdf
 393 to zero outside of 95% probability intervals for each component of θ_{RC} .
 394 This strategy strongly limits the possible interactions between θ_{RC} and
 395 other inferred parameters. It guarantees that after calibration of the
 396 RS model, the RC parameters will still be within the 95% credibility
 397 intervals derived by calibrating the RC to gaugings. Note that bluntly
 398 truncating the prior as done here makes the resulting distribution un-
 399 normalized; however this is not problematic in the Bayesian-MCMC
 400 context of this paper since the posterior only needs to be known up to
 401 a normalizing constant.

402 5. Similarly, strategy NoS* is a variation of the NoS strategy, with a
403 truncated prior for θ_{RC} .

404 4. Case study: the Ardèche river at Meyras

405 4.1. Ardèche catchment

406 The river Ardèche is a right tributary of the River Rhône and has its
407 sources in the Massif Central in France (Figure 2). The gauging station
408 Meyras, located at 318 m a.s.l., controls an area of 98.43 km². The mean
409 elevation of this catchment is 899 m a.s.l., with the highest point located at
410 1467 m a.s.l. The catchment is quite steep with an average slope of 23.4 %
411 and it is in 68% covered by forests [56]. The average annual precipitation,
412 estimated based on fifty years of observations at the station Péreyres (840
413 m a.s.l.), is 1774 mm/yr in this region, whereof approximately 40% is lost
414 to evaporation. With the yearly mean daily temperature equal to 9.25°C
415 and the snowfall ratio of less than 3% of the annual precipitation, the snow
416 processes can be neglected to model this catchment.

417 4.2. Rating curve

418 As a RC model (eq. 1), we use a piecewise combination of power functions
419 of the form $q = a(h - b)^c$. This combination is defined by the succession of
420 hydraulic controls governing the stage-discharge relationship, as explained in
421 more details by Le Coz [23]. At the Meyras gauging station, three controls
422 can be identified (Figure 3). Low flows are first governed by a natural gravel
423 riffle (control 1). When the stage gets above a certain level, this riffle is
424 drowned and a channel control takes over (control 2). Finally, for very high
425 stage values, the main channel may be full and some flow may also occur in
426 the floodplain (control 3). This configuration leads to the following rating
427 curve equation:

$$f_{RC}(h_t, \theta_{RC}) = \begin{cases} a_1 (h_t - b_1)^{c_1} & \text{if } \kappa_1 < h_t \leq \kappa_2 \text{ (control 1)} \\ a_2 (h_t - b_2)^{c_2} & \text{if } \kappa_2 < h_t \leq \kappa_3 \text{ (control 2)} \\ a_2 (h_t - b_2)^{c_2} + a_3 (h_t - b_3)^{c_3} & \text{if } \kappa_3 < h_t \text{ (control 2 + 3)} \end{cases} \quad (26)$$

428 where $(\kappa_1, \kappa_2, \kappa_3)$ are the (unknown) activation stages for each control. Note
429 that parameters b_1 , b_2 and b_3 do not need to be inferred because they can be
430 deduced by continuity of the RC as shown in eq. 27 below. Consequently,

431 the parameters of the rating curve are $\boldsymbol{\theta}_{\text{RC}} = (\kappa_1, a_1, c_1, \kappa_2, a_2, c_2, \kappa_3, a_3, c_3)$
 432 where the relationships between κ and b are as follows:

$$b_1 = \kappa_1; b_2 = \kappa_2 - \left(\frac{a_1}{a_2} \cdot (\kappa_2 - b_1)^{c_1} \right)^{\frac{1}{c_2}}; b_3 = \kappa_3 \quad (27)$$

433 The parameters $\boldsymbol{\theta}_{\text{RC}}$ are related to physical characteristics of the gauging
 434 section, which opens the possibility to specify informative priors. For in-
 435 stance, the first control by a natural riffle can be approximated using a rect-
 436 angular weir formula, as shown in Table 2. This formula indicates that the
 437 exponent c_1 should be close to 1.5. Moreover, the parameter a_1 is linked
 438 to the weir width B_w and to a discharge coefficient C_r . The width can be
 439 approximated at ¹ $B_w = (8 \pm 2)$ m, while literature suggests values of the
 440 coefficient $C_r = 0.4 \pm 0.1$ (see [48, 23]). These two uncertainties can be com-
 441 bined by using the uncertainty propagation formula recommended by the
 442 Guide to the Expression of Uncertainty in Measurement [57]. This yields
 443 the Gaussian prior distribution for a_1 shown in Table 2. Lastly, the eleva-
 444 tion of the weir crest, which defines the activation stage κ_1 , is estimated at
 445 $\kappa_1 = (-0.05 \pm 0.05)$ m.

446 A similar approach can be used to specify priors for parameters of controls
 447 2 and 3, using the Manning-Strickler formula for wide rectangular channels
 448 (see Table 2). For the main channel, the Strickler coefficient is set to $K_S =$
 449 $(25 \pm 2.5) \text{ m}^{1/3} \cdot \text{s}^{-1}$, the channel width to $B_w = (15 \pm 2.5)$ m and the slope to
 450 $S = (3 \pm 1) \text{ m} \cdot \text{km}^{-1}$. For the floodplain, we use $K_S = (15 \pm 2.5) \text{ m}^{1/3} \cdot \text{s}^{-1}$,
 451 $B_w = (30 \pm 5)$ m and $S = (3 \pm 1.25) \text{ m} \cdot \text{km}^{-1}$. This completes the prior
 452 specification shown in Table 2.

453 4.3. Rainfall-runoff model (HBV)

454 The rainfall-runoff process within the Ardèche catchment is modelled with
 455 a HBV model [58, 59, 60]. The HBV consists of four main routines responsible
 456 for modelling snow dynamics, soil moisture, runoff response, and flow routing
 457 in the channel. Because snow processes can be neglected in this catchment,
 458 we use a simplified version of the HBV model, i.e., with an inactive snow
 459 component. To further simplify the model, we model the catchment as a
 460 single subcatchment without any elevation-dependent correction factors for
 461 inputs. This further reduces the number of inferred parameters to 6 (Table 3).

¹in the notation $x \pm s$, s is the standard deviation.

462 Such a simplified HBV model requires mean areal precipitation and long
 463 term evaporation estimates as input, while temperature data responsible for
 464 modeling the snow component are not strictly required. In this study, the
 465 HBV model is run at hourly time steps. Since the HBV model was not
 466 applied before in this catchment, no previous knowledge was available for its
 467 parameters. Thus, we formulate prior for each HBV parameter as a uniform
 468 distribution restricted to possible ranges that were defined for each parameter
 469 independently (Table 3).

470 4.4. Calibration data

471 4.4.1. stage 1: rating curve calibration

472 To infer the RC parameters θ_{RC} and γ , we use 41 gaugings made between
 473 2001 and 2008, for a period with no noticeable shift of the RC. For each
 474 gauged discharge, we assume a constant relative uncertainty of $\pm 3.5\%$, i.e.
 475 for a gauged discharge equal to q_t , the standard deviation δ_t in eq. 5 is set to
 476 $\delta_t = 0.035 \cdot q_t$. The gaugings and their uncertainty can be seen in Figure 4b.

477 4.4.2. stage 2: rainfall-stage model calibration

478 The RS model described with the Eq. 13 requires mean areal precipita-
 479 tion at the hourly time step as input. Yet, the stage observations at the
 480 gauging station are recorded by the limnigraph with unequal time steps ad-
 481 equate to the current dynamics of flow processes (i.e., between 1 hour and
 482 10/15 days). Hence, we chose to use directly these data instead of convert-
 483 ing them into the hourly estimates, which would yield additional errors due
 484 to the stage approximation. Note that this involves interpolating the HBV-
 485 discharge simulations on the temporal (irregular) grid used for stage values.
 486 Using irregularly spaced data is possible with the correlated error term on
 487 the hydrological model introduced (Eqs. 8 and 9).

