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A comparative study of different reliability methods for high dimensional stochastic problems related to earth dam stability analyses

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Abstract

This article presents a probabilistic stability analysis of an existing earth dam including uncertainty quantification of soil properties and a reliability analysis of the dam sliding stability. The analyses are conducted by exploiting the available field measurements, and then by performing the Monte Carlo Simulation (MCS). Random fields and random variables approaches are both used to model the soil variabilities. Two left-and-right-bounded distributions, beta and truncated normal, are considered for the input random variables in the reliability analysis, and the influence of the horizontal autocorrelation distance on the failure probability is investigated.

A comparative study of different reliability methods is also carried out by comparing with the results of the MCS. The considered reliability methods are: the Subset Simulation (SS), the Moment Method (MM), the Sparse Polynomial Chaos Expansion in combination with the Global Sensitivity Analysis (SPCE/GSA) and the Sparse Polynomial Chaos Expansion in combination with the Sliced Inverse Regression (SPCE/SIR). The comparative study shows that all these methods can give accurate results in term of the dam failure probability with small errors. It is also found that the most accurate method is the SPCE/GSA and the most efficient method is the SS.

Keywords: Earth Dam; Dam Factor of Safety; Reliability analysis; Random fields; Monte Carlo Simulations; Polynomial chaos expansions.

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1 Introduction

Various uncertainties of soil properties (inherent spatial variability and measurement error) exist in earth dam engineering. Accounting for these uncertainties with specific methods and understanding their effects are of great value for dam design and safety assessment. In the literature, some studies related to probabilistic stability analyses of earth dams can be found [1–5]. However, most of the previous works simulated the uncertainties of soil properties by hypothetical or statistical considerations [1–4]. Another limitation of these existed studies is that soil spatial variabilities were generally ignored, such as in [1,3,4]. Additionally, only one reliability analysis result (reliability index or failure probability) was provided and no information about the distribution or the statistical moments for the factor of safety (FoS) was available in these studies [1–5].

This paper is dedicated to address the problems mentioned above by presenting a comprehensive reliability analysis of an earth dam. It includes quantification of soil properties, soil variability modelling from real field data, uncertainties quantification and failure probability estimation. The reliability analysis is based on the sliding stability analyses of the dam under steady state flow conditions. The uncertainties of three soil properties (dry density ($\gamma_d$), effective cohesion ($C'$) and effective friction angle ($\phi'$)) are considered and simulated by random fields or random variables. The selected soil properties are the most relevant for a slope stability analysis (as showed in [5,6]) and they are sufficient for a probabilistic study under the present design scenario (with a steady state flow condition). An advantage of the studied dam is that it was well documented and there are a large number of measurements available. More importantly, the $\gamma_d$ measurements are geo-localized during the embankment compaction, which allows a data geostatistical analysis and leads to a representation
of $\gamma_d$ by random fields. The random fields of $\phi'$ can then be obtained by transforming the ones of $\gamma_d$ using a physical relation (Caquot’s relation as the works of [5]) and the $C'$ is simulated by means of random variables. The propagation of these uncertainties is quantified by performing a classical MCS in combination with a mechanical model based on the limit equilibrium method which focuses on computing the dam FoS.

The second objective of the paper is to investigate the performance of four reliability methods. Considering these methods, the results are compared with the ones of MCS. Since small values of autocorrelation distance are obtained for the considered dam by analyzing the measurements, a large number of random variables (around 2000) is needed to represent accurately the random fields of $\gamma_d$. The present study becomes thus a very high dimensional stochastic problem. Therefore, the comparison study is focused on evaluating the performance of different reliability methods for very high dimensional stochastic problems. Few studies exist for the comparison of different reliability methods in real engineering problems and no study has been done for the stochastic problems with more than 1000 random variables in the geotechnical field. The selected four reliability methods to be assessed are explained as follows.

For a reliability analysis, the MCS is always considered as a standard reference to test other methods [7,8]. However, it suffers from a very low computational efficiency. Based on the MCS, two advanced sampling methods (Importance Sampling (IS) [7] and Subset Simulation (SS) [9]) were proposed to reduce the variance of the MCS estimator with a limited number of deterministic model calls. The SS can be used in a reliability analysis with both random variables approach and random fields approach as shown in [10], whereas the IS is not applicable for some cases with random fields if the involved random variables have no physical meaning. Another sampling-based technique is the Point Estimate Method (PEM) [11] which uses specific samplings to
estimate first moments of a system response and then to approximate reliability index by the estimated moments. Alternatively, the first moments can also be determined by performing an MCS until the convergence is reached. Furthermore, the First-Order Reliability Method (FORM) and the Second-Order Reliability Method (SORM) are also commonly used in the field of reliability [3,12]. They are usually employed in combination with the Response Surface Method (RSM). The aim is to seek a so-called design point by solving a constrained problem. Unfortunately, the FORM, SORM and RSM are not able to handle too many random variables [13]. During the last decades, meta-modelling techniques have received much attention in the reliability analysis due to their efficiency and accuracy [14]. This technique allows constructing a surrogate model (a.k.a. meta-model) to an original mechanical model. The constructed meta-model is usually expressed in an analytical form. The computational burden is thus quasi-negligible, which enables an MCS with respect to the meta-model. There are several mathematical tools available to reach the goal of a meta-modelling, such as Artificial Neural Networks (ANN) [15,16], Kriging model [17–19] and Polynomial Chaos Expansions (PCE) [20,21]. In the context of high dimensional stochastic problems, some dimension reduction techniques were introduced and combined with the meta-modelling to improve its performance, such as the SPCE/GSA [22] and the SPCE/SIR [23] which were proposed in recent years. They use respectively the GSA and the SIR to reduce the number of the involved random variables at first stage, and then to construct an accurate SPCE meta-model based on the reduced dimension. In summary, the selected reliability methods for the comparative study in the article are thus: a variance reduced MCS (the SS), an MCS-based moment method (the MM), and two meta-modelling methods for high dimensional stochastic problems (the SPCE/GSA and the SPCE/SIR).

The studied dam was investigated in a probabilistic framework by Guo et al. in [24]. The authors studied the dam reliability by using the SPCE with the field data. The soil
variabilities of $C', \phi'$ and $\gamma_d$ were simulated by means of random variables. Two
deterministic models were developed in [24] for evaluating the dam FoS. The present
study is dedicated to extend the studies in [24] by conducting a variogram analysis on
the geo-localised $\gamma_d$ measurements to consider the soil spatial variabilities, and to
compare different reliability methods in related to the considered dam reliability. The
main objectives of the article are to present a thorough probabilistic stability analysis
of an earth dam, and to conduct a comparative study on the performance of different
reliability methods in a context of high dimensional stochastic problems. The
presented procedure and obtained results could help designers to better understand
reliability analyses of earth dams using real data, and to choose more specifically
reliability methods for future problems.
2 Random fields and Reliability analysis methods

This section aims at presenting all the reliability analysis tools used in the article. It includes the method for generating random fields, the variogram analysis and the selected reliability methods.

