Coupling experimental and computational fluid dynamics: Synopsis of approaches, issues and perspectives

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Coupling experimental and computational fluid dynamics: Synopsis of approaches, issues and perspectives

Dominique Heitz

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ACTA team leader, Irstea, Rennes, France

Confronting EFD and CFD is inherent of fluid mechanics approach

TomoPIV (Irstea)
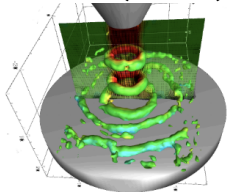
DNS (Dairy *et al.*,2015)

Experiments

- ▶ LDV as a reference
- ▶ HWA → very good
- ▶ PIV → good

Numerical simulations

- ▶ DNS as a reference → numerical wind tunnel
- ▶ A priori parameter calibration
- ▶ A posteriori simulation validation

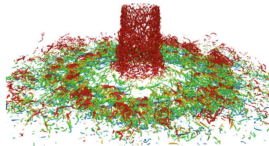# EFD and CFD limitations

TomoPIV (Irstea)
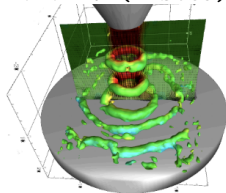
DNS (Dairy *et al.*,2015)
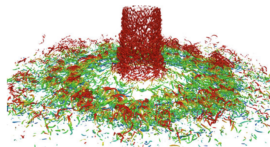
Experiments

- HWA and LDV → pointwise
- PIV → large scale
- TomoPIV → very large scale

  ⇒ sparse data

Numerical simulations

- Initial conditions
- Boundary conditions
- Turbulence model and parameters

  ⇒ non "realistic" simulations

Coupling EFD and CFD with data assimilation

TomoPIV (Irstea)
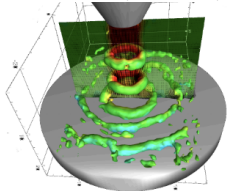
DNS (Dairy *et al.*,2015)
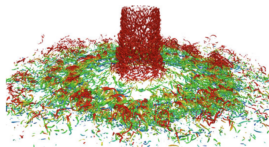
Objective

- Estimation of the unknown true state of interest $\mathbf{x}(t, x)$
- Recover as accurately as possible the state of the fluid flow using all available information

Question: how to do that ?

Outline

Data assimilation ingredients

TomoPIV (Irstea)
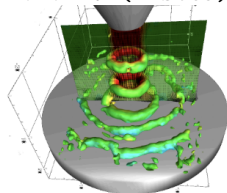
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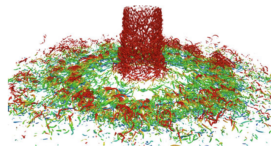
Experiments

- Observation model
  $\boldsymbol{\mathcal{Y}}(t,x) = \mathbb{H}(\mathbf{x}(t,x)) + \boldsymbol{\varepsilon}(t,x)$

Numerical model

- Dynamical model
  $\partial_t \mathbf{x}(t,x) + \mathbb{M}(\mathbf{x}(t,x)) = \mathbf{q}(t,x)$

- Prior knowledge model
  $\mathbf{x}(t_0,x) = \mathbf{x}_0^b + \boldsymbol{\eta}(x)$

Inria   IRMAR   irstea

Data and dynamics dimensions

TomoPIV (Irstea)
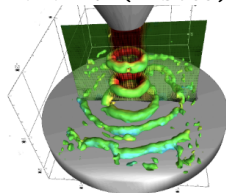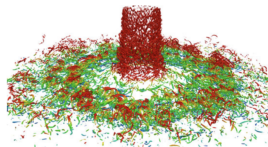
DNS (Dairy *et al.*,2015)

Data and model resolution: d vs m

- Geosciences $d << m$

- PIV $d \leq m$ or $d << m$
    - Model resolution: ROM vs DNS
    - Laboratory vs Industrial processes
    - 2D vs 3D
    - Reynolds

Data assimilation: observation and dynamics models

TomoPIV (Irstea)
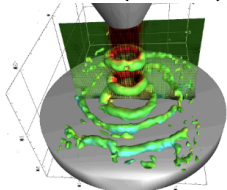
DNS (Dairy *et al.*,2015)
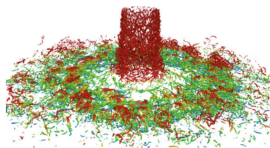
$$\mathcal{Y}(t,x) = \mathbb{H}(\mathbf{x}(t,x)) + \varepsilon(t,x)$$

$$\partial_t \mathbf{x}(t,x) + \mathbb{M}(\mathbf{x}(t,x)) = \mathbf{q}(t,x)$$

- Pseudo observation → velocity, vorticity, lagrangian acceleration, thus $\mathcal{Y}(t,x) = \hat{\mathbf{x}}(t,x)$ and $\mathbb{H} = \mathbb{I}$

- Observation → images of particles, scalar (smoke, gaz, temperature), thus $\mathcal{Y}(t,x) = I(t,x)$ and $\mathbb{H}$ can be nonlinear

- Eulerian or Lagrangian

- Eulerian: ROM, Vortex particle, Lattice Boltzman, RANS, LES, DNS

- Lagrangian: Smooth Particule Hydrodynamics (SPH)

Data assimilation ideal case

Papadakis & Mémin (2008) - Heitz *et al.* (2010)

$$\mathcal{Y}(t,x) = \mathbb{H}(\mathbf{x}(t,x)) + \varepsilon(t,x)$$

- ► Observation → particle images, thus $\mathcal{Y}(t,x) = I(t,x)$ and $\mathbb{H}$ linear

- ► Pseudo observation → velocity, thus $\mathcal{Y}(t,x) = \hat{\mathbf{x}}(t,x)$ and $\mathbb{H} = \mathbb{I}$

$$\partial_t \mathbf{x}(t,x) + \mathbb{M}(\mathbf{x}(t,x)) = 0$$

- ► DNS of 2D IHT at $Re = 256$

- ► Resolution : 256×256

Data assimilation ideal case

Papadakis & Mémin (2008) - Heitz *et al.* (2010)

