# A state-space representation of the GR4J rainfall-runoff model

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## Objectives

- ✗ Harmonize the mathematical equations of the GR4J model (Perrin *et al.* , 2003) within a state-space representation
- ✗ Replace the sequential resolution of equations by a global resolution with adaptative time-stepping
- ✗ Represent the unit hydrographs as a state variables model

### Constraints

- ✗ Keep similar performances
- ✗ Keep a lumped model with four free parameters

## 1. Structural modifications

The two unit hydrographs cannot be written as state space variable model. The following options were chosen:

- ✗ Substitute the two unit hydrographs by a "Nash Cascade" (Nash, 1957)
- ✗ Place it before the split of flow components

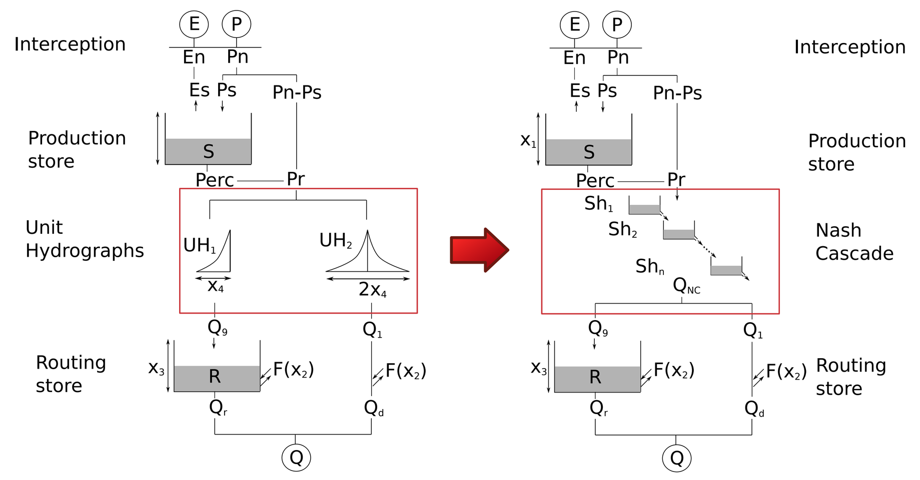


Fig. 1: Substitution of the unit hydrographs of the original GR4J model (left) by a "Nash cascade" for the state-space model (right)

## 2. Mathematical modifications

- ✗ All the state variables are expressed as $\dot{v} = f(v, u)$ ($v$ is the states vector and $u$ the inputs) and merged in one system
- ✗ The "Nash cascade" is formulated with eleven stores and an outflow coefficient dependent on the GR4J time base ($x_4$) parameter (to easily compare it with $UH_2$)

The state-space formulation can be written as:

$$\begin{pmatrix} \dot{S} \\ \dot{S}_{h1} \\ \dot{S}_{h2} \\ \vdots \\ \dot{S}_{hn} \\ \dot{R} \end{pmatrix} = \begin{pmatrix} -\frac{x_1^{1-\beta}}{U_t(\beta-1)}\nu^{\beta-1}S(t)^\beta + (E_n - P_n)\left(\frac{S(t)}{x_1}\right)^\alpha - 2E_n\frac{S(t)}{x_1} + P_n \\ P_n\left(\frac{S(t)}{x_1}\right)^\alpha + \frac{x_1^{1-\beta}}{(\beta-1)U_t}\nu^{\beta-1}S(t)^\beta - \frac{n-1}{x_4}S_{h,1}(t) \\ \frac{n-1}{x_4}S_{h,1}(t) - \frac{n-1}{x_4}S_{h,2}(t) \\ \vdots \\ \frac{n-1}{x_4}S_{h,n-1}(t) - \frac{n-1}{x_4}S_{h,n}(t) \\ \Phi\frac{n-1}{x_4}S_{h,n}(t) - \frac{x_3^{1-\gamma}}{(\gamma-1)U_t}R(t)^\gamma + \frac{x_2}{x_3^\omega}R(t)^\omega \end{pmatrix}$$

Greek letters are fixed parameters, latin letters are model states and inputs, $x_n$ are free parameters

## 3. Evaluation methodology

- ✗ 650 French catchments to get general conclusions
- ✗ Calibration of the models using the $KGE'$ as an objective function
- ✗ Comparison of performances, output hydrographs, parameter values and internal fluxes
- ✗ Tests at daily and hourly time steps