2.1 Simulation of random fields by the Karhunen–Loève expansion (K-L)

In this study, the K-L expansion method is adopted to simulate random fields. Let us consider a stationary Gaussian random field \( H(x, \xi) \) in a bounded domain \( \Omega \).

Following the principles of the K-L expansion, \( H(x, \xi) \) can be expressed as [25]:

\[
H(x, \xi) = \mu + \sigma \sum_{i=1}^{\infty} \sqrt{\lambda_i} \theta_i(x) \xi_i \approx \mu + \sigma \sum_{i=1}^{S} \sqrt{\lambda_i} \theta_i(x) \xi_i
\]  

(1)

where \( x \) represents the coordinates of an arbitrary point in \( \Omega \), \( \mu \) and \( \sigma \) are respectively the mean value and the standard deviation of the random field, \( \xi \) is a vector of standard uncorrelated random variable, \( \lambda_i \) and \( \theta_i \) are respectively the eigenvalues and the eigenfunctions of the autocovariance functions of the random field, and \( S \) is the size of the series expansion for the truncated form. An autocovariance function is defined as the product of the variance and the autocorrelation function which gives a correlation value between two arbitrary points \((x, y)\) and \((x', y')\) in \( \Omega \). In this study, an exponential autocorrelation function is used [8].

The value of \( S \) depends on the desired accuracy, the autocorrelation distance \((L_x, L_y)\) and the dimension of the random field. It can be determined by evaluating the error estimation of the truncated series expansion. The error estimate based on the variance of the truncated error for a K-L expansion with \( S \) terms is given by [26]:

\[
\varepsilon = \frac{1}{\Omega} \int_{\Omega} \left[ 1 - \sum_{i=1}^{S} \lambda_i \theta_i^2(x) \right] d\Omega
\]  

(2)
In order to obtain a sufficient accuracy in terms of the variance error for random fields, Li and Der Kiureghian [27] recommended that the stochastic grid size of a random field can be set as 0.2 times the autocorrelation distance. For the cases of non-Gaussian random fields, it can be achieved by an isoprobabilistic transformation once a Gaussian random field is obtained using the K-L expansion [28,29].

2.2 Variograms

A variogram is a function which provides a description of how data are correlated. The first step of a variogram analysis is to construct an experimental variogram which describes the correlation between any two values of the observation data separated by a distance $h$. The experimental semivariogram $\gamma^*(h)$ is defined as [25]:

$$\gamma^*(h) = \frac{1}{2N_h} \sum_h (g_i - g_j)^2$$

(3)

where $g_i$ and $g_j$ represent all the possible pairs of samples which are separated with a distance of $h$, and $N_h$ is the number of the pairs of $g_i$ and $g_j$. This calculation should be repeated for as many different values of $h$ as the observation data will support. Then a mathematical model is applied to the experimental semivariogram in order to represent an autocorrelation structure over the whole study area and to estimate autocorrelation distances. One of the most common variogram models is the exponential model, which is used in this study and whose equation is [25]:

$$\gamma(h) = C [1 - e^{-(3h/a)}]$$

(4)

The parameter $a$ represents the range of the variogram (also called autocorrelation distance), and $C$ is the sill value at which the variogram levels off. Figure 1 shows the characteristics of a variogram analysis.
2.3 Presentation of the reliability methods used in the study

2.3.1 Monte Carlo Simulation (MCS): the reference method for the study

The MCS method has been widely employed in reliability analyses [30]. For an MCS with $N_{MCS}$ model runs, the failure probability is given as:

$$ Pf = \frac{1}{N_{MCS}} \sum_{j=1}^{N_{MCS}} I \quad (I = 1 \text{ if } G < 0; \ I = 0 \text{ else}) $$

(5)

where $I$ is an index of failure and $G$ presents a performance function. The number of $N_{MCS}$ should be large enough in order to obtain an accurate failure probability. The coefficient of variation (CoV) of $Pf$ for a MCS can be calculated by [26]:

$$ CoV_{Pf} = \sqrt{(1 - Pf)/(N_{MCS} * Pf)} * 100\% $$

(6)

Although this method suffers from low computational efficiency, it often serves as a standard reference to test other reliability methods because of its versatility and robustness. In this article, the reliability analysis of the studied dam is performed by
the MCS and the obtained results are used to evaluate the accuracy and the efficiency of the selected reliability methods which are presented in next sub-sections.

2.3.2 Subset simulation (SS)

In order to tackle the problem of using the MCS especially for low failure probability cases, the SS was developed by [9]. The principle is to decompose a failure event \( E \) into a sequence of intermediate events \( [F_1, F_2, ..., F_m] \) with larger probabilities of occurrence. The target failure probability is written as [31]:

\[
P_f = P(E) = P(F_1) \prod_{i=2}^{m} P(F_i|F_{i-1})
\]

where \( P(F_i|F_{i-1}) \) is the conditional failure probability of the event \( F_i|F_{i-1} \). A key element of successfully using the SS is the generation of the conditional samples in each intermediate event. This is achieved by using the modified Metropolis-Hasting algorithm (MMH) in this article.

2.3.3 Moment method approximation (MM)

The MM was introduced by [32] for structural reliability analyses. A well-known MM is the second-moment approximation (SM). It assumes that a system response follows a normal distribution and uses the first two moments to estimate the reliability index. In the present study, a fourth-moment approximation (FM) is also used to estimate the dam failure probability since it can give more accurate results compared to the SM as reported in [33]. The formulas of the two adopted MM methods (SM and FM) are given as follows [32]:

\[
\beta_{SM} = \frac{\mu_G}{\sigma_G}; \quad Pf_{SM} = \Phi(-\beta_{SM})
\]

\[
\beta_{FM} = \frac{3(\alpha_{4G} - 1)\beta_{SM} + \alpha_{3G}(\beta_{SM}^2 - 1)}{\sqrt{(9\alpha_{4G} - 5\alpha_{3G}^2 - 9)(\alpha_{4G} - 1)}}; \quad Pf_{FM} = \Phi(-\beta_{FM})
\]
where \((\beta_{SM},P_{SM})\) and \((\beta_{FM},P_{FM})\) are respectively the (reliability index, failure probability) estimated by the SM and FM, \(\mu_G,\sigma_G^2,\alpha_{3G}\) and \(\alpha_{4G}\) are the first four statistical moments of the performance function, and \(\Phi(\cdot)\) represents the cumulative distribution function (CDF) of a standard normal variable. The values of the required statistical moments are determined by an MCS in this study. The deterministic model is repeatedly run for different sets of input parameters generated from a specific PDF, until all the desired moments are converged.