$\partial_t I(t, x) + \mathbf{x} \cdot \nabla I(t, x) = \varepsilon(t, x)$

- ▶ Observation $\rightarrow$ particle images, thus $\mathcal{Y}(t, x) = I(t, x)$ and $\mathbb{H}$ linear
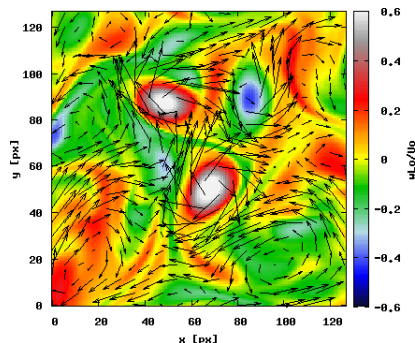
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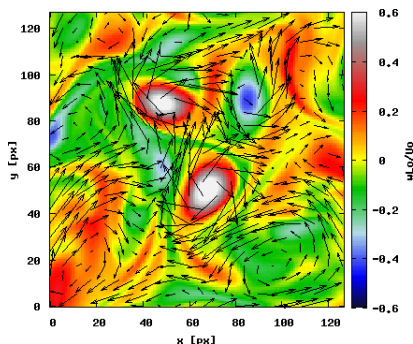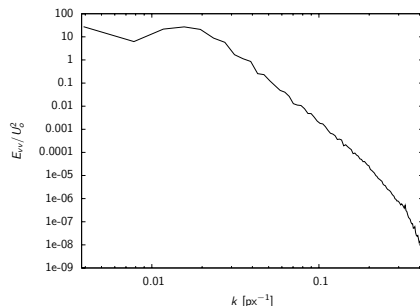$\partial_t \mathbf{x}(t, x) + \mathbb{M}(\mathbf{x}(t, x)) = 0$

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Outline

Data assimilation: the state estimation problem

Ingredients

- Observation model
  $$\boldsymbol{\mathcal{Y}}(t,x) = \mathbb{H}(\mathbf{x}(t,x)) + \boldsymbol{\varepsilon}(t,x)$$

- Dynamical model
  $$\partial_t \mathbf{x}(t,x) + \mathbb{M}(\mathbf{x}(t,x)) = \mathbf{q}(t,x)$$

- Prior knowledge model
  $$\mathbf{x}(t_0,x) = \mathbf{x}_0^b + \boldsymbol{\eta}(x)$$

$\rightarrow$ Random nature of observation, dynamic and knowledge errors described in term of pdf

Bayesian formulation

$$p(\mathbf{x}|\boldsymbol{\mathcal{Y}}) = \frac{p(\boldsymbol{\mathcal{Y}}|\mathbf{x})p(\mathbf{x})}{p(\boldsymbol{\mathcal{Y}})}$$

$$p(\mathbf{x}|\boldsymbol{\mathcal{Y}}) \propto p(\boldsymbol{\mathcal{Y}}|\mathbf{x})p(\mathbf{x})$$

posterior $\propto$ likelihood $\times$ prior

estimation $\propto$ observations $\times$ knowledge

Prior distribution:

- Prevents from over-fitting
- Introduce past information
- Good prior not straightforward

Data assimilation: the state estimation problem

Carassi *et al.* (2017)

Information: past
$\rightarrow$ For the control?

Bayesian formulation

$$p(\mathbf{x}|\mathcal{Y}) = \frac{p(\mathcal{Y}|\mathbf{x})p(\mathbf{x})}{p(\mathcal{Y})}$$

$$p(\mathbf{x}|\mathcal{Y}) \propto p(\mathcal{Y}|\mathbf{x})p(\mathbf{x})$$

posterior $\propto$ *likelihood* $\times$ *prior*

estimation $\propto$ *observations* $\times$ *knowledge*

Prior distribution:

► Prevents from over-fitting

► Introduce past information

► Good prior not straightforward

Data assimilation: the state estimation problem

Carassi *et al.* (2017)

Information: past and present
→ Sequential processing providing
   discontinuous trajectories

Bayesian formulation

$$p(\mathbf{x}|\mathcal{Y}) = \frac{p(\mathcal{Y}|\mathbf{x})p(\mathbf{x})}{p(\mathcal{Y})}$$

$$p(\mathbf{x}|\mathcal{Y}) \propto p(\mathcal{Y}|\mathbf{x})p(\mathbf{x})$$

posterior $\propto$ *likelihood* $\times$ *prior*

estimation $\propto$ *observations* $\times$ *knowledge*

Prior distribution:

▶ Prevents from over-fitting

▶ Introduce past information

▶ Good prior not straightforward

Data assimilation: the state estimation problem

Carassi *et al.* (2017)

Information: past, present and future
$\rightarrow$ Relevant for reconstruction or reanalysis and for model parameters estimation

Bayesian formulation

$$p(\mathbf{x}|\mathcal{Y}) = \frac{p(\mathcal{Y}|\mathbf{x})p(\mathbf{x})}{p(\mathcal{Y})}$$

$$p(\mathbf{x}|\mathcal{Y}) \propto p(\mathcal{Y}|\mathbf{x})p(\mathbf{x})$$

posterior $\propto$ *likelihood* $\times$ *prior*

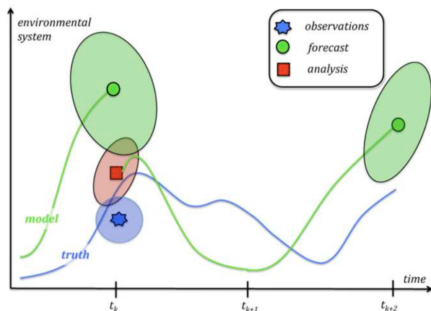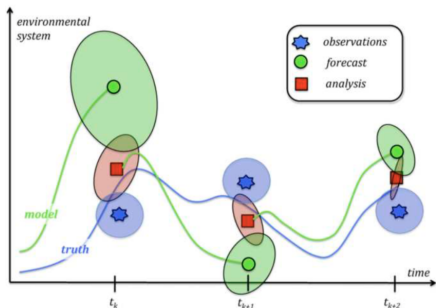*estimation* $\propto$ *observations* $\times$ *knowledge*

Prior distribution:

► Prevents from over-fitting

► Introduce past information

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Data assimilation: the state estimation problem

Computational problem

- ▶ Huge dimension of data and models prevent use of fully Bayesian approach
- ▶ Difficulty to define and transport the pdfs

Solution to overcome this issue

- ▶ Uncertainties of observations, model and prior are assumed Gaussian

- ▶ Pdfs completely described by first and second moments (i.e mean and covariance matrix)

