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Coupling experimental and computational fluid dynamics: Synopsis of approaches, issues and perspectives

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Coupling experimental and computational fluid dynamics: Synopsis of approaches, issues and perspectives

Dominique Heitz

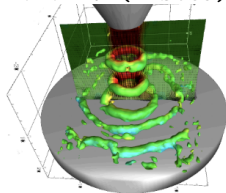
Fluminance team, Irstea, IRMAR and Inria of Rennes, France
ACTA team leader, Irstea, Rennes, France

2nd Workshop on Data Assimilation and CFD Processing Techniques
December 14, 2017, Delft, Netherland

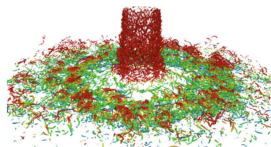


Confronting EFD and CFD is inherent of fluid mechanics approach

TomoPIV (Irstea)



DNS (Dairy *et al.*, 2015)



Experiments

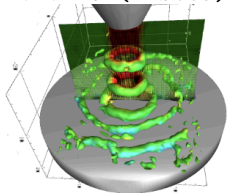
- ▶ LDV as a reference
- ▶ HWA → very good
- ▶ PIV → good

Numerical simulations

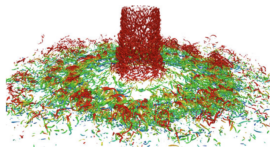
- ▶ DNS as a reference → numerical wind tunnel
- ▶ A priori parameter calibration
- ▶ A posteriori simulation validation

EFD and CFD limitations

TomoPIV (Irstea)



DNS (Dairy *et al.*, 2015)



Experiments

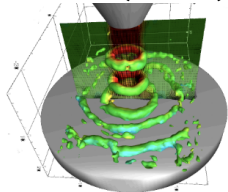
- ▶ HWA and LDV → pointwise
 - ▶ PIV → large scale
 - ▶ TomoPIV → very large scale
- ⇒ sparse data

Numerical simulations

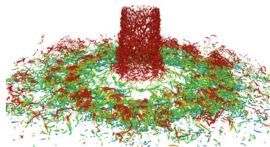
- ▶ Initial conditions
 - ▶ Boundary conditions
 - ▶ Turbulence model and parameters
- ⇒ non "realistic" simulations

Coupling EFD and CFD with data assimilation

TomoPIV (Irstea)



DNS (Dairy et al., 2015)



Objective

- ▶ Estimation of the unknown true state of interest $\mathbf{x}(t, \mathbf{x})$
- ▶ Recover as accurately as possible the state of the fluid flow using all available information

Question: how to do that ?

Outline

Data assimilation ingredients

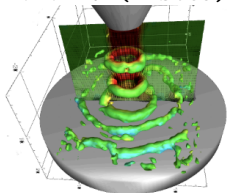
Data assimilation tools

Overview of significant achievements

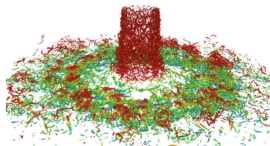
Some applications

Data assimilation ingredients

Tomographic PIV (Irstea)



DNS (Dairy et al., 2015)



Experiments

- ▶ Observation model

$$\mathcal{Y}(t, \mathbf{x}) = \mathbb{H}(\mathbf{x}(t, \mathbf{x})) + \varepsilon(t, \mathbf{x})$$

Numerical model

- ▶ Dynamical model

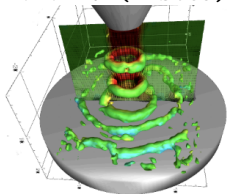
$$\partial_t \mathbf{x}(t, \mathbf{x}) + \mathbb{M}(\mathbf{x}(t, \mathbf{x})) = \mathbf{q}(t, \mathbf{x})$$

- ▶ Prior knowledge model

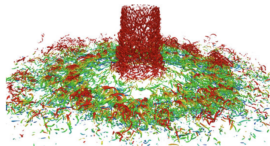
$$\mathbf{x}(t_0, \mathbf{x}) = \mathbf{x}_0^b + \boldsymbol{\eta}(\mathbf{x})$$

Data and dynamics dimensions

TomoPIV (Irstea)



DNS (Dairy *et al.*, 2015)

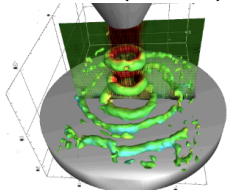
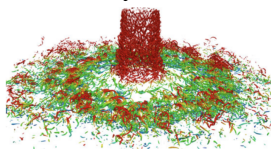


Data and model resolution: d vs m

- ▶ Geosciences $d \ll m$
- ▶ PIV $d \leq m$ or $d \ll m$
 - ▶ Model resolution: ROM vs DNS
 - ▶ Laboratory vs Industrial processes
 - ▶ 2D vs 3D
 - ▶ Reynolds

Data assimilation: observation and dynamics models

TomoPIV (Irstea)

DNS (Dairy *et al.*, 2015)

$$\mathcal{Y}(t, \mathbf{x}) = \mathbb{H}(\mathbf{x}(t, \mathbf{x})) + \varepsilon(t, \mathbf{x})$$

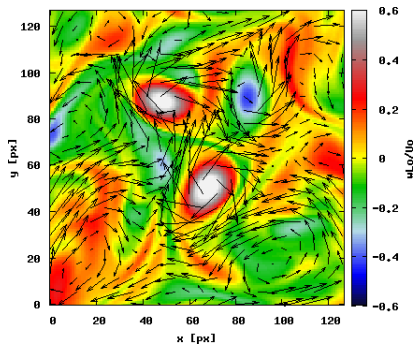
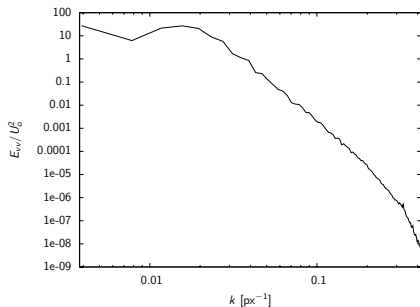
- ▶ Pseudo observation \rightarrow velocity, vorticity, lagrangian acceleration, thus $\mathcal{Y}(t, \mathbf{x}) = \hat{\mathbf{x}}(t, \mathbf{x})$ and $\mathbb{H} = \mathbb{I}$
- ▶ Observation \rightarrow images of particles, scalar (smoke, gaz, temperature), thus $\mathcal{Y}(t, \mathbf{x}) = I(t, \mathbf{x})$ and \mathbb{H} can be nonlinear
- ▶ Eulerian or Lagrangian

$$\partial_t \mathbf{x}(t, \mathbf{x}) + \mathbb{M}(\mathbf{x}(t, \mathbf{x})) = \mathbf{q}(t, \mathbf{x})$$