2.3.4 Spares polynomial chaos expansion/Global sensitivity analysis (SPCE/GSA)

The SPCE/GSA was proposed by Sudret (2008) [34] and improved by Al-Bittar et al. [22] for high dimensional stochastic problems. The SPCE presents a suitable sparse basis of the PCE. The sparse basis can be built by a stepwise regression algorithm as described in [21,35,36]. By using the SPCE, a model response can be expanded as [36]:

\[ Y \cong \sum_{\alpha \in \mathbb{N}^M} k_\alpha \Psi_\alpha(\xi) \tag{10} \]

where \(\xi = \{\xi_1, \xi_2, \ldots, \xi_M\}\) are independent random variables, \(\Psi_\alpha(\xi)\) are multivariate polynomials, \(k_\alpha\) are unknown coefficients to be computed and \(\alpha = \{\alpha_1, \ldots, \alpha_M\}\) is a multidimensional index. In this paper, the hyperbolic truncation scheme proposed in [21] is used to truncate the series expansion and the unknown coefficients \(k_\alpha\) are computed by using the least-regression method [36].

Concerning the GSA, it allows quantifying contributions of an input variable to the response variance of a physical model [34]. Sudret (2008) [34] introduced an analytical way to compute the Sobol index (a sensitivity index) by post-processing the SPCE coefficients. The Sobol index of one variable can be calculated as:
$S(\xi_i) = \frac{\sum_{\alpha \in A} (k_j)^2 E[(\Psi_\alpha)^2]}{\sum_{\alpha \in A} (k_j)^2 E[(\Psi_\alpha)^2]}$ (11)

where $k_j$ are PCE coefficients, $A$ is a truncation set, $A_{\xi_i}$ is a subset of $A$ in which the multivariate polynomials $\Psi_\alpha$ are only functions of the random variable $\xi_i$ (i.e., they only contain the variable $\xi_i$), and $E[(\Psi_\alpha)^2]$ is the expectation of $(\Psi_\alpha)^2$.

As a summary, the SPCE/GSA implementation for a reliability analysis consists of 3 steps:

1. Select significant input variables by performing a GSA based on a 2-order SPCE. It should be noted that the SPCE order has almost no influence on the Sobol index, so an SPCE with the order 2 can accurately provide contributions of each input variable to system response variabilities [22,34],
2. Construct a meta-model using a high-order SPCE with the selected variables (effective dimension),
3. Perform an MCS using the obtained meta-model to compute the system response PDF and the failure probability.

2.3.5 Spare polynomial chaos expansion/Sliced inverse regression (SPCE/SIR)

The SPCE/SIR was proposed by [23]. The principle remains the same to the SPCE/GSA which lies on a dimension reduction before construction of an accurate SPCE meta-model. The SPCE/GSA utilizes the GSA to reduce the number of the involved random variables, while it is achieved by another technique named SIR in the SPCE/SIR. This approach is based on the principle that a few linear combinations of original input variables could capture essential information of a model response [37]. It aims to find an effective dimension reduction (EDR) space by considering an inverse regression relation which regresses input variables against model responses. The algorithm presented in [23] is adopted in this study to find the EDR. By performing this algorithm, a new input vector can be obtained which is a linear combination of original input variables, and the dimension is reduced. Once the new input vector is determined, an accurate SPCE model can be constructed and then the failure probability can be estimated with an MCS.
Study case presentation: soil properties available and variability modelling

This section focuses firstly on presenting the studied earth dam and describing the available field data. Then, the statistical parameters of the three soil properties \( (C', \phi' \text{ and } \gamma_d) \) required for generating random fields or random variables are determined using the field data.

3.1 Presentation of the studied dam

Figure 2 presents the main cross section of the considered dam. It is a 170 m long and 23.8 m high earth-filled dam located in the west of France. It closes a valley covered with alluvial deposits and can retain a reservoir of about 5 hm\(^3\). The normal and maximal reservoir water level is respectively 20 and 21.6 m. In the downstream part, two filter drains were installed for the purpose of lowering the phreatic surface [38]. In the foundation, a waterproof grout curtain was realized with a depth of 15 m.

As presented in Figure 2, the dam is formed by three different zones including a core and two backfill zones in respectively the upstream and downstream part of the core. These three zones are respectively named as Core, Shell-1 and Shell-2 in this article. The materials constituting the dam were collected from the vicinity of the dam site. Two different types of soils can be identified in the valley. The first type is gravelly sands resulting from alteration of shales on the slopes and uplands which dominate the
valley. This material is used for the construction of the Shell zone. The second soil type, sandy silts, can be found on the bottom of the valley and on the slopes. It was used for the construction of the Core zone. The foundation is composed of altered schists whose superficial layers have been purged. Its location is very close to the Shell zone according to the site investigation and granulometric analyses.

3.2 The available field data

Different data are available on the studied earth dam in several phases: design studies, construction controls and structure monitoring. This article presents only the field data which are relevant to the stability analysis of dams: dry density measurements collected during the construction and the results of the triaxial tests performed in laboratories. The former is directly related to the $\gamma_d$ and the latter allows estimating $C'$ and $\phi'$.  

3.2.1 Embankment compaction

During the construction, the dry density and the soil water content after compaction were monitored in-situ using a gammadensimeter. This leads to a large number of $\gamma_d$ data. An advantage of these data is that they were collected following a grid monitoring system. This makes it possible to localize the measurements in space (along three axes). The grid system consists of 10 profiles in the longitudinal direction (Y axis) and 13 profiles in the transversal direction (X axis). Such a grid system allows determining the location of the measurements on an X-Y plan and the knowledge of the construction layer gives the elevation of the measurements along the Z axis. In total, the number of effective geo-localized $\gamma_d$ measurements is 381 for the Core zone, 248 for the Shell-1 zone and 272 for the Shell-2 zone.

3.2.2 Triaxial shear tests

The shear strength parameters were determined by triaxial shear tests. For a long-term stability analysis of earth dams, the effective cohesion and friction angle are required. Concerning these parameters, totally 8 consolidated-undrained triaxial shear tests with
pore water pressure measurement are available. Among the 8 tests, 5 tests are for the Shell zone and the other 3 for the Core zone. By plotting the Mohr circles of the effective stress at failure, the values of $C'$ and $\tan \phi'$ can be estimated using the Coulomb line (approximately tangent to all the circles). Using this method, the results of each test can be exploited to compute the values of $C'$ and $\phi'$. According to the 8 available tests, the average of $C'$ is estimated to be equal to 9.4 kPa for the Shell zone and to 10 kPa for the Core zone. For $\phi'$, a value of $34.2^\circ$ was obtained for the Shell zone and of $34.3^\circ$ for the Core zone. It can be found that the shear strength parameters for long term of the two soils are very close to each other. In fact, the two materials are relatively similar as they derive from the schists alteration composing the bedrock. The considered dam is actually a pseudo-zoned earth dam.

### 3.3 Variability modelling of the soil properties

In the present study, soil variability modelling consists of two steps. Firstly, an appropriate distribution type is assumed and then the relating distribution parameters are determined by fitting the measurements to the assumed distribution. Secondly, autocorrelation structures are estimated through a variogram analysis based on the geo-localized measurements. It is noted that the first step is directly related to the works in [24]. Therefore, only a brief description of the first step is given in the following parts.