488 4.5. Results: rating curve calibration (stage 1)

489 Figure 4a shows the prior RC resulting from the hydraulic analysis of
 490 the gauging station (Table 2). Figure 4b shows the posterior RC and illus-
 491 trates the uncertainty reduction resulting from the information brought by
 492 the gaugings. The posterior RC is overall quite precise, especially for stages
 493 smaller than 1 m. For such relatively small stages, parametric uncertainty
 494 is only a small part of the total uncertainty, which is hence dominated by
 495 structural uncertainty. For stage values beyond 1 m, total uncertainty in-
 496 creases, mostly due to an increase of parametric uncertainty which becomes

497 dominant for such high stages. In particular the parameter κ_3 representing
 498 the activation stage of the third control is not precisely estimated (between
 499 1 m and 1.5 m, see green band in Figure 4b).

500 The posterior distribution of RC parameters θ_{RC} and γ obtained after this
 501 first stage is now being used as a prior distribution for the second stage. Note
 502 that the posterior on RC is in fact represented with Monte Carlo samples.
 503 Hence, to specify the prior distribution for the second stage of calibration,
 504 we fit a multivariate Gaussian distribution to the Monte Carlo samples from
 505 the first stage. The resulting corresponding marginal distributions can be
 506 seen as gray boxplots in Figure 5.

507 4.6. Results: parameter estimates (stage 2)

508 Posteriors for the RS model for all six calibration strategies are plotted
 509 as boxplots against prior information (obtained from stage 1) in Figure 5.
 510 For parameters of the RR and RC sub-models and of the structural error
 511 of the RR model, we observe that parameters tend to form three groups
 512 in terms of their posterior behaviours. These groups are shaped as follows:
 513 (1) calibration strategies FULL and NoS, (2) NoP and NoPNoS, and (3)
 514 FULL* and NoS*, as seen in the figure. It appears that this grouping is
 515 driven by the way of accounting for RC parametric uncertainty, i.e.: fully
 516 accounting (group 1), non-accounting (group 2), and accounting but within
 517 the constrained truncated prior (group 3). The grouping effect is obviously
 518 not visible for parameters responsible for the RC structural uncertainty (γ)
 519 as these parameters are excluded from the inference in the strategies NoS,
 520 NoPNoS and NoS*.

521 4.6.1. Hydrological model

522 With respect to the HBV parameters (θ_{RR} , i.e., *PERC:MAXBAS*, top
 523 two rows in Figure 5), which mainly control the response and routing func-
 524 tion, we specifically observe that posterior parameters vary among three pat-
 525 tern groups and particularly between FULL and NoPNoS strategy (*standard*
 526 *uncertainty estimation approach*). We observe that using a simplified de-
 527 scription of errors as in NoPNoS leads to different values of inferred model
 528 parameters than when explicitly representing all major contributing sources.
 529 Such modified parameters of the RR model should mostly transfer to altered
 530 discharge simulations (being an intermediate step within the RS model) and
 531 might lead to biased estimates. Confronting posterior ranges of different cal-
 532 ibration strategies indicates that generally parametric uncertainty of the RR

533 model is very similar for most parameters in all strategies. The interpretation
 534 of individual parameter uncertainty is however difficult due to their interactions.
 535 Therefore, not the uncertainty of individual parameters but rather the
 536 resulting parametric uncertainty in predicted discharge is our major interest
 537 (see Sect. 4.8.2).

538 Diagnosis of the structural error model of the RR model (ϕ) show that the
 539 error standard deviation (ϕ_1) is the smallest for the FULL and the largest for
 540 NoPNoS strategy (Fig. 5). This seems logical as in the FULL strategy the to-
 541 tal residual variance is decomposed into two contributing sources originating
 542 from the RR model (ϕ) and from the RC model (γ , see further below), while
 543 in the strategy NoPNoS all variance is explained with ϕ only. Hence, only
 544 this error can be increased to capture the mismatch between the observed
 545 and the simulated stage. Posterior error standard deviations of all other cal-
 546 ibration strategies lie between these two strategies. This seems reasonable as
 547 they represent transitional steps between FULL and NoPNoS strategies in
 548 terms of the level of the variance decomposition from the simplest strategy
 549 (NoPNoS) towards the most complex strategy (FULL). As it also seems logi-
 550 cal, excluding RC parameters (θ_{RC}) from the inference results in an increased
 551 error ϕ_1 (NoP and NoPNoS) in comparison to strategies which include θ_{RC}
 552 into the inference (NoS) even if restricted prior is used (FULL* or NoS*).
 553 The error autocorrelation length (ϕ_2) generally follows the behavior of the
 554 error standard deviation and is the longest for strategy NoP and NoPNoS,
 555 and the shortest for NoS* and FULL*.

556 4.6.2. Rating curve model

557 Posterior RC parameters (θ_{RC}) are presented in three bottom rows in
 558 Figure 5 (parameters k_1 till c_3). Note that RC parameters in strategies NoP
 559 and NoPNoS are not altered during the inference and are kept at the values
 560 of maxpost from the calibration stage 1 from section 4.5. For other four
 561 strategies, again two groups of parameter behaviors can be observed.

562 Specifically, using informative but unbounded prior for RC parameters
 563 θ_{RC} during the inference (FULL and NoS) results in a significant shift of
 564 posteriors often outside of the 95% prior bounds. This effect appears to be
 565 a result of a possible compensation for other uncertainty sources, specifically
 566 for the one originating from the RR model parameters. As it appears, nine
 567 parameters of the RC in addition to six parameters of the RR model gives
 568 a higher level of freedom for modifying model simulations to match stage
 569 observations. It is worth recalling that although RC parameters are related

570 to physical characteristics of the gauging station and informative prior is
 571 used, prior information on the third control is very imprecise as it is con-
 572 strained with only very few gaugings (see section 4.5). Indeed, posteriors on
 573 parameters of the third control are strongly modified during the inference.
 574 As all RS parameters are inferred at the same time, the possible compensa-
 575 tion between parameters of the RR and the RC sub-model cannot be avoided
 576 given unbounded priors on all parameters being inferred. This shifting of the
 577 RC parameters outside of hydraulically reasonable boundaries is expected to
 578 have a consequence on the shape of an updated RC (see section 4.7).

579 Using a truncated prior on RC parameters indeed prevents from a strong
 580 modification of RC parameters (FULL* and NoS*). As it is visible in the
 581 Figure 5, posteriors attempt to move towards the values from unbounded
 582 strategies but remain within the 95% limits set. This also results in a smaller
 583 RC parametric uncertainty in strategies FULL* and NoS* than in unbounded
 584 strategies FULL and NoS.

585 Finally, the structural error of the rating curve (γ) varies in different cal-
 586 ibration strategies. As it is represented with two parameters, the combined
 587 structural error of the RC cannot be easily quantified from estimated poste-
 588 riors. We observe, however, an inverse relationship between its behavior and
 589 the behavior of the RR structural error. This relationships seems also logical
 590 as both structural and parametric uncertainties of the RC are decomposed
 591 from the total uncertainty in FULL and FULL* strategies.

592 4.7. Results: updated rating curve (stage 2)

593 Updated RCs for four strategies accounting for RC parametric uncertainty
 594 (i.e., FULL, NoS, FULL* and NoS*) are plotted in Figure 6 with uncertainty
 595 bands (blue polygons) against the prior (red polygons). RCs for strategies
 596 NoP and NoPNoS are not plotted as their parameters are not altered during
 597 the calibration. As expected, a strong shift in the RC posterior parameters
 598 observed for strategies which use non-bounded prior (FULL and NoS) leads
 599 to a strong modification of the RC shape. This effect is especially visible in
 600 the range of the third control for which the updated RC distinctly transcends
 601 the prior ranges (red polygons) by pushing the RC towards assigning smaller
 602 discharge values for the same stages. As previously mentioned, the prior
 603 for parameter inference on the third RC control is established with very few
 604 measures and thus is very uncertain (see Figure 4), which allows for freely
 605 modifying these parameters. It is clear that setting bounded priors on RC

606 parameters in strategies FULL* and NoS* prevents from destroying the RC
607 shape which remains within the 95% prior limits.

