Variational Data Assimilation with Turbulence Modelling

Pranav Chandramouli, Etienne Mémin, Dominique Heitz

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Pranav Chandramouli[1], Etienne Memin[1], Dominique Heitz[2]

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Motivation

To assimilate observations and optimise the analysis trajectory for turbulent flows using:

- Turbulence modelling[1, 2]
- Volumetric observations [3]
- Accurate background condition [3]
- Background covariance estimation
- Optimised model coefficient

Mathematical Formulation [4]

Cost

$$J(\delta x_0, \delta u) = \underbrace{\frac{1}{2}||\delta x_0||^2_{B^{-1}}}_{\text{Background Error}} + \underbrace{\frac{1}{2}\int_{t_0}^{t_f}||\delta u_t||^2_{B_c^{-1}} dt}_{\text{Control Error}} + \underbrace{\frac{1}{2}\int_{t_0}^{t_f}||\mathbb{H}(x_t) - \mathcal{Y}(t)||^2_{R^{-1}} dt}_{\text{Observation Error}}$$

Gradient

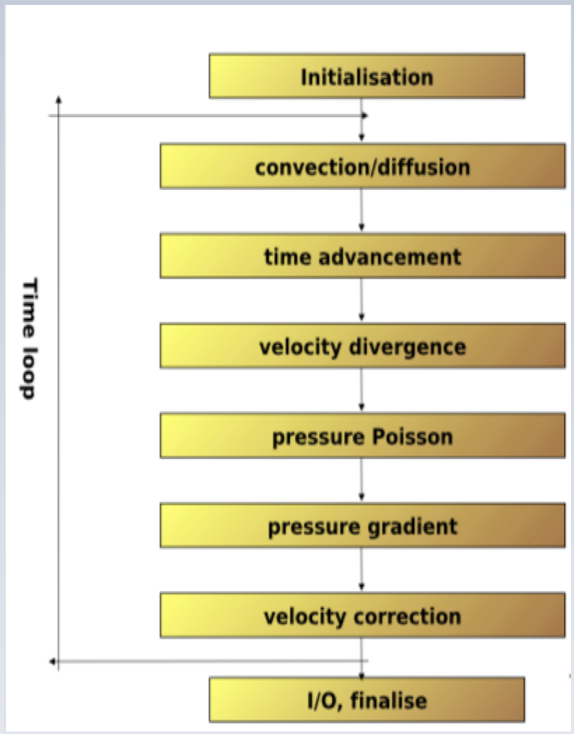
$$\frac{\partial J}{\partial(\delta x_0)} = -\lambda(t_0) + B^{-1}\delta x_0 \qquad \frac{\partial J}{\partial(\delta u)} = -\lambda(t_0) + B_c^{-1}\delta u + (\partial_u \mathbb{M})^* \lambda.$$

Glossary:

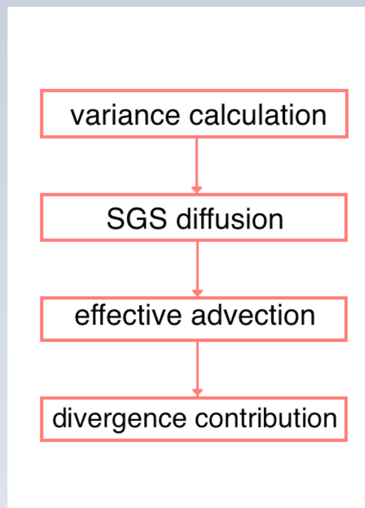
x_0 – Initial state (x) of the system
u – Control parameters
B – background covariance matrix
R – observation covariance matrix
λ – adjoint variable
$\mathbb{H}$ - observation operator
$\mathcal{Y}$ – set of observations
$\mathbb{M}$ – dynamical evolution model
$(\partial_u \mathbb{M})^*$ - adjoint of the control dynamical model