- ▶ Eulerian: ROM, Vortex particle, Lattice Boltzman, RANS, LES, DNS
- ▶ Lagrangian: Smooth Particle Hydrodynamics (SPH)

Data assimilation ideal case

Papadakis & Mémin (2008) - Heitz *et al.* (2010)



$$\mathcal{Y}(t, \mathbf{x}) = \mathbb{H}(\mathbf{x}(t, \mathbf{x})) + \varepsilon(t, \mathbf{x})$$

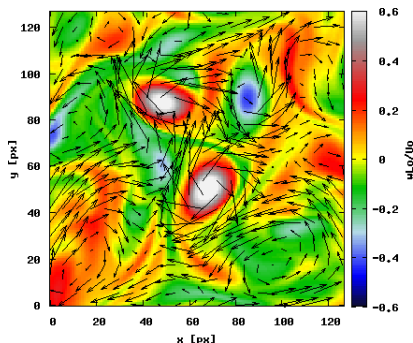
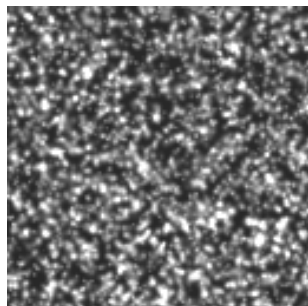
- ▶ Observation \rightarrow particle images, thus $\mathcal{Y}(t, \mathbf{x}) = I(t, \mathbf{x})$ and \mathbb{H} linear
- ▶ Pseudo observation \rightarrow velocity, thus $\mathcal{Y}(t, \mathbf{x}) = \hat{\mathbf{x}}(t, \mathbf{x})$ and $\mathbb{H} = \mathbb{I}$

$$\partial_t \mathbf{x}(t, \mathbf{x}) + \mathbb{M}(\mathbf{x}(t, \mathbf{x})) = 0$$

- ▶ DNS of 2D IHT at $Re = 256$
- ▶ Resolution : 256×256

Data assimilation ideal case

Papadakis & Mémin (2008) - Heitz *et al.* (2010)



$$\partial_t I(t, \mathbf{x}) + \mathbf{x} \cdot \nabla I(t, \mathbf{x}) = \varepsilon(t, \mathbf{x})$$

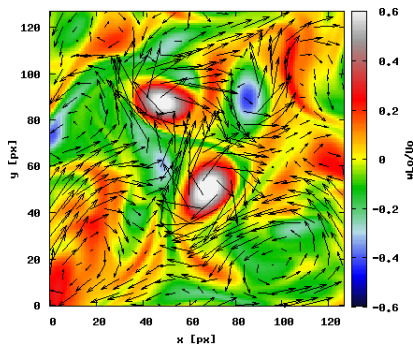
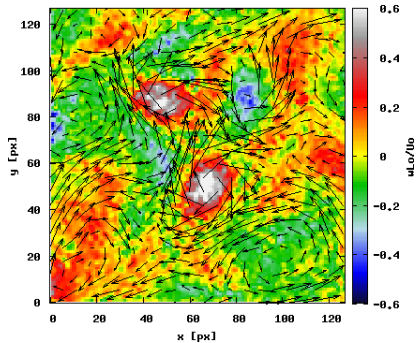
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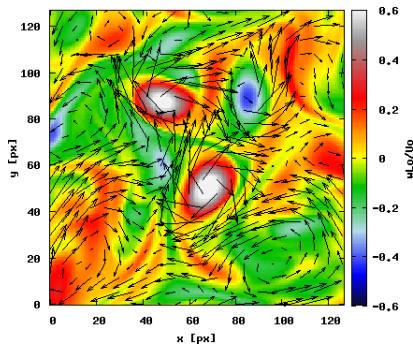
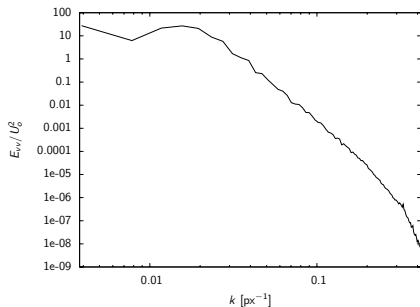
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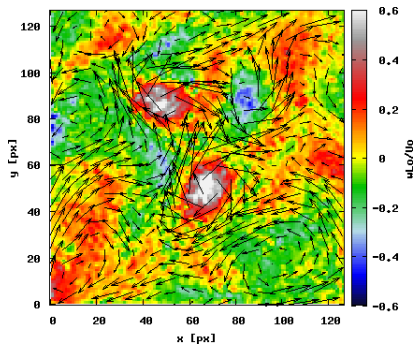
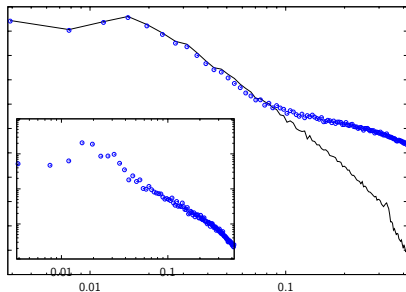
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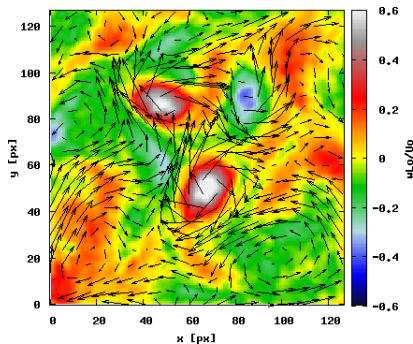
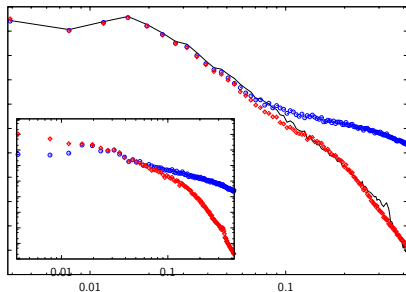
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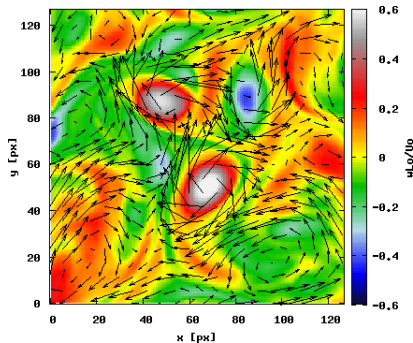
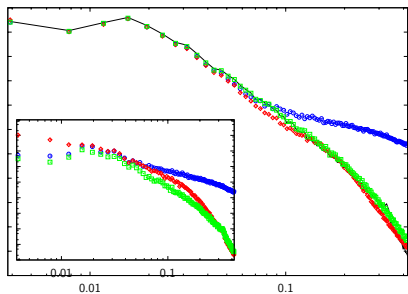
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- ▶ DNS of 2D IHT at $Re = 256$
- ▶ Resolution : 256×256