Bayesian formulation

$$p(\mathbf{x}|\mathcal{Y}) = \frac{p(\mathcal{Y}|\mathbf{x})p(\mathbf{x})}{p(\mathcal{Y})}$$

$$p(\mathbf{x}|\mathcal{Y}) \propto p(\mathcal{Y}|\mathbf{x})p(\mathbf{x})$$

posterior $\propto$ likelihood $\times$ prior

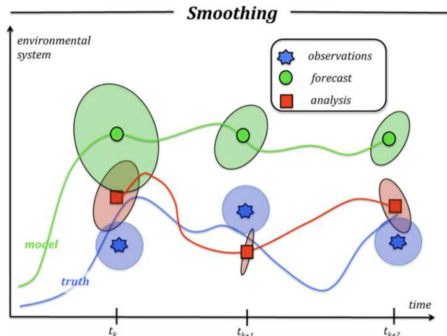*estimation $\propto$ observations $\times$ knowledge*

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- ▶ Prevents from over-fitting
- ▶ Introduce past information
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Data assimilation: Kalman filter
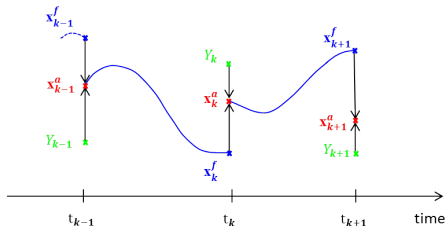
Main algorithm

1. Forecast step
$$\mathbf{x}_k^{\mathrm{f}} = \mathbf{M}_{k:k-1}\mathbf{x}_{k-1}^{\mathrm{a}},$$

$$\mathbf{P}_k^{\mathrm{f}} = \mathbf{M}_{k:k-1}\mathbf{P}_{k-1}^{\mathrm{a}}\mathbf{M}_{k:k-1}^{\mathrm{T}} + \mathbf{Q}_k.$$

2. Analysis step
$$\mathbf{K}_k = \mathbf{P}_k^{\mathrm{f}}\mathbf{H}_k^{\mathrm{T}}(\mathbf{H}_k\mathbf{P}_k^{\mathrm{f}}\mathbf{H}_k^{\mathrm{T}} + \mathbf{R}_k)^{-1},$$

$$\mathbf{x}_k^{\mathrm{a}} = \mathbf{x}_k^{\mathrm{f}} + \mathbf{K}_k(\mathbf{y}_k - \mathbf{H}_k\mathbf{x}_k^{\mathrm{f}}),$$

$$\mathbf{P}_k^{\mathrm{a}} = (\mathbf{I}_k - \mathbf{K}_k\mathbf{H}_k)\mathbf{P}_k^{\mathrm{f}}.$$
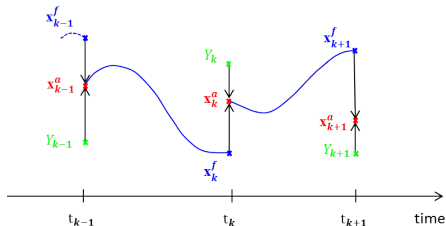
Properties

- Obs. and dynamics linear
- Noises Gaussian, unbiased, white-in-time
- $\rightarrow$ Time dependent prior (mean, cov.)
- $\rightarrow$ Comput. cost. of K and P

Data assimilation: Kalman filter

Properties

- Obs. and dynamics linear

- Noises Gaussian, unbiased, white-in-time

$\rightarrow$ Time dependent prior (mean, cov.)

$\rightarrow$ Comput. cost. of K and P

Alternative approaches

- Extended Kalman Filter (EKF)

  $\rightarrow$ H and M linearized

- Sub Optimal Filter (SOS)

  $\rightarrow$ Reduce comput. cost H

Data assimilation: Kalman filter
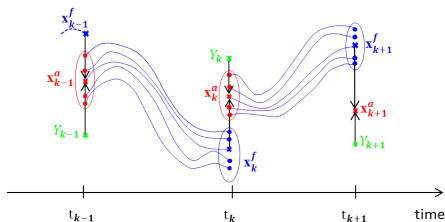
Properties

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Alternative approaches

- Extended Kalman Filter (EKF)

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- Sub Optimal Filter (SOS)

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- Ensemble Kalman Filter (EnKF)

  $\rightarrow$ Empirical estimation of P
  $\rightarrow$ H and M non linear

Data assimilation: Kalman filter
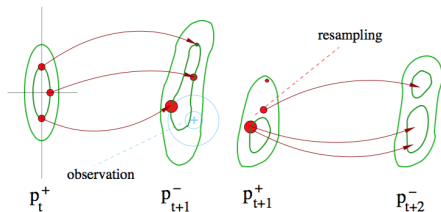
From Boquet's lecture notes (2014-2015)

Properties

- Obs. and dynamics linear

- Noises Gaussian, unbiased, white-in-time

$\rightarrow$ Time dependent prior (mean, cov.)

$\rightarrow$ Comput. cost. of K and P

Alternative approaches

- Extended Kalman Filter (EKF)

    $\rightarrow$ H and M linearized

- Sub Optimal Filter (SOS)

    $\rightarrow$ Reduce comput. cost H

- Particle Filter (PF)

    $\rightarrow$ H and M non linear
    $\rightarrow$ Noises: non-Gaussian, biased, multimodal
    $\rightarrow$ Sampling issues due to high dimensions

Data assimilation: Variationnal 4DVar

Properties

- ▶ Obs. and dynamics non-linear

- ▶ Noises Gaussian, unbiased, white-in-time

- → Time independent prior (B)

- → Derivation of the adjoint model
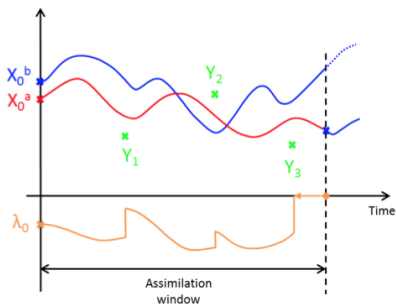
Energy function

$$J(\mathbf{x}_0) = \tfrac{1}{2}\|\mathbf{x}_0 - \mathbf{x}_0^b\|_B^2 + \frac{1}{2}\int_{t_0}^{t_f} \|\mathbb{H}(\mathbf{x}) - \boldsymbol{\mathcal{Y}}\|_R^2 dt,$$
$$\text{s.t.} \quad \partial_t \mathbf{x}(t,x) + \mathbb{M}(\mathbf{x}(t,x), u) = 0.$$

- ▶ Computing the gradient of $J(\mathbf{x}_0)$ is very expensive!