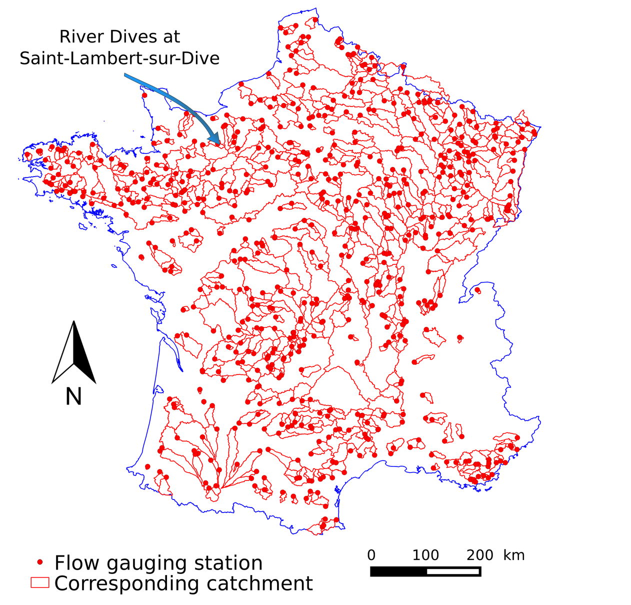


Fig. 2: Locations of the 650 test catchments

## 4. Daily models results

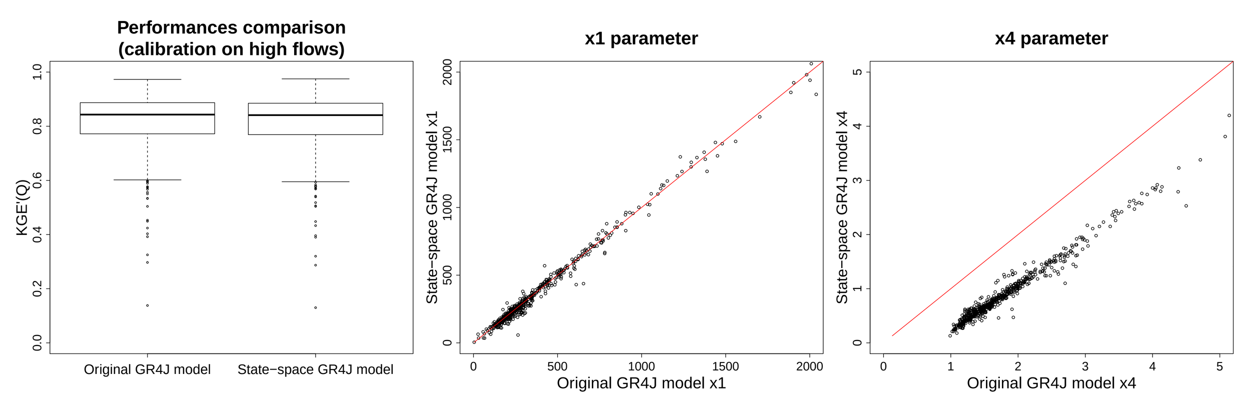


Fig. 3: $KGE'$ and parameter values of the state-space model compared to the original GR4J

- ✗ Very similar performances and parameter values (except $x_4$)
- ✗ On all the catchments hydrographs, the peak flows are lower for the state-space representation
- ✗ Discrepancies in the internal fluxes (see figure 6)

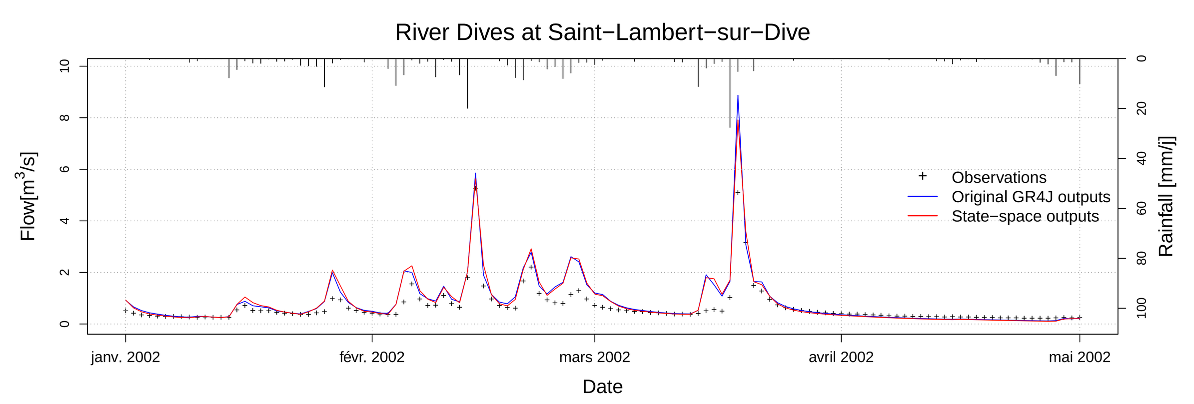


Fig. 4: Simulated hydrographs of the River Dives in winter and spring 2002

### References


Ficchì, Andrea, Perrin, Charles, & Andréassian, Vazken. 2016. Impact of temporal resolution of inputs on hydrological model performance: An analysis based on 2400 flood events. *Journal of Hydrology*, **538**(Jul), 454–470.

Nash, J. E. 1957. The form of the instantaneous unit hydrograph. *Int. Assoc. Sci. Hydrol. Publ.*, **45**(3), 114–121.

Perrin, Charles, Michel, Claude, & Andréassian, Vazken. 2003. Improvement of a parsimonious model for streamflow simulation. *Journal of Hydrology*, **279**(1-4), 275–289.


