Coupling experimental and computational fluid dynamics: Synopsis of approaches, issues and perspectives

Dominique Heitz

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Fluminance team, Irstea, IRMAR and Inria of Rennes, France
ACTA team leader, Irstea, Rennes, France

2nd Workshop on Data Assimilation and CFD Processing Techniques
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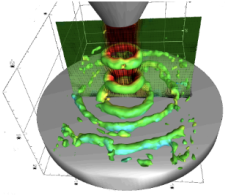
Confronting EFD and CFD is inherent of fluid mechanics approach

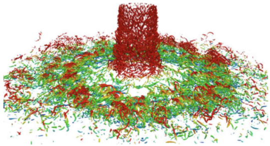
TomoPIV (Irstea)



Experiments

- LDV as a reference
- HWA → very good
- PIV → good

DNS (Dairy *et al.*,2015)

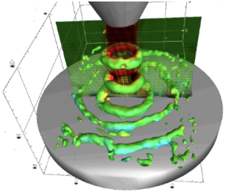


Numerical simulations

- DNS as a reference → numerical wind tunnel
- A priori parameter calibration
- A posteriori simulation validation

EFD and CFD limitations

TomoPIV (Irstea)

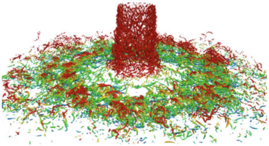


Experiments

- HWA and LDV → pointwise
- PIV → large scale
- TomoPIV → very large scale

⇒ sparse data

DNS (Dairy *et al.*,2015)



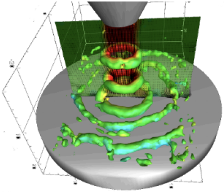
Numerical simulations

- Initial conditions
- Boundary conditions
- Turbulence model and parameters

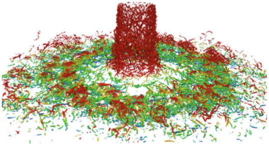
⇒ non "realistic" simulations

Coupling EFD and CFD with data assimilation

`TomoPIV (Irstea)`



`DNS (Dairy` *et al.*,2015`)`



Objective

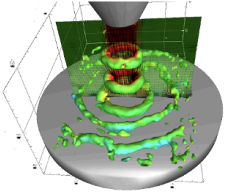
- Estimation of the unknown true state of interest $\mathbf{x}(t, x)$
- Recover as accurately as possible the state of the fluid flow using all available information

Question: how to do that ?

Outline

Data assimilation ingredients

TomoPIV (Irstea)



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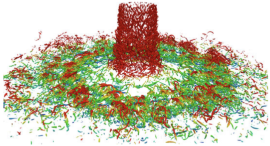


Experiments

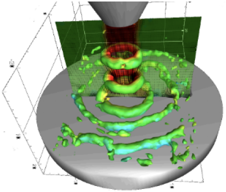
- Observation model
  $\mathcal{Y}(t,x) = \mathbb{H}(\mathbf{x}(t,x)) + \varepsilon(t,x)$

Numerical model

- Dynamical model
  $\partial_t \mathbf{x}(t,x) + \mathbb{M}(\mathbf{x}(t,x)) = \mathbf{q}(t,x)$
- Prior knowledge model
  $\mathbf{x}(t_0,x) = \mathbf{x}_0^b + \boldsymbol{\eta}(x)$

Data and dynamics dimensions

TomoPIV (Irstea)



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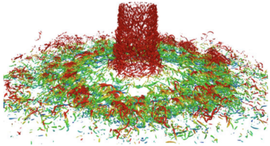
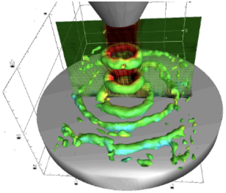


Data and model resolution: d vs m

- Geosciences $d << m$
- PIV $d \leq m$ or $d << m$
  - Model resolution: ROM vs DNS
  - Laboratory vs Industrial processes
  - 2D vs 3D
  - Reynolds

Data assimilation: observation and dynamics models

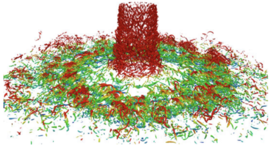
TomoPIV (Irstea)



$$\mathcal{Y}(t,x) = \mathbb{H}(\mathbf{x}(t,x)) + \varepsilon(t,x)$$

- Pseudo observation → velocity, vorticity, lagrangian acceleration, thus $\mathcal{Y}(t,x) = \hat{\mathbf{x}}(t,x)$ and $\mathbb{H} = \mathbb{I}$
- Observation → images of particles, scalar (smoke, gaz, temperature), thus $\mathcal{Y}(t,x) = I(t,x)$ and $\mathbb{H}$ can be nonlinear
- Eulerian or Lagrangian

DNS (Dairy *et al.*,2015)

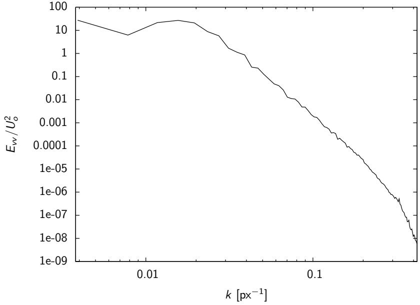


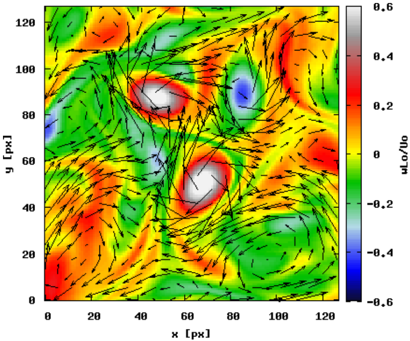
$$\partial_t \mathbf{x}(t,x) + \mathbb{M}(\mathbf{x}(t,x)) = \mathbf{q}(t,x)$$

- Eulerian: ROM, Vortex particle, Lattice Boltzman, RANS, LES, DNS
- Lagrangian: Smooth Particule Hydrodynamics (SPH)

Data assimilation ideal case

Papadakis & Mémin (2008) - Heitz *et al.* (2010)





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- Pseudo observation $\rightarrow$ velocity, thus $\mathcal{Y}(t,x) = \hat{\mathbf{x}}(t,x)$ and $\mathbb{H} = \mathbb{I}$

$$\partial_t \mathbf{x}(t,x) + \mathbb{M}(\mathbf{x}(t,x)) = 0$$

- DNS of 2D IHT at $Re = 256$
- Resolution : 256×256

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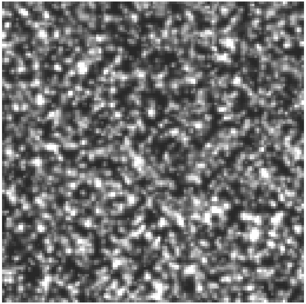