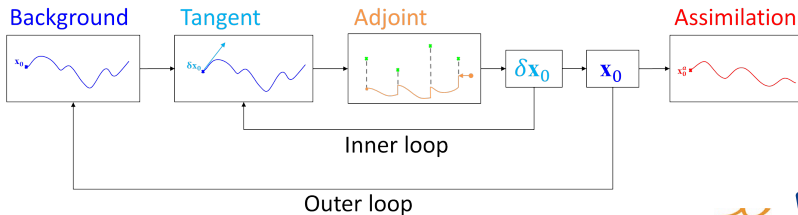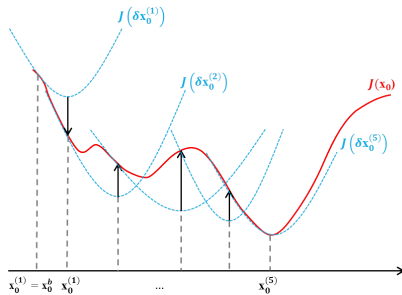- ▶ Deduced by solving the backwards adjoint equation

$$-\partial_t \lambda(t) + (\partial_X \mathbb{M})^* \lambda(t) = (\partial_X \mathbb{H})^* R^{-1}(Y(t) - H(X(t)$$
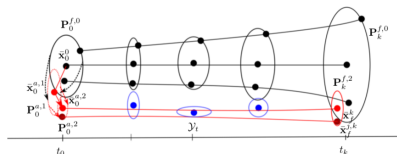$$\lambda(t_f) = 0$$

Data assimilation: 4DVar implementation

Data assimilation: Ensemble Variationnal EnVar

Energy function

$$J(\mathbf{x}_0) = \tfrac{1}{2}\|\mathbf{x}_0 - \mathbf{x}_0^b\|_B^2 + \frac{1}{2}\int_{t_0}^{t_f} \|\mathbb{H}(\mathbf{x}) - \boldsymbol{\mathcal{Y}}\|_R^2 dt,$$

$$\text{s.t.} \quad \partial_t \mathbf{x}(t,x) + \mathbb{M}(\mathbf{x}(t,x), u) = 0.$$

▶ Change cost function in terms of weighting vector

▶ Propagation of $B^{\frac{1}{2}}$ projected into observation space

→ Based on optimization theory

→ Fast operational implementation

→ Uncertainty sample-based or from optimization procedure

→ Localization and inflation

Properties

▶ Obs. and dynamics non-linear

▶ Noises Gaussian, unbiased, white-in-time

→ Sample based covariance (B)

→ Time dependent prior (B)

→ No derivation of the adjoint model

Outline

Data assimilation: data-driven vs model-driven

Different modelling
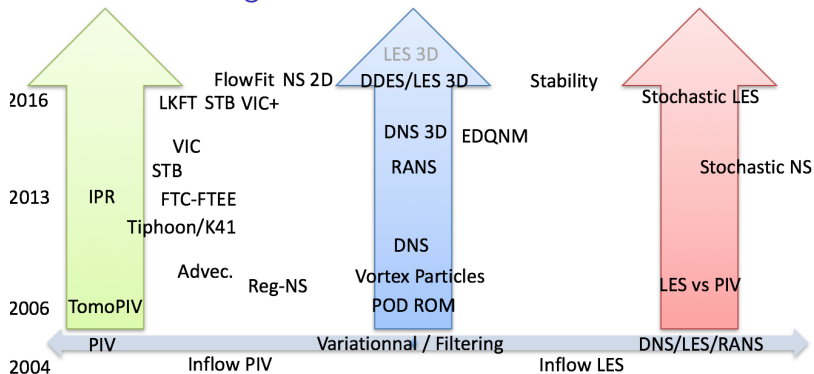
Exp Fluid Dynamics   **Data Assimilation**   **Comp. Fluid Dynamics**

Data assimilation: data-driven vs model-driven

Non exhaustive state of the art

FlowFit Yegavian (2017) Meldi Poux (2017)
LKFT STB VIC+
Robinson (2015)
Mons et al. (2015,2016)
Chandramouli et al. (2016,2017)
VIC
Foures et al. (2014)
STB
Gronskis et al. (2013)
Mémin (2014)
FTC-FTEE
Zuzuki et al. (2012)
Derian et al. (2013)
Colburn et al. (2011)
Héas et al. (2012)
Papadakis et al. (2008)
Scarano et al. (2012)
Cuzol et al. (2007)
Parnaudeau et al. (2008)
Heitz et al. (2008)
D'Adamo et al. (2007)
Elsinga et al. (2006)
Perret et al. (2006)
Perret et al. (2006)
IPR
LES 3D
Seddine (2017)

PIV   Variationnal / Filtering   DNS/LES/RANS
Druault et al. (2004)

Exp Fluid Dynamics   **Data Assimilation**   **Comp. Fluid Dynamics**
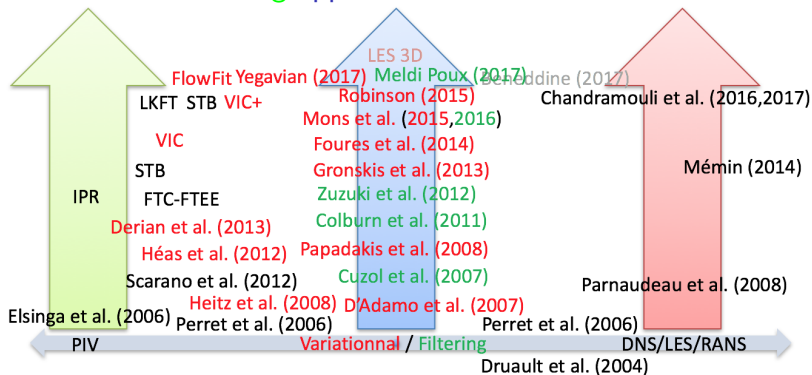
Data assimilation: data-driven vs model-driven

