Variational Data Assimilation with Turbulence Modelling

Pranav Chandramouli, Etienne Mémin, Dominique Heitz

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Pranav Chandramouli[1], Etienne Memin[1], Dominique Heitz[2]

1 Inria informatics mathematics

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Motivation

To assimilate observations and optimise the analysis trajectory for turbulent flows using:

- Turbulence modelling[1, 2]
- Volumetric observations [3]
- Accurate background condition [3]
- Background covariance estimation
- Optimised model coefficient

Mathematical Formulation [4]

Cost

$$J(\delta \boldsymbol{x}_0, \delta \boldsymbol{u}) = \frac{1}{2}\left|\left|\delta \boldsymbol{x}_0\right|\right|^2_{\boldsymbol{B^{-1}}} + \frac{1}{2}\int_{t_0}^{t_f}\left|\left|\delta \boldsymbol{u}_t\right|\right|^2_{\boldsymbol{B_c^{-1}}} dt + \frac{1}{2}\int_{t_0}^{t_f}\left|\left|\mathbb{H}\left(\boldsymbol{x}_t\right) - \mathcal{Y}(t)\right|\right|^2_{\boldsymbol{R^{-1}}} dt$$

Background Error — Control Error — Observation Error

Gradient

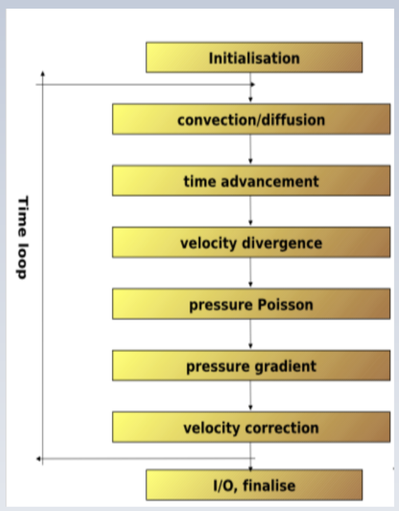
$$\frac{\partial J}{\partial(\delta \boldsymbol{x}_0)} = -\boldsymbol{\lambda}(t_0) + \boldsymbol{B}^{-1}\delta \boldsymbol{x}_0 \qquad \frac{\partial J}{\partial(\delta \boldsymbol{u})} = -\boldsymbol{\lambda}(t_0) + \boldsymbol{B_c}^{-1}\delta \boldsymbol{u} + (\partial_{\boldsymbol{u}}\mathbb{M})^*\boldsymbol{\lambda}.$$

Glossary:

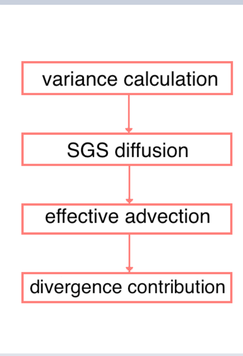
- $\boldsymbol{x}_0$ – Initial state $(\boldsymbol{x})$ of the system
- $\boldsymbol{u}$ – Control parameters
- $\boldsymbol{B}$ – background covariance matrix
- $\boldsymbol{R}$ – observation covariance matrix
- $\boldsymbol{\lambda}$ – adjoint variable
- $\mathbb{H}$ - observation operator
- $\mathcal{Y}$ – set of observations
- $\mathbb{M}$ – dynamical evolution model
- $(\partial_{\boldsymbol{u}}\mathbb{M})^*$ - adjoint of the control dynamical model