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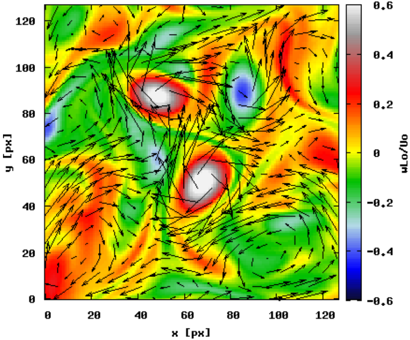

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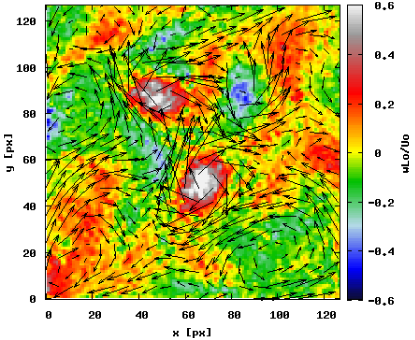

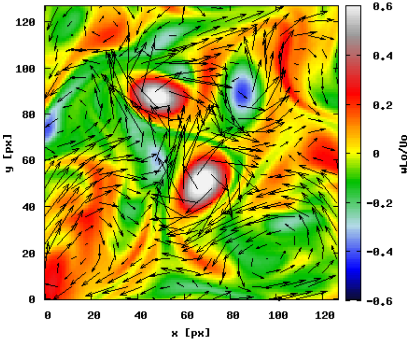

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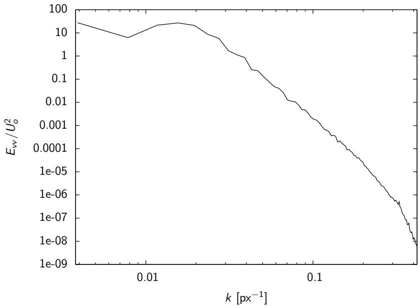

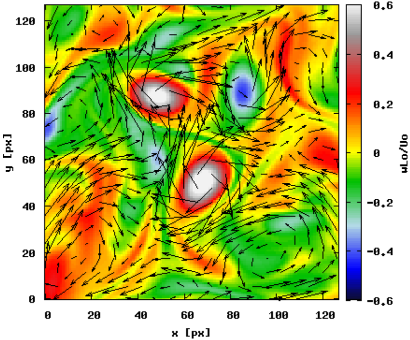

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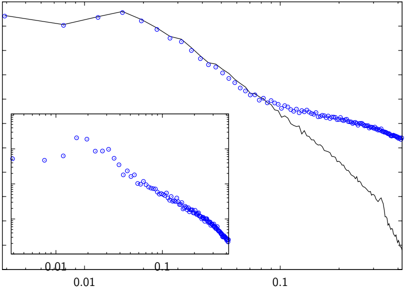

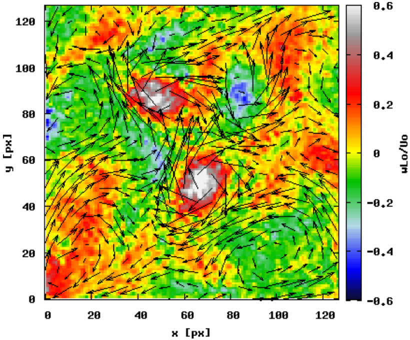

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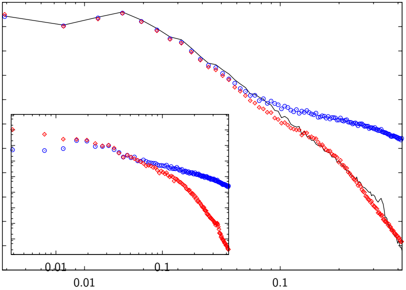

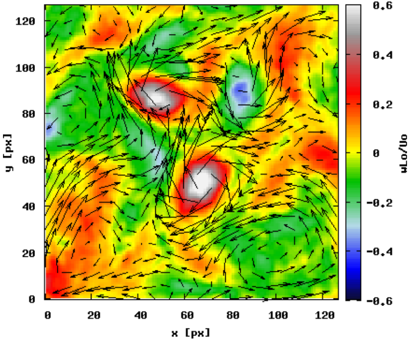

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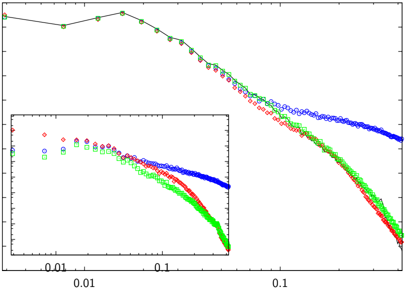

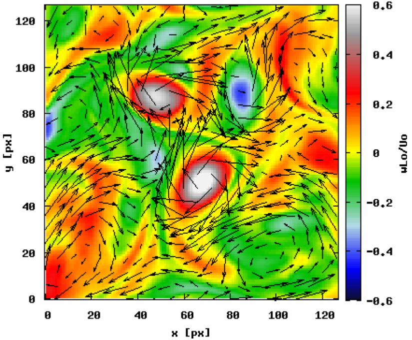

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Outline

Data assimilation: the state estimation problem

Ingredients

- Observation model
  $\mathcal{Y}(t,x) = \mathbb{H}(\mathbf{x}(t,x)) + \varepsilon(t,x)$
- Dynamical model
  $\partial_t \mathbf{x}(t,x) + \mathbb{M}(\mathbf{x}(t,x)) = \mathbf{q}(t,x)$
- Prior knowledge model
  $\mathbf{x}(t_0,x) = \mathbf{x}_0^b + \boldsymbol{\eta}(x)$

→ Random nature of observation, dynamic and knowledge errors described in term of pdf

Bayesian formulation

$$p(\mathbf{x}|\mathcal{Y}) = \frac{p(\mathcal{Y}|\mathbf{x})p(\mathbf{x})}{p(\mathcal{Y})}$$

$$p(\mathbf{x}|\mathcal{Y}) \propto p(\mathcal{Y}|\mathbf{x})p(\mathbf{x})$$

$$\textit{posterior} \propto \textit{likelihood} \times \textit{prior}$$

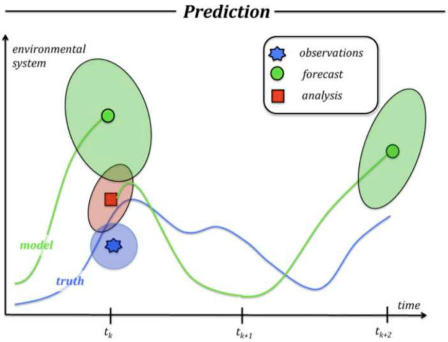
$$\textit{estimation} \propto \textit{observations} \times \textit{knowledge}$$

Prior distribution:

- Prevents from over-fitting
- Introduce past information
- Good prior not straightforward

Data assimilation: the state estimation problem

Carassi *et al.* (2017)



Information: past

→ For the control?