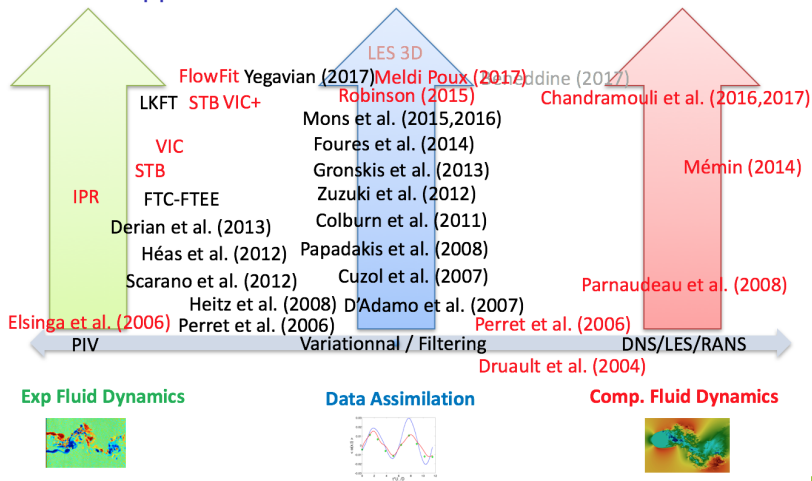**Variational** vs **Filtering** approaches

Data assimilation: data-driven vs model-driven

3D vs 2D approaches

FlowFit Yegavian (2017) Meldi Poux (2017) Cordier Laddine (2012)
LKFT STB VIC+   Robinson (2015)   Chandramouli et al. (2016,2017)
                                         Mons et al. (2015,2016)
            VIC   Foures et al. (2014)
           STB   Gronskis et al. (2013)   Mémin (2014)
         FTC-FTEE   Zuzuki et al. (2012)
   IPR   Colburn et al. (2011)
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PIV   Variationnal / Filtering   DNS/LES/RANS
                                    Druault et al. (2004)

LES 3D

Exp Fluid Dynamics   **Data Assimilation**   **Comp. Fluid Dynamics**

Data assimilation: data-driven vs model-driven

Focus

LES 3D

FlowFit Yegavian (2017) Meldi Poux (2017) Beneddine (2017)
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Mons et al. (2015,2016)
VIC Foures et al. (2014) Mémin (2014)
STB Gronskis et al. (2013)
FTC-FTEE Zuzuki et al. (2012)
Derian et al. (2013) Colburn et al. (2011)
Héas et al. (2012) Papadakis et al. (2008)
Scarano et al. (2012) Cuzol et al. (2007) Parnaudeau et al. (2008)
IPR **Heitz et al. (2008)** D'Adamo et al. (2007)
Elsinga et al. (2006) Perret et al. (2006) **Perret et al. (2006)**

PIV Variationnal / Filtering DNS/LES/RANS

Druault et al. (2004)

Exp Fluid Dynamics **Data Assimilation** **Comp. Fluid Dynamics**

Data assimilation: data-driven vs model-driven

Generation of Three-Dimensional Turbulent Inlet Conditions for Large-Eddy Simulation

P. Druault[*]
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and
J.-P. Bonnet,[‡] F. Coiffet,[§] J. Delville,[¶] E. Lamballais,[**] J. F. Largeau,[§] and L. Perret[§]
Université de Poitiers, 86962 Futuroscope Chasseneuil CEDEX, France

A method for generating realistic (i.e., reproducing in space and time the large-scale coherence of the flows)
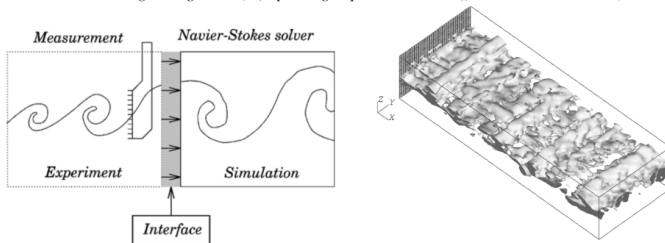
Data assimilation: data-driven vs model-driven

Turbulent inflow conditions for large-eddy simulation based on low-order empirical model

Laurent Perret,[1,a)] Joël Delville,[2] Rémi Manceau,[2] and Jean-Paul Bonnet[2]

[1]*Laboratoire de Mécanique des Fluides (LMF), UMR CNRS 6598, Ecole Centrale de Nantes, 1 rue de la Noë BP 92101, F-44321 Nantes Cedex 3, France*
[2]*Laboratoire d'Etudes Aérodynamiques (LEA), ENSMA, CNRS, CEAT, Université de Poitiers, 43, route de l'aérodrome, F-86036 Poitiers, France*
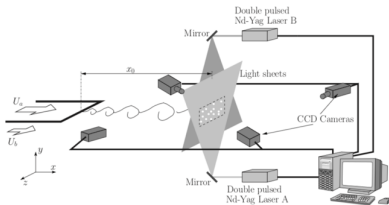
FIG. 1. DT-SPIV setup.
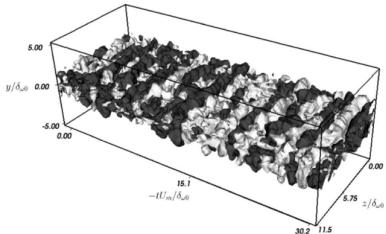
Data assimilation: data-driven vs model-driven

RESEARCH ARTICLE

Dynamic consistent correlation-variational approach for robust optical flow estimation

D. Heitz · P. Héas · E. Mémin · J. Carlier

$$J(\mathbf{u}, I) = J_d(\mathbf{u}, I) + J_r(\mathbf{u}) + J_p(\mathbf{u}, \mathbf{u}_p) + J_c(\mathbf{u}, \mathbf{u}_c), \quad (12)$$

where $J_p(\cdot)$ is an energy function constraining displacements $\mathbf{u}$ to be consistent with a physically sound prediction $\mathbf{u}_p$ relying on Navier–Stokes equations. As proposed in Héas et al. (2007), we define this functional as a quadratic distance between the estimated field $\mathbf{u}$ and the dense propagated field $\mathbf{u}_p = (u_p, v_p)$:

$$J_p(\mathbf{u}, \mathbf{u}_p) = \beta \int_\Omega \| \mathbf{u}_p(\mathbf{s}) - \mathbf{u}(\mathbf{s}) \|^2 \, d\mathbf{s}, \quad (13)$$

where β denotes a weighting factor. This approach constitutes an alternative to the spatio-temporal smoother defined in Weickert and Schnörr (2001) and is to some extent similar to the temporal constraint introduced in Rhunau et al. (2007). It is important to distinguish this
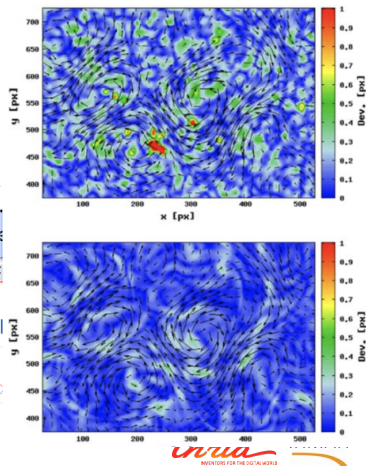
Data assimilation: data-driven vs model-driven