608 This issue of using the RC parameters to compensate for limitations of
609 the RR model has clearly serious implications for using such updated RCs
610 and will be further discussed in section 5.1.

611 4.8. Results: predictive uncertainty (stage 2)

612 4.8.1. Total uncertainty bands

613 Total uncertainty bands (TUB) for the FULL strategy are plotted in
614 Figure 7 for stages and in Figure 8 for discharges (top panels). For both
615 variables TUB appear to be reasonable as they cover most of the data points
616 and are smaller for low flow and higher for high flow conditions (assessed
617 visually). The smaller uncertainty during low flows is more apparent for
618 discharges than for stages.

619 Widths of TUB for all other strategies are plotted in Figure 9 for both
620 stages (top) and discharges (bottom). The TUB width in the FULL strategy
621 is used as a reference. Widths of TUB for all other strategies are represented
622 with respect to the FULL TUB width and thus are plotted as curves (a
623 value larger than one representing a TUB width larger than that of the
624 FULL strategy). The top panel of Figure 9 shows that for stage, the TUB
625 widths of all strategies are larger than that of the FULL strategy for almost
626 the entire calibration and validation periods. Specifically, during low flow
627 periods (e.g. around the vertical red line), TUB widths are larger than that
628 of the FULL strategy by a factor of up to 2, while during high flows this
629 factor decreases to about 1.25. Similar patterns are observed with respect to
630 discharges apart for the strategy NoS*, for which TUB width is similar to
631 the FULL strategy on average.

632 Although the effect of obtaining the smallest TUB width for the FULL
633 strategy is visible for both stage and discharge, it has greater implications for
634 modeling discharge. Much smaller TUB for FULL strategy clearly demon-
635 strates a benefit compared to the strategy NoPNoS (*standard uncertainty*
636 *estimation approach*). This finding indicates that accounting for both (struc-
637 tural and parametric) RC uncertainties allows for removing these uncertainty
638 parts from the total discharge uncertainty and this results in narrower TUB
639 in comparison to strategies which do not present such ability (NoPNoS, NoP,
640 NoS). Using bounded prior on RC parameters results in wider TUB in com-
641 parison to their respective unbounded strategies, which confirms that the

642 structural error of the RR model is used for compensation of other unrepre-
643 sented uncertainty components.

644 4.8.2. *Uncertainty contributors*

645 An explicit representation of different uncertainty components within the
646 TUB (i.e. of rating curve and of hydrological model) allows for their relative
647 assessment. Depending on the strategy, these are parametric and structural
648 uncertainty of the RR model and/or the RC model. Clearly, the most inter-
649 esting is the FULL strategy which makes it possible to assess all four uncer-
650 tainty components in predictions of stages and two components in prediction
651 of discharges. Note that by an explicit representation of the parametric and
652 structural uncertainties of RC in the FULL strategy, these uncertainty com-
653 ponents can be decomposed from the TUB and thus are not propagated on
654 the discharge (since the aim is to predict the “true” discharge and not the
655 RC-estimated one). On the contrary, not accounting for structural or para-
656 metric uncertainty of the RC does not allow for removing these uncertainty
657 parts from the total uncertainty and thus they will be implicitly propagated
658 on the discharge simulations. Uncertainty contributions for the FULL strat-
659 egy are presented visually in Figure 7 for stages and in Figure 8 for discharges
660 (bottom panels).

661 With respect to stages, it can be seen that the structural error of the
662 RR model (ϕ) represents the majority of the total uncertainty while the
663 next major contributor is the structural error of the RC model. Parametric
664 uncertainty of the RR model and of the RC model are both less relevant.
665 These contributions vary slightly over time and the contribution of the RC
666 structural error is slightly higher during recession periods, whereas the con-
667 tribution of the RC parametric error increases during high flows. The con-
668 tribution of the parametric uncertainty of the RR model is higher during high
669 flows and successive recession periods, while it is smaller during low flows.
670 In a similar fashion, the structural error of the RR model accounts for the
671 majority of the total uncertainty of discharge prediction.

672 The visual assessment of uncertainty contributions is accompanied by
673 the time-averaged relative contributions of each uncertainty source for all six
674 strategies and these are presented in Table 4. Generally, we observe quite
675 stable uncertainty contributions of different error sources in all six calibration
676 schemes. For all calibration strategies, the structural error of the RR model
677 explains the majority of the total uncertainty which is ranging from 81% in
678 the FULL strategy to 94% in NoPNoS for stages, and from 92% in FULL to

679 94% for NoP, NoPNoS, FULL* and NoS* for discharges. Both the parametric
680 uncertainties of the RR and of the RC model vary only insignificantly and
681 are much less relevant than other two uncertainty components. Hence, the
682 change in contribution shares is thus mainly due to structural uncertainties
683 of both the RC but mostly the RR model. The latter component is thus used
684 to compensate for all unrepresented uncertainty source(s).

685 5. Discussion

686 5.1. Feasibility of accounting for rating curve uncertainty through a rainfall- 687 stage model

688 The approach proposed in this paper to explicitly account for RC un-
689 certainty in the calibration of a RR model is to include both RC and RR
690 parameters within a rainfall-stage (RS) model. However, the results of the
691 case study show that the initial RC (established using gaugings) is strongly
692 modified after calibration of the RS model (strategies FULL and NoS), unless
693 a restrictive truncated prior is used for RC parameters (strategies FULL* and
694 NoS*). We consider that the extent to which the RC is modified is hardly
695 defensible; we therefore do not consider this modification as a meaningful
696 improvement of the RC, but rather as a sign that the results produced by
697 strategies FULL and NoS should be taken with caution.

698 It is of interest to further discuss this issue in terms of the information
699 content used in the two successive calibration stages. The initial RC is estab-
700 lished using the information brought by 41 independent gaugings, along with
701 the prior information derived from the hydraulic analysis (the latter being
702 informative but still quite imprecise). The posterior distribution of calibra-
703 tion stage 1 reflects this quantity of information. During calibration stage 2,
704 this posterior is used as a prior, but the RC can be further modified by the
705 information brought by more than 1000 stage values used for calibration of
706 the RS model. At first sight, the information imbalance between 41 gaugings
707 vs. 1000 stage values may explain why the RC is strongly modified by the
708 calibration of the RS model (well beyond the prior constraint induced by the
709 gaugings). However, one should keep in mind the following points:

- 710 1. Since the RR structural error uses an autocorrelation component, the
711 information content of these 1000 stage values does not correspond to
712 that of 1000 independent data;

713 2. The information contained in the 41 gaugings is only used to estimate
714 the RC, while the information contained in the 1000 stage values is
715 used to infer both the RC and the RR model.

716 These clarifications notwithstanding, the strong modification of the RC is a
717 sign that the error models we used do not convincingly weight the information
718 brought by the gaugings and the stage time series. We see at least two
719 avenues to improve this:

- 720 1. Improve the error models, as discussed in the next section 5.2;
- 721 2. Do not re-estimate the RC during calibration stage 2. This can be
722 achieved by means of a propagation approach, as discussed in section
723 5.3

724 5.2. Limitation of the error models

725 The RC structural error model in Eq. 3 assumes independent errors, which
726 is questionable at least for time steps close to each other. In principle, it is
727 feasible to avoid this independence assumption e.g. by using an autocorrela-
728 tion component. However, identifying an autocorrelation structure based on
729 gaugings is difficult in practice, if not impossible, because gaugings are made
730 too sporadically. Typically two successive gaugings are separated by weeks
731 or months, which makes shorter autocorrelation structures non-identifiable.
732 While implementing dedicated high-frequency gauging strategies might be
733 feasible, we do not see any obvious solution with existing operational gaug-
734 ing datasets.

