Variational Data Assimilation with Turbulence Modelling

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Motivation

To assimilate observations and optimise the analysis trajectory for turbulent flows using:

- Turbulence modelling[1, 2]
- Volumetric observations [3]
- Accurate background condition [3]
- Background covariance estimation
- Optimised model coefficient

Mathematical Formulation [4]

Cost

$$J(\delta \boldsymbol{x}_0, \delta \boldsymbol{u}) = \underbrace{\frac{1}{2}||\delta \boldsymbol{x}_0||^2_{\boldsymbol{B}^{-1}}}_{\text{Background Error}} + \underbrace{\frac{1}{2}\int_{t_0}^{t_f} ||\delta \boldsymbol{u}_t||^2_{\boldsymbol{B}_c^{-1}} dt}_{\text{Control Error}} + \underbrace{\frac{1}{2}\int_{t_0}^{t_f} ||\mathbb{H}(\boldsymbol{x}_t) - \mathcal{Y}(t)||^2_{\boldsymbol{R}^{-1}} dt}_{\text{Observation Error}}$$

Gradient

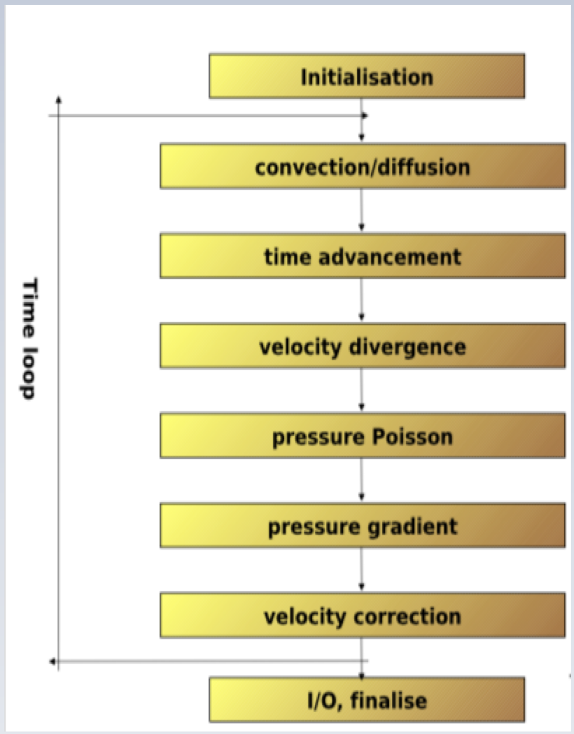
$$\frac{\partial J}{\partial(\delta \boldsymbol{x}_0)} = -\boldsymbol{\lambda}(t_0) + \boldsymbol{B}^{-1}\delta \boldsymbol{x}_0 \qquad \frac{\partial J}{\partial(\delta \boldsymbol{u})} = -\boldsymbol{\lambda}(t_0) + \boldsymbol{B}_c^{-1}\delta \boldsymbol{u} + (\partial_u \mathbb{M})^* \boldsymbol{\lambda}.$$

Glossary:

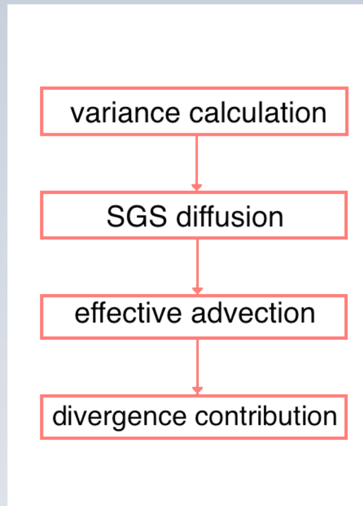
$\boldsymbol{x}_0$ – Initial state ($\boldsymbol{x}$) of the system
$\boldsymbol{u}$ – Control parameters
$\boldsymbol{B}$ – background covariance matrix
$\boldsymbol{R}$ – observation covariance matrix
$\boldsymbol{\lambda}$ – adjoint variable
$\mathbb{H}$ - observation operator
$\mathcal{Y}$ – set of observations
$\mathbb{M}$ – dynamical evolution model
$(\partial_u \mathbb{M})^*$ - adjoint of the control dynamical model