CrossMark

RESEARCH ARTICLE

Dense velocity reconstruction from tomographic PTV with material derivatives

Jan F. G. Schneiders[1] · **Fulvio Scarano**[1]

$$J = J_u + \alpha^2 J_{Du}, \tag{6}$$

where α is a weighting coefficient (Sect. 2.3.3), J_u is given by Eq. (7) and J_{Du} is given by Eq. (8),

$$J_u = \sum_p \left\| u_h(x_p) - u_m(x_p) \right\|^2, \tag{7}$$

$$J_{Du} = \sum_p \left\| \frac{Du_h}{Dt}(x_p) - \frac{Du_m}{Dt}(x_p) \right\|^2, \tag{8}$$

where u_h and Du_h/Dt are calculated from Eqs. (1) and (2) and are evaluated at the particle locations, x_p, by linear interpolation from the computational grid. The cost function penalizes the difference between the PTV measurements and the velocity and material derivative at a single measurement time-instant calculated from the optimization variables. The optimization problem does not include time-integration of the vorticity transport equation.
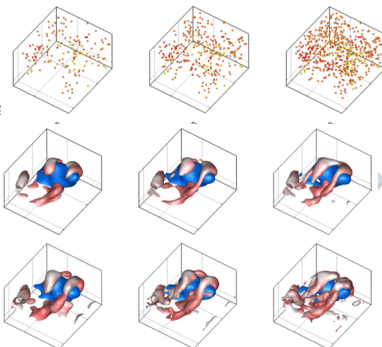
Outline

Wave in a rectangular flat bottom tank

Reconstruct unobserved state from depth camera

WEnKF approach from Combès et al. (2015)
EnVar approach from Yang et al. (2015)

Depth observations

Data assimilation

- Reconstruct unobserved surface velocity

- Error model

Wave in a rectangular flat bottom tank
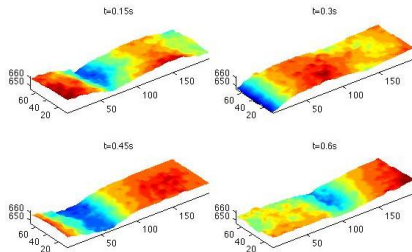
Flow configuration

- $Lx \times Ly = 250$ mm $\times$ 100 mm
- Initial free surface height difference $h_0 = 1$ cm
- Observations every $10\Delta t\, u_0/L_x$ leading to $St_{\mathrm{obs}} \simeq 24$, that was rather high !

Simulation parameters

- $n_x \times n_y = 222 \times 88$
- $\Delta t\, u_0/L_x = 0.0042$

Assimilation parameters

- particle number $N = 100$
- $\mathbf{X}_0 \sim \mathcal{N}(\mathbf{x}_{\mathrm{init}}, \mathbf{R}_0)$
- $\mathbf{W}_t^f \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_t)$
- $\mathbf{W}_t^g \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_t)$
- $\mathbf{R}_0$ $(0.05\, h_0; 0.25\, u_0; r_h)$
- $\mathbf{R}_t$ $(0.04\, h_0; 0.06\, u_0; r_h)$
- $\mathbf{Q}_t$ $(0.013\, h_0^2; diag.)$
- localization $h_{correl} = 0.6 h_0$
- $\mathbf{x}_{\mathrm{init}} = (0, 0, 0)$

Suddenly expanding flume
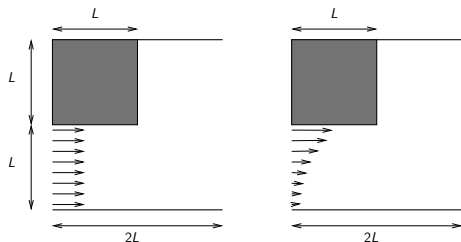
Flow configurations

- $L = 10$ cm
- Inflow velocity and elevation oscillatory in phase at 1 Hz with $H_{\text{in}} = 1$ cm and $V_{\text{in}} = 0.22$ m/s
- $Fr = U_{\text{in}}/\sqrt{g\, H_{\text{in}}} = 0.7$

Simulation parameters

- $n_x \times n_y = 200 \times 200$
- $\Delta t\, u_0/L = 0.006$

Assimilation parameters

- particle number $N = 100$
- $\mathbf{X}_0 \sim \mathcal{N}(\mathbf{x}_{\text{init}}, \mathbf{R}_0)$
- $\mathbf{W}_t^f \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_t)$
- $\mathbf{W}_t^g \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_t)$
- $\mathbf{R}_0$ $(0.05\, h_0; 0.25\, u_0; r_h)$
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- $\mathbf{Q}_t$ $(0.013\, h_0^2; diag.)$
- localization $h_{correl} = 0.6 h_0$
- $\mathbf{x}_{\text{init}} = (0, 0, 0)$

Suddenly expanding flume

Non-uniform inlet velocity profile (with spatial complexity)