735 Unlike RC structural errors, RR structural errors are not assumed inde-
736 pendent and instead an explicit autocorrelation component is used (Eq. 8).
737 This autocorrelation structure is identifiable because the stage time series is
738 sampled at a high frequency. However, due to the particular dynamics of the
739 RR model, even this autocorrelation structure is too simplistic. In partic-
740 ular, autocorrelation properties are likely very different during dry periods
741 and rainy periods, when quick-flow components are activated. As input er-
742 ror is implicitly encompassed into the model bias, these different properties
743 cannot be distinguished with the RR error model used here and the inferred
744 bias is “averaged” over dry and wet conditions. More flexibility should hence
745 be added to the autocorrelation component to allow distinguishing these dis-
746 tinct properties, for instance by making a bias dry/wet period dependent or
747 input-related (for further discussion on input error see Sect. 5.5).

748 Moreover, a common limitation of both RC and RR structural error mod-
 749 els is the lack of a systematic component. A structural error is indeed defined
 750 as the difference between the model prediction (forced with perfect inputs)
 751 and the unknown truth. For a given set of inputs, this error is likely to
 752 have a non-zero mean, because it is (at least partly) due to model struc-
 753 tural deficits that will systematically manifest themselves when the model is
 754 forced with similar inputs. Such a non-zero mean can also be interpreted as
 755 a “conditional bias” (conditional to the inputs and initial conditions). The
 756 fact that the structural error models we used ignore this conditional bias (as
 757 do the error models we are aware of in the literature) probably explains the
 758 undesired modification of the RC discussed in section 5.1: the calibration can
 759 only use parameters θ_{RR} and θ_{RC} (whose modification induces a systematic
 760 difference in model prediction) to minimize this conditional bias. Deriving
 761 an error model that explicitly describes the conditional bias is an important
 762 perspective in our opinion, but also a challenging one: its formulation and
 763 identifiability from the data in the absence of prior knowledge are open ques-
 764 tions. Finally, we note that this discussion has some links with the problem
 765 of describing epistemic errors with statistical models, which motivated the
 766 development of “informal” likelihoods for hydrological [61] and rating curve
 767 [39] models.

768 5.3. An alternative: propagating RC uncertainty

769 An alternative to the approach used in this paper is to propagate RC
 770 uncertainty by performing many calibrations of the RS model, with each
 771 calibration using a distinct RC. As an illustration, consider the box-plots
 772 shown in Figure 10. They have been obtained by performing 5 calibrations
 773 using the strategy NoP, where parameters θ_{RC} are fixed to 5 distinct values
 774 randomly chosen in the MCMC simulations of calibration stage 1. For the
 775 sake of simplicity, we demonstrate this approach on the basis of only one RR
 776 and RC parameter (θ_{RR} and θ_{RC} respectively). One given box-plot represents
 777 the uncertainty in RR parameter θ_{RR} , conditional on one particular RC.
 778 Merging all 5 box-plots together allows “unconditioning”, i.e. representing
 779 the total uncertainty in RR parameter θ_{RR} , given all plausible RCs. In a
 780 similar fashion, “unconditional” estimates are derived for all RR parameters
 781 θ_{RR} . This propagation approach has been used for instance by Steinbakk [55]
 782 or Petersen-Overleir [34] in a flood frequency analysis context.

783 Formally, the propagation approach leads to the following pdf represent-
 784 ing uncertainty in RR parameters θ_{RR} :

$$p_{propa}(\boldsymbol{\theta}_{RR}|\mathbf{h}) = \int p(\boldsymbol{\theta}_{RR}|\mathbf{h}, \boldsymbol{\theta}_{RC}) p(\boldsymbol{\theta}_{RC}) d\boldsymbol{\theta}_{RC} \quad (28)$$

785 By contrast, the approach used in this paper represents uncertainty in RR
786 parameters $\boldsymbol{\theta}_{RR}$ using its marginal posterior distribution, defined as follows:

$$\begin{aligned} p(\boldsymbol{\theta}_{RR}|\mathbf{h}) &= \int p(\boldsymbol{\theta}_{RR}, \boldsymbol{\theta}_{RC}|\mathbf{h}) d\boldsymbol{\theta}_{RC} \\ &= \int p(\boldsymbol{\theta}_{RR}|\mathbf{h}, \boldsymbol{\theta}_{RC}) p(\boldsymbol{\theta}_{RC}|\mathbf{h}) d\boldsymbol{\theta}_{RC} \end{aligned} \quad (29)$$

787 The difference between the two approaches appears clearly in these equations:
788 the latter uses the information contained in the stage calibration data to
789 update the inference of RC parameters (term $p(\boldsymbol{\theta}_{RC}|\mathbf{h})$ in Eq. 29), while the
790 former ignores this information and only uses the prior RC estimates (term
791 $p(\boldsymbol{\theta}_{RC})$ in Eq. 28), i.e. the RC inferred with gaugings only.

792 Future work should investigate the pros and cons of each approach. The
793 propagation approach is akin to repeating the NoP approach many times,
794 except that the RC parameters are not fixed at their maxpost estimate, but
795 are rather sampled from the prior distribution derived from the analysis of
796 rating curve data. The advantage compared to NoP is that RC parametric
797 uncertainty is not ignored. However, an obvious drawback of the propagation
798 approach is its computational cost, since a potentially costly calibration has
799 to be repeated many times.

800 5.4. Limitation of the proposed approach in terms of time steps

801 The approach proposed in this paper uses the inverse of the RC to de-
802 rive a RS model. This only makes sense if the RC is invertible, or in other
803 words, if the stage-discharge relationship can be represented by a bijective
804 function. This may not be the case under some particular circumstances
805 (e.g. hydraulic hysteresis or variable backwater effects). But even more gen-
806 erally, the stage-discharge relationship can only be represented by a bijective
807 function at a nearly-instantaneous time step. Consider for instance a given
808 daily-averaged discharge value: for this particular day, an infinity of stage
809 time series could lead to the same daily discharge. Consequently, it is not
810 possible to relate this daily discharge to a single daily stage indicator (e.g.
811 daily mean/median/etc.).

812 Consequently, the approach proposed here is restricted to time steps for
813 which the within-step variability of stage can be neglected. Whether and how
814 the approach can be extended for larger time steps remains unclear yet. A
815 possible strategy would be to define a RC between e.g. daily-averaged stage
816 and discharge, equipped with a stochastic component in order to account
817 for the non-uniqueness of the daily discharge associated with a given daily
818 stage. The variability of this stochastic component would directly depend of
819 the within-step variability of stage.

820 *5.5. Towards a complete decomposition of input/output/structural errors*

821 Deriving a complete uncertainty framework that allows explicitly repre-
822 senting all uncertainty sources remains a major challenge of hydrologic mod-
823 eling. Several methodological frameworks have been proposed for this pur-
824 pose, e.g., SODA [62], BATEA [63], Kalman and particle filters (e.g. [64, 65])
825 and many more. These methodological frameworks need to be equipped with
826 specific error models to describe the various sources of uncertainty (input,
827 output and structural errors), and realistic error models are a prerequisite
828 for a meaningful uncertainty analysis. Moreover, specifying precise and ac-
829 curate prior distributions to characterize input and output errors is another
830 prerequisite to limit the interactions between the various error sources (e.g.,
831 [6, 15]). Consequently, studies focusing on a specific uncertainty component
832 are valuable to derive realistic error models and investigate their properties.
833 For instance, previous research was devoted to investigate properties of in-
834 put errors (e.g., [13, 15]) and their impact on model calibration. Following
835 the same line of thought, we focus in this paper on rating curve output
836 errors, and their impact on model calibration. The specific error models
837 we propose could later be included into a more general framework such as
838 SODA, BATEA or a Kalman/particle filter. Finally, we stress that there is no
839 unique solution to uncertainty estimation in hydrologic modelling. Instead,
840 varied and flexible error models are necessary to adapt to the objective of
841 the study, the available information, etc. As an illustration, we note that the
842 output error model we propose in this paper requires a significant amount
843 of information (hydraulic analysis of the gauging station, gaugings and their
844 uncertainty). While this allows making valuable use of local information, it is
845 primarily adapted to the detailed analysis of a small number of catchments.
846 This information may not be available for larger-scale analyses that may in-
847 volve hundreds or thousands of catchments. In this case, an alternative error

848 model would need to be considered (see, e.g., the nonparametric discharge
849 uncertainty estimate of Vrugt et al. [62]).