Outline

Data assimilation ingredients

Data assimilation tools

Overview of significant achievements

Some applications

Data assimilation: the state estimation problem

Ingredients

- ▶ Observation model
 $\mathcal{Y}(t, \mathbf{x}) = \mathbb{H}(\mathbf{x}(t, \mathbf{x})) + \varepsilon(t, \mathbf{x})$
 - ▶ Dynamical model
 $\partial_t \mathbf{x}(t, \mathbf{x}) + \mathbb{M}(\mathbf{x}(t, \mathbf{x})) = \mathbf{q}(t, \mathbf{x})$
 - ▶ Prior knowledge model
 $\mathbf{x}(t_0, \mathbf{x}) = \mathbf{x}_0^b + \boldsymbol{\eta}(\mathbf{x})$
- Random nature of observation, dynamic and knowledge errors described in term of pdf

Bayesian formulation

$$p(\mathbf{x}|\mathcal{Y}) = \frac{p(\mathcal{Y}|\mathbf{x})p(\mathbf{x})}{p(\mathcal{Y})}$$

$$p(\mathbf{x}|\mathcal{Y}) \propto p(\mathcal{Y}|\mathbf{x})p(\mathbf{x})$$

$$\text{posterior} \propto \text{likelihood} \times \text{prior}$$

$$\text{estimation} \propto \text{observations} \times \text{knowledge}$$

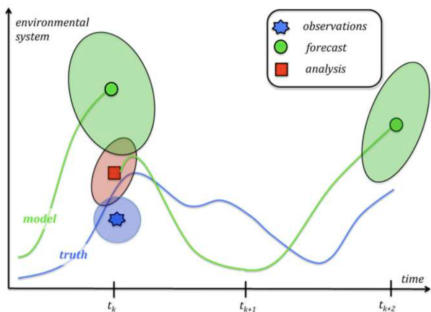
Prior distribution:

- ▶ Prevents from over-fitting
- ▶ Introduce past information
- ▶ Good prior not straightforward

Data assimilation: the state estimation problem

Carassi *et al.* (2017)

Prediction



Information: past
→ For the control?

Bayesian formulation

$$p(\mathbf{x}|\mathcal{Y}) = \frac{p(\mathcal{Y}|\mathbf{x})p(\mathbf{x})}{p(\mathcal{Y})}$$

$$p(\mathbf{x}|\mathcal{Y}) \propto p(\mathcal{Y}|\mathbf{x})p(\mathbf{x})$$

posterior \propto *likelihood* \times *prior*

estimation \propto *observations* \times *knowledge*

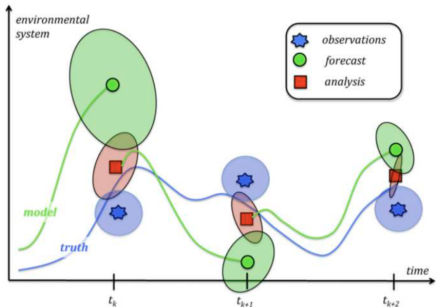
Prior distribution:

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- ▶ Introduce past information
- ▶ Good prior not straightforward

Data assimilation: the state estimation problem

Carassi et al. (2017)

Filtering



Information: past and present

→ Sequential processing providing discontinuous trajectories

Bayesian formulation

$$p(\mathbf{x}|\mathcal{Y}) = \frac{p(\mathcal{Y}|\mathbf{x})p(\mathbf{x})}{p(\mathcal{Y})}$$

$$p(\mathbf{x}|\mathcal{Y}) \propto p(\mathcal{Y}|\mathbf{x})p(\mathbf{x})$$

$$\text{posterior} \propto \text{likelihood} \times \text{prior}$$

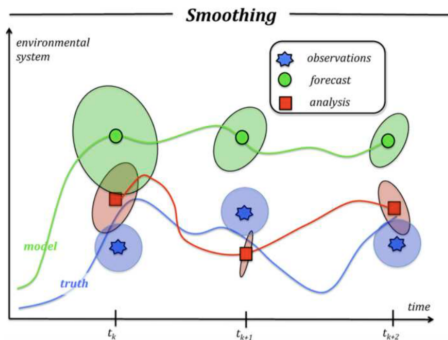
$$\text{estimation} \propto \text{observations} \times \text{knowledge}$$

Prior distribution:

- ▶ Prevents from over-fitting
- ▶ Introduce past information
- ▶ Good prior not straightforward

Data assimilation: the state estimation problem

Carassi *et al.* (2017)



Information: past, present and future
 → Relevant for reconstruction or reanalysis and for model parameters estimation

Bayesian formulation

$$p(\mathbf{x}|\mathcal{Y}) = \frac{p(\mathcal{Y}|\mathbf{x})p(\mathbf{x})}{p(\mathcal{Y})}$$

$$p(\mathbf{x}|\mathcal{Y}) \propto p(\mathcal{Y}|\mathbf{x})p(\mathbf{x})$$

$$\text{posterior} \propto \text{likelihood} \times \text{prior}$$

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Prior distribution:

- ▶ Prevents from over-fitting
- ▶ Introduce past information
- ▶ Good prior not straightforward

Data assimilation: the state estimation problem

Computational problem

- ▶ Huge dimension of data and models prevent use of fully Bayesian approach
- ▶ Difficulty to define and transport the pdfs

Solution to overcome this issue

- ▶ Uncertainties of observations, model and prior are assumed Gaussian
- ▶ Pdfs completely described by first and second moments (i.e mean and covariance matrix)

Bayesian formulation

$$p(\mathbf{x}|\mathcal{Y}) = \frac{p(\mathcal{Y}|\mathbf{x})p(\mathbf{x})}{p(\mathcal{Y})}$$

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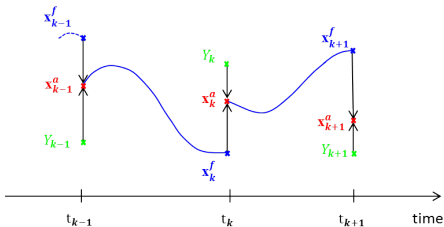
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$$\text{estimation} \propto \text{observations} \times \text{knowledge}$$

Prior distribution:

- ▶ Prevents from over-fitting
- ▶ Introduce past information
- ▶ Good prior not straightforward

Data assimilation: Kalman filter



Properties

- ▶ Obs. and dynamics linear
- ▶ Noises Gaussian, unbiased, white-in-time
- Time dependent prior (mean, cov.)
- Comput. cost. of K and P