Bayesian formulation

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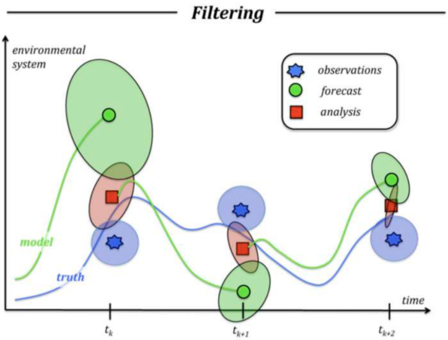

Information: past and present

$\rightarrow$ Sequential processing providing discontinuous trajectories

Bayesian formulation

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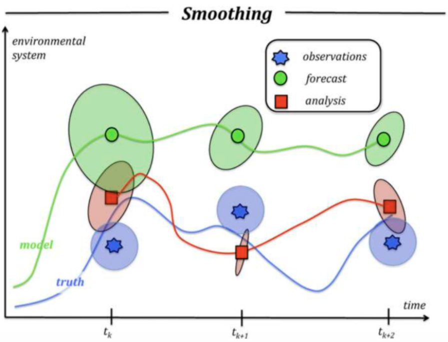

Information: past, present and future

→ Relevant for reconstruction or reanalysis and for model parameters estimation

Bayesian formulation

$$p(\mathbf{x}|\mathcal{Y}) = \frac{p(\mathcal{Y}|\mathbf{x})p(\mathbf{x})}{p(\mathcal{Y})}$$

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Data assimilation: the state estimation problem

Computational problem

- Huge dimension of data and models prevent use of fully Bayesian approach
- Difficulty to define and transport the pdfs

Solution to overcome this issue

- Uncertainties of observations, model and prior are assumed Gaussian
- Pdfs completely described by first and second moments (i.e mean and covariance matrix)

Bayesian formulation

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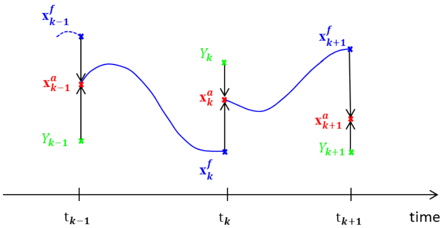
Data assimilation: Kalman filter



Properties

- Obs. and dynamics linear
- Noises Gaussian, unbiased, white-in-time

$\rightarrow$ Time dependent prior (mean, cov.)

$\rightarrow$ Comput. cost. of K and P

Main algorithm

1. Forecast step

$$\mathbf{x}_k^{\mathrm{f}} = \mathbf{M}_{k:k-1}\mathbf{x}_{k-1}^{\mathrm{a}},$$

$$\mathbf{P}_k^{\mathrm{f}} = \mathbf{M}_{k:k-1}\mathbf{P}_{k-1}^{\mathrm{a}}\mathbf{M}_{k:k-1}^{\mathrm{T}} + \mathbf{Q}_k.$$

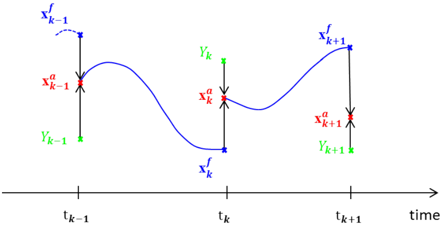
2. Analysis step

$$\mathbf{K}_k = \mathbf{P}_k^{\mathrm{f}}\mathbf{H}_k^{\mathrm{T}}(\mathbf{H}_k\mathbf{P}_k^{\mathrm{f}}\mathbf{H}_k^{\mathrm{T}} + \mathbf{R}_k)^{-1},$$

$$\mathbf{x}_k^{\mathrm{a}} = \mathbf{x}_k^{\mathrm{f}} + \mathbf{K}_k(\mathbf{y}_k - \mathbf{H}_k\mathbf{x}_k^{\mathrm{f}}),$$

$$\mathbf{P}_k^{\mathrm{a}} = (\mathbf{I}_k - \mathbf{K}_k\mathbf{H}_k)\mathbf{P}_k^{\mathrm{f}}.$$

Data assimilation: Kalman filter



Properties

- Obs. and dynamics linear
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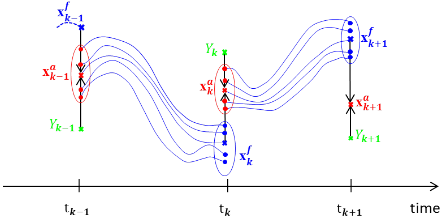
$\rightarrow$ Time dependent prior (mean, cov.)

$\rightarrow$ Comput. cost. of K and P

Alternative approaches

- Extended Kalman Filter (EKF)
  $\rightarrow$ H and M linearized
- Sub Optimal Filter (SOS)
  $\rightarrow$ Reduce comput. cost H

Data assimilation: Kalman filter



Properties

- Obs. and dynamics linear
- Noises Gaussian, unbiased, white-in-time

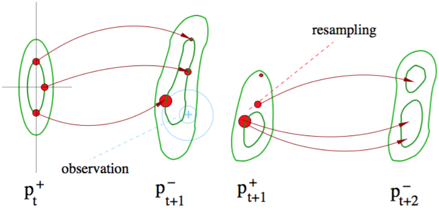
→ Time dependent prior (mean, cov.)

→ Comput. cost. of K and P

Alternative approaches

- Extended Kalman Filter (EKF)
  → H and M linearized
- Sub Optimal Filter (SOS)
  → Reduce comput. cost H
- Ensemble Kalman Filter (EnKF)
  → Empirical estimation of P
  → H and M non linear

Data assimilation: Kalman filter



From Boquet's lecture notes (2014-2015)

Properties

- Obs. and dynamics linear
- Noises Gaussian, unbiased, white-in-time

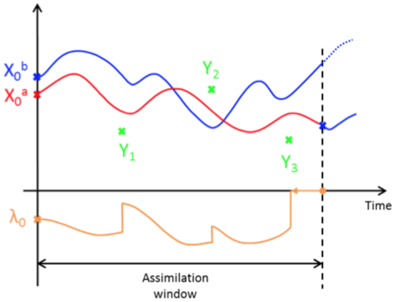
→ Time dependent prior (mean, cov.)