850 6. Conclusions

851 In this work, we develop a Bayesian approach to probabilistically repre-
852 sent parametric and structural uncertainties of the rating curve in the esti-
853 mation of the hydrological model. To achieve this, we couple the hydrological
854 model with the inverse rating curve yielding the *rainfall-stage model* that is
855 calibrated in the stage and not in the discharge space. Such a model de-
856 scription enables us for explicitly representing and quantifying uncertainties
857 associated with both the hydrological and the rating curve model in the total
858 uncertainty of stage and discharge predictions. For a case study in France,
859 we consider six different calibration strategies with a different representation
860 level of rating curve uncertainties (parametric and/or structural). Our re-
861 sults show that a) ignoring rating curve uncertainty leads to visible changes
862 in hydrological model parameters, and b) structural uncertainty of the hy-
863 drological model dominates other uncertainty sources. The major limitation
864 of the current method arises from a strong modification of the rating curve
865 shape if rating curve parameters are re-estimated during the calibration of
866 the rainfall-stage model and unbounded prior is used. We see this problem
867 to be related to the shortcomings of the error models used to describe cor-
868 related errors of the hydrological model and structural errors of the rating
869 curve. Thus, the next step should be to test the method with a more ad-
870 vanced description of errors and/or to explore the proposed alternative of
871 propagating rating curve parametric uncertainty in more detail.

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Table 1: Calibration strategies.

| Strategy | θ_{RC} | | E_t | | $\pi(\theta_{RC})$ | |
|----------|---------------|-----|--------|------|--------------------|-----------|
| | infer | fix | active | zero | full | truncated |
| FULL | ✓ | | ✓ | | ✓ | |
| NoS | ✓ | | | ✓ | ✓ | |
| NoP | | ✓ | ✓ | | | N/A |
| NoPNoS | | ✓ | | ✓ | | N/A |
| FULL* | ✓ | | ✓ | | | ✓ |
| NoS* | ✓ | | | ✓ | | ✓ |

Table 2: Prior distributions for rating curve parameters. The Manning–Strickler equation is a simplified version valid for wide rectangular channels.

| Control | Idealized formula | $\pi(\kappa)$ | $\pi(a)$ | $\pi(c)$ |
|-----------------------------|--|--------------------|----------------|--------------------|
| | Rectangular weir | | | |
| Control 1 natural riffle | $q = \underbrace{C_r B_w \sqrt{2g}}_a (h - \underbrace{h_0}_b) \underbrace{1.5}_c$ | $N(-0.05, 0.05^2)$ | $N(14, 5^2)$ | $N(1.5, 0.025^2)$ |
| | Manning–Strickler equation | | | |
| Control 2 main channel | $q = \underbrace{K_S B_w \sqrt{S}}_a (h - \underbrace{h_0}_b) \underbrace{5/3}_c$ | $N(0.1, 0.05^2)$ | $N(20, 5^2)$ | $N(1.67, 0.025^2)$ |
| Control 3 floodplain | Manning–Strickler equation | $N(1.2, 2^2)$ | $N(25, 7.5^2)$ | $N(1.67, 0.025^2)$ |

Table 3: HBV parameters being inferred during calibration and their prior.

| Parameter | Significance [unit] | Prior min | Prior max |
|-----------|---|-----------|-----------|
| PERC | Percolation threshold parameter [$mm h^{-1}$] | 0 | 2 |
| UZL | Groundwater runoff threshold parameter [mm] | 0 | 100 |
| K0 | Recession coefficient of the 1st storage [h^{-1}] | 0 | 0.4 |
| K1 | Recession coefficient of the 2nd storage [h^{-1}] | 0 | 0.2 |
| K2 | Recession coefficient of the 3rd storage [h^{-1}] | 0 | 0.1 |
| MAXBAS | Length of the triangular weighing function [h] | 1 | 10 |

Table 4: Time-averaged relative contribution (in %) of each source of uncertainty.

| Prediction of Calibration strategy | stage | | | | discharge | |
|---------------------------------------|---------------|-------|---------------|-------|---------------|-------|
| | θ_{RR} | B_t | θ_{RC} | E_t | θ_{RR} | B_t |
| FULL | 6 | 81 | 5 | 8 | 8 | 92 |
| NoS | 7 | 89 | 4 | 0 | 7 | 93 |
| NoP | 6 | 88 | 0 | 6 | 6 | 94 |
| NoPNoS | 6 | 94 | 0 | 0 | 6 | 94 |
| FULL* | 5 | 87 | 2 | 6 | 6 | 94 |
| NoS* | 6 | 92 | 2 | 0 | 6 | 94 |

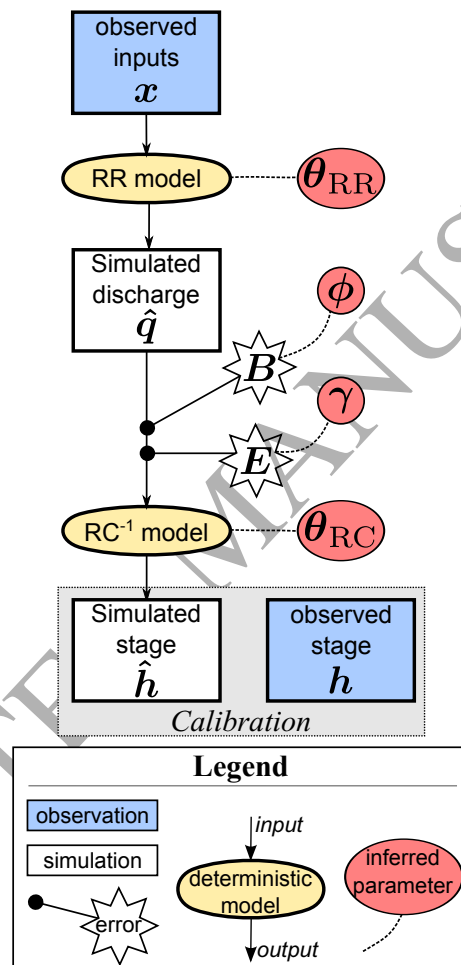


Figure 1: Schematic of the full calibration strategy.

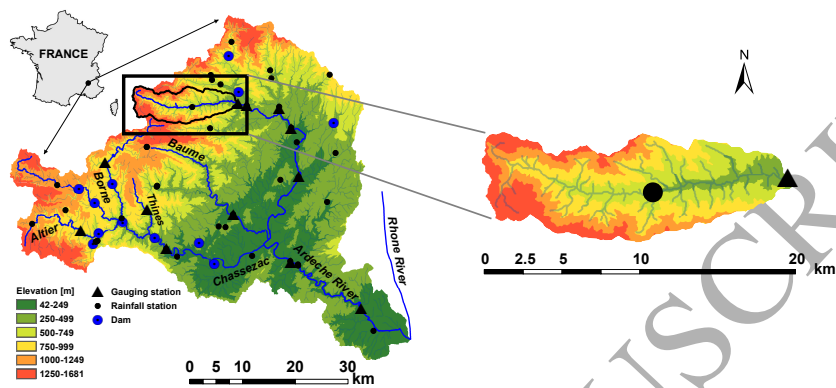


Figure 2: Overview of the catchment Ardèche. Left panel: entire catchment until its confluence with the Rhone River; right panel: Ardèche catchment at Meyras gauging station. Modified from Adamivic et al. [56].