Main algorithm

1. Forecast step

$$\mathbf{x}_k^f = \mathbf{M}_{k:k-1} \mathbf{x}_{k-1}^a,$$

$$\mathbf{P}_k^f = \mathbf{M}_{k:k-1} \mathbf{P}_{k-1}^a \mathbf{M}_{k:k-1}^T + \mathbf{Q}_k.$$

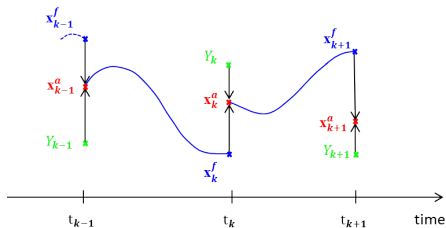
2. Analysis step

$$\mathbf{K}_k = \mathbf{P}_k^f \mathbf{H}_k^T (\mathbf{H}_k \mathbf{P}_k^f \mathbf{H}_k^T + \mathbf{R}_k)^{-1},$$

$$\mathbf{x}_k^a = \mathbf{x}_k^f + \mathbf{K}_k (\mathbf{y}_k - \mathbf{H}_k \mathbf{x}_k^f),$$

$$\mathbf{P}_k^a = (\mathbf{I}_k - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_k^f.$$

Data assimilation: Kalman filter



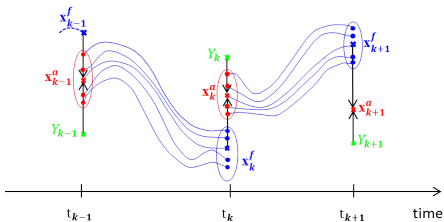
Properties

- ▶ Obs. and dynamics linear
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- Time dependent prior (mean, cov.)
- Comput. cost. of K and P

Alternative approaches

- ▶ Extended Kalman Filter (EKF)
 - H and M linearized
- ▶ Sub Optimal Filter (SOS)
 - Reduce comput. cost H

Data assimilation: Kalman filter



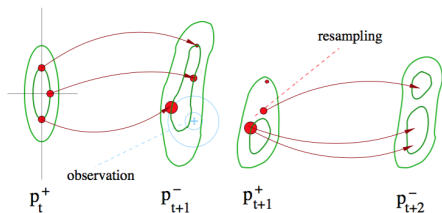
Properties

- ▶ Obs. and dynamics linear
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- Comput. cost. of K and P

Alternative approaches

- ▶ Extended Kalman Filter (EKF)
 - H and M linearized
- ▶ Sub Optimal Filter (SOS)
 - Reduce comput. cost H
- ▶ Ensemble Kalman Filter (EnKF)
 - Empirical estimation of P
 - H and M non linear

Data assimilation: Kalman filter



From Boquet's lecture notes (2014-2015)

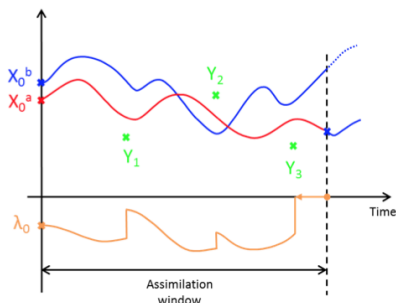
Properties

- ▶ Obs. and dynamics linear
- ▶ Noises Gaussian, unbiased, white-in-time
- Time dependent prior (mean, cov.)
- Comput. cost. of K and P

Alternative approaches

- ▶ Extended Kalman Filter (EKF)
 - H and M linearized
- ▶ Sub Optimal Filter (SOS)
 - Reduce comput. cost H
- ▶ Particle Filter (PF)
 - H and M non linear
 - Noises: non-Gaussian, biased, multimodal
 - Sampling issues due to high dimensions

Data assimilation: Variational 4DVar



Energy function

$$J(\mathbf{x}_0) = \frac{1}{2} \|\mathbf{x}_0 - \mathbf{x}_0^b\|_B^2 + \frac{1}{2} \int_{t_0}^{t_f} \|\mathbb{H}(\mathbf{x}) - \mathcal{Y}\|_R^2 dt,$$

s.t. $\partial_t \mathbf{x}(t, \mathbf{x}) + \mathbb{M}(\mathbf{x}(t, \mathbf{x}), u) = 0.$

- ▶ Computing the gradient of $J(\mathbf{x}_0)$ is very expensive!
- ▶ Deduced by solving the backwards adjoint equation

$$-\partial_t \lambda(t) + (\partial_{\mathbf{x}} \mathbb{M})^* \lambda(t) = (\partial_{\mathbf{x}} \mathbb{H})^* R^{-1} (\mathcal{Y}(t) - H(\mathbf{x}(t)))$$

$$\lambda(t_f) = 0$$

Properties

- ▶ Obs. and dynamics non-linear
- ▶ Noises Gaussian, unbiased, white-in-time
- Time independent prior (B)
- Derivation of the adjoint model