→ Comput. cost. of K and P

Alternative approaches

- Extended Kalman Filter (EKF)
  → H and M linearized
- Sub Optimal Filter (SOS)
  → Reduce comput. cost H
- Particle Filter (PF)
  → H and M non linear
  → Noises: non-Gaussian, biased, multimodal
  → Sampling issues due to high dimensions

Data assimilation: Variationnal 4DVar



Energy function

$$J(\mathbf{x}_0) = \frac{1}{2}\|\mathbf{x}_0 - \mathbf{x}_0^b\|_B^2 + \frac{1}{2}\int_{t_0}^{t_f} \|\mathbb{H}(\mathbf{x}) - \boldsymbol{\mathcal{Y}}\|_R^2 dt,$$

$$\text{s.t.} \quad \partial_t \mathbf{x}(t,x) + \mathbb{M}(\mathbf{x}(t,x),u) = 0.$$

- Computing the gradient of $J(\mathbf{x}_0)$ is very expensive!
- Deduced by solving the backwards adjoint equation

$$-\partial_t\lambda(t) + (\partial_X\mathbb{M})^*\lambda(t) = (\partial_X\mathbb{H})^*R^{-1}(Y(t) - H(X(t)$$
$$\lambda(t_f) = 0$$

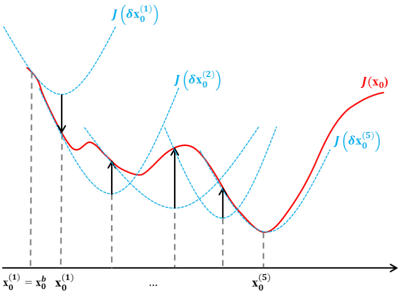
Properties

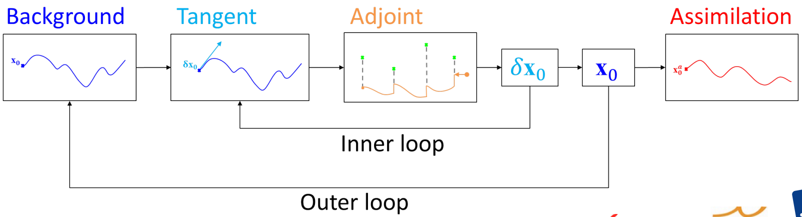
- Obs. and dynamics non-linear
- Noises Gaussian, unbiased, white-in-time

→ Time independent prior (B)

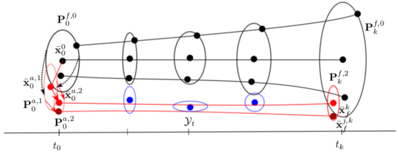
→ Derivation of the adjoint model

Data assimilation: 4DVar implementation





Data assimilation: Ensemble Variationnal EnVar



Properties

- Obs. and dynamics non-linear
- Noises Gaussian, unbiased, white-in-time

→ Sample based covariance (B)

→ Time dependent prior (B)

→ No derivation of the adjoint model

Energy function

$$J(\mathbf{x}_0) = \tfrac{1}{2}\|\mathbf{x}_0 - \mathbf{x}_0^b\|_B^2 + \frac{1}{2}\int_{t_0}^{t_f} \|\mathbb{H}(\mathbf{x}) - \boldsymbol{\mathcal{Y}}\|_R^2 dt,$$

$$\text{s.t.} \quad \partial_t \mathbf{x}(t,x) + \mathbb{M}(\mathbf{x}(t,x),u) = 0.$$

- Change cost function in terms of weighting vector
- Propagation of $B^{\frac{1}{2}}$ projected into observation space

→ Based on optimization theory

→ Fast operational implementation

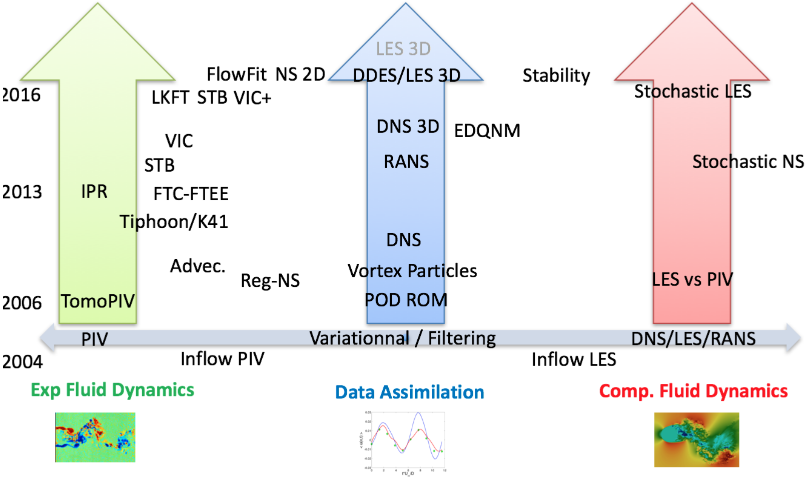
→ Uncertainty sample-based or from optimization procedure

→ Localization and inflation

Outline

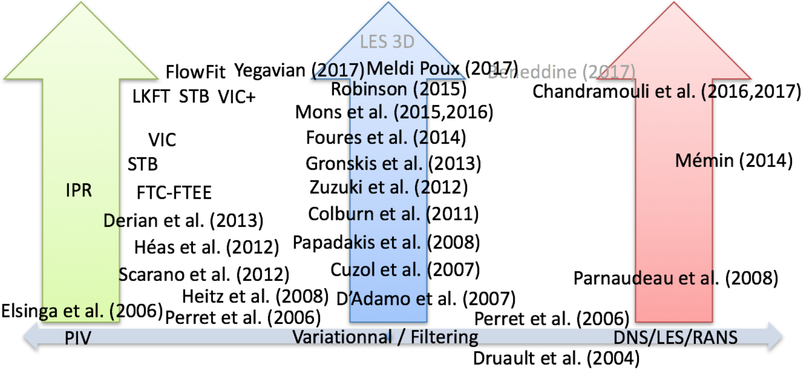
Data assimilation: data-driven vs model-driven