Numerical Formulation

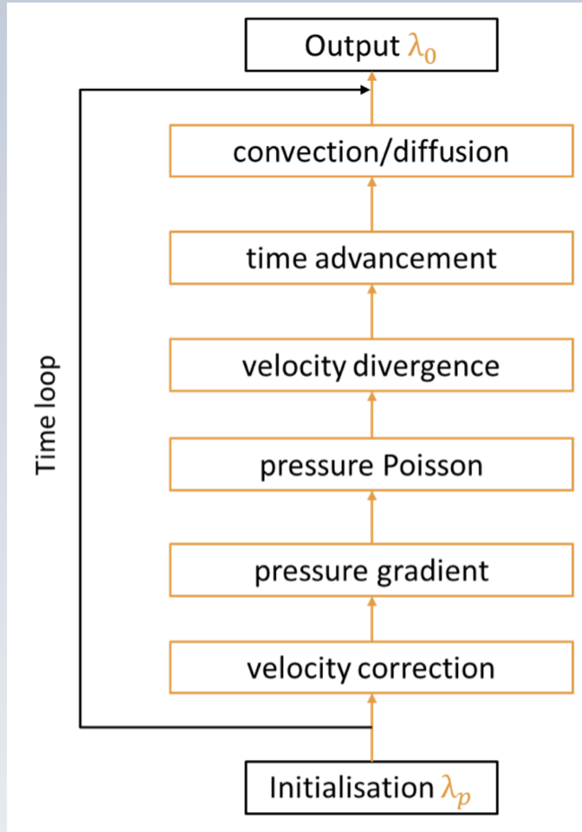
Forward/Tangent Iteration*



Sub-grid model

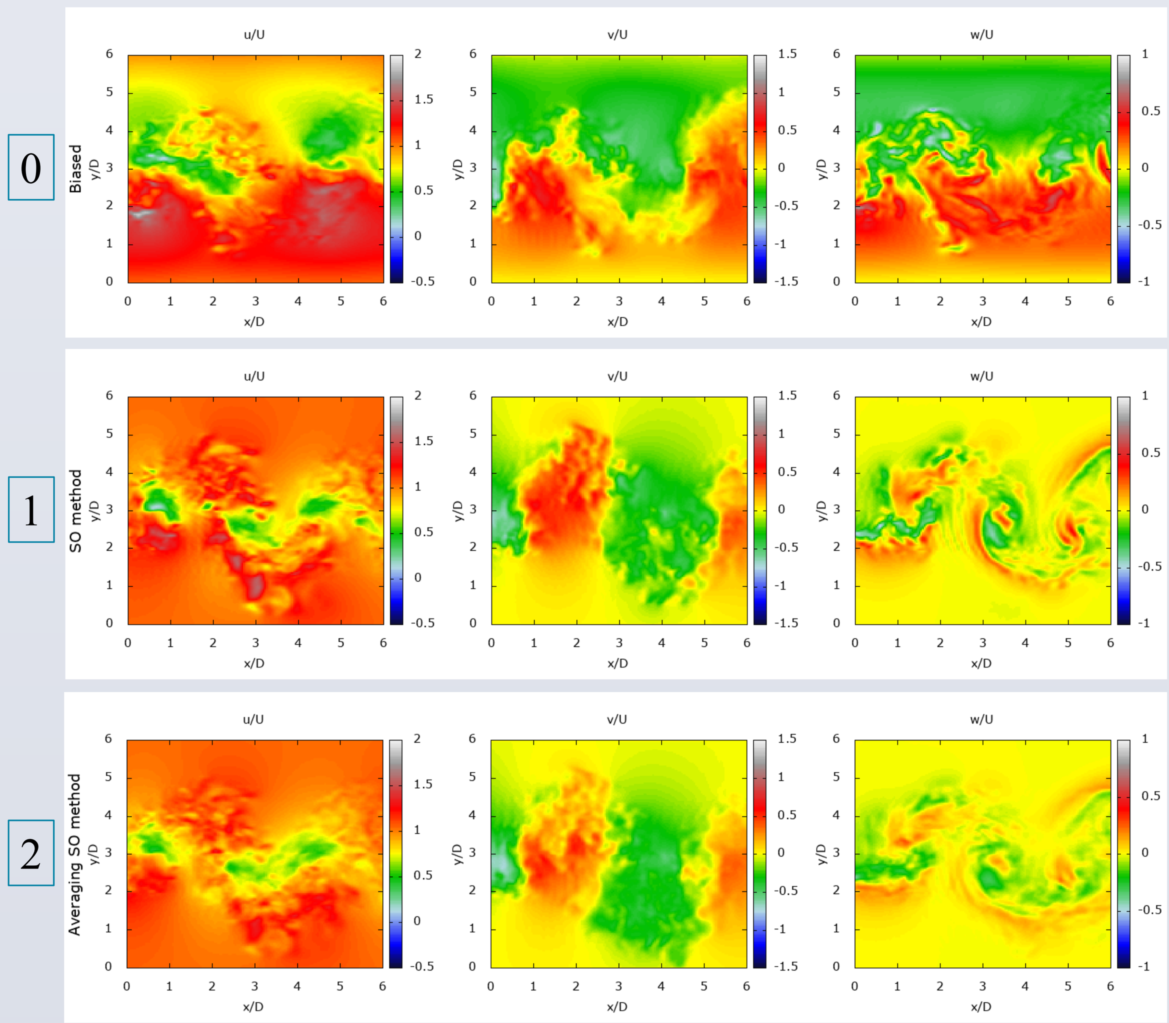