Outline

Data assimilation ingredients

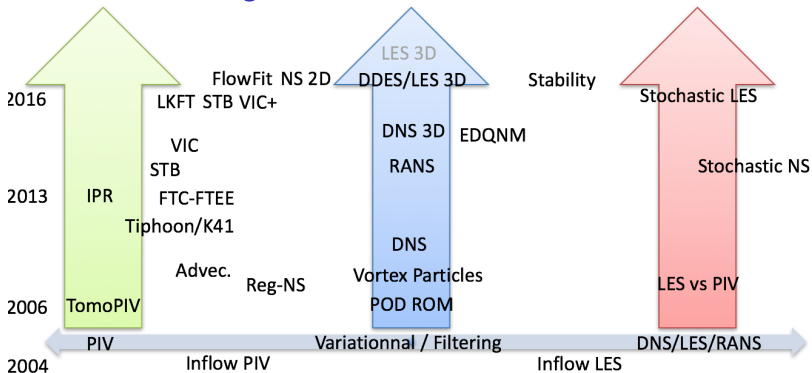
Data assimilation tools

Overview of significant achievements

Some applications

Data assimilation: data-driven vs model-driven

Different modelling



Exp Fluid Dynamics



Data Assimilation

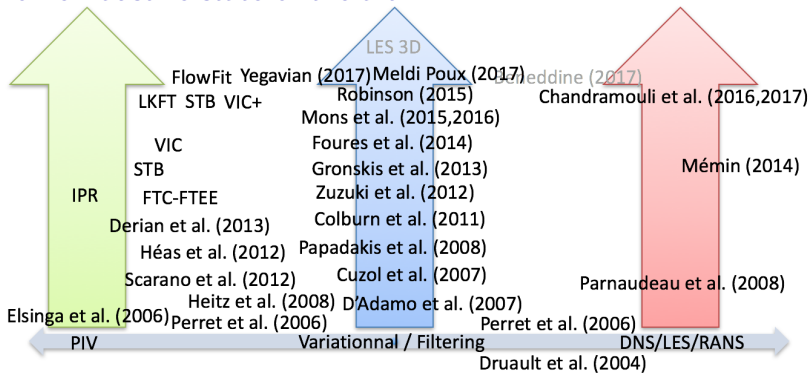


Comp. Fluid Dynamics



Data assimilation: data-driven vs model-driven

Non exhaustive state of the art



Exp Fluid Dynamics



Data Assimilation

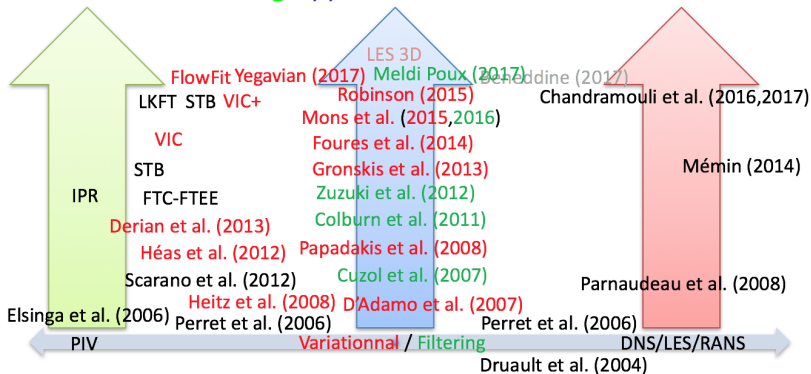


Comp. Fluid Dynamics



Data assimilation: data-driven vs model-driven

Variational vs Filtering approaches



Exp Fluid Dynamics



Data Assimilation

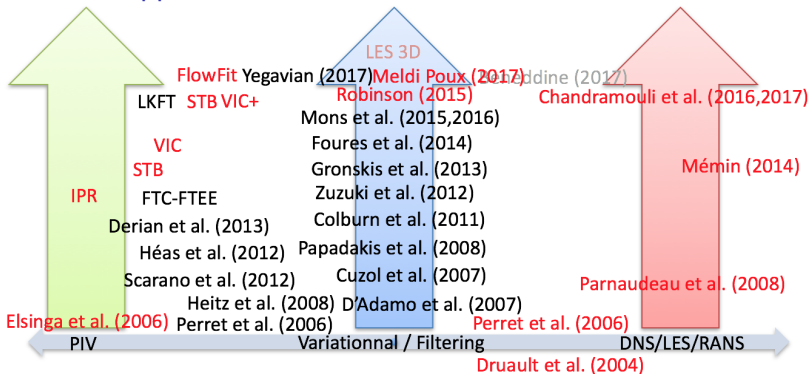


Comp. Fluid Dynamics



Data assimilation: data-driven vs model-driven

3D vs 2D approaches



Exp Fluid Dynamics



Data Assimilation

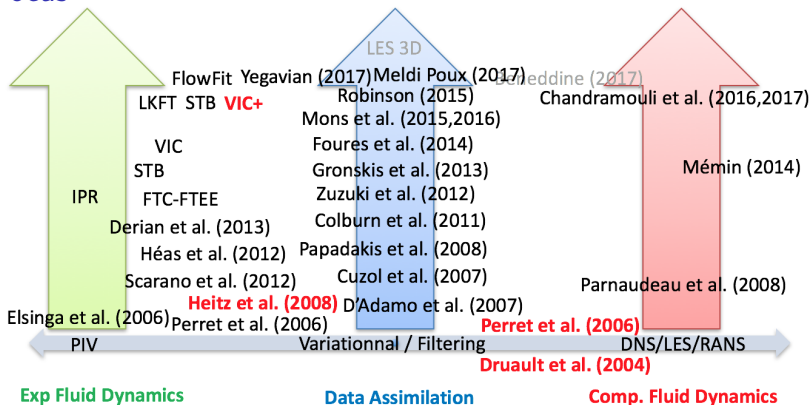


Comp. Fluid Dynamics



Data assimilation: data-driven vs model-driven

Focus



Exp Fluid Dynamics



Data Assimilation



Comp. Fluid Dynamics



Data assimilation: data-driven vs model-driven

AIAA JOURNAL
Vol. 42, No. 3, March 2004

Generation of Three-Dimensional Turbulent Inlet Conditions for Large-Eddy Simulation

P. Druault*

Université Pierre-et-Marie-Curie, 78210 Saint Cyr l'Ecole, France

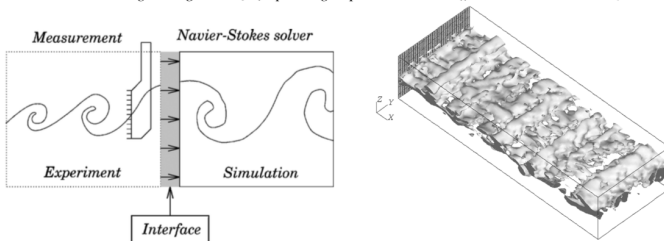
S. Lardeau†

Imperial College of Science, Technology, and Medicine, London, England SW7 2BY, United Kingdom
and

J.-P. Bonnet,‡ F. Coiffet,§ J. Delville,¶ E. Lamballais,** J. F. Largeau,§ and L. Perret§

Université de Poitiers, 86962 Futuroscope Chasseneuil CEDEX, France

A method for generating realistic (i.e., reproducing in space and time the large-scale coherence of the flows)



Data assimilation: data-driven vs model-driven

PHYSICS OF FLUIDS 20, 075107 (2008)

Turbulent inflow conditions for large-eddy simulation based on low-order empirical model

Laurent Perret,^{1,a)} Joël Delville,² Rémi Manceau,² and Jean-Paul Bonnet²

¹Laboratoire de Mécanique des Fluides (LMF), UMR CNRS 6598, Ecole Centrale de Nantes, 1 rue de la Noë BP 92101, F-44321 Nantes Cedex 3, France

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(Received 30 October 2007; accepted 3 June 2008; published online 22 July 2008)

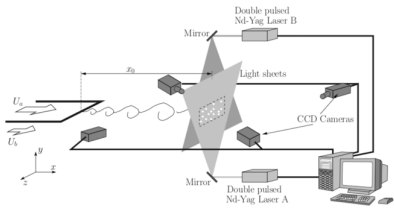
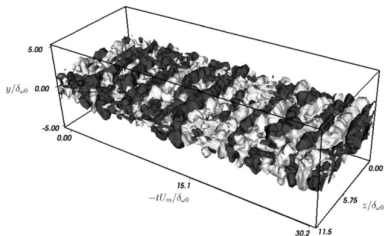


FIG. 1. DT-SPIV setup.