Numerical Formulation

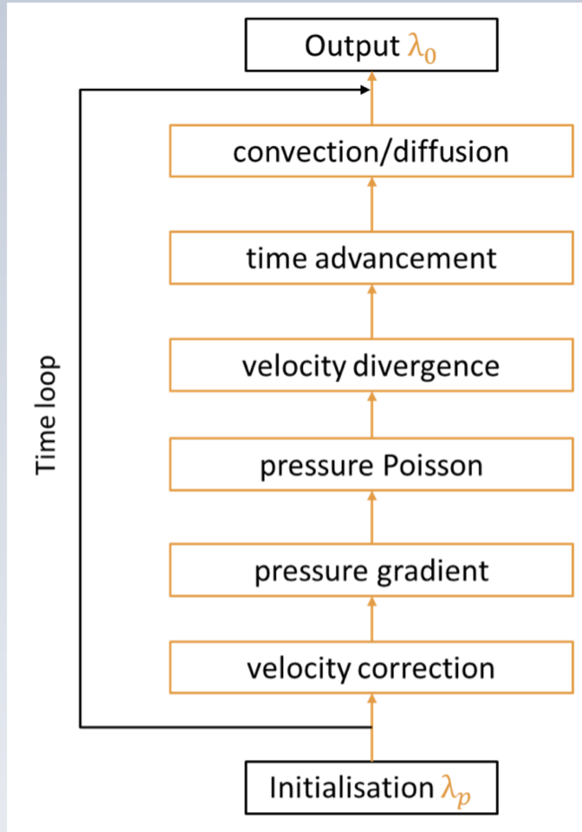
Forward/Tangent Iteration*



Sub-grid model



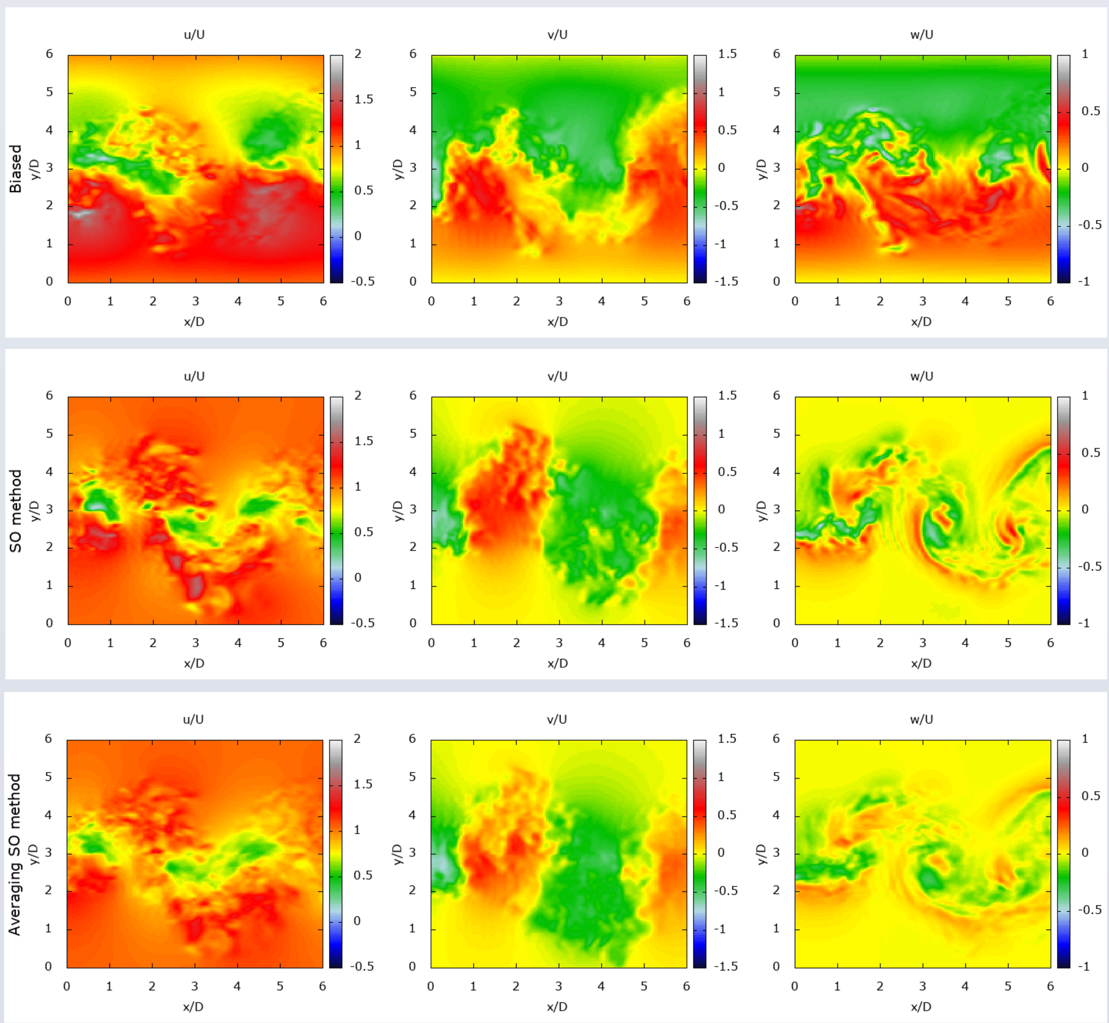
Backward/Adjoint Iteration



* Based on Incompact3d [5]