Numerical Formulation

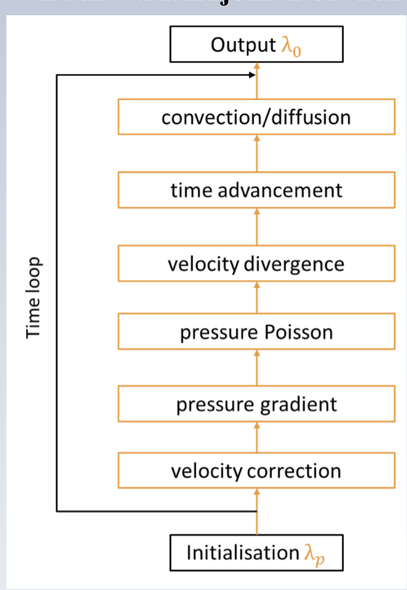
Forward/Tangent Iteration*



Sub-grid model

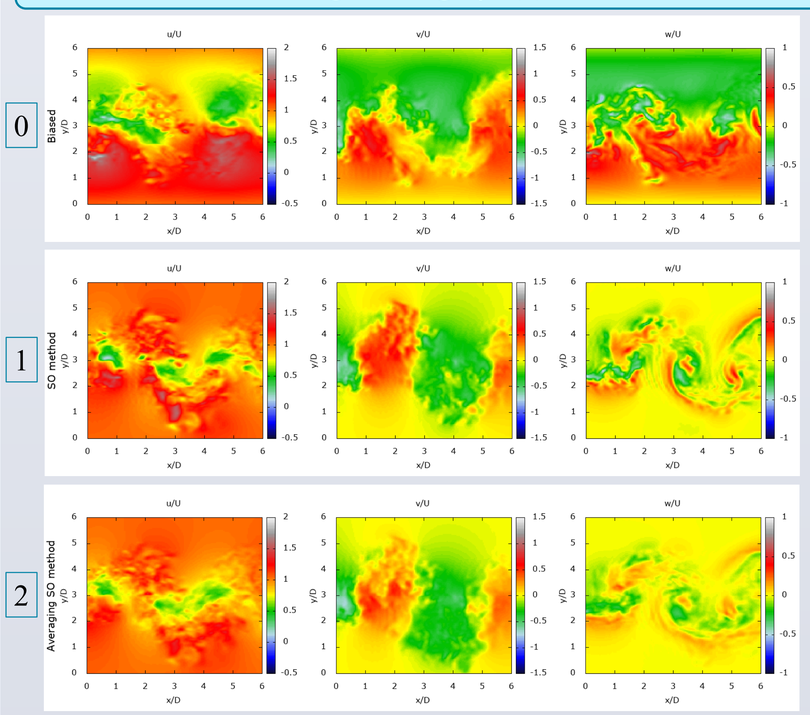


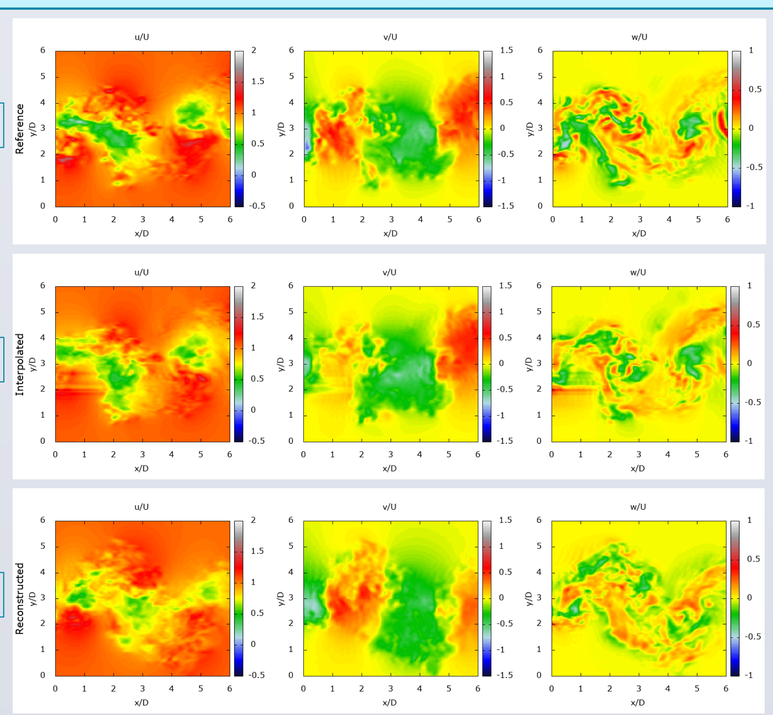
Backward/Adjoint Iteration

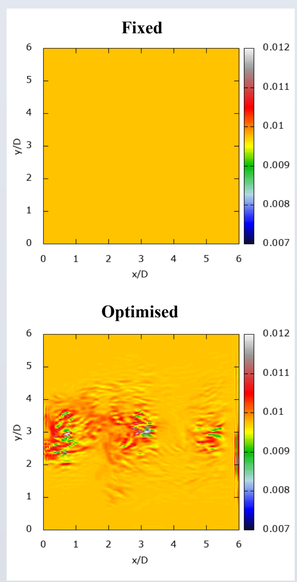


* Based on Incompact3d [5]

Background @ t_0 - (1)234 | **Observations @ t_0 - 1(2)34** | **Coefficient - 12(3)4** | **Background Covariance - 123(4)**