## 5. Temporal consistency

- ✗ For the original GR4J model, time step changes are made possible by the changes in parameters values (Ficchì *et al.* , 2016)
- ✗ State space model parameter values are stable, time step changes are managed by the integration of the differential equations

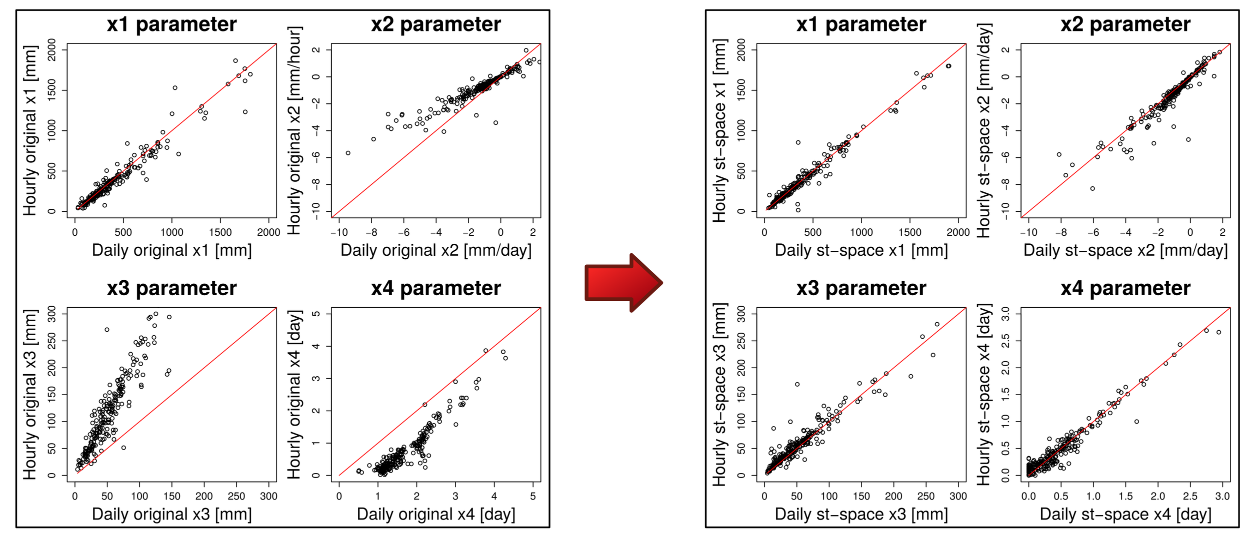


Fig. 5: Scatter plots of the four daily and hourly parameters of the original GR4J (left) and the state-space representation (right)

## 6. Discrepancies in internal fluxes

- ✗ High exchange values occur sooner after rainfall in the state-space model
- ✗ Calibrated $x_4$ parameters create faster and higher response from the "Nash cascade" than with the unit hydrograph in the original GR4J
- ✗ These two patterns seem related
- ✗ No real consequences on performances

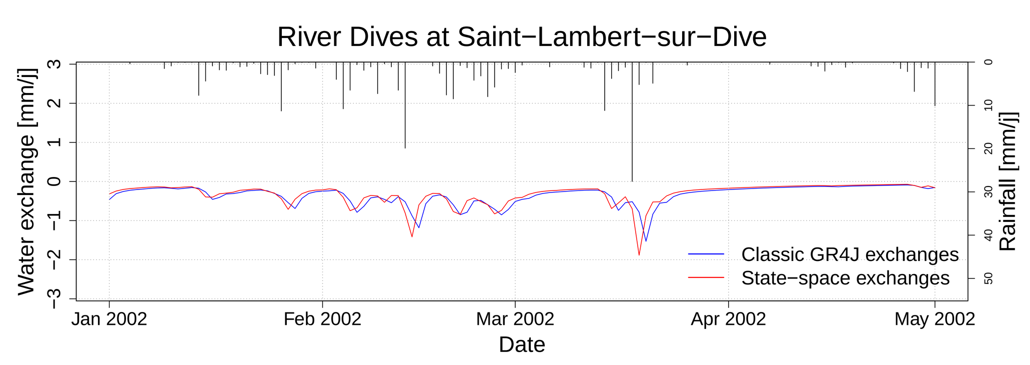


Fig. 6: Simulated water exchanges of the River Dives in winter and spring 2002

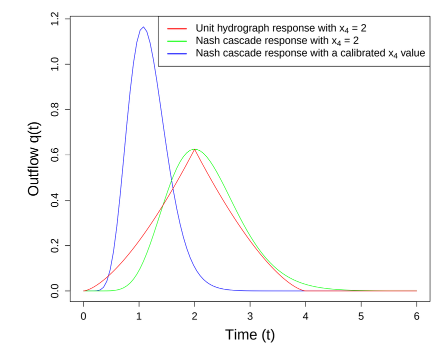

Fig. 7: Comparison of the impulse responses of $UH_2$ and a 11-stores "Nash cascade"

## Conclusion

- ✗ Mathematically more uniform and continuous version of the GR4J model
- ✗ This version should not substitute the original GR4J model, as it does not outperform the original model and have a higher computational time
- ✗ It could be useful for specific applications like time variable modelling, data assimilation or multimodel approaches (see poster EGU2017-4093 Friday, 28 Apr, 17:30–19:00)