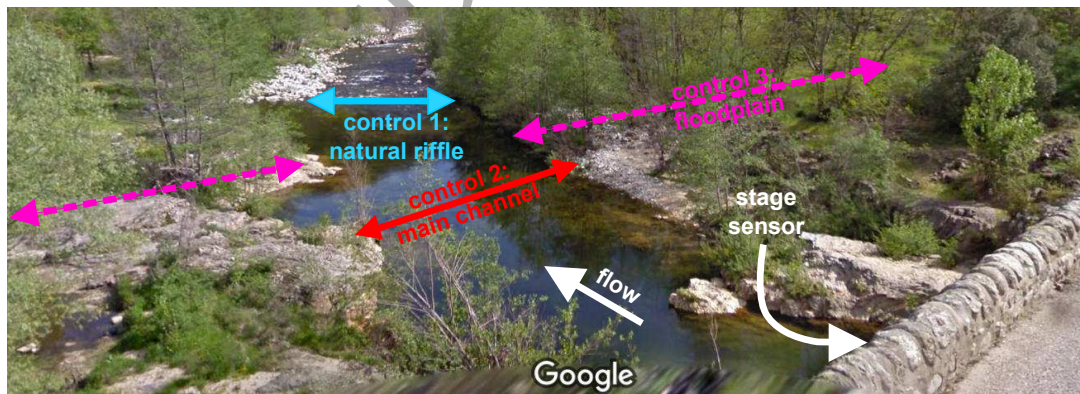


Figure 3: View of the gauging station for the Ardèche at Meyras. Picture from Google Maps, taken in April 2010.

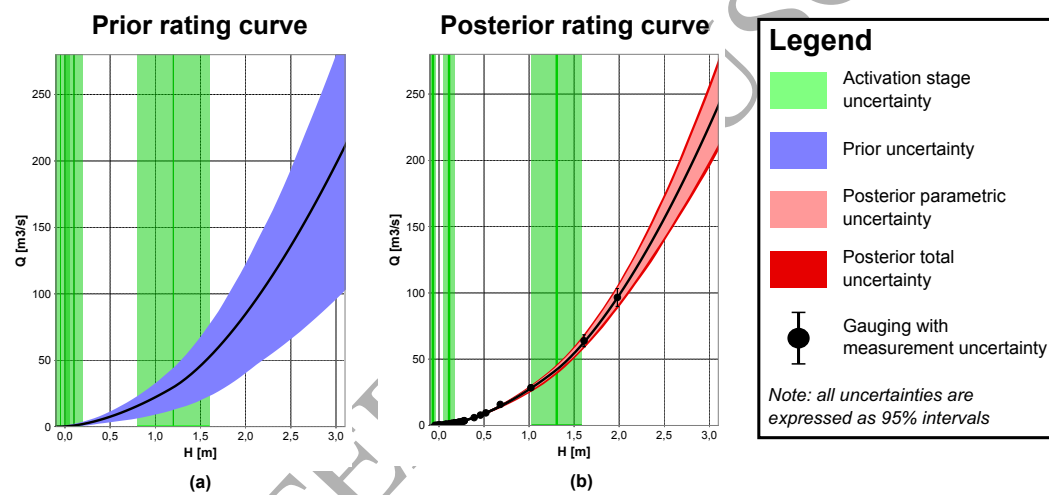


Figure 4: Prior (a) and posterior (b) rating curves in the 1st stage of calibration.

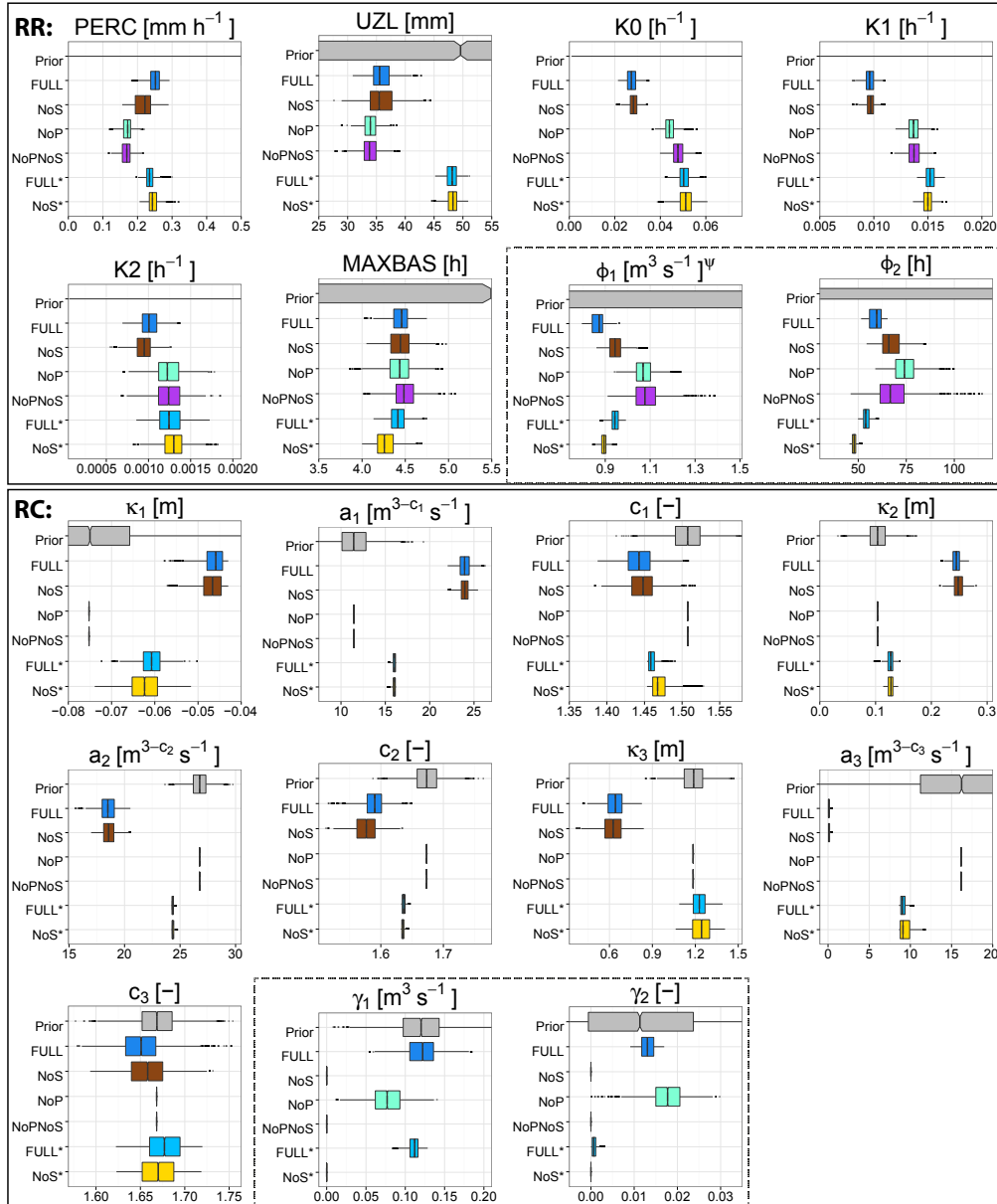


Figure 5: Calibration of the rainfall-stage model (stage 2): boxplots of prior (gray) and posterior (colored) distributions obtained with the six calibration strategies for the RR model (top panel) and RC model (bottom panel). Both error models (of RR and RC) are marked in the dashed boxes.