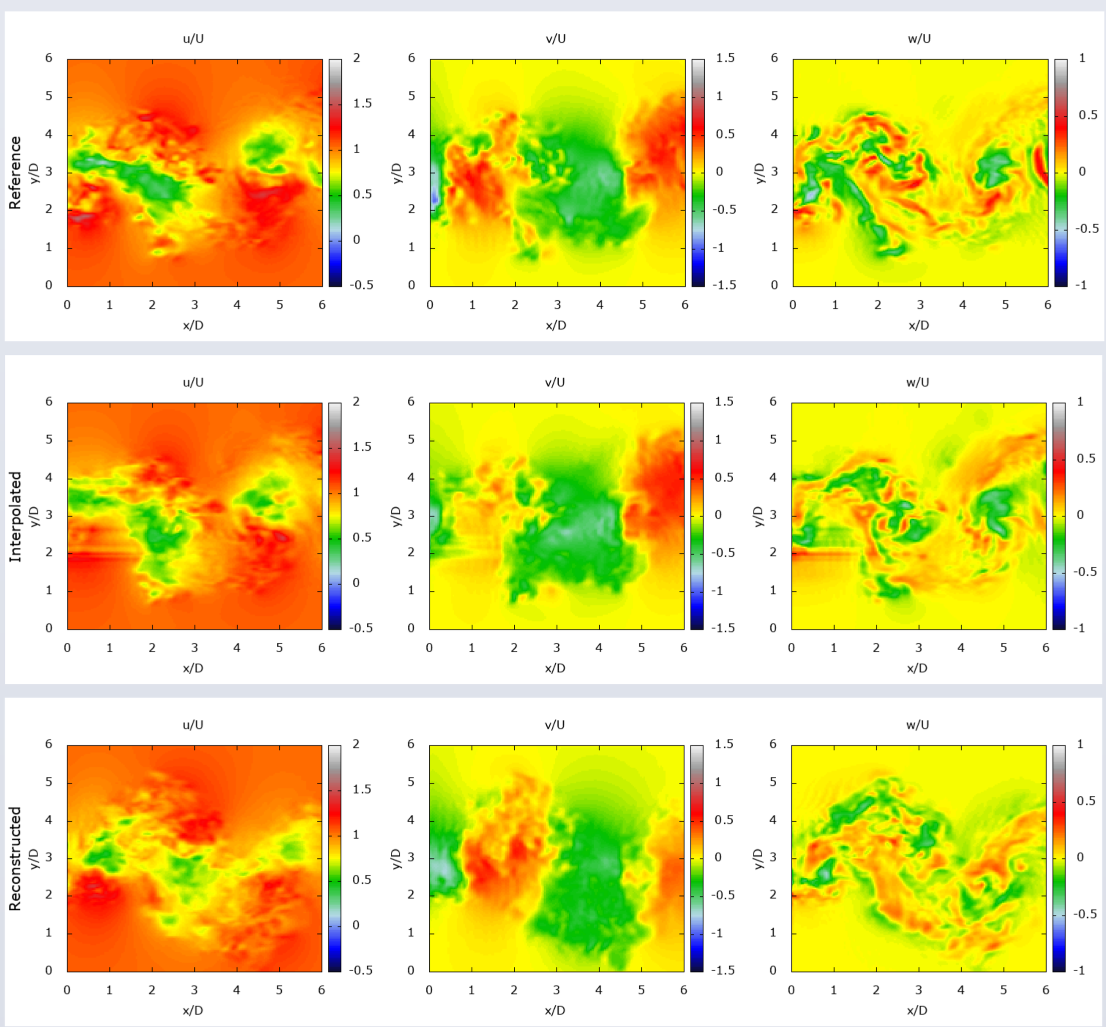
Background @ t_0 - (1)234

0 1 2



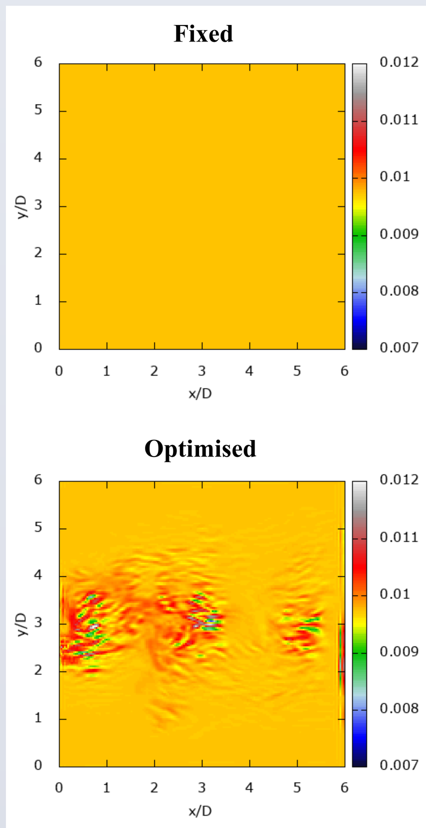
Observations @ t_0 - 1(2)34

0 1 2



Coefficient - 12(3)4

0 1



Background Covariance - 123(4)

