Variational Data Assimilation with Turbulence Modelling
P. Chandramouli, E. Mémin, D. Heitz

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**Variational Data Assimilation with Turbulence Modelling**

Pranav Chandramouli\(^1\), Etienne Memin\(^1\), Dominique Heitz\(^2\)

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**To assimilate observations and optimise the analysis trajectory for turbulent flows using:**

- Turbulence modelling\(^{[1, 2]}\)
- Volumetric observations\(^{[3]}\)
- Accurate background condition\(^{[3]}\)
- Background covariance estimation
- Optimised model coefficient

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**Mathematical Formulation**\(^{[4]}\)

\[
\begin{align*}
\text{Cost} & = \frac{1}{2} \| \delta x_0 \|_{a^{-1}}^2 + \frac{1}{2} \int_0^T \| \delta u \|_{R^{-1}}^2 \, dt + \frac{1}{2} \int_0^T \| \mathbf{H} (x(t)) - \mathbf{y}(t) \|_{a^{-1}}^2 \, dt \\
\frac{\partial}{\partial (\delta x_0)} &= -\lambda (t_0) + B^{-1} \delta x_0 \\
\frac{\partial}{\partial (\delta u)} &= -\lambda (t_0) + B^{-1} \delta u + (\partial_a \mathbf{M}) \lambda
\end{align*}
\]

**Glossary:**

- \(x_0\) - Initial state \((x)\) of the system
- \(u\) - Control parameters
- \(B\) - Background covariance matrix
- \(R\) - Observation covariance matrix
- \(\lambda\) - Adjoint variable

**Numerical Formulation**

- **Forward/Tangent Iteration**
  - \(\mathbf{H}\) - Observation operator
  - \(\mathbf{M}\) - Dynamical evolution model
- **Backward/Adjoint Iteration**
  - \(\partial_a \mathbf{M}\) - Adjoint of the control dynamical model

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**Case**

**Analysis at \(t_3\)**

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**Reference**


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**Conclusion**

- Turbulence modelling facilitates assimilation of turbulent flows
- Well-estimated background significantly improves analysis
- Physically relevant coefficient estimation is feasible via data assimilation
- Fully-defined background covariance matrix reduces computational time significantly at minor loss of accuracy
- Reconstructed volumetric observations are sufficient to perform assimilation and achieve meaningful results

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**Contact:** cpranav93@gmail.com