Different modelling

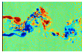


Data assimilation: data-driven vs model-driven

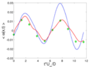
Non exhaustive state of the art



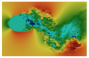
Exp Fluid Dynamics



Data Assimilation

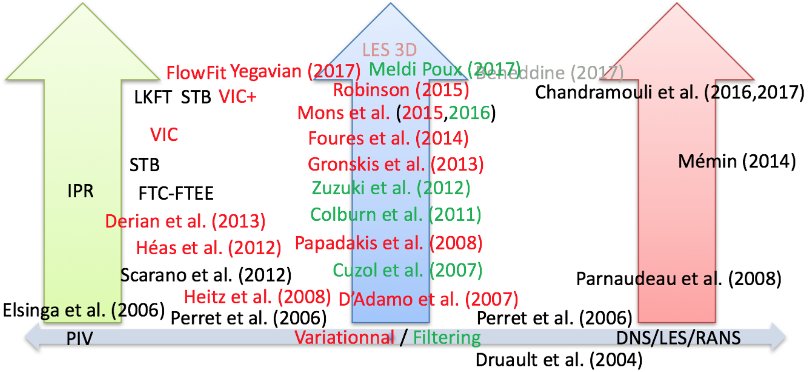


Comp. Fluid Dynamics

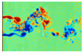


Data assimilation: data-driven vs model-driven

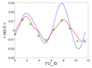
Variational vs Filtering approaches



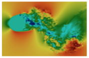
Exp Fluid Dynamics



Data Assimilation

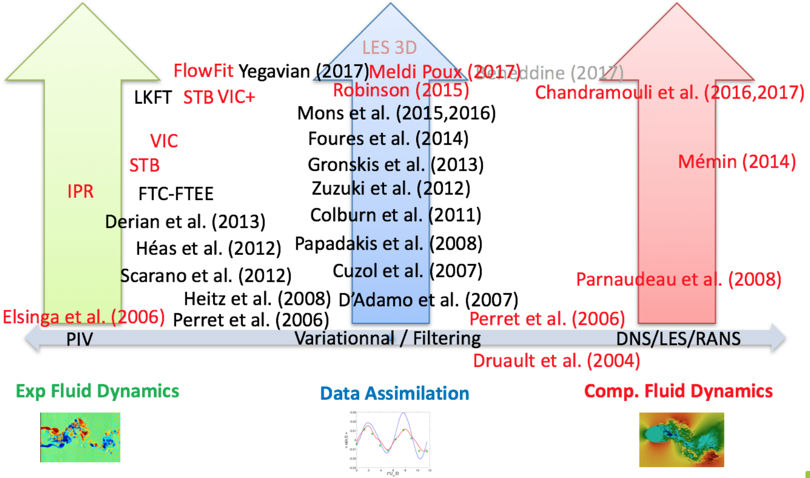


Comp. Fluid Dynamics



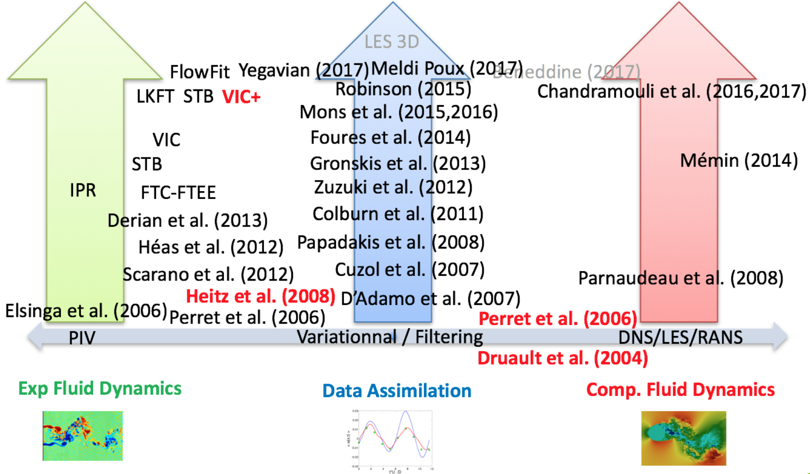
Data assimilation: data-driven vs model-driven

3D vs 2D approaches



Data assimilation: data-driven vs model-driven

Focus



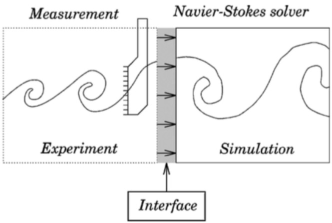
Data assimilation: data-driven vs model-driven

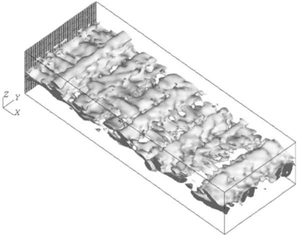
AIAA Journal
Vol. 42, No. 3, March 2004

Generation of Three-Dimensional Turbulent Inlet Conditions for Large-Eddy Simulation

P. Druault*
Université Pierre-et-Marie-Curie, 78210 Saint Cyr l'Ecole, France
S. Lardeau†
Imperial College of Science, Technology, and Medicine, London, England SW7 2BY, United Kingdom
and
J.-P. Bonnet,‡ F. Coiffet,§ J. Delville,¶ E. Lamballais,** J. F. Largeau,§ and L. Perret§
Université de Poitiers, 86962 Futuroscope Chasseneuil CEDEX, France

A method for generating realistic (i.e., reproducing in space and time the large-scale coherence of the flows)





Data assimilation: data-driven vs model-driven

PHYSICS OF FLUIDS **20**, 075107 (2008)

Turbulent inflow conditions for large-eddy simulation based on low-order empirical model

Laurent Perret,[1,a)] Joël Delville,[2] Rémi Manceau,[2] and Jean-Paul Bonnet[2]
[1]*Laboratoire de Mécanique des Fluides (LMF), UMR CNRS 6598, Ecole Centrale de Nantes, 1 rue de la Noë BP 92101, F-44321 Nantes Cedex 3, France*
[2]*Laboratoire d'Etudes Aérodynamiques (LEA), ENSMA, CNRS, CEAT, Université de Poitiers, 43, route de l'aérodrome, F-86036 Poitiers, France*

(Received 30 October 2007; accepted 3 June 2008; published online 22 July 2008)

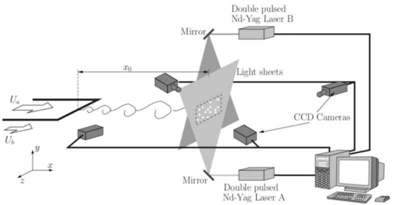
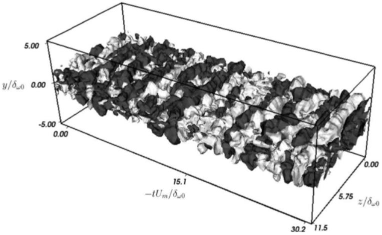
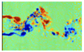
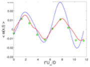
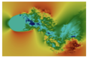


FIG. 1. DT-SPIV setup.









Data assimilation: data-driven vs model-driven

Exp Fluids (2008) 45:595–608
DOI 10.1007/s00348-008-0567-4

RESEARCH ARTICLE

Dynamic consistent correlation-variational approach for robust optical flow estimation

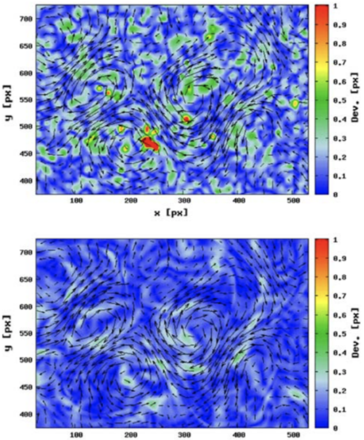
D. Heitz · P. Héas · E. Mémin · J. Carlier

$$J(\mathbf{u}, I) = J_{\mathrm{d}}(\mathbf{u}, I) + J_{\mathrm{r}}(\mathbf{u}) + J_{\mathrm{p}}(\mathbf{u}, \mathbf{u}_{\mathrm{p}}) + J_{\mathrm{c}}(\mathbf{u}, \mathbf{u}_{\mathrm{c}}), \qquad (12)$$

where $J_{\mathrm{p}}(\cdot)$ is an energy function constraining displacements $\mathbf{u}$ to be consistent with a physically sound prediction $\mathbf{u}_{\mathrm{p}}$ relying on Navier–Stokes equations. As proposed in Héas et al. (2007), we define this functional as a quadratic distance between the estimated field $\mathbf{u}$ and the dense propagated field $\mathbf{u}_{\mathrm{p}} = (u_{\mathrm{p}}, v_{\mathrm{p}})$:

$$J_{\mathrm{p}}(\mathbf{u}, \mathbf{u}_{\mathrm{p}}) = \beta \int_{\Omega} \| \mathbf{u}_{\mathrm{p}}(\mathbf{s}) - \mathbf{u}(\mathbf{s}) \|^2 \, \mathrm{d}\mathbf{s}, \qquad (13)$$

where β denotes a weighting factor. This approach constitutes an alternative to the spatio-temporal smoother defined in Weickert and Schnörr (2001) and is to some extent similar to the temporal constraint introduced in Rhunau et al. (2007). It is important to distinguish this