Data assimilation: data-driven vs model-driven

Exp Fluids (2008) 45:595–608
DOI 10.1007/s00348-008-0567-4

RESEARCH ARTICLE

Dynamic consistent correlation-variational approach for robust optical flow estimation

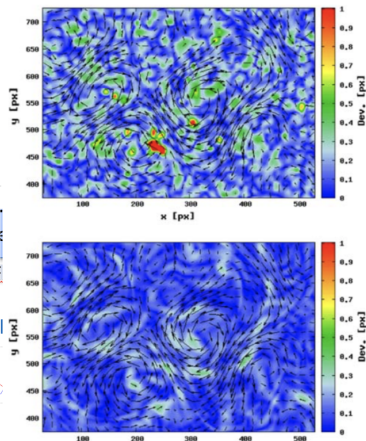
D. Heitz · P. Héas · E. Mémin · J. Carlier

$$J(\mathbf{u}, I) = J_d(\mathbf{u}, I) + J_r(\mathbf{u}) + J_p(\mathbf{u}, \mathbf{u}_p) + J_c(\mathbf{u}, \mathbf{u}_c), \quad (12)$$

where $J_p(\cdot)$ is an energy function constraining displacements \mathbf{u} to be consistent with a physically sound prediction \mathbf{u}_p relying on Navier–Stokes equations. As proposed in Héas et al. (2007), we define this functional as a quadratic distance between the estimated field \mathbf{u} and the dense propagated field $\mathbf{u}_p = (u_p, v_p)$:

$$J_p(\mathbf{u}, \mathbf{u}_p) = \beta \int_{\Omega} \|\mathbf{u}_p(\mathbf{s}) - \mathbf{u}(\mathbf{s})\|^2 ds, \quad (13)$$

where β denotes a weighting factor. This approach constitutes an alternative to the spatio-temporal smoother defined in Weickert and Schnörr (2001) and is to some extent similar to the temporal constraint introduced in Rhunau et al. (2007). It is important to distinguish this



Data assimilation: data-driven vs model-driven

Exp Fluids (2016) 57:139
 DOI 10.1007/s00348-016-2225-6



RESEARCH ARTICLE

Dense velocity reconstruction from tomographic PTV with material derivatives

Jan F. G. Schneiders¹ · Fulvio Scarano¹

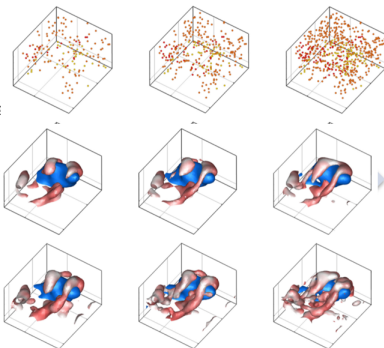
$$J = J_u + \alpha^2 J_{Du}, \quad (6)$$

where α is a weighting coefficient (Sect. 2.3.3), J_u is given by Eq. (7) and J_{Du} is given by Eq. (8),

$$J_u = \sum_p \|\mathbf{u}_h(\mathbf{x}_p) - \mathbf{u}_m(\mathbf{x}_p)\|^2, \quad (7)$$

$$J_{Du} = \sum_p \left\| \frac{D\mathbf{u}_h}{Dt}(\mathbf{x}_p) - \frac{D\mathbf{u}_m}{Dt}(\mathbf{x}_p) \right\|^2, \quad (8)$$

where \mathbf{u}_h and $D\mathbf{u}_h/Dt$ are calculated from Eqs. (1) and (2) and are evaluated at the particle locations, \mathbf{x}_p , by linear interpolation from the computational grid. The cost function penalizes the difference between the PTV measurements and the velocity and material derivative at a single measurement time-instant calculated from the optimization variables. The optimization problem does not include time-integration of the vorticity transport equation.



Outline

Data assimilation ingredients

Data assimilation tools

Overview of significant achievements

Some applications

Wave in a rectangular flat bottom tank

Reconstruct unobserved state from depth camera

WEnKF approach from Combès et al. (2015)

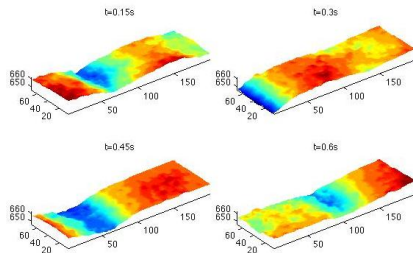
EnVar approach from Yang et al. (2015)

Depth observations

Data assimilation

- ▶ Reconstruct unobserved surface velocity
- ▶ Error model

Wave in a rectangular flat bottom tank



Flow configuration

- ▶ $L_x \times L_y = 250 \text{ mm} \times 100 \text{ mm}$
- ▶ Initial free surface height difference $h_0 = 1 \text{ cm}$
- ▶ Observations every $10\Delta t u_0/L_x$ leading to $St_{\text{obs}} \simeq 24$, that was rather high !

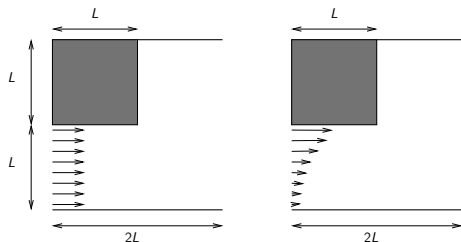
Simulation parameters

- ▶ $n_x \times n_y = 222 \times 88$
- ▶ $\Delta t u_0/L_x = 0.0042$

Assimilation parameters

- ▶ particle number $N = 100$
- ▶ $\mathbf{X}_0 \sim \mathcal{N}(\mathbf{x}_{\text{init}}, \mathbf{R}_0)$
- ▶ $\mathbf{W}_t^f \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_t)$
- ▶ $\mathbf{W}_t^g \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_t)$
- ▶ \mathbf{R}_0 (0.05 h_0 ; 0.25 u_0 ; r_h)
- ▶ \mathbf{R}_t (0.04 h_0 ; 0.06 u_0 ; r_h)
- ▶ \mathbf{Q}_t (0.013 h_0^2 ; *diag.*)
- ▶ localization $h_{\text{correl}} = 0.6h_0$
- ▶ $\mathbf{x}_{\text{init}} = (0, 0, 0)$

Suddenly expanding flume



Flow configurations

- ▶ $L = 10$ cm
- ▶ Inflow velocity and elevation oscillatory in phase at 1 Hz with $H_{in} = 1$ cm and $V_{in} = 0.22$ m/s
- ▶ $Fr = U_{in}/\sqrt{g H_{in}} = 0.7$