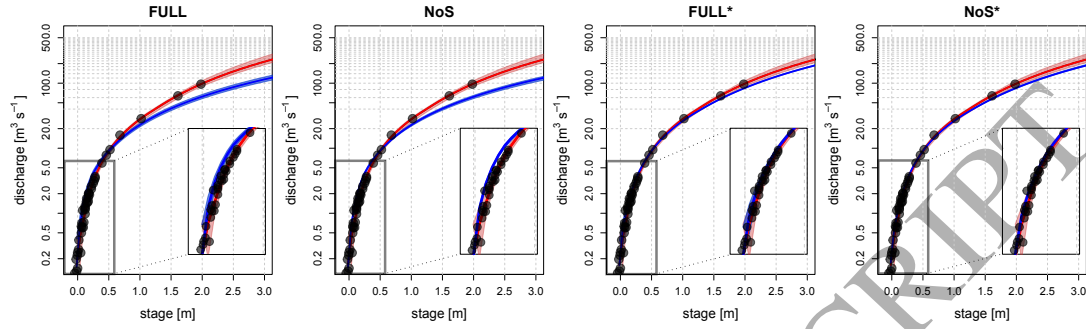


Figure 6: Comparison of rating curves before (red) and after (blue) calibration of the rainfall-stage model (stage 2) for the four calibration strategies accounting for RC parametric uncertainty.

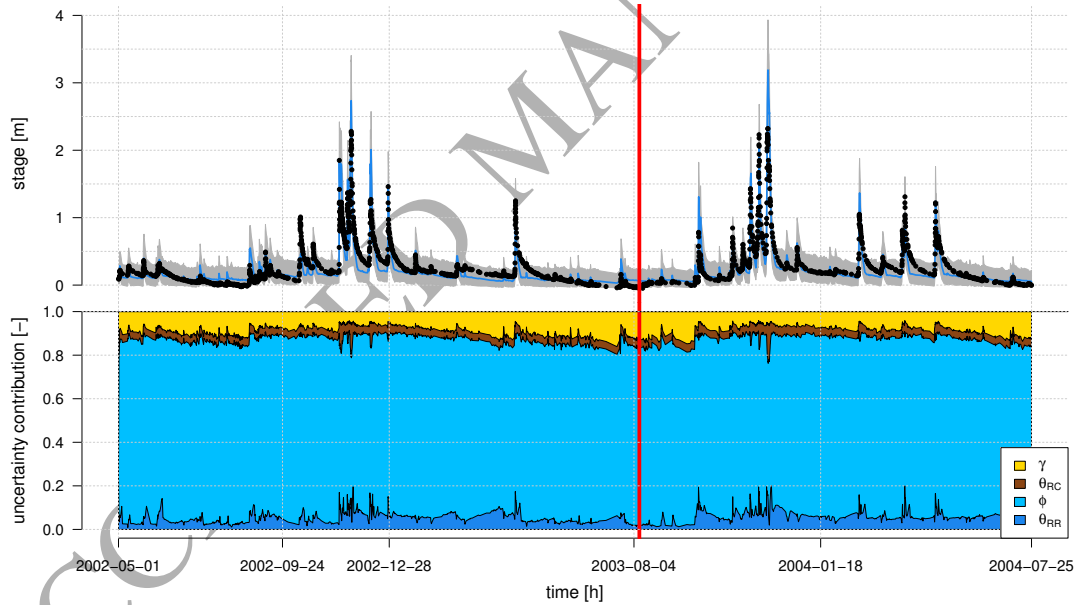


Figure 7: Stage prediction using the FULL calibration strategy. Top panel shows predicted vs. observed stage along with 95% intervals representing the total uncertainty. Bottom panel shows the relative contribution of each source of uncertainty. The calibration period is before the vertical line. Note the irregular time step of observed stages as demonstrated on the x-axis.

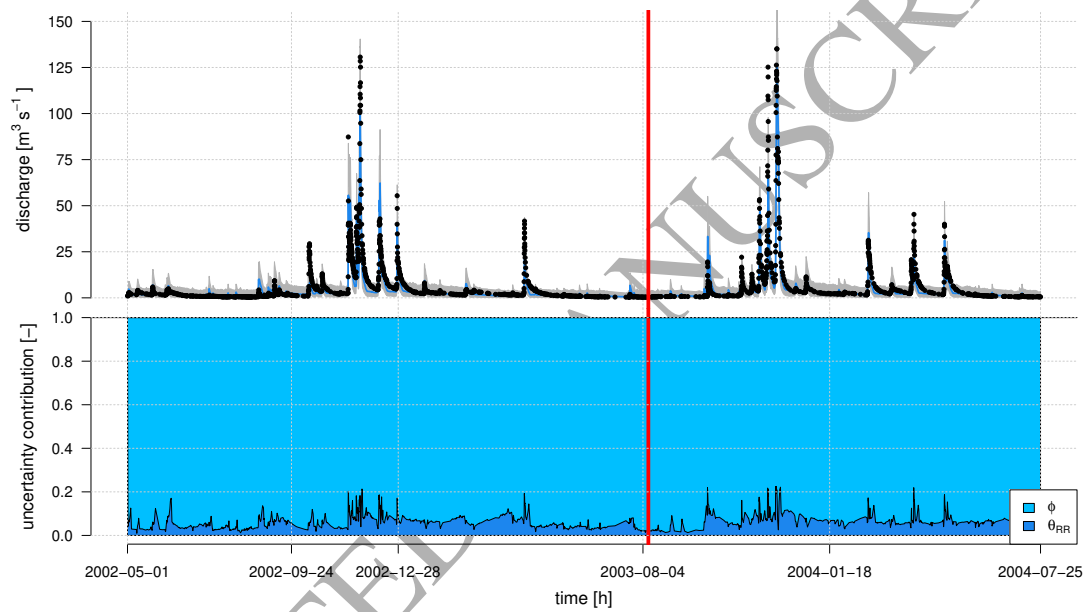


Figure 8: Discharge prediction using the FULL calibration strategy. Top panel shows predicted vs. observed discharge along with 95% intervals representing the total uncertainty. Bottom panel shows the relative contribution of each source of uncertainty. The calibration period is before the vertical line. Note that the irregular time step of discharges results from the irregular time step of observed stages.

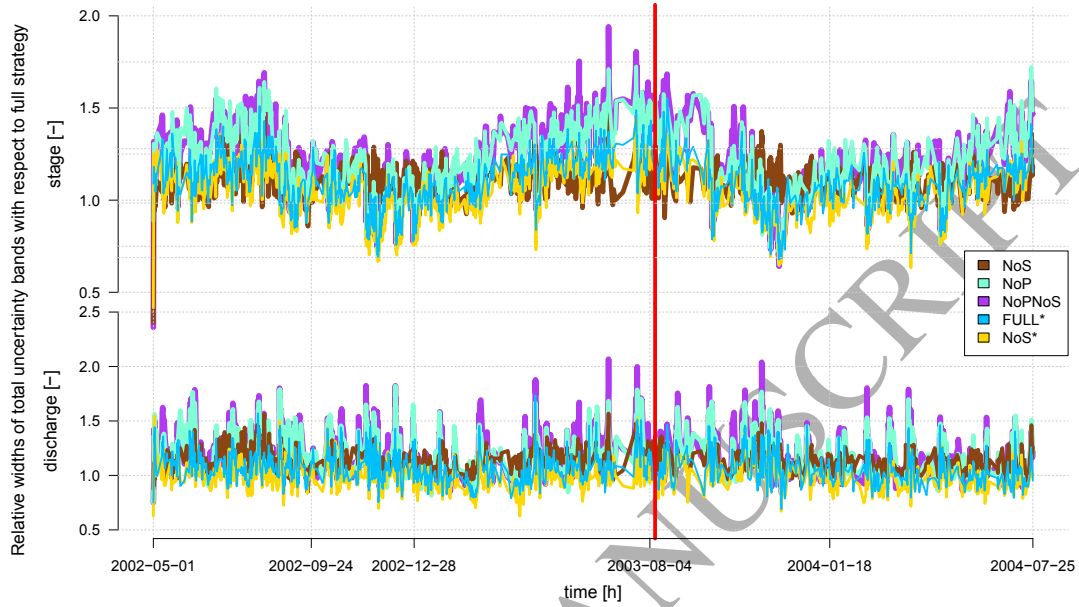


Figure 9: Comparison of total uncertainty for six calibration strategies. Each curve shows the ratio of 95% interval widths between the considered strategy and the reference strategy (FULL) in stage (top) and discharge (bottom) space. The calibration period is before the vertical line. Note the irregular time steps of both stages and discharges.