Data assimilation: data-driven vs model-driven

Exp Fluids (2016) 57:139
DOI 10.1007/s00348-016-2225-6

CrossMark

RESEARCH ARTICLE

Dense velocity reconstruction from tomographic PTV with material derivatives

Jan F. G. Schneiders[1] · Fulvio Scarano[1]

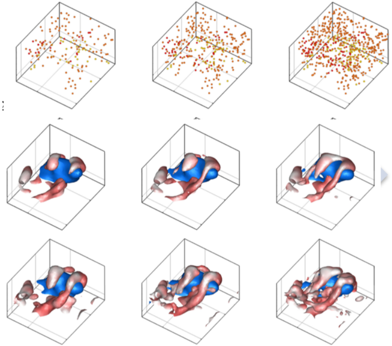
$$J = J_{\boldsymbol{u}} + \alpha^2 J_{D\boldsymbol{u}}, \tag{6}$$

where α is a weighting coefficient (Sect. 2.3.3), $J_{\boldsymbol{u}}$ is given by Eq. (7) and $J_{D\boldsymbol{u}}$ is given by Eq. (8),

$$J_{\boldsymbol{u}} = \sum_p \left\| \boldsymbol{u}_h(\boldsymbol{x}_p) - \boldsymbol{u}_m(\boldsymbol{x}_p) \right\|^2, \tag{7}$$

$$J_{D\boldsymbol{u}} = \sum_p \left\| \frac{D\boldsymbol{u}_h}{Dt}(\boldsymbol{x}_p) - \frac{D\boldsymbol{u}_m}{Dt}(\boldsymbol{x}_p) \right\|^2, \tag{8}$$

where $\boldsymbol{u}_h$ and $D\boldsymbol{u}_h/Dt$ are calculated from Eqs. (1) and (2) and are evaluated at the particle locations, $\boldsymbol{x}_p$, by linear interpolation from the computational grid. The cost function penalizes the difference between the PTV measurements and the velocity and material derivative at a single measurement time-instant calculated from the optimization variables. The optimization problem does not include time-integration of the vorticity transport equation.



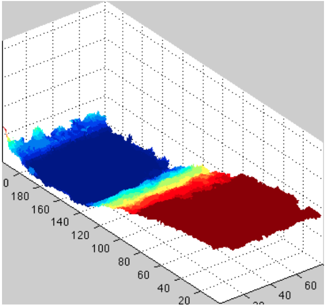
Outline

Wave in a rectangular flat bottom tank

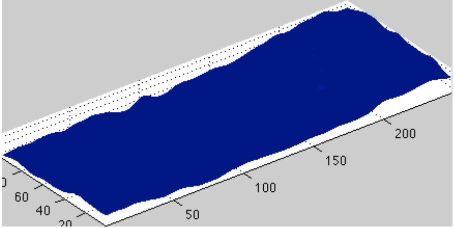
Reconstruct unobserved state from depth camera

WEnKF approach from Combès et al. (2015)
EnVar approach from Yang et al. (2015)

Depth observations

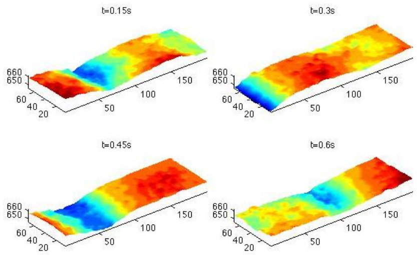


Data assimilation



- Reconstruct unobserved surface velocity
- Error model

Wave in a rectangular flat bottom tank



Flow configuration

- $Lx \times Ly = 250$ mm $\times$ 100 mm
- Initial free surface height difference $h_0 = 1$ cm
- Observations every $10\Delta t\, u_0/L_x$ leading to $St_{\mathrm{obs}} \simeq 24$, that was rather high !

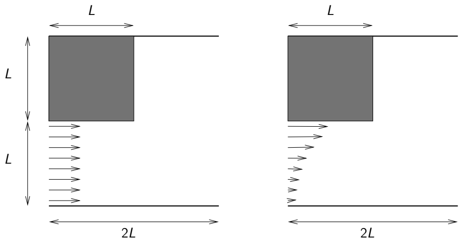
Simulation parameters

- $n_x \times n_y = 222 \times 88$
- $\Delta t\, u_0/L_x = 0.0042$

Assimilation parameters

- particle number $N = 100$
- $\mathbf{X}_0 \sim \mathcal{N}(\mathbf{x}_{\mathrm{init}}, \mathbf{R}_0)$
- $\mathbf{W}^f_t \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_t)$
- $\mathbf{W}^g_t \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_t)$
- $\mathbf{R}_0$ $(0.05\, h_0; 0.25\, u_0; r_h)$
- $\mathbf{R}_t$ $(0.04\, h_0; 0.06\, u_0; r_h)$
- $\mathbf{Q}_t$ $(0.013\, h_0^2; diag.)$
- localization $h_{correl} = 0.6 h_0$
- $\mathbf{x}_{\mathrm{init}} = (0, 0, 0)$

Suddenly expanding flume



Flow configurations

- $L = 10$ cm
- Inflow velocity and elevation oscillatory in phase at 1 Hz with $H_{\mathrm{in}} = 1$ cm and $V_{\mathrm{in}} = 0.22$ m/s
- $Fr = U_{\mathrm{in}} / \sqrt{g\, H_{\mathrm{in}}} = 0.7$

Simulation parameters

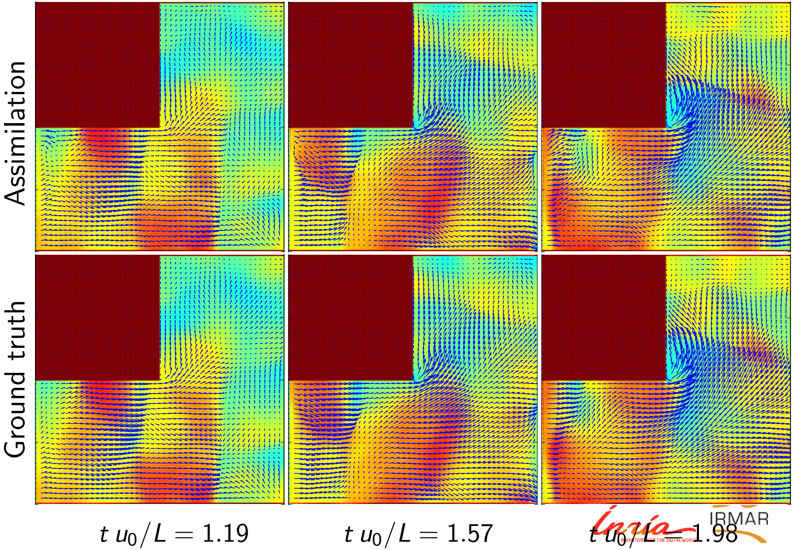
- $n_x \times n_y = 200 \times 200$
- $\Delta t\, u_0 / L = 0.006$