0 Identity

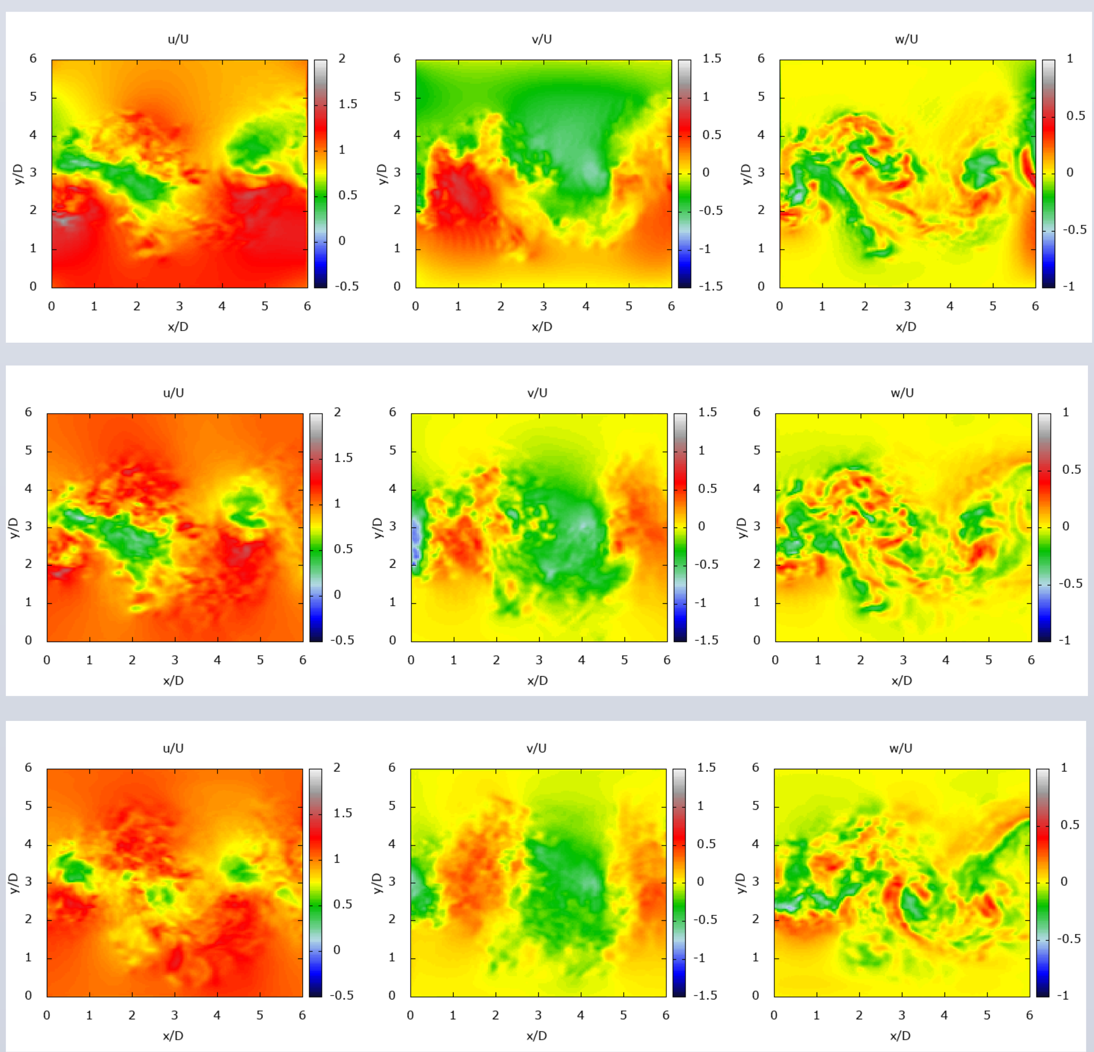
$$\begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & 1 & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix}$$

1 Fully defined

$$\begin{bmatrix} b_{11} & \cdots & b_{i1} & \cdots & b_{n_x 1} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ b_{1j} & \ddots & b_{ij} & \ddots & b_{n_x j} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ b_{1n_y} & \cdots & b_{in_y} & \cdots & b_{n_x n_y} \end{bmatrix}$$