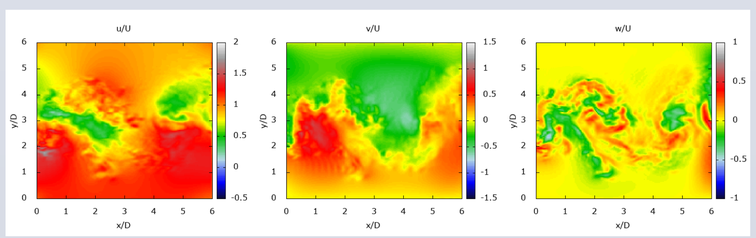
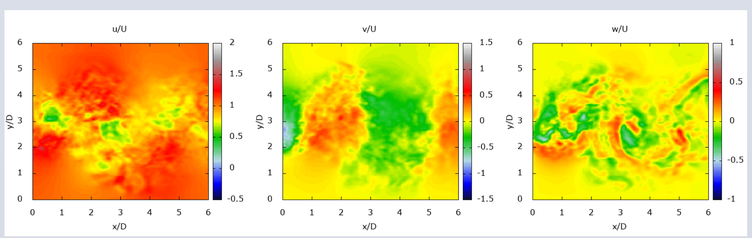
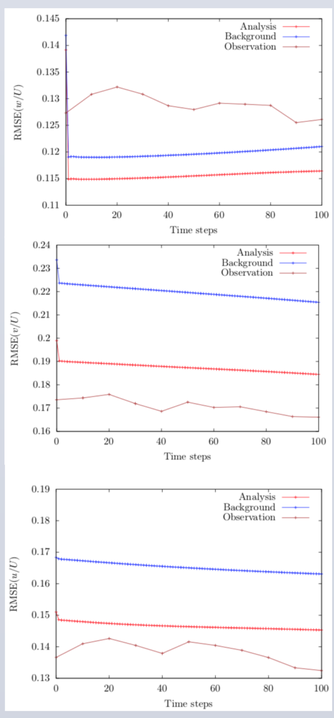
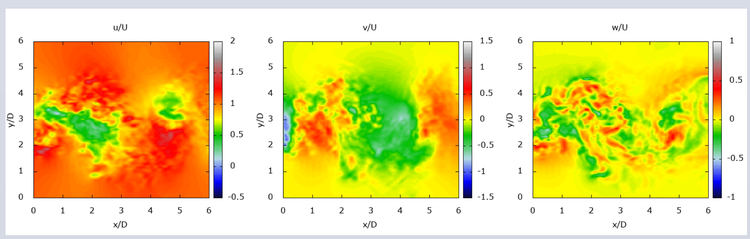
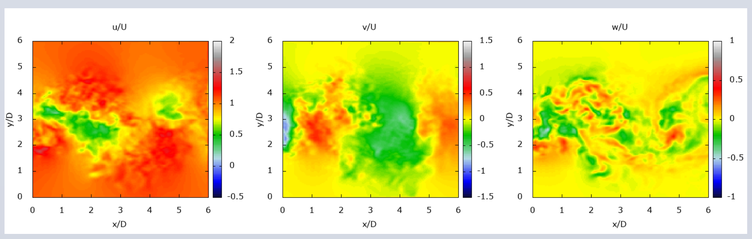
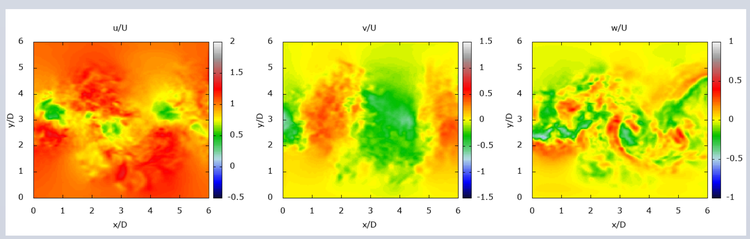
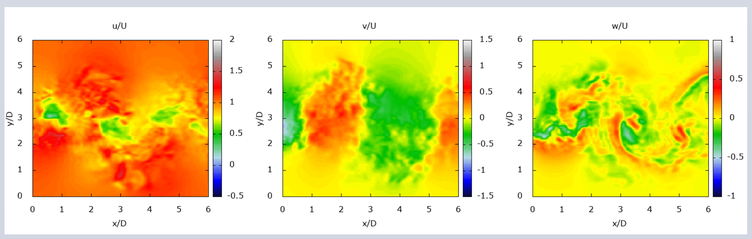


Identity

$$\begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & 1 & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix}$$

Fully defined

$$\begin{bmatrix} b_{11} & \cdots & b_{i1} & \cdots & b_{n_x 1} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ b_{1j} & \ddots & b_{ij} & \ddots & b_{n_x j} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ b_{1n_y} & \cdots & b_{in_y} & \cdots & b_{n_x n_y} \end{bmatrix}$$

Case	Analysis @ t_0	Case	Analysis @ t_0	RMSE - 2211
0000		1200		
2000		2001		
1100		2211		

Conclusion

- ✓ Turbulence modelling facilitates assimilation of turbulent flows
- ✓ Well-estimated background significantly improves analysis
- ✓ Physically relevant coefficient estimation is feasible via data assimilation
- ✓ Fully-defined background covariance matrix reduces computational time significantly at minor loss of accuracy
- ✓ Reconstructed volumetric observations are sufficient to perform assimilation and achieve meaningful results

Reference

1. E. Mémin. Fluid flow dynamics under location uncertainty. Geophys. Astro. Fluid, 108(2):119–146, 2014.
2. P. Chandramouli, D. Heitz, S. Laizet, and E. Mémin. Coarse large-eddy simulations in a transitional wake flow with flow models under location uncertainty. Comput. Fluids, 168:170–189, 2018.
3. P. Chandramouli, E. Mémin, D. Heitz, and L. Fiabane. Fast 3D flow reconstructions from PIV cross-plane observations. (Under Review) Exp. Fluids.
4. A. Gronskis, D. Heitz, and E. Mémin. Inflow and initial conditions for direct numerical simulation based on adjoint data assimilation. J. Comput. Phys., 242:480–497, 2013.
5. Laizet, S. & Lamballais, E. High-order compact schemes for incompressible flows: A simple and efficient method with quasi-spectral accuracy. J. Comput. Phys., 228(16):5989–6015, 2009.

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