Assimilation parameters

- particle number $N = 100$
- $\mathbf{X}_0 \sim \mathcal{N}(\mathbf{x}_{\mathrm{init}}, \mathbf{R}_0)$
- $\mathbf{W}_t^f \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_t)$
- $\mathbf{W}_t^g \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_t)$
- $\mathbf{R}_0$ $(0.05\, h_0; 0.25\, u_0; r_h)$
- $\mathbf{R}_t$ $(0.04\, h_0; 0.06\, u_0; r_h)$
- $\mathbf{Q}_t$ $(0.013\, h_0^2; diag.)$
- localization $h_{correl} = 0.6 h_0$
- $\mathbf{x}_{\mathrm{init}} = (0, 0, 0)$

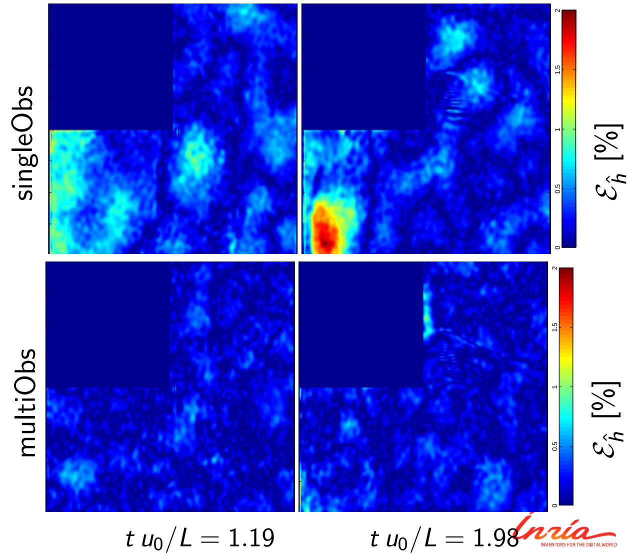
Suddenly expanding flume

Non-uniform inlet velocity profile (with spatial complexity)



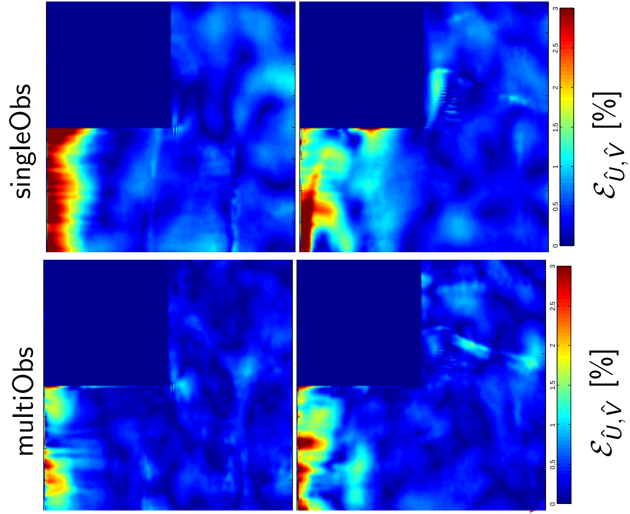
Suddenly expanding flume

Elevation error maps for singleObs and multiObs



Suddenly expanding flume

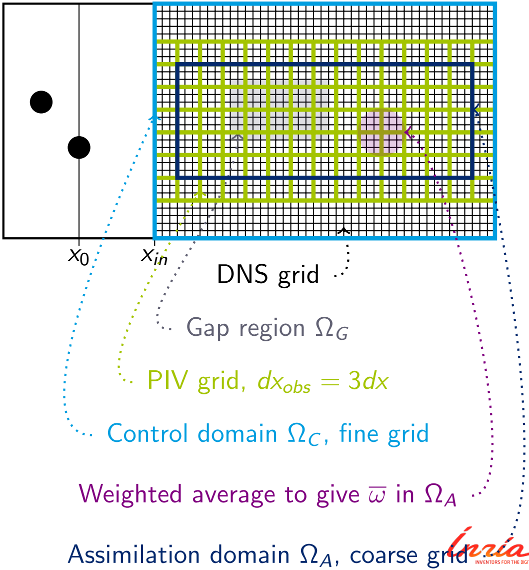
Velocity error maps for singleObs and multiObs



$t\, u_0/L = 1.19$ $t\, u_0/L = 1.98$

Cylinder wakes at Re=112

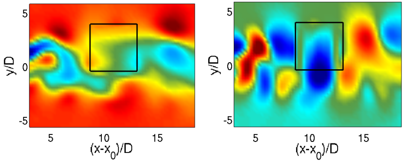
Gronskis et al. (2013, 2015)



Cylinder wakes at $Re = 112$

Initial condition

$u_x^{obs}(\mathbf{x}, t_0)$ $u_y^{obs}(\mathbf{x}, t_0)$

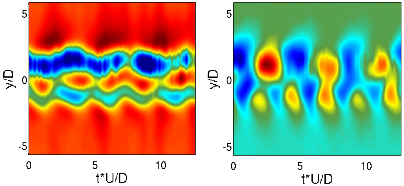


Initial condition in Ω_G (**IC**)

1. Uniform stagnant flow
2. Velocity interpolation

Inflow condition

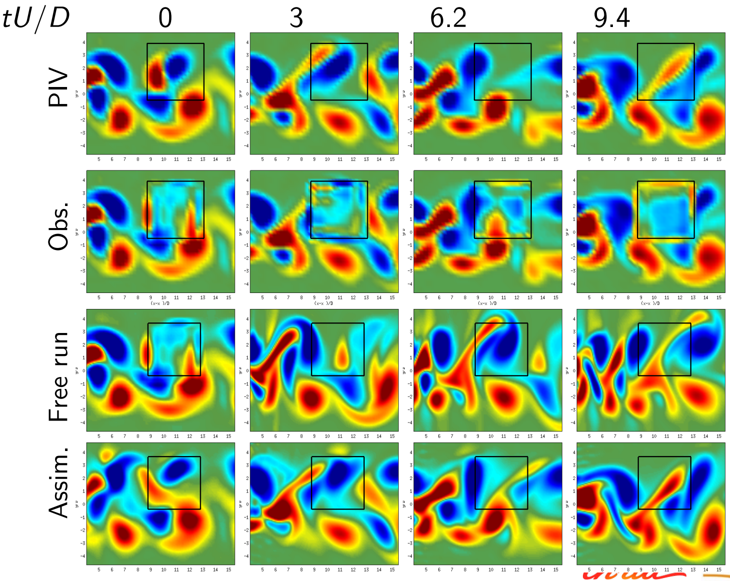
$u_x^{k=0}(\mathbf{x}_{in}, t)$ $u_y^{k=0}(\mathbf{x}_{in}, t)$