#### 3.3.1 First step: Distribution type and relating parameters

Two distribution types (beta and truncated normal distribution) are adopted as a candidate, in this article, to describe the measured data of $C'$, $\phi'$ and $\gamma_d$. The reason of choosing the two distribution types is that they can avoid unreasonable values by considering a physical range of soil properties. By fitting the measurements with the beta or truncated normal distribution, the corresponding soil parameter can be described as a random variable. The best fitted parameters are estimated using the
maximum likelihood estimation method, and the bounded values are determined according to the soil type and the reference values recommend in [39,40].

The procedure of the first step mentioned above can be easily applied to the measurements of $\gamma_d$ since many data exist for this soil property in each zone of the dam. However, the number of the available triaxial tests is only 8 and it does not allow a meaningful statistical estimation of the distribution parameters for $C'$ and $\phi'$.

In order to address this problem, [5] introduced a method which can generate a large number of artificial data for $C'$ and $\phi'$ with limited triaxial test results. This method is also adopted in the works of [24] to determine the distribution parameters with the beta or truncated normal distribution for $C'$ and $\phi'$ of the studied dam. With this method, the distribution parameters for all the three soil properties ($C'$, $\phi'$ and $\gamma_d$) can be obtained. Table 1 gives a summary of the distribution parameters for each zone. As an illustration, Figure 3 and Figure 4 present respectively the histogram and the two fitted CDF curves for the $\gamma_d$ measurements in the Core zone, and for the generated $C'$ in the Shell zone. The method introduced by [5] for determining the distribution parameters of $C'$ and $\phi'$ is based on a linear regression performed on the top of the Mohr circles. This method assumes that the intermediate parameters (the y-intercept and the slope of the Kf line) which are used for estimating the values of $C'$ and $\phi'$, are two uncorrelated normal variables. Therefore, no correlation is considered between the shear strength parameters in this paper. According to previous works [41] on slope reliability analyses, ignoring the correlation between $C'$ and $\phi'$ which is usually negative [42], leads to conservative estimates of failure probabilities.
Figure 3. Histogram and fitted CDF for the $\gamma_d$ measurements in the Core zone
361 Figure 4. Histogram and CDF of the generated $C'$ in the Shell zone

Table 1. Distribution parameters of the soil properties

<table>
<thead>
<tr>
<th>Zones</th>
<th>Soil property</th>
<th>Beta $a^1$</th>
<th>Beta $b^1$</th>
<th>Truncated normal Mean</th>
<th>Truncated normal CoV($%$)</th>
<th>Extreme values</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\gamma_d$ (g/cm$^3$)</td>
<td>15.7</td>
<td>18.0</td>
<td>1.99</td>
<td>3.21</td>
<td>1.63</td>
</tr>
<tr>
<td>Shell-1</td>
<td>$C'$ (kPa)</td>
<td>1.48</td>
<td>2.78</td>
<td>10.55</td>
<td>57.63</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$\phi'$ (°)</td>
<td>28.71</td>
<td>29.61</td>
<td>34.85</td>
<td>3.72</td>
<td>25</td>
</tr>
<tr>
<td>Core</td>
<td>$\gamma_d$ (g/cm$^3$)</td>
<td>22.4</td>
<td>27.5</td>
<td>1.83</td>
<td>3.33</td>
<td>1.44</td>
</tr>
<tr>
<td></td>
<td>$C'$ (kPa)</td>
<td>4.07</td>
<td>5.22</td>
<td>13.23</td>
<td>34.21</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$\phi'$ (°)</td>
<td>231.16</td>
<td>192.28</td>
<td>34.11</td>
<td>2.48</td>
<td>15</td>
</tr>
<tr>
<td>Shell-2</td>
<td>$\gamma_d$ (g/cm$^3$)</td>
<td>26.7</td>
<td>22.2</td>
<td>2.05</td>
<td>2.65</td>
<td>1.63</td>
</tr>
<tr>
<td></td>
<td>$C'$ (kPa)</td>
<td>1.48</td>
<td>2.78</td>
<td>10.55</td>
<td>57.63</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>$\phi'$ (°)</td>
<td>28.71</td>
<td>29.61</td>
<td>34.85</td>
<td>3.72</td>
<td>25</td>
</tr>
</tbody>
</table>

Note: $^1$Beta distribution parameters; $^2$Coefficient of variation

363 3.3.2 Second step: Autocorrelation structure

The second step aims at determining the autocorrelation structure of the simulated soil property for each dam zone. The method [5] adopted for generating values of $C'$ and $\phi'$ cannot provide the location information. Therefore, only the autocorrelation structure of $\gamma_d$ is estimated. It could be realized by a variogram analysis on the geo-localized $\gamma_d$ measurements. Taking the Shell-2 zone as an example, an experimental semivariogram is firstly obtained by applying Eq. (3) to all the $\gamma_d$ measurements of this zone. Then, the autocorrelation distances can be estimated by fitting a mathematical model (exponential one in this paper) to the experimental semivariogram. Figure 5 shows the experimental semivariogram together with the fitted exponential model for both horizontal and vertical directions. It can be observed that the variance between two measurements increases with the increase of its separation distance. The variance roughly reaches a constant value after the distance beyond 5-7m for the horizontal directions. For the vertical direction, it converges
when the distance is bigger than 1.5m (lower value than for the horizontal direction).

The black points in Figure 5 represent the points which reach 95% of the sill value. It is considered that the abscissa of these points is the autocorrelation distance. For the cases in Figure 5, the horizontal and vertical distances are respectively 4.9 m and 1.9 m. It indicates that the soil is less homogeneous in the vertical direction than in the horizontal direction. This finding is consistent with the observations of [42,43]. By repeating the same procedure to the $\gamma_d$ measurements in the other two zones Core and Shell-1, all the necessary autocorrelation distances are obtained and presented in Table 2.

Table 2 indicates that a considerable homogeneity can be found in the Shell-1 zone, while the $\gamma_d$ in the Shell-2 and Core zones are more spatially variable. This difference can be explained by the better selection of the material composing the upstream zone and the greater attention which has given to its construction. The nugget effect corresponds to about a half of the variance for the upstream shoulder and to a slightly lower fraction for the downstream shoulder and the core. The nugget effect can be attributed to the mixture of the materials during their excavation from the borrow pits. In our case, it is considered as a short dimension structure whose scale is less than the sampling step.
Figure 5. Variogram analysis for the $\gamma_d$ measurements in the Shell-2 zone

Table 2. Results of the geostatistical analysis for the $\gamma_d$ measurements

<table>
<thead>
<tr>
<th>Zones</th>
<th>Autocorrelation distance (m)</th>
<th>Nugget effect</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Horizontal ($L_x$)</td>
<td>Vertical ($L_y$)</td>
</tr>
<tr>
<td>Shell-1</td>
<td>78.1 m</td>
<td>7.8 m</td>
</tr>
<tr>
<td>Core</td>
<td>13.0 m</td>
<td>1.5 m</td>
</tr>
<tr>
<td>Shell-2</td>
<td>4.9 m</td>
<td>1.9 m</td>
</tr>
</tbody>
</table>

As presented in section 2.1, a K-L expansion should be truncated to a limited number of series terms $S$ for practical applications. The value of $S$ can be determined by evaluating the error defined by Eq. (2) with a prescribed accuracy. This error depends on the autocorrelation distances and size of a random field. For an accuracy between 10% and 9%, the $S$ is estimated to be equal to 30, 368 and 1710 for respectively the Shell-1, Core and Shell-2 zones (considering their site dimension and the relating
autocorrelation distances presented in Table 2). Therefore, it needs 1710 random variables to represent accurately a random field of $\gamma_d$ in the Shell-2 zone. Such small values of the $L_x$ and $L_y$ in the Shell-2 zone lead to the present study become a very high dimensional stochastic problem.