$t\, u_0/L = 1.19$ $t\, u_0/L = 1.57$ $t\, u_0/L = 1.98$
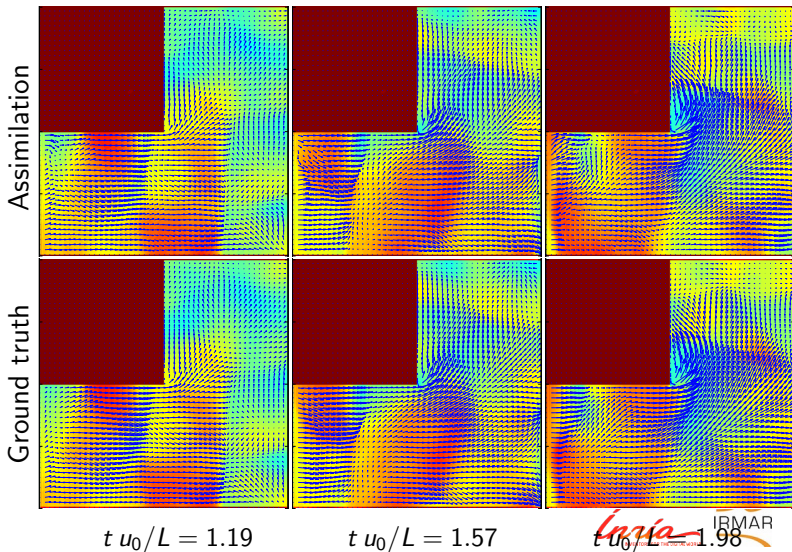
Suddenly expanding flume

Elevation error maps for singleObs and multiObs

$t\,u_0/L = 1.19$ $t\,u_0/L = 1.98$
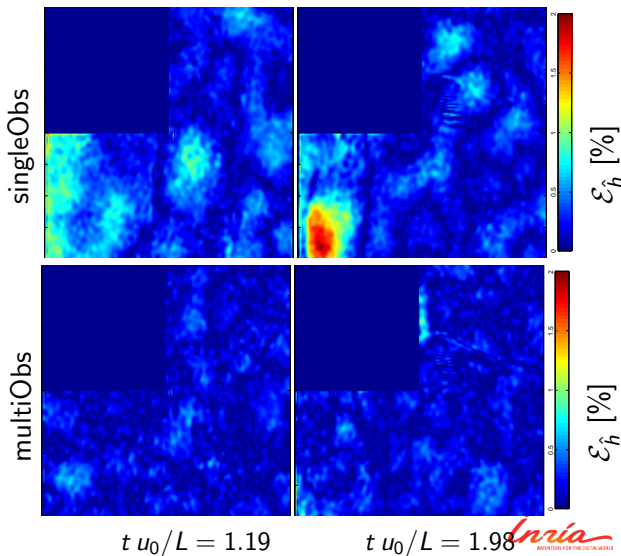
Suddenly expanding flume

Velocity error maps for singleObs and multiObs

singleObs

multiObs

$\mathcal{E}_{\hat{U},\hat{V}}$ [%]

$t\,u_0/L = 1.19$ $t\,u_0/L = 1.98$
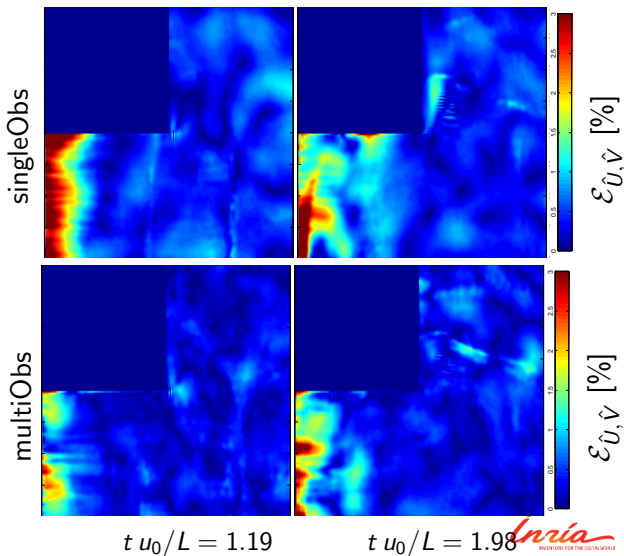
Cylinder wakes at $Re=112$

Gronskis et al. (2013, 2015)
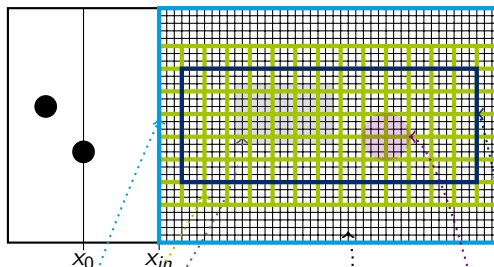
DNS grid

Gap region Ω_G

PIV grid, $dx_{obs} = 3dx$

Control domain Ω_C, fine grid

Weighted average to give $\overline{\omega}$ in Ω_A

Assimilation domain Ω_A, coarse grid

Cylinder wakes at Re=112

Initial condition

$u_x^{obs}(\mathbf{x}, t_0)$ $u_y^{obs}(\mathbf{x}, t_0)$
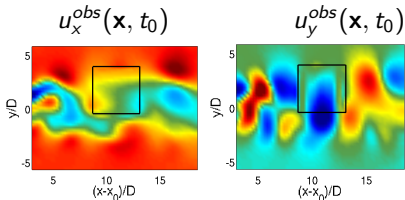
Initial condition in Ω_G (**IC**)

1. Uniform stagnant flow
2. Velocity interpolation

Inflow condition

$u_x^{k=0}(\mathbf{x}_{in}, t)$ $u_y^{k=0}(\mathbf{x}_{in}, t)$
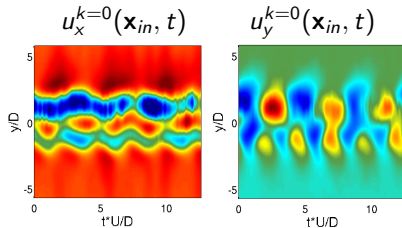
▶ From PIV sequence with Taylor's hypothesis