Simulation parameters

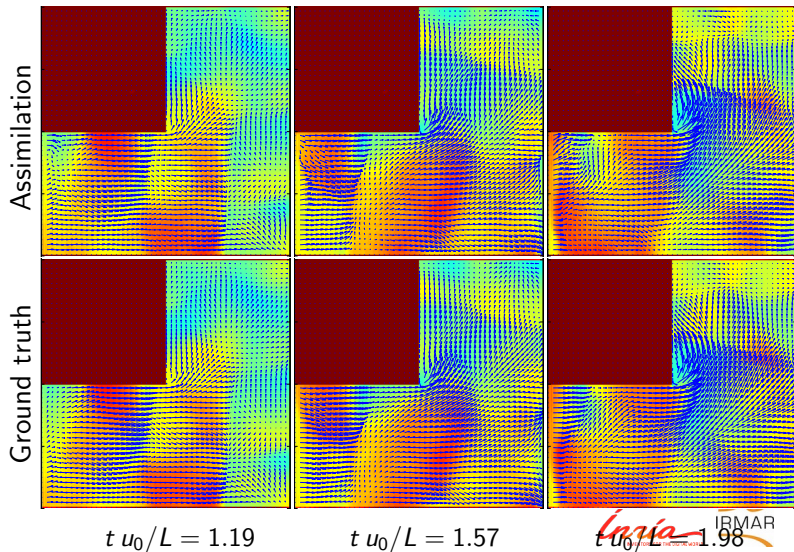
- ▶ $n_x \times n_y = 200 \times 200$
- ▶ $\Delta t u_0/L = 0.006$

Assimilation parameters

- ▶ particle number $N = 100$
- ▶ $\mathbf{X}_0 \sim \mathcal{N}(\mathbf{x}_{init}, \mathbf{R}_0)$
- ▶ $\mathbf{W}_t^f \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_t)$
- ▶ $\mathbf{W}_t^g \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_t)$
- ▶ \mathbf{R}_0 (0.05 h_0 ; 0.25 u_0 ; r_h)
- ▶ \mathbf{R}_t (0.04 h_0 ; 0.06 u_0 ; r_h)
- ▶ \mathbf{Q}_t (0.013 h_0^2 ; *diag.*)
- ▶ localization $h_{correl} = 0.6 h_0$
- ▶ $\mathbf{x}_{init} = (0, 0, 0)$

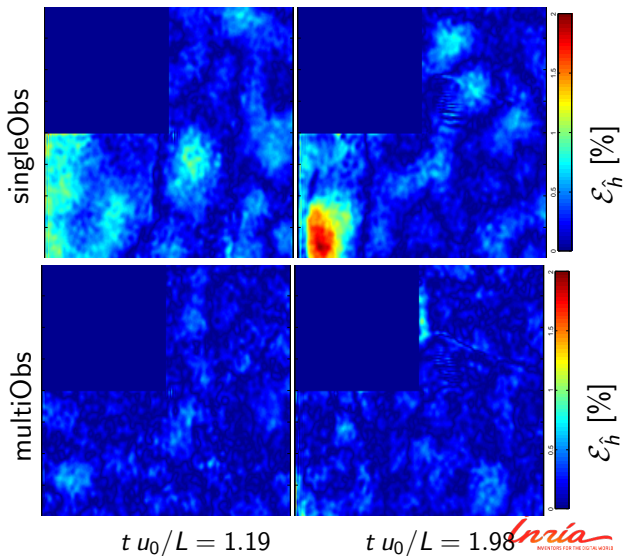
Suddenly expanding flume

Non-uniform inlet velocity profile (with spatial complexity)



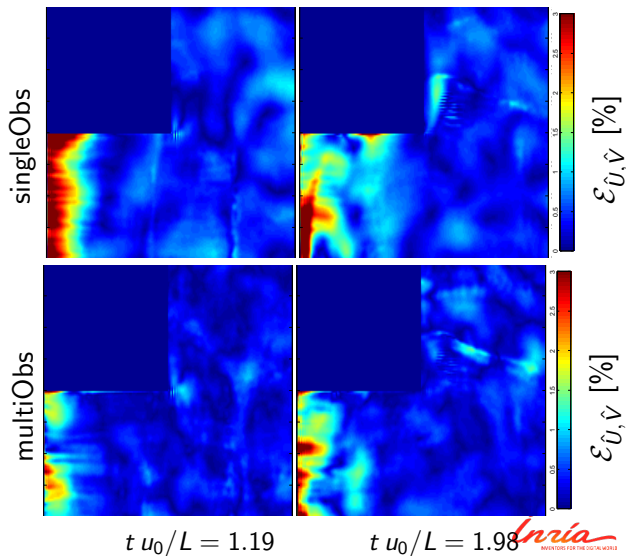
Suddenly expanding flume

Elevation error maps for singleObs and multiObs



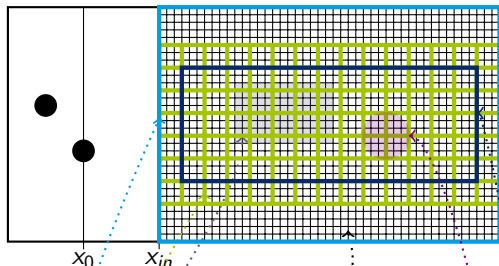
Suddenly expanding flume

Velocity error maps for singleObs and multiObs



Cylinder wakes at $Re=112$

Gronskis et al. (2013, 2015)

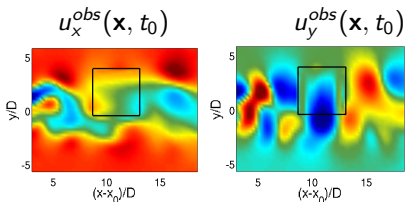


DNS grid

Gap region Ω_G PIV grid, $dx_{obs} = 3dx$ Control domain Ω_C , fine gridWeighted average to give \bar{w} in Ω_A Assimilation domain Ω_A , coarse grid

Cylinder wakes at $Re=112$

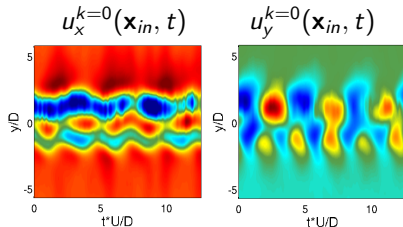
Initial condition



Initial condition in Ω_G (IC)