3.4 Seismic loading condition

A pseudo-static acceleration is considered in this study in order to take into account seismic loading conditions. The value of the acceleration is set equal to 2.4 m.s$^{-2}$. It is determined according to the location of the considered dam with respect to the seismic zones in France and the category of the dam [44]. The seismic acceleration used in the calculation is considered to be related to a return period of 5000 years. This means that the failure probability directly obtained under such a pseudo-static acceleration should be multiplied by 1/5000 to consider the seismic occurrence probability. The two types of failure probability are respectively noted as $P_{f\text{direct}}$ and $P_{f\text{con}}$ in the study.

4 Presentation of the deterministic models

Two deterministic models were developed in [24] for computing the dam FoS. The first one is a numerical model based on the strength reduction method, and the second one is an analytical model based on the limit equilibrium theory. The latter is employed in this article to perform the deterministic calculations in the reliability analyses since it can give similar FoS values compared to the numerical model but with a lower computational time. Such an advantage is very significant and important for a reliability analysis which needs usually a large number of calls to a deterministic model. Concerning the numerical model, it was developed for providing the pore water pressure distribution inside the dam and validating the analytical model.
This section aims at presenting the two models briefly. In the end, a comparison study between the two models is conducted in order to validate the analytical model in the context of random fields.

4.1 The numerical model

The numerical model in [24] was created using Flac2D which is a two-dimensional explicit finite difference program [45]. The boundary conditions used in this model are the following ones: the displacements are blocked following the horizontal and vertical axis on the base of the model; the horizontal displacements are blocked on the lateral edges of the model. Figure 6 presents a mesh used for the following calculations. The mesh includes around 18000 4-node quadrilateral plane elements. The selected number of the elements was determined by a mesh refinement study [24]. The created model allows calculating the pore water pressure distribution inside the dam by applying a hydrostatic head in the upstream. The dam FoS is computed based upon the strength reduction method [6].

4.2 The analytical model

The analytical model proposed in [24] is based on the limit equilibrium theory in combination with a genetic algorithm (GA) [46]. The principle is to generate a number of trial slip surfaces as an initial population at first, and then to search the minimum FoS value by simulating natural process along generations including reproduction, crossover, mutation and survivors’ selection. The FoS of a given slip surface is computed by using the procedure of Zhu et al. [23] which is based on the
Morgenstern Price method, and the slip surface generation method described in [24] is adopted which allows generating non-circular slip surfaces. It is also noted that the pore water pressures at the base of each slice are determined using the ones obtained by the numerical model. For more details about the model, readers are referred to [24].

4.3 Validation of the analytical model

It was shown in [24] that the analytical model is able to give similar FoS values in a deterministic calculation and similar reliability results in a probabilistic calculation compared to the numerical model. However, the comparison and validation studies conducted in [24] are only related to the cases of random variables. The performance of the analytical model in the context of random fields is thus still unknown. In order to address this issue, a comparison study is carried out and presented in this section. The idea is to generate $N_{\text{com}}$ random fields for $\phi'$ and $\gamma_d$, and $N_{\text{com}}$ random variables for $C'$. For each set of input parameters, the two deterministic models are both performed. The obtained FoS values are then compared with each other to evaluate the accuracy of the analytical model. The numerical model is adopted as a reference to assess the performance of the analytical model for the following reasons: 1) no assumptions are needed concerning the failure surface, 2) no assumptions on inter-slice side forces are needed, since there is no concept of slices, and 3) no optimization procedure is needed since the minimum FoS and the critical slip surface are obtained automatically.

In total 150 sets of input parameters are considered in the comparison study. Each set of input parameters is obtained randomly and is composed of three random fields of $\gamma_d$, three random fields of $\phi'$ and two random variables of $C'$. The other soil parameters, except $C', \phi'$ and $\gamma_d$, required for the numerical model are taken from the values given in [24]. The values of the GA parameters in the analytical model are the
same to the ones in [24]. As an illustration, a realization of three random fields of $\gamma_d$
are mapped to the numerical model and presented in Figure 7. Using the Caquot’s relation, three random fields of $\phi'$ are also obtained and shown in Figure 8.

Figure 9 presents a direct comparison of the FoS values computed with the two models for the 150 different parameter sets. It is shown that the results are close to the unit line and relative errors are smaller or around 5%. These observations indicate that the analytical model is able to estimate an accurate FoS value for the studied dam considering random fields. Therefore, the analytical model is validated and can be used for deterministic calculations for the following reliability analyses.

![Figure 7. Example of a realization of three random fields of $\gamma_d$](image1)

![Figure 8. Example of a realization of three random fields of $\phi'$](image2)
Using the analytical model, rather than the numerical one, can reduce the computational time of a stability analysis for the studied dam from 20 minutes to 10 seconds in an Intel Xeon CPU E5-1620 3.5 GHz PC. Such a reduction in computational time is very significant for a reliability analysis which needs usually a large number of calls to a deterministic model. Given that the analytical model can give reasonable FoS values compared to the numerical one but with a reduced time, the following analyses are all based on the analytical model.

5 Reliability analysis results by the reference method MCS

This section presents the reliability analysis for the studied dam using the reference method MCS. The uncertainties in the soil properties $C', \phi'$ and $\gamma_d$ are quantified using random fields or random variables with the parameters presented in the sections.
3.2 and 3.3. In addition, the effect of autocorrelation distances on the dam failure probability is discussed.

5.1 Reliability analysis results

20,000 deterministic calculations are performed in the reliability analysis by an MCS with the Latin Hypercube sampling technique. For each calculation, 2,110 independent standard random variables $\xi_i$ are generated firstly. The first two $\xi_i$ are transformed to physical values of $C'$ using the iso-probabilistic method with the specific PDF presented in Table 1. The rest $\xi_i$ are used in the K-L expansion for the generation of the three random fields of $\gamma_d$. The three random fields of $\phi'$ are then obtained by a transformation from those of $\gamma_d$ using the Caquot's relation.

Figure 10 shows the PDF of the obtained 20,000 FoS values for the two distribution types, and Table 3 gives the reliability results.