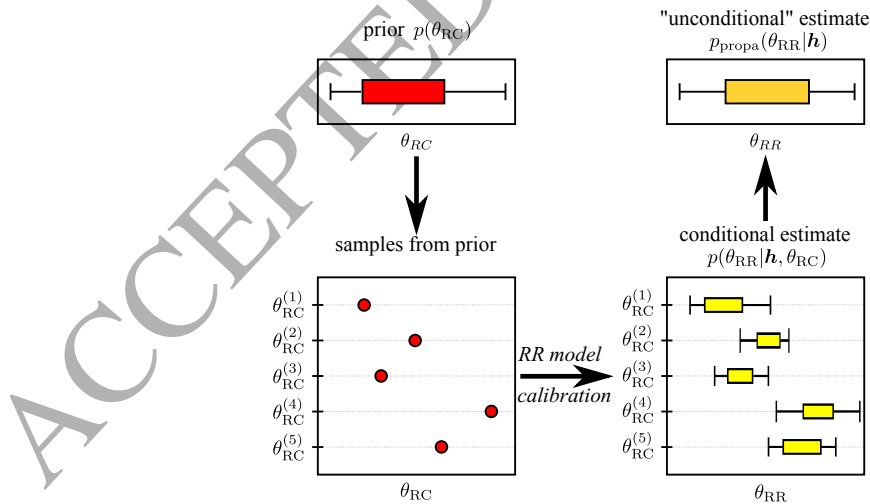


Figure 10: Schematic overview of the RC parametric uncertainty propagation approach.

1089 **Appendix A. Box-Cox transformation $\psi(\cdot)$**

1090 The Box-Cox transformation [66] with parameters λ_1 and λ_2 can be writ-
1091 ten as follows:

$$\psi(y) = \begin{cases} \frac{(y+\lambda_2)^{\lambda_1}-1}{\lambda_1} & \text{if } \lambda_1 \neq 0 \\ \ln(y+\lambda_2) & \text{if } \lambda_1 = 0 \end{cases} \quad (\text{A.1})$$

1092 Parameter $\lambda_2 \geq 0$ aims at ensuring that the term $y + \lambda_2$ remains positive.
1093 Note that for $(\lambda_1 = 1, \lambda_2 = 1)$, the Box-Cox transformation is the identity,
1094 while for $(\lambda_1 = 0, \lambda_2 = 0)$ it simplifies to a logarithmic transformation.
1095 Typically parameter λ_1 is taken between 0 and 1.

1096 The inverse of the Box-Cox transform and its derivative can be written as
1097 follows:

$$\psi^{-1}(\hat{y}) = \begin{cases} (\lambda_1 \cdot \hat{y} + 1)^{1/\lambda_1} - \lambda_2 & \text{if } \lambda_1 \neq 0 \\ \exp(\hat{y}) - \lambda_2 & \text{if } \lambda_1 = 0 \end{cases} \quad (\text{A.2})$$

$$\psi'(y) = (y + \lambda_2)^{\lambda_1 - 1} \quad (\text{A.3})$$

1098 **Appendix B. Likelihood computation for the RS model**

1099 The task is to derive the joint pdf of $(h_{t_1}, \dots, h_{t_N})$, where h_t is given by
1100 eq. 13 (recalled below in a simplified form):

$$h_t = f_{\text{RC}}^{-1}(\psi^{-1}[\psi(\hat{q}_t) - B_t] + E_t) \quad (\text{B.1})$$

1101 The first step is to use a first-order approximation of the backward transform
1102 ψ^{-1} based on a first-order Taylor expansion, whose general form can be
1103 written as:

$$f(x + e) \approx f(x) + f'(x) \times e \quad (\text{B.2})$$

1104 Applied to the function ψ^{-1} in eq. B.1, this yields:

$$\psi^{-1}[\psi(\hat{q}_t) - B_t] \approx \psi^{-1}[\psi(\hat{q}_t)] - (\psi^{-1})'[\psi(\hat{q}_t)] \times B_t \quad (\text{B.3})$$

1105 We then use here the inverse-derivative rule:

$$(\psi^{-1})'(z) = \frac{1}{\psi'(\psi^{-1}(z))} \quad (\text{B.4})$$

1106 Plugging this back into eq. B.3 yields:

$$\psi^{-1} [\psi(\hat{q}_t) - B_t] \approx \hat{q}_t - \frac{B_t}{\psi'(\psi^{-1}[\psi(\hat{q}_t)])} = \hat{q}_t - \frac{B_t}{\psi'(\hat{q}_t)} \quad (\text{B.5})$$

1107 Finally, eq. B.1 becomes:

$$h_t \approx f_{\text{RC}}^{-1} \left(\underbrace{\hat{q}_t - \frac{B_t}{\psi'(\hat{q}_t)}}_{Z_t} + E_t \right) \quad (\text{B.6})$$

1108 The second step is to deduce the joint pdf of $(h_{t_1}, \dots, h_{t_N})$ from that of
 1109 $(Z_{t_1}, \dots, Z_{t_N})$. We use the change-of-variables formula for this purpose, which
 1110 can be written in general terms as follows. Let $\mathbf{y} = (y_1, \dots, y_N) = \mathbf{r}(x_1, \dots, x_N)$,
 1111 where \mathbf{r} is a one-to-one transformation. The pdf of \mathbf{y} can be deduced from
 1112 the pdf of \mathbf{x} using the following formula:

$$p_{\mathbf{y}}(y_1, \dots, y_N) = p_{\mathbf{x}}(\mathbf{r}^{-1}(\mathbf{y})) |\det(J_{\mathbf{r}^{-1}}(\mathbf{y}))| \quad (\text{B.7})$$

1113 where $J_{\mathbf{r}^{-1}}(\mathbf{y})$ is the $N \times N$ Jacobian matrix (partial derivatives) of the in-
 1114 verse transform \mathbf{r}^{-1} .

1115 Applying the change-of-variables formula above to the transformation $(h_{t_1}, \dots, h_{t_N}) =$
 1116 $(f_{\text{RC}}^{-1}(Z_{t_1}), \dots, f_{\text{RC}}^{-1}(Z_{t_N}))$ yields the following formula:

$$\begin{aligned} p_{\mathbf{h}}(h_{t_1}, \dots, h_{t_N}) &= p_{\mathbf{Z}}(f_{\text{RC}}(h_{t_1}), \dots, f_{\text{RC}}(h_{t_N})) \left| \det \begin{pmatrix} f'_{\text{RC}}(h_{t_1}) & & 0 \\ & \dots & \\ 0 & & f'_{\text{RC}}(h_{t_N}) \end{pmatrix} \right| \\ &= p_{\mathbf{Z}}(f_{\text{RC}}(h_{t_1}), \dots, f_{\text{RC}}(h_{t_N})) \prod_{k=1}^N |f'_{\text{RC}}(h_{t_k})| \end{aligned} \quad (\text{B.8})$$

1117 which corresponds to the likelihood function from section 3.2 (eq. 24).