Backward/Adjoint Iteration

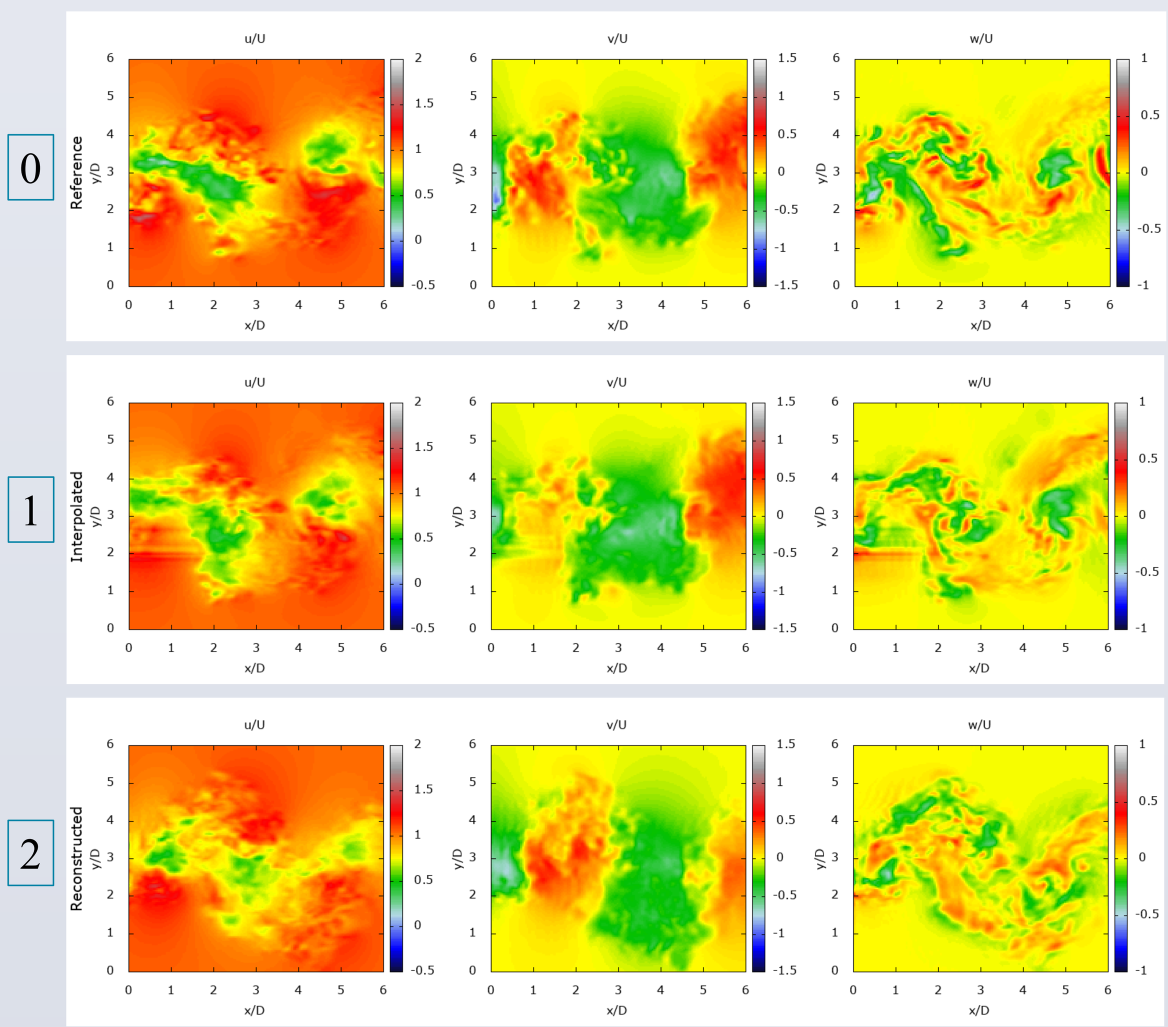


* Based on Incompact3d [5]

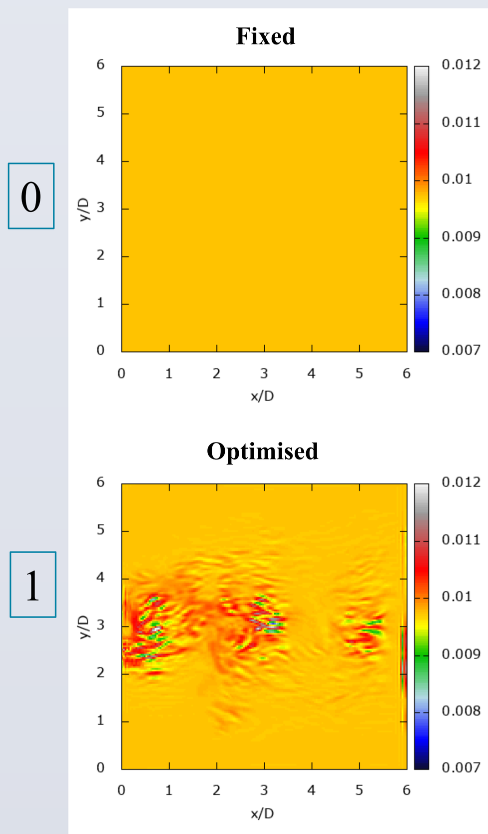
Background @ t_0 - (1)234



Observations @ t_0 - 1(2)34



Coefficient - 12(3)4



Background Covariance - 123(4)