![Figure 10. PDF of the FoS values obtained by the MCS with two distributions](image)
Table 3. Reliability results obtained by the MCS with two distributions

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Failure probability</th>
<th>Statistical moments of FoS</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_{fdirect}$</td>
<td>$P_{fcon}$</td>
<td>$CoV_{PF}$</td>
<td>Mean</td>
</tr>
<tr>
<td>Beta</td>
<td>0.022</td>
<td>4.4×10^{-6}</td>
<td>4.71%</td>
<td>1.232</td>
</tr>
<tr>
<td>Truncated normal</td>
<td>0.016</td>
<td>3.2×10^{-6}</td>
<td>5.61%</td>
<td>1.248</td>
</tr>
</tbody>
</table>

From Figure 10, it can be observed that the PDF curve obtained by the truncated normal distribution ($PDF_N$) is taller and narrower than the one of beta distribution ($PDF_B$). It means that the FoS values are less dispersive if a truncated normal distribution is assumed for the input random variables. More precisely, the two PDF curves are almost superposed for relative high FoS values (bigger than 1.4), while the $PDF_N$ is significantly lower than the $PDF_B$ for relative small FoS values (smaller than 1.2). This is because of the probability of generating a small value of $C'$ in the Shell zone drawn from the fitted beta distribution is higher than the truncated normal distribution, as presented in Figure 4. In addition, the two curves are not symmetric. They are considered to be negatively skewed with a relatively bigger tail at the left. It can be explained by the fact that the distribution of the input variables is not symmetric and that some variables have more small values, such as $C'$ in the Shell zone as shown in Figure 4.

The direct failure probability $P_{fdirect}$ of the dam under a pseudo-static acceleration of 2.4 m.s\(^{-2}\) is estimated to be equal to 0.022 and 0.016 respectively by the two distributions. These values are then multiplied by a coefficient of 1/5000 to consider the seismic occurrence probability and become equal to 4.4×10^{-6} and 3.2×10^{-6} respectively. As for the statistical moments of the FoS values, the beta assumption gives a slightly lower value for the mean but a bigger value for the standard deviation,
compared to the truncated normal assumption. A big value of standard deviation means a high level of data scatter. This is consistent with the observation in Figure 10.

In conclusion, the dam failure probability under a pseudo-static loading condition is estimated to be around $4 \times 10^{-6}$. The two distribution assumptions lead to similar results with the same order of magnitude. The beta distribution gives slightly more conservative results in term of the failure probability. As the beta distribution describes better the variability of the soil properties as shown in Figure 4 and is conservative in the design, this type of distribution is adopted for the next analyses.

5.2 Influence of the autocorrelation distance

One of the factors which can influence reliability results is autocorrelation distance. It defines by means of an autocorrelation function, the autocorrelation structure of a random field. According to a literature review given by El-Ramly et al. [47], the autocorrelation distance for soils is usually within a range of 10-40 m in the horizontal direction, while it ranges between 1 and 3 m in the vertical direction. It is found that the $L_x$ and $L_y$ in the Shell-1 zone (Table 2) are bigger than the values indicated in [47] while the $L_x$ in the Shell-2 zone is smaller than expected values. Therefore, careful attention must be done to these parameters and their induced influence on the reliability analysis.

Finally, the impact investigation is focused on the value of $L_x$ in the Shell-2 zone while the estimated autocorrelation distances in the Shell-1 zone are accepted for the values in Table 2. The reasons are as follows: 1) the obtained large values of $L_x$ and $L_y$ are expected for the Shell-1 zone since the materials are better selected and more attention are given to its construction; 2) the upstream part of the backfill embankment is considered to have a very limited influence on the dam stability under
steady state flow conditions; 3) large values of autocorrelation distances may lead to bigger failure probabilities, so conservative designs as pointed in [13,48,49].

For the $L_x$ value in the Shell-2 zone, a first improvement is made by fitting the experimental semivariogram with other theoretical variogram models such as the Gaussian and the spherical models [50]. The $L_x$ is estimated to be equal to 6.7m for the Gaussian model, and to 4.7m for the spherical model. These values are both different to the one estimated with the exponential model (4.9m) as shown in the section 3.3, and these differences may induce an impact on the dam failure probability.

In order to quantify the influence induced by different values of $L_x$, a parametric study is conducted. Several values of $L_x$ in the Shell-2 zone are tested using the MCS. The objective is to investigate the evolution of the dam failure probability with the $L_x$ value. For the sake of simplicity and clarity, the other values of autocorrelation distance are rounded to integer values (see details in Table 4). In addition, the $L_x$ value in the Core zone is also varied. Totally, four cases are selected for the parametric study. The $L_x$ value in the Core and Shell-2 zones are decreased from 80 to 10m. These four cases allow investigating the influence of the horizontal autocorrelation distance in the Core and Shell-2 zones on the dam failure probability. The reference case in Table 4 refers to the values estimated by the variogram analysis with the exponential model (see Table 2).

<table>
<thead>
<tr>
<th>Case</th>
<th>Shell-1</th>
<th>Core</th>
<th>Shell-2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$L_x$ (m)</td>
<td>$L_y$ (m)</td>
<td>$L_x$ (m)</td>
</tr>
<tr>
<td>Case1</td>
<td>80</td>
<td>8</td>
<td>80</td>
</tr>
<tr>
<td>Case2</td>
<td>80</td>
<td>8</td>
<td>40</td>
</tr>
<tr>
<td>Case3</td>
<td>80</td>
<td>8</td>
<td>20</td>
</tr>
</tbody>
</table>
By adopting different values of autocorrelation distance as shown in Table 4, four complementary MCSs are performed. Figure 11 plots the four obtained failure probabilities together with the one of the reference case. The first two moments of the dam FoS for each case are also given in Figure 11.

![Figure 11. Influence of the $L_x$ on the dam failure probability](image)

It can be observed from Figure 11 that the horizontal autocorrelation distance in the Core and Shell-2 zones have an influence on the dam failure probability. As the $L_x$ decreases, the $P_f$ decreases. For example, a decrease of $L_x$ from 80 to 10 m results in a reduction of about 10% for the $P_f$ (from 0.0295 to 0.0259). This finding has already been confirmed by many researchers for different geotechnical engineering [13,48,49]. Concerning the statistical moments, the mean value remains almost
constant whereas the standard deviation increases when increasing the $L_x$. This indicates that the $L_x$ has no impact on the mean value of the dam FoSs, whereas it affects the FoSs dispersion of the dam. The reference case corresponds to the lowest failure probability and the smallest standard deviation.

6 A comparative study of different reliability methods

This section presents the results and the relating interpretation of the comparative study of the four selected approximated reliability methods. The objective is to evaluate the performance of the considered methods for very high dimensional stochastic problems.

The parametric study on the autocorrelation distance (presented in section 5.2) is re-performed by the four reliability methods (SS, MM, SPCE/GSA and SPCE/SIR) which are mentioned in the section Introduction and presented in section 2.3. The obtained results in term of the failure probability for each case are plotted in Figure 12, and the numbers of calls to the deterministic model ($N_{call}$) for each case are summarized in Table 5. Additionally, the results of the MCS are also provided and considered as a standard reference for the comparison. In Table 5, the number of required random variables ($N_{RV} = N_{RV,KL} + 2$) for representing $C', \phi'$ and $\gamma_d$ by means of random fields or random variables is given as well. The $N_{RV,KL}$ is the number of random variables needed for generating relatively accurate random fields of $\gamma_d$ by using the K-L expansion method, and the number 2 represents the two random variables of $C'$ in the Shell and the Core zones. The information in Table 5 helps to visualize the efficiency of each method by comparing the $N_{RV}$ with the $N_{call}$.
Figure 12. Comparison of the failure probability obtained by the five reliability methods.

