A conceptual snow model with an analytic resolution of the heat and phase change equations

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Context

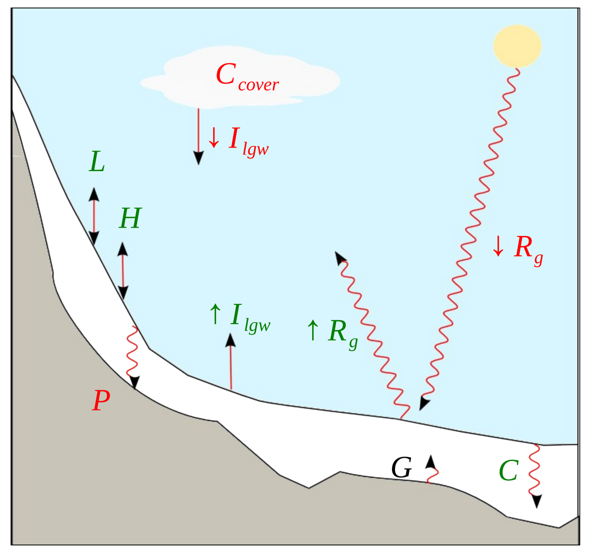
Snow models are often designed to simulate snowmelt and river discharge when coupled to a rainfall-runoff model, **but few of them simulate correctly the snow water equivalent (SWE) at point scale**. Tackling this flaw could have several advantages: improving the model reliability and performance for short- and long-term prediction, permitting spatial regionalization, and allowing data assimilation of observed snow measurements.

The model developed in this study is designed to have the following characteristics:

1. Be **parsimonious** and be coupleable to different hydrological models, using only **temperature and precipitation** as inputs.
2. Be able to simulate **the snowpack (SWE and snow depth)** without worsening discharge simulations when coupled to a hydrological model.
3. Have state variables directly comparable to observations.

The model structure is based on a **simplified energy balance**. It is structured in two separate sections:

- **Atmospheric forcing**: parameterization of the atmospheric downward radiation
- **The snowpack response**: modelling of the conduction of energy into the snowpack, snowpack compaction and albedo, modelling of the surface temperature



Schematic representation of energy balance for snow, divided in two main sections: atmospheric forcings (red) and snowpack retro action (green)

Dataset

- **The snow model has been tested on the Col de Porte experimental station** located in the French Alps (Morin et al., 2012).
- Meteorological hourly measurements: air temperature, humidity, direct and scattered solar radiation, downward longwave radiation, atmospheric pressure, precipitations, wind speed.
- Snow measurements: snow surface temperature, snow profile temperature, snow depth, SWE, surface albedo

View of the Col de Porte experimental station (Morin et al., 2012)

Conduction into the snowpack

About snow properties:

- Snow is a **poor thermal conductor**
- This induces **strong vertical temperature gradients** in the snowpack
- Snowmelt is determined by the snowpack ripening that depends on these temperature gradients

Analytical solutions for heat equation and phase change have been used in order to estimate temperature in the snowpack. It has the advantage of giving simulations for any time step and any depth steps.