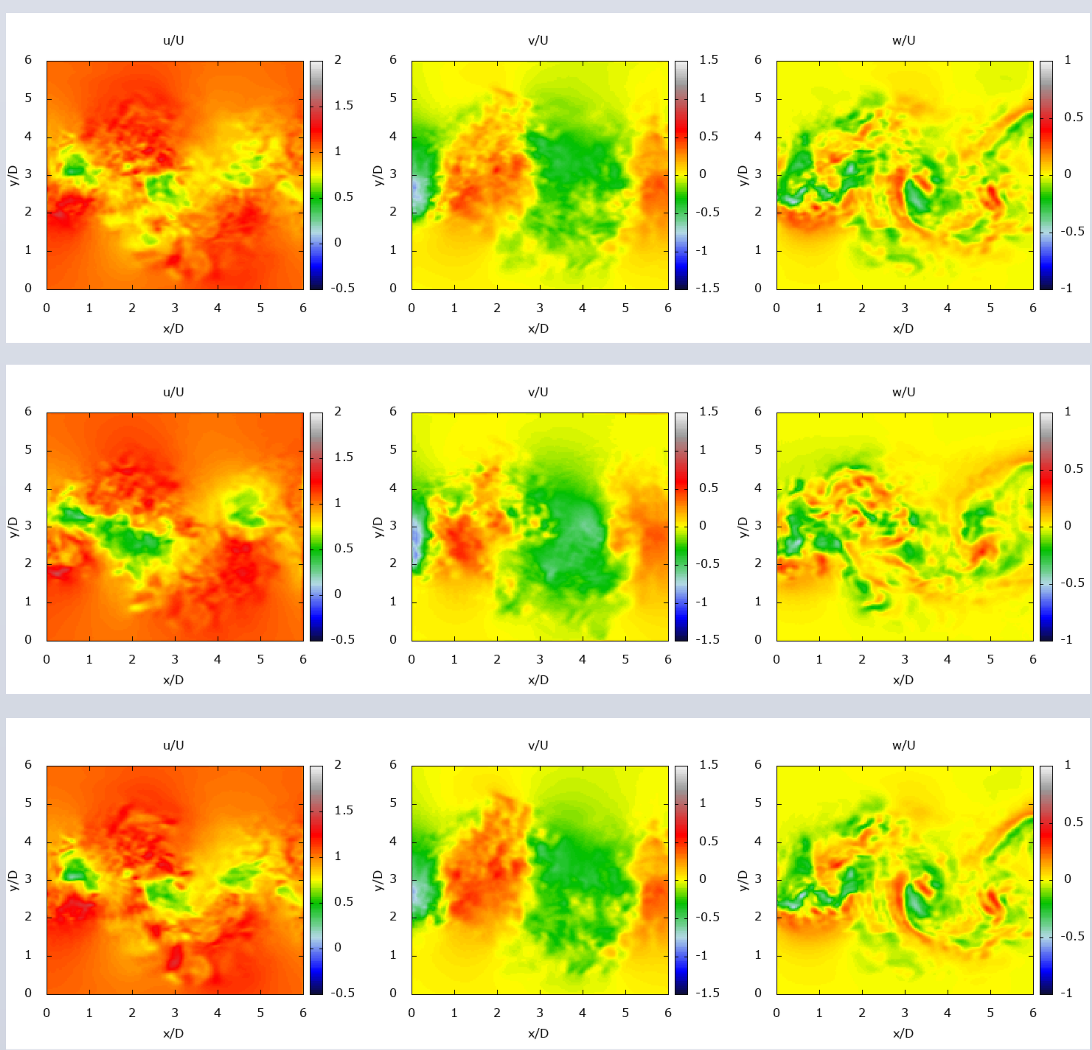
Case | **Analysis @ t_0**

0000 2000 1100

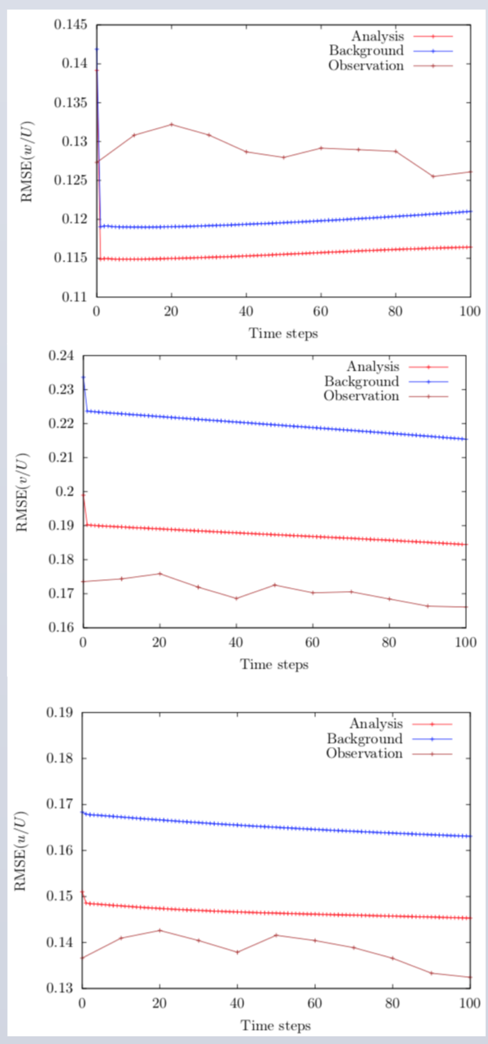


Case | **Analysis @ t_0**

1200 2001 2211



RMSE - 2211



Conclusion

- ✓ Turbulence modelling facilitates assimilation of turbulent flows
- ✓ Well-estimated background significantly improves analysis
- ✓ Physically relevant coefficient estimation is feasible via data assimilation
- ✓ Fully-defined background covariance matrix reduces computational time significantly at minor loss of accuracy
- ✓ Reconstructed volumetric observations are sufficient to perform assimilation and achieve meaningful results

Reference

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