Table 5. Comparison of the necessary run numbers of the deterministic model for the five reliability methods.

<table>
<thead>
<tr>
<th>Case</th>
<th>Case 1</th>
<th>Case 2</th>
<th>Case 3</th>
<th>Case 4</th>
<th>Reference case</th>
</tr>
</thead>
<tbody>
<tr>
<td>(N_{RV}^1)</td>
<td>225</td>
<td>370</td>
<td>647</td>
<td>1207</td>
<td>2110</td>
</tr>
<tr>
<td>MCS</td>
<td>20000</td>
<td>20000</td>
<td>20000</td>
<td>20000</td>
<td>20000</td>
</tr>
<tr>
<td>SS</td>
<td>600</td>
<td>600</td>
<td>600</td>
<td>600</td>
<td>600</td>
</tr>
<tr>
<td>SPCE/SIR</td>
<td>1000</td>
<td>5000</td>
<td>8000</td>
<td>10000</td>
<td>15000</td>
</tr>
<tr>
<td>SPCE/GSA</td>
<td>3000</td>
<td>5000</td>
<td>8000</td>
<td>13000</td>
<td>18000</td>
</tr>
<tr>
<td>MM</td>
<td>1946</td>
<td>1755</td>
<td>2081</td>
<td>2026</td>
<td>1981</td>
</tr>
</tbody>
</table>

Note: 1Number of required random variables for representing \(C', \phi'\) and \(\gamma_d\) by means of random fields or random variables for each case.
A quick review of Figure 12 reveals that the four methods can all give relatively accurate failure probabilities compared to the results of MCS. The values of $P_f$ are within a same order of magnitude for different methods. For example, the $P_f$ varies between 0.015 and 0.027 for the Reference case according to the four methods.

Concerning the efficiency comparison presented in Table 5, it is found that all the approximated methods need fewer calls of the deterministic model than the MCS. This is the reason why these methods are an alternative to the MCS for reliability analyses. Besides, the value of $N_{RV}$ increased from Case1 to Case4, and the $N_{RV}$ of the Reference Case is the biggest one. This is because the autocorrelation distance $L_x$ value in the Core and Shell-2 zones are decreased from 80 to 10m. Smaller value of the autocorrelation distance means that it needs more random variables $N_{RV, K L}$ to represent a random field with a specific error variance. By comparing the $N_{RV}$ with the $N_{call}$ of each method, it is observed that the $N_{call}$ of the methods MCS, SS and MM is almost changeless to the $N_{RV}$. In other words, the efficiency of these three methods is not related to the number of input random variables, but depends on, in fact, the value of the target failure probability. However, the $N_{call}$ of the two meta-modelling methods (SPCE/GSA and SPCE/SIR) increases rapidly with increasing the $N_{RV}$. This indicates that the efficiency of the meta-modelling method depends strongly on the number of input random variables. Indeed, more input variables means that more information is needed. Thus, a higher $N_{call}$ will be required for constructing a meta-model which will be used to replace an original mechanical model.

The following subsections give a detailed interpretation of the comparative study for each reliability method. In the end, some concluding remarks of the comparative study are summarised.

6.1 The SS
This method is the most efficient one, according to Table 5, which requires only 600 calls of the deterministic model. The $N_{\text{call}}$ of the SS is much less than the MCS one (about 3%) and is constant with the $N_{RV}$ variation. This finding is not surprising since the target failure probability is relatively high (around 0.022) and changes slightly between the different proposed cases in the present problem. If a conditional probability $P_c$ of 0.2 is adopted for each simulation level, only 3 simulation levels are needed to reach the final failure domain. In this study, the $P_c$ is set to 0.2 and the sample numbers in each simulation level ($N_{\text{level}}$) is set to 200. However, it is found that the SS cannot produce a consistent evolution of $P_f$ with $L_x$. The obtained values of $P_f$ are fluctuated from the Case 1 to the Reference case, while they are expected to be monotone decreasing as shown by the MCS. This limitation originates from the generation of the conditional samples in the SS. As a large number of random variables are considered and more importantly those used for random field generation have no physical meanings, it is thus very difficult to generate effective conditional samples. This results in a large number of repeated samples in the SS. Given that the number of these repeated samples is not constant (i.e. random) for each SS, the obtained results are thus not steady.

### 6.2 The SPCE/SIR

According to Figure 12, this method gives always lower failure probability than the MCS. This can be explained by the fact that a dimension reduction technique is employed. As the dimension is reduced, the variability of the input parameters is reduced. The estimated failure probability is thus smaller. Compared to the SS, this method has a better performance in the parametric study i.e. the obtained values show a clear reduction trend of $P_f$ with decreasing $L_x$. Concerning the efficiency, the $N_{\text{call}}$ is found not constant for different cases but increases from the Case 1 to the Reference case. The $N_{\text{call}}$ for the Reference case is even very close to the MCS one. An explanation is given as follows. The required number of input random variables...
for the random field generation increased since the $L_x$ becomes smaller. Therefore, the construction of a meta-model needs more training points i.e. more deterministic simulation. As a result, it is not recommended to use this method if the $N_{RV}$ is large (e.g. $>2500$) from a point of view of efficiency.

6.3 The SPCE/GSA

This method is the most accurate one based on Figure 12. The obtained values of $P_f$ are extremely close to the ones of the MCS. Except to this remark, similar observations to the SPCE/SIR can also be noted: 1) the estimated values of $P_f$ are all lower than those of the MCS; 2) the parametric study can be correctly conducted; 3) the $N_{call}$ increases with decreasing the $L_x$ and the $N_{call}$ for the Reference case is even very close to the one of the MCS. The interpretation to these observations given above remains valid as well for this method. In addition, it is found that this method is always less efficient than the SPCE/SIR. This difference originates from the different dimension reduction techniques employed in the two methods. For the SPCE/GSA, it should always construct a 2-order meta-model with a full dimension. On the contrary, the dimension is reduced before constructing meta-models in the SPCE/SIR. For very high dimensional stochastic problems, considerable deterministic simulations are required even for constructing a 2-order SPCE meta-model.

