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Abstract

In civil engineering, the utilisation of expert judgement is important for evaluating the reliability of structures whose failure behaviour is little known and difficult to quantify. This article presents the application of the approaches of elicitation, calibration and aggregation of expert opinions to evaluate the reliability of river levees. These approaches have seldom been used in the field of French hydraulic structures whereas there is growing interest for them in other countries, concerning structures reliability and other fields such as aerospace industry, nuclear industry, hydrology, statistics, economics, psychology etc. This article proposes a quantitative approach to the use of expert judgement for evaluating river levee failure probability. An application to the case of an existing river levee is presented. The approach developed assesses the expert opinions according to their calibration (statistical accuracy) and their informativeness, then aggregates them in a single calibrated opinion. Applying the proposed weighting and aggregation procedure reduces variability in the probability of failure estimate, then it provides a better probabilistic distribution of aggregated expert opinion than of individual expert opinions. The results allow identifying a trend of aggregated expert opinions that point towards over- or lack of confidence.

Keywords: *structural reliability, subjective probability, levees, expert judgement, elicitation, calibration*

1 Introduction

River levees intended for flood protection are long linear structures of critical importance for people and property. Obtaining better knowledge of their risks and their structural reliability is a major challenge for engineers and the managers of these structures. However, the levees in France and abroad are often old, little known, and poorly maintained and documented [1, 2]. Evaluating their risks and reliability is complex due to the small quantities of data available in relation to the size and heterogeneity of these structures, and the complexity of failure mechanisms. Indeed, there appears to be a general lack of statistical data on levee characteristics (geometric, geotechnical and hydraulic data).

Furthermore some failure mechanisms such as internal erosion, overtopping and scouring are subject to many stability criteria that can be found in the literature according to the type of erosion, materials and levee profiles [2, 3, 4, 5, 6, and 7]. However it is difficult to make formulation choices given the large number of physical laws and parameters potentially combinable as well as uncertainties resulting from these modelling assumptions. There is not really consensus on the limit state conditions for internal erosion, overtopping and scouring in the national regulations, standards and professional recommendations. Only the sliding mechanism is subject to a well-established limit state condition in the regulations and recommendations [8].

In this context, evaluation using different quantitative approaches for risks and the structural reliability of river levees regarding different failure mechanisms – sliding mechanism, internal erosion, overtopping and scouring – is complex and often unfeasible [9]. This leads to calling on expert judgement in risk analysis studies. Expert engineers must interpret the available data and voice their uncertainties, in order to then evaluate the reliability of levees in terms of failure probability. Moreover, during the assessment of the safety of the French levees, it is necessary to mobilize various areas of expertise and complementary: hydraulic, morpho dynamic, geotechnical, expertise of hydraulic structures, civil engineering, etc. Thus it is necessary to call upon several experts, in charge of deciding on a probability of failure, in a context of high uncertainties: about the physics of the phenomena considered, the representativeness and the variability of the available data.

The ILH (International Levees Handbook) [2] underlines the advantage and importance of using expert judgement for analysing the risks and reliability of levees when evaluation by quantitative approaches is difficult to perform. However, it does not recommend any specific approach allowing the efficient utilisation of expert judgement in an uncertain context. In the scientific literature, there are few works focused on the use of expert judgement for analysing the risks of levee failures. Perlea and Ketchum [10] proposed the use of expert judgment to build fragility curves, assessing impact of vegetation, rodents' activity, encroachments, utility penetrations, and erosion of the river bank and levee slopes. Serre *et al.* [11] proposed a deterministic model of evaluating the performance of levees based on a multi-criteria decision aid method: the rule-based assignment model. Vuillet *et al.* [12, 13] proposed a levee evaluation model based on performance indicators, using the judgement of an expert or a consensus of experts and including uncertainties on the data available on the levees. This model is semi-quantitative and does not allow obtaining the failure probability of structures or combining several expert opinions. It appears important to employ a methodology capable of dealing with the latter two points, especially in France where the regulations relating to structural hazard studies systematically require engineers to declare on the failure probability of important structures.