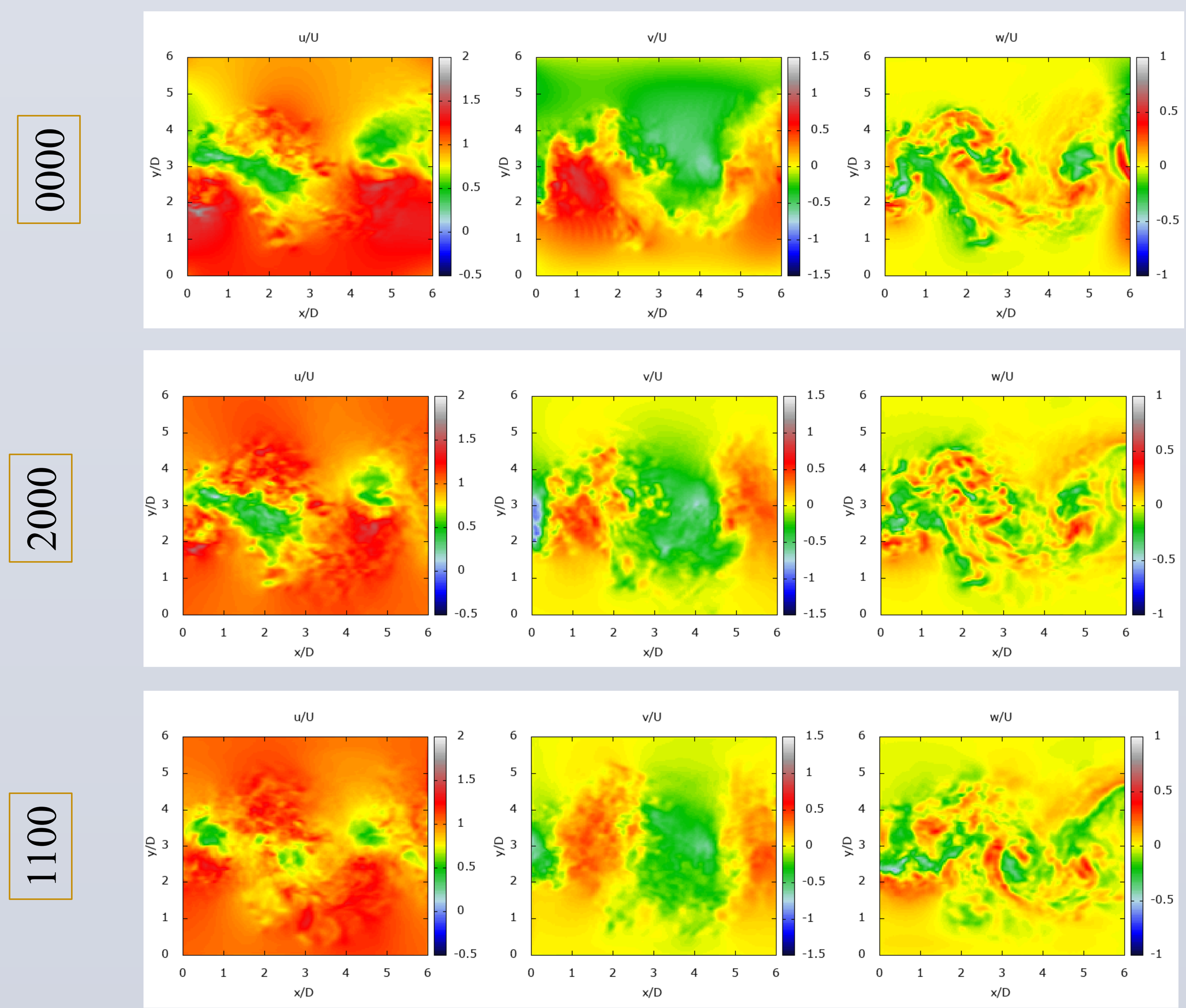
0 — Identity

$$\begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & 1 & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix}$$