6.4 The MM

This method is the second efficient one according to Table 5 and it shows also a good performance in estimating the value of $P_f$. Given its simplicity and easy implementation procedure, it is a good alternative to the MCS for such a very high dimensional stochastic problem. However, this article only evaluates the MM for the cases of relative high failure probability. Careful attention should be paid when applying this method to calculate low failure probabilities since it may lead to large errors as pointed out in [33]. Besides, this method is not able to carry out a parametric study of $L_x$ as expected since the obtained $P_f$ results fluctuate. Theoretically, the
collection of all the moments (of all orders, from 0 to $\infty$) uniquely describes a
bounded distribution. Then, failure probabilities can be determined by estimating the
tail area of the distribution. In the present study, only four moments are collected and
the tail area is estimated by an approximated way (Eq. 8 and 9). The induced errors
are not related to the $N_{RV}$ or $L_x$ of the problem but depend on the complexity of the
FoS distribution and the target value of the dam failure probability (tail area). It may
lead to a large error for a lower $Pf$ but a small error for a higher $Pf$. As a result, the
obtained $Pf$ values in the parametric study are not monotonously decreasing.

6.5 Concluding remarks
Here gives a summary of the remarks observed in Figure 12 and Table 5.
a) The most accurate method is the SPCE/GSA and the most efficient one is the SS.
b) The efficiency of the two meta-modelling methods strongly depends on the
number of input variables, while the $N_{call}$ of the sampling-based methods (the SS
and the MM) is related to the target value $Pf$. As the $Pf$ doesn’t vary significantly
from Case1 to the Reference Case, the $N_{call}$ of SS and MM is more or less
changeless.
c) The sampling-based methods cannot produce a consistent evolution of $Pf$ in the
parametric study of the $L_x$ whereas the meta-modelling methods perform well in
such a study.
d) The sampling-based methods are more efficient than the two considered meta-
modelling methods for very high dimensional stochastic problems.
e) The two meta-modelling methods estimate smaller values of $Pf$ compared to the
MCS. This is because that dimension reduction techniques are employed in these
two methods.

It should be noted that the conducted comparative study is related to a case of a
relatively high failure probability (order of $10^{-2}$). The performance of the four
methods for estimating low failure probabilities (e.g. $<10^{-4}$) in the context of very high dimensional stochastic problems is not investigated and thus unknown. This is a difficult issue in the field of reliability analyses to assess an approximated method for very low $P_f$, since the consuming time of running an MCS is very high even with a simplified deterministic model. The present study provides first insights into the performance of the four reliability methods in the context of very high stochastic problems, and some concluding remarks (e.g. the points b, c and e mentioned above) can be extended to the cases of low $P_f$.

7 Conclusions and perspectives

In this article, a probabilistic stability analysis of an earth dam is presented. The uncertainties in three soil properties ($C'$, $\phi'$ and $\gamma_d$) are considered in the analysis and quantified by exploiting the project-specific data. A large number of available geolocalized $\gamma_d$ measurements allow accounting for the soil spatial variability by estimating the autocorrelation structure with a variogram analysis. The MCS is adopted for performing the reliability analysis. Two distribution types for the input random variables are considered and compared in the article. Besides, the effect of the $L_x$ on the dam $P_f$ is investigated. Such a study is original because it uses real dam construction data, proposes using the benefits of geostatistics (a very high dimensional stochastic problem) and presents a procedure on how to produce meaningful statistical estimations of soil variability with limited measurements. This study can then be used as a part of a global dam safety assessment combined with a risk analysis as reported in [51].

By benefiting to the results of the deterministic simulations collected in the performed MCS, a comparative study is carried out. It aims at evaluating the performance of different reliability methods for very high dimensional stochastic problems. Both the accuracy and efficiency are considered in the comparison. The results show that the
most accurate method is the SPCE/GSA and the most efficient method is the SS. The efficiency of the methods SS and MM are independent to the number of input variables while the necessary $N_{\text{call}}$ of the methods SPCE/GSA and SPCE/SIR can be very important (close to the $N_{\text{call}}$ of an MCS) when a large number of random variables are involved. For a first order estimate, the methods SS and MM are sufficient to give relatively accurate results. Nevertheless, it should be noted that these two methods were not sufficiently accurate for the parametric study of $L_x$ since the obtained values of $P_f$ fluctuate.

This study also has some weakness points which will allow possible improvements for future works:

- It is commonly recognized that a negative correlation exists between $C'$ and $\phi'$ [42]. However, no correlation is considered in this study for the two soil properties due to the limited number of available triaxial test results and the employed method [5] for generating the values of $C'$ and $\phi'$.

- The $C'$ is represented by means of random variables. Its spatial variability is thus ignored in the present study,

- For the sake of simplicity and consistency with other studies, only stationary unconditional random fields are considered in this article. The effects of more complex random fields (non-stationary or conditional) could be investigated in future studies,

- The performance of the four reliability methods is only assessed for the cases of relatively high failure probability. For the cases with low $P_f$, the accuracy and efficiency of the four methods remain unknown for very high dimensional stochastic problems.

8 List of symbols

---

Soil properties
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_d$ (g/cm$^3$)</td>
<td>Dry density</td>
</tr>
<tr>
<td>$C'$ (kPa)</td>
<td>Effective cohesion</td>
</tr>
<tr>
<td>$\phi'$ (°)</td>
<td>Friction angle</td>
</tr>
</tbody>
</table>

**Some important symbols used in the statistical models**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>Mean value</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Standard deviation</td>
</tr>
<tr>
<td>$S$</td>
<td>Truncation term of the PCE series expansion</td>
</tr>
<tr>
<td>$L_x$ and $L_y$</td>
<td>Horizontal and vertical autocorrelation distance</td>
</tr>
<tr>
<td>$N_{MCS}$</td>
<td>Number of MCS population</td>
</tr>
<tr>
<td>$G$</td>
<td>Performance function</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Reliability index</td>
</tr>
<tr>
<td>$P_f$</td>
<td>Probability of Failure</td>
</tr>
<tr>
<td>$a$ and $b$</td>
<td>Beta distribution parameters</td>
</tr>
<tr>
<td>$\xi$</td>
<td>A vector of standard uncorrelated random variable</td>
</tr>
<tr>
<td>$N_{RV}$</td>
<td>Number of random variables</td>
</tr>
<tr>
<td>$N_{call}$</td>
<td>Number of calls to the deterministic model</td>
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**Abbreviation**

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tbody>
<tr>
<td>MCS</td>
<td>Monte Carlo simulation</td>
</tr>
<tr>
<td>SS</td>
<td>Subset Simulation</td>
</tr>
<tr>
<td>MM</td>
<td>Moment method</td>
</tr>
<tr>
<td>SPCE</td>
<td>Sparse Polynomial Chaos Expansions</td>
</tr>
<tr>
<td>GSA</td>
<td>Global Sensitivity Analysis</td>
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<tr>
<td>SIR</td>
<td>Sliced Inverse Regression</td>
</tr>
<tr>
<td>FoS</td>
<td>Factor of Safety</td>
</tr>
<tr>
<td>CoV</td>
<td>Coefficient of variation</td>
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<tr>
<td>PDF</td>
<td>Probability Density Function</td>
</tr>
<tr>
<td>K-L</td>
<td>Karhunen–Loève expansions</td>
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</tbody>
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### Acknowledgement

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10 References


[37] LI KC. High dimensional data analysis via the SIR/PHD approach. 2000.