- From PIV sequence with Taylor's hypothesis

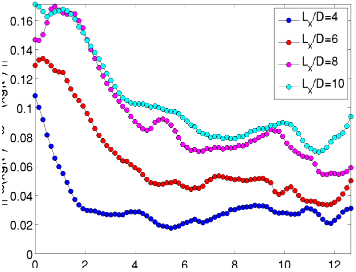
Cylinder wakes at *Re*=112

Gap reconstruction



Cylinder wakes at Re=112

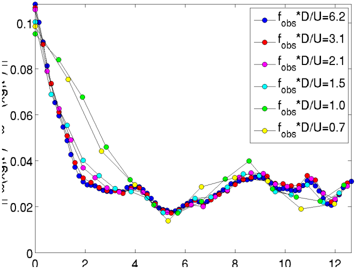
Influence of gap size



IC=1, $f_{obs}D/U$=6.2

Method's accuracy was strongly related to the size of the gap.

Influence of obs. frequency

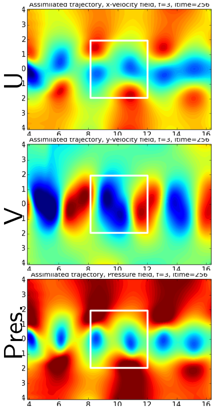


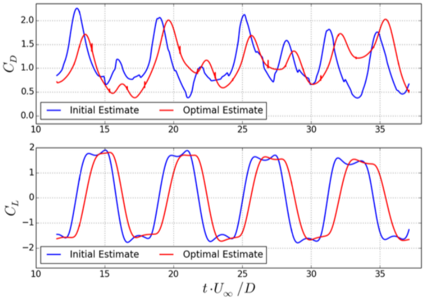
L_x/D=4, IC=1

Error decreased with increasing observations frequency $St_{obs} = f_{obs}\, D/U$.

Cylinder wakes at Re=112

Pressure, Drag and Lift reconstruction via 4DVar

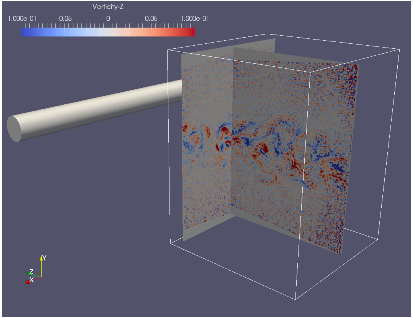


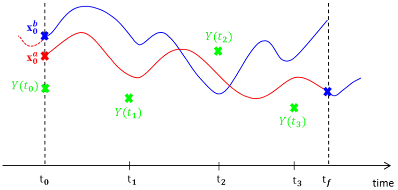


- Reconstruct unobserved pressure
- Lift and Drag via control volume

How to build the background ?

Real orthogonal-plane SPIV observations at $Re = 300$ and 4DVar (Robinson, 2015)

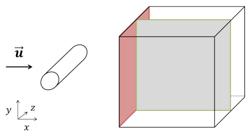


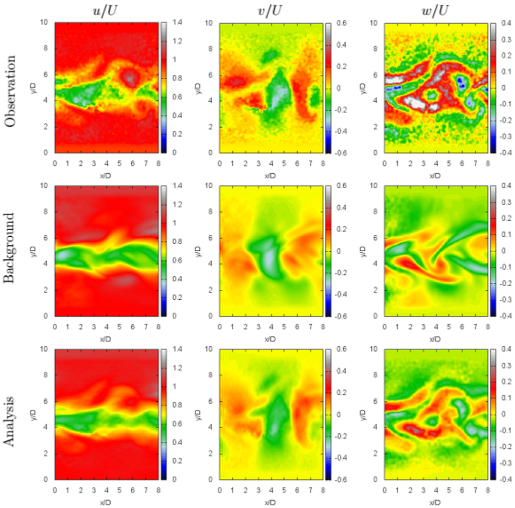


- Run a simulation from inlet observations

3D initial condition correction

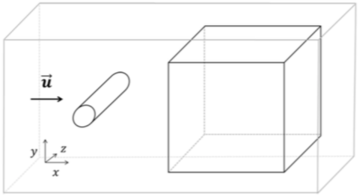
Real orthogonal-plane SPIV observations at $Re = 300$ and 4DVar (Robinson, 2015)





LES coefficient 4DVar data assimilation

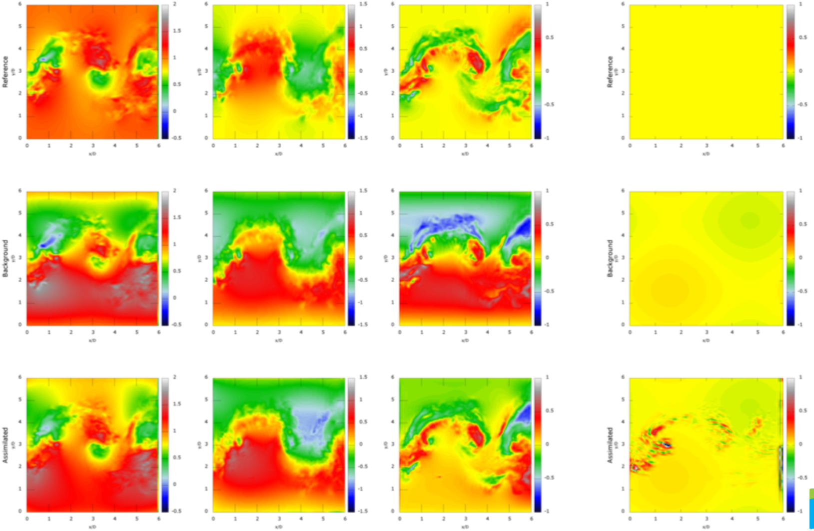
Chandramouli (2017, CFDforPIV)



	Re	$n_x \times n_y \times n_z$	$l_x/D \times l_y/D \times l_z/D$	$U\Delta t/D$	Duration
FD	3900	361×361×48	20×20×3.14	0.003	$40100\Delta t$
4DVAR	3900	145×145×48	6×6×3.14	0.003	$100\Delta t$

LES coefficient 4DVar data assimilation

Chandramouli (2017, CFDforPIV)



Sumary

- Data assimilation is a powerful technique to combine observations and models
- Data driven vs model driven (d vs m)
- When the amount of available data is insufficient to fully describe the system one cannot rely on data-driven approaches $\rightarrow$ model and regularization are paramount
- Data assimilation for prediction, filtering or smoothing
- History of use is the search for suitable approximation that, even sub-optimal, works with non-linear, non Gaussian and high dimensional settings

Outlooks

- Dynamics model (large scale, uncertainties)
- Control BC (inflow, outflow, ...) and model parameters
- From pseudo-observations to observations