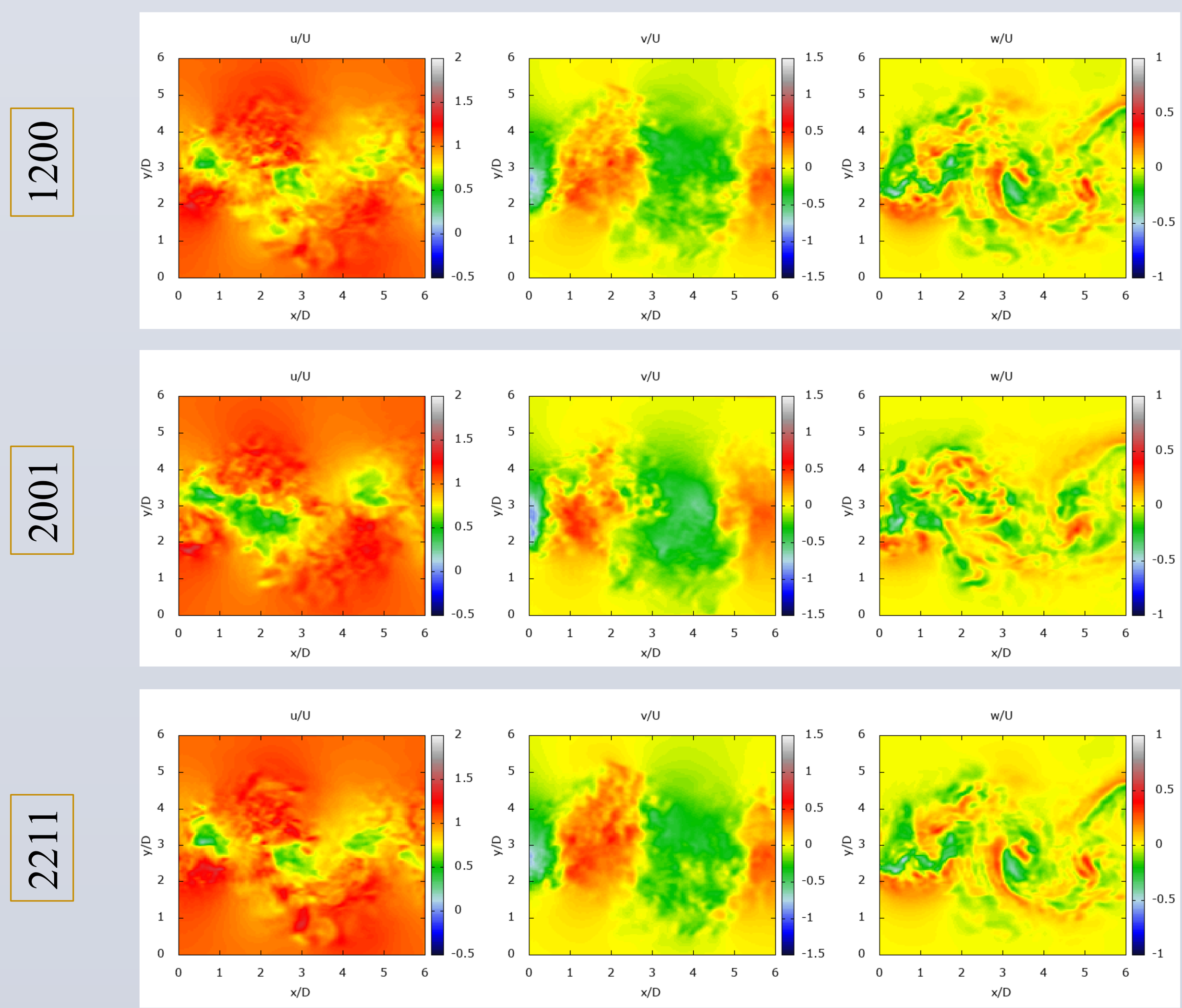
1 — Fully defined

$$\begin{bmatrix} b_{11} & \cdots & b_{i1} & \cdots & b_{n_x 1} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ b_{1j} & \ddots & b_{ij} & \ddots & b_{n_x j} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ b_{1n_y} & \cdots & b_{in_y} & \cdots & b_{n_x n_y} \end{bmatrix}$$

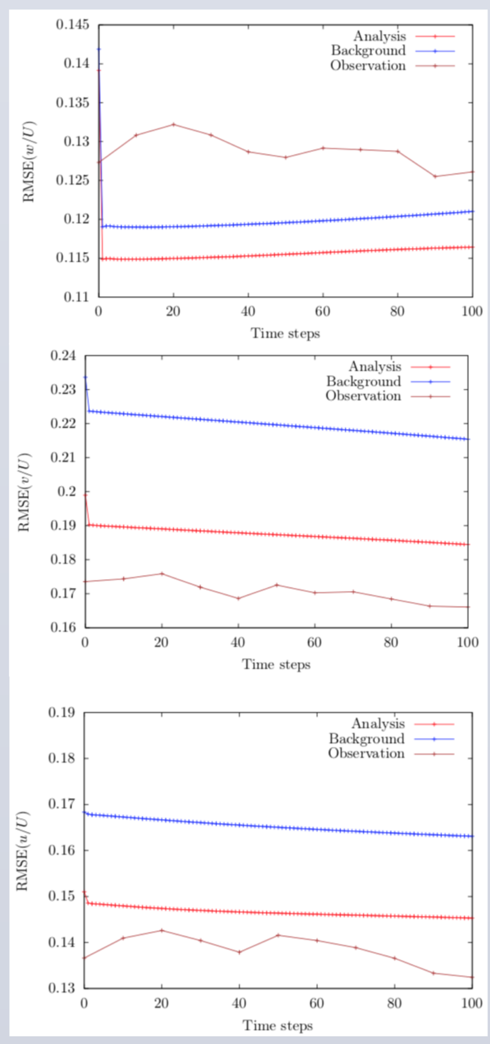
Case | **Analysis @ t_0**



Case | **Analysis @ t_0**



RMSE - 2211



Conclusion

- ✓ Turbulence modelling facilitates assimilation of turbulent flows
- ✓ Well-estimated background significantly improves analysis
- ✓ Physically relevant coefficient estimation is feasible via data assimilation
- ✓ Fully-defined background covariance matrix reduces computational time significantly at minor loss of accuracy
- ✓ Reconstructed volumetric observations are sufficient to perform assimilation and achieve meaningful results

Reference

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Contact: cpranav93@gmail.com