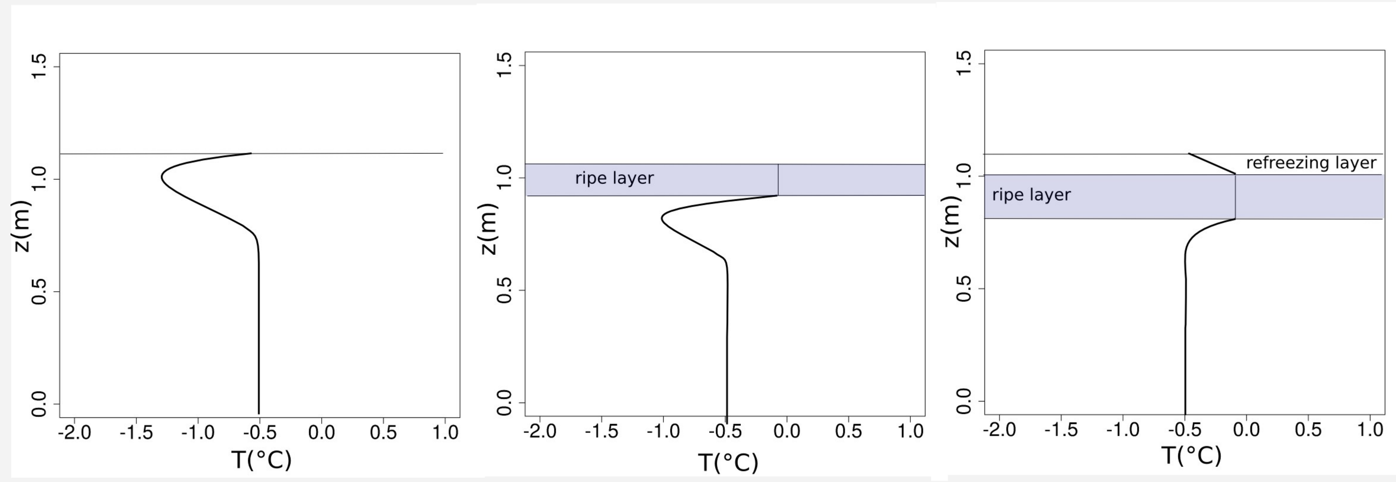
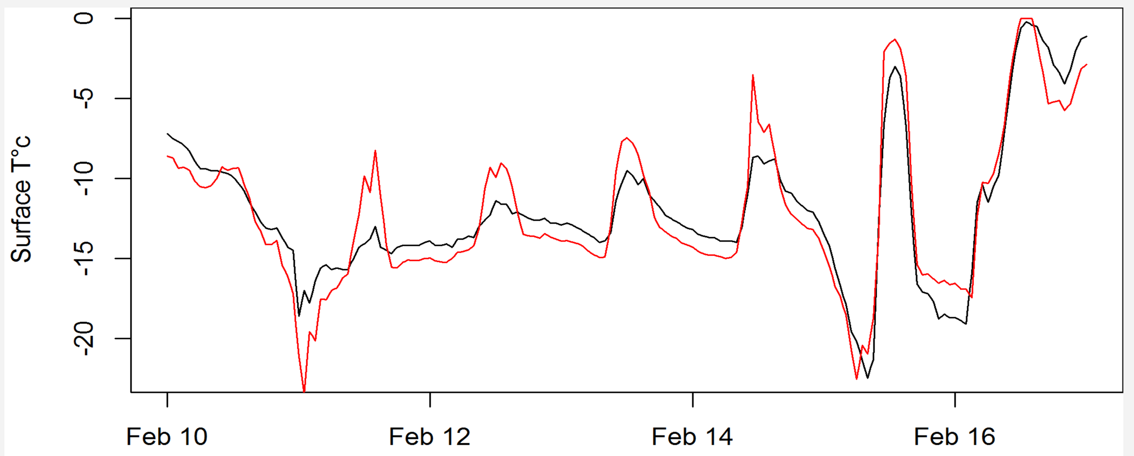


Diagram of the three possible patterns of the model: pure conduction, ripe layer, and refreezing + ripe layer

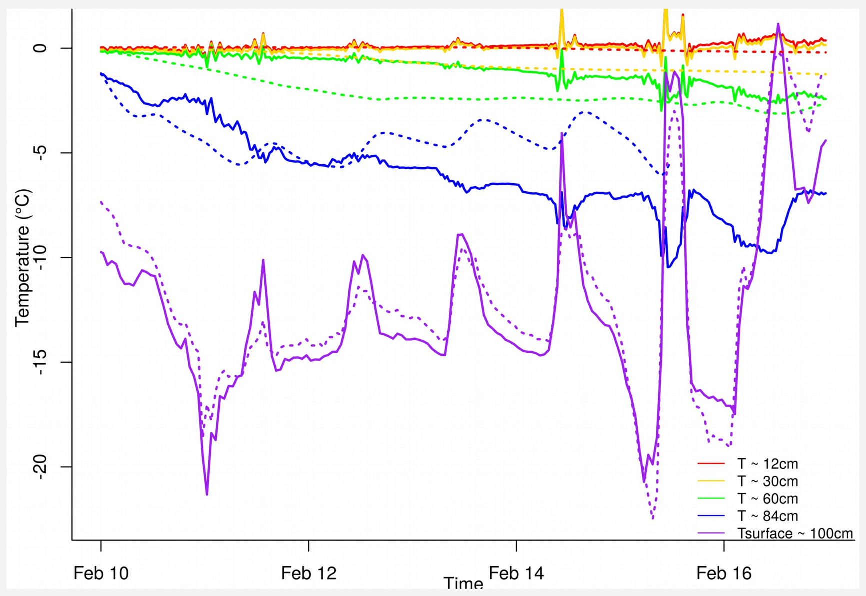
The solution to the heat equation between the boundaries of the snowpack (base and surface) has been found by **linearizing radiative fluxes** and using **dimensionless variables**. The first step for linearizing the radiative fluxes is to **estimate the surface temperature**. At first, the conductive flux is neglected and the albedo is fixed at a reference value (α_{ref}). The value of the surface temperature is solved in order to allow the incoming radiative fluxes to compensate the outgoing energy fluxes (which depend on the surface temperature).