1. Uniform stagnant flow
2. Velocity interpolation

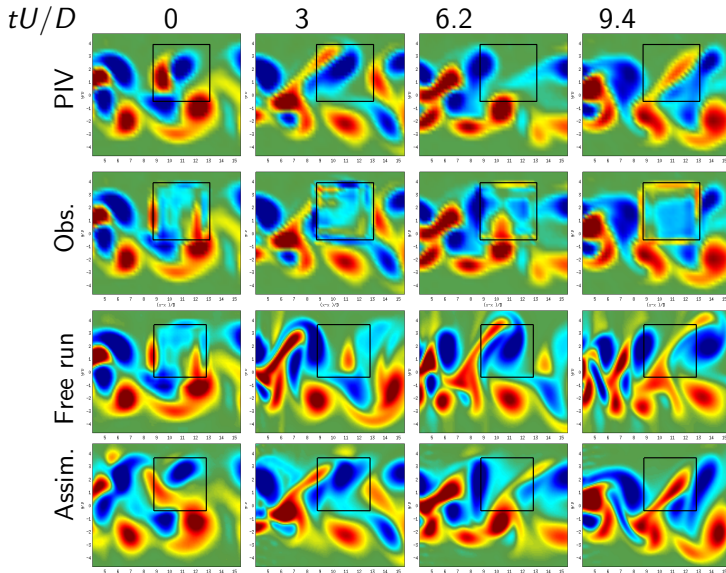
Inflow condition



- From PIV sequences with Taylor's hypothesis

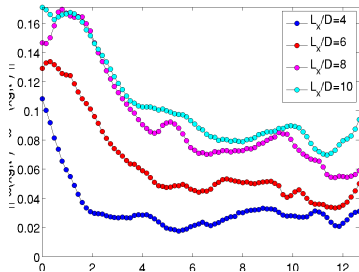
Cylinder wakes at $Re=112$

Gap reconstruction



Cylinder wakes at $Re=112$

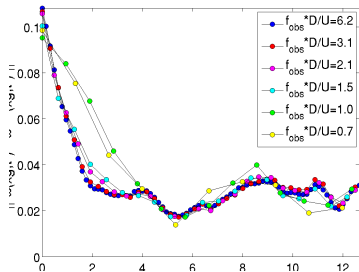
Influence of gap size



$IC=1, f_{obs} D/U=6.2$

Method's accuracy was strongly related to the size of the gap.

Influence of obs. frequency



$L_x/D=4, IC=1$

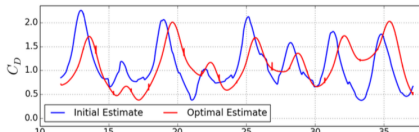
Error decreased with increasing observations frequency

$St_{obs} = f_{obs} D/U.$

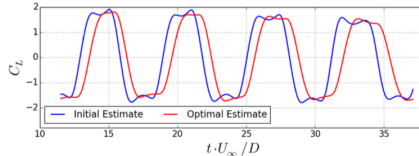
Cylinder wakes at $Re=112$

Pressure, Drag and Lift reconstruction via 4DVar

U



V

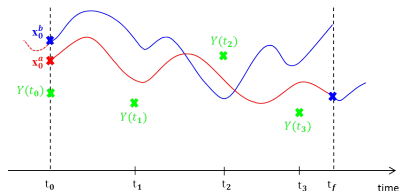


Pres.

- ▶ Reconstruct unobserved pressure
- ▶ Lift and Drag via control volume

How to build the background ?

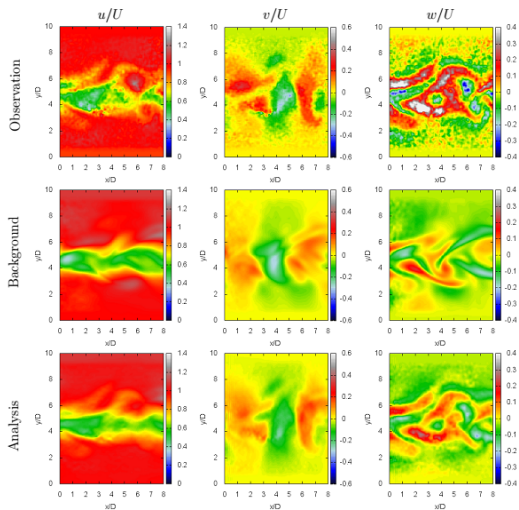
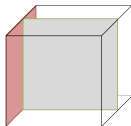
Real orthogonal-plane SPIV observations at $Re = 300$ and 4DVar (Robinson, 2015)



- ▶ Run a simulation from inlet observations

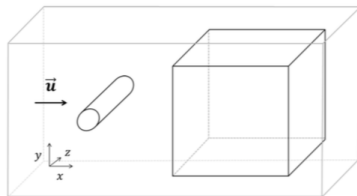
3D initial condition correction

Real orthogonal-plane SPIV observations at $Re = 300$ and 4DVar (Robinson, 2015)



LES coefficient 4DVar data assimilation

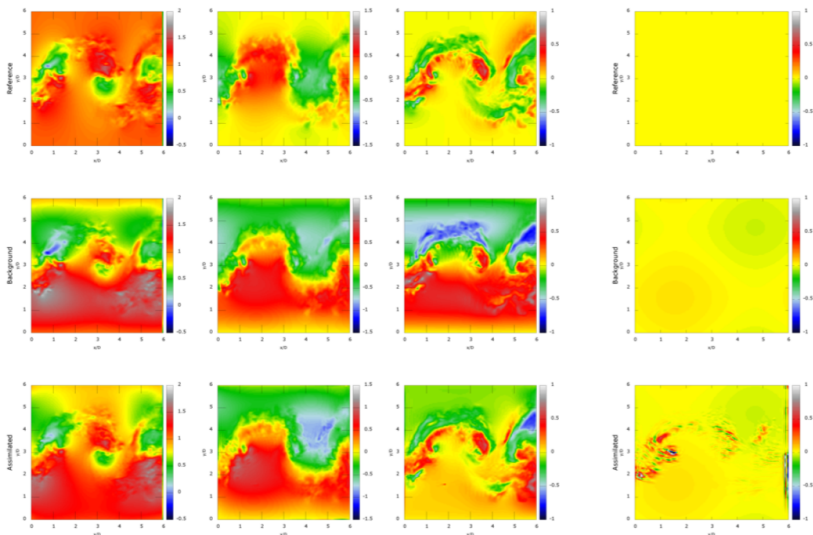
Chandramouli (2017, CFDforPIV)



	Re	$n_x \times n_y \times n_z$	$l_x/D \times l_y/D \times l_z/D$	$U\Delta t/D$	Duration
FD	3900	$361 \times 361 \times 48$	$20 \times 20 \times 3.14$	0.003	$40100\Delta t$
4DVAR	3900	$145 \times 145 \times 48$	$6 \times 6 \times 3.14$	0.003	$100\Delta t$

LES coefficient 4DVar data assimilation

Chandramouli (2017, CFDforPIV)



Summary

- ▶ Data assimilation is a powerful technique to combine observations and models
- ▶ Data driven vs model driven (d vs m)
- ▶ When the amount of available data is insufficient to fully describe the system one cannot rely on data-driven approaches → model and regularization are paramount
- ▶ Data assimilation for prediction, filtering or smoothing
- ▶ History of use is the search for suitable approximation that, even sub-optimal, works with non-linear, non Gaussian and high dimensional settings

Outlooks

- ▶ Dynamics model (large scale, uncertainties)
- ▶ Control BC (inflow, outflow, ...) and model parameters
- ▶ From pseudo-observations to observations