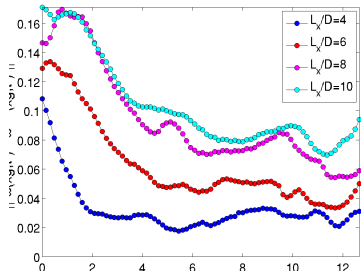# Cylinder wakes at $Re=112$

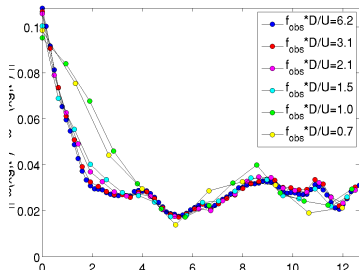## Gap reconstruction
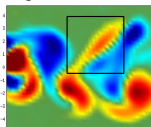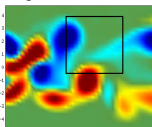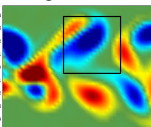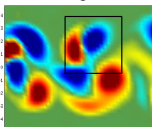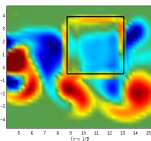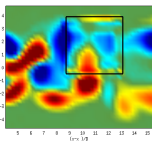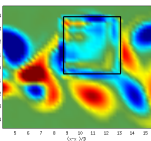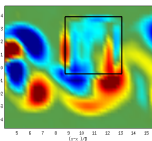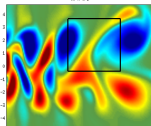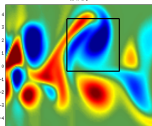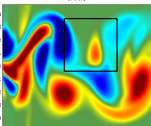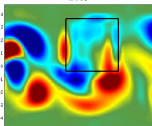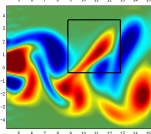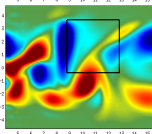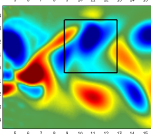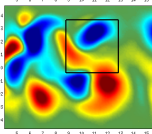
Cylinder wakes at Re=112

Influence of gap size

IC=1, $f_{obs}D/U$=6.2

Method's accuracy was strongly related to the size of the gap.

Influence of obs. frequency

L_x/D=4, IC=1

Error decreased with increasing observations frequency
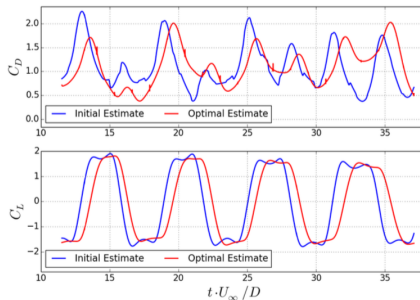$St_{obs} = f_{obs}\, D/U$.

Cylinder wakes at Re=112

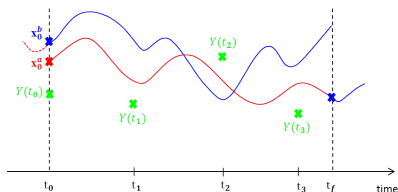## Pressure, Drag and Lift reconstruction via 4DVar

U

V

Pres.

- Reconstruct unobserved pressure
- Lift and Drag via control volume
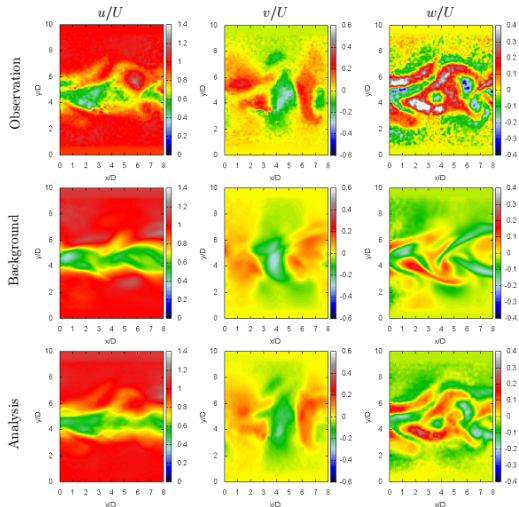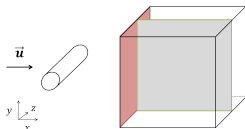
How to build the background ?

Real orthogonal-plane SPIV observations at $Re = 300$ and 4DVar (Robinson, 2015)

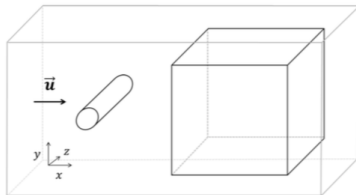▶ Run a simulation from inlet observations

3D initial condition correction

Real orthogonal-plane SPIV observations at $Re = 300$ and 4DVar (Robinson, 2015)
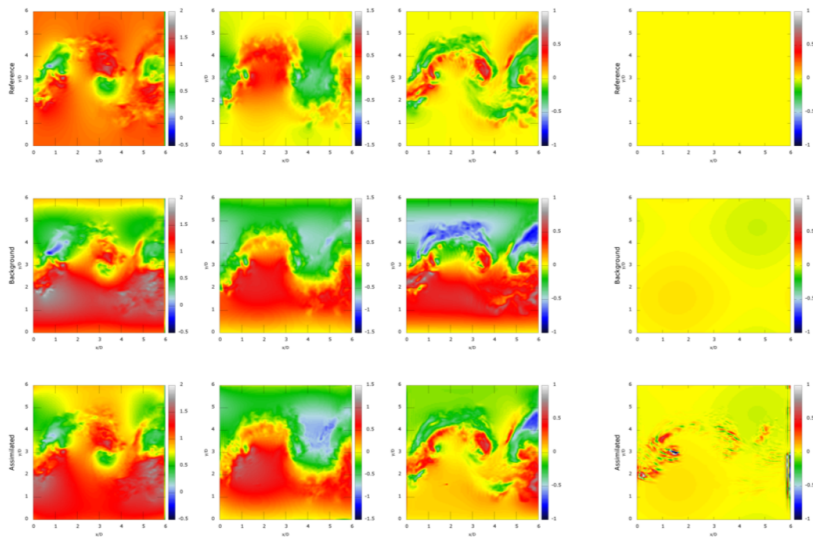
LES coefficient 4DVar data assimilation

Chandramouli (2017, CFDforPIV)

	Re	$n_x \times n_y \times n_z$	$l_x/D \times l_y/D \times l_z/D$	$U\Delta t/D$	Duration
FD	3900	361×361×48	20×20×3.14	0.003	40100Δt
4DVAR	3900	145×145×48	6×6×3.14	0.003	100Δt

LES coefficient 4DVar data assimilation

Chandramouli (2017, CFDforPIV)

Sumary

- Data assimilation is a powerful technique to combine observations and models
- Data driven vs model driven (d vs m)
- When the amount of available data is insufficient to fully describe the system one cannot rely on data-driven approaches $\rightarrow$ model and regularization are paramount
- Data assimilation for prediction, filtering or smoothing
- History of use is the search for suitable approximation that, even sub-optimal, works with non-linear, non Gaussian and high dimensional settings

Outlooks

- Dynamics model (large scale, uncertainties)
- Control BC (inflow, outflow, ...) and model parameters
- From pseudo-observations to observations