$$\phi_\downarrow(t)=\phi_\uparrow(T_{guess},\alpha_{ref},t)$$



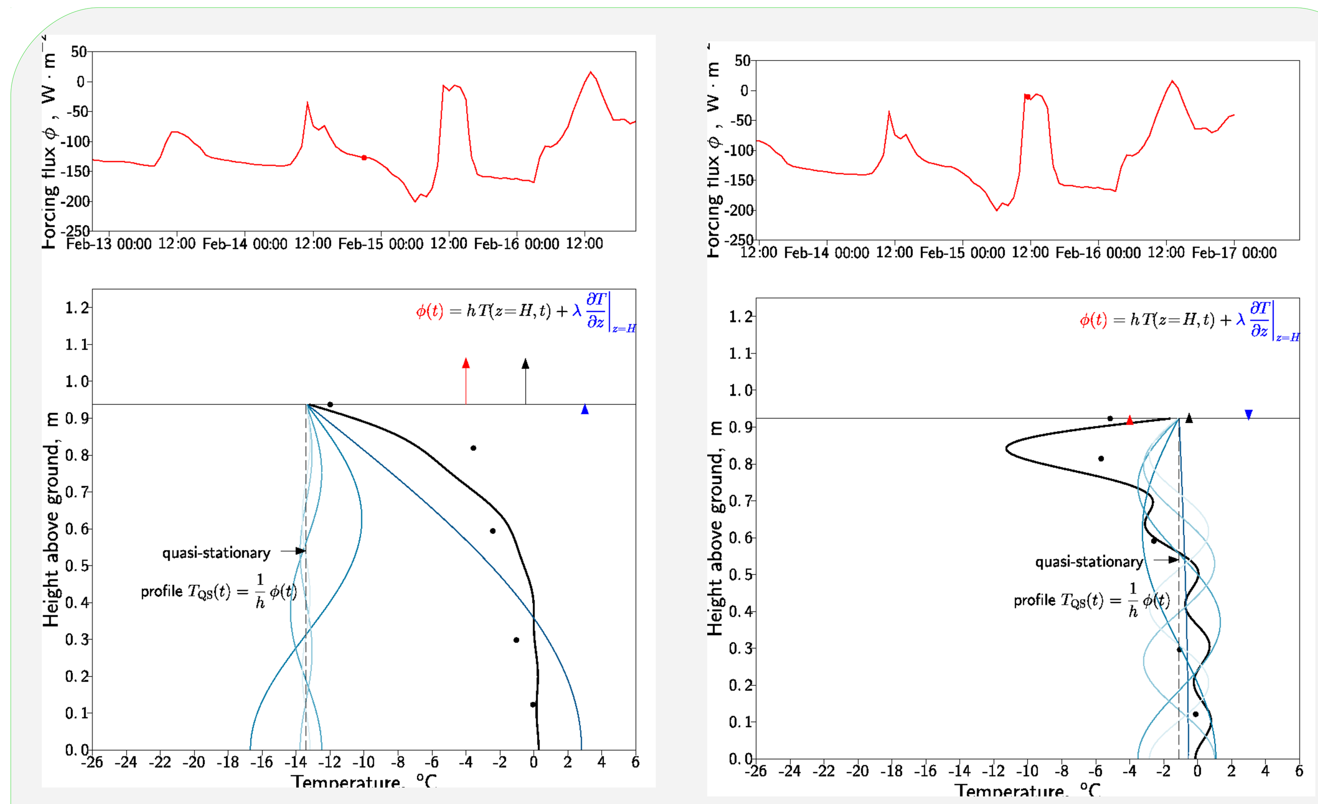
Estimation of the surface temperature (in red) and observed surface temperature (in black) between February 10 and 17 at Col de Porte.

Then, the simulated albedo and the conductive fluxes are used to correct the surface temperature value. The heat diffusion has been modelled using eigenvalues and eigenfunctions from Sturm-Liouville theory. The state variables of this model are: **SWE**, vertically-averaged density, thickness of ripe layer (=**latent heat content**), and **sensible heat content** (=temperature pattern).



Time series simulations of the snowpack temperature between February 10 and 17 2010. Simulated values are in solid lines, measured in dotted lines

Temperature profile of the snowpack



Temperature profile of the snowpack for February 14 at 2 pm (left) and February 15 at 11 am (right). Dots correspond to measured temperatures. The time series in red correspond to the surface atmospheric forcing, independent of snowpack state (and especially of surface temperature).

The linear combination of a **small number** of eigenfunctions allows to compute an **approximate temperature profile** of the snowpack (black). The amplitude of each eigenfunction becomes a state variable, thus leading to a parsimonious representation of temperature patterns in the snowpack. Values are close to the observations (RMSE ~ 1°C). These simulations show that the model is **able to reproduce strong temperature gradients.**

Conclusion and perspectives

In this case study at Col de Porte, the snowpack thermal model has been forced with observed forcing fluxes (LWdown, SWnet, Qair).
Coming work:

- **The modeled atmospheric forcing will be coupled with the snowpack response model.**
- **A complete analysis of the model response to the forcing will be done.**

The complete snow model should run at basin scale, using snow and discharge measured data for calibration.

References

Morin, S., Y. Lejeune, B. Lesaffre, J.-M. Panel, D. Poncet, P. David, and M. Sudul (2012), An 18-yr long (1993–2011) snow and meteorological dataset from a mid-altitude mountain site (Col de Porte, France, 1325 m alt.) for driving and evaluating snowpack models, Earth System Science Data, 4(1), 13–21, doi:10.5194/essd-4-13-2012.

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