Numerous approaches have been developed proposing probabilistic modelling of expert assessments to study the reliability of structures [14, 15], in various fields such as the aerospace industry, the nuclear industry, resistance of buildings to earthquakes [16] and hydraulic structures [17]. Cooke [18] and Cooke and Goossens [19] distinguished three approaches for the implementation of expert judgement: i) elicitation, ii) calibration and iii) the aggregation of expert opinions.

The elicitation of expert opinion is a process of collecting expert judgements using specialised verbal and written communication methods that allow quantifying risk [20]. There is general scientific consensus regarding the main steps to be followed for the elicitation of expert opinions, shared by several researchers in the domain like Morgan and Henrion [21]; Cooke [18]; Clemen and Reilly [22], Ayyub [23]; Cooke and Goossens [19], Garthwaite et *al.* [24] O'Hagan *et al* [14] and Burgman [25]. These steps generally correspond to the definition of the problem, the identification and recruitment of experts and then the elicitation of expert judgement.

To help the elicitation of experts in a probabilistic framework, economics researchers have revealed the importance of the expert having access to data describing the context studied and on which a probabilistic estimation can be made [26]. These data can be theoretical or experimental. Furthermore, the authors show the importance of implementing the precise modalities of eliciting expert opinions in a probabilistic format and recommend distinguishing a range of values during elicitation and specifying that which seems most likely [27]. Relying on O'Hagan *et al* [14], Mason *et al* [28] says that with individual elicitation, each expert gives their view independently, and these views are expressed numerically and then appropriately aggregated. The alternative consisting in an elicitation group in the form of an expert panel allows experts to debate and reach consensus about the elicited values, but is more time consuming and limits the number and range of experts included.

The calibration of expert opinions permits evaluating the consistency between the information supplied by the expert and the known values (experimental / observed) [18]. The approach to calibrating the expert opinions consists in evaluating and weighting the expert opinions in relation to calibration variables whose real values are known. The approach entails performing a quantitative evaluation of the calibration, or statistical accuracy, which refers to how often experts capture the realization into their uncertainty interval and informativeness of the expert elicitations through the use of calibration variables whose true values are known by the analyst. The mathematical calibration models such as the classical model of Cooke [18] allow determining a performance weight for expert proportional to the product of statistical accuracy and informativeness. The classical model has been used in several applications [15, 29, 30], such as nuclear, banking, volcanoes, underground gas pipelines reliability [31, 32, 33, 34], to quantify internal erosion process of dams [35] and levees failure frequency in Netherland [36]. One of the objectives of the calibration models is to aggregate expert opinions onto a single opinion, however several authors recommend combining and aggregating expert opinions using a simple average of the expert elicitated values, which does not require calibration [37, 38]. In a probabilistic framework, Lichtendahl et *al*. [39] proposed a method for aggregating expert opinions based on the mean of quantiles elicited by the experts, permitting aggregation in the form of an uncertainty interval and a most probable value. Colson and Cooke [40] also proposed aggregating expert elicited distributions constructed using the elicited values values by performance weighting.

The objective of this article is to present an approach for the elicitation, calibration and aggregation of expert opinions for evaluating the probabilities of river levee failures due to failure mechanisms. We wish to propose a method: i) to unify the estimates of several experts and reduce the variability of the expert estimates, ii) to assess the global uncertainty of the expert estimates and the degree of confidence of each expert, iii) and to assess the uncertainties that each expert estimates and finally the degrees of confidence that can be given to the expert estimates given their calibration scores.

The article first presents the development of an approach for eliciting expert opinions in terms of failure probability represented by the slope sliding failure mechanism. It then presents the application of Cooke's [18] calibration model to levees in order to weight the experts' probabilistic opinions. Lastly, it presents the result of applying the protocol for aggregating expert opinions developed by Lichtendahl et *al.* [39] to determine, on the basis of expert judgement, a calibrated and aggregated probabilistic distribution of failure probabilities for several profiles of an existing levee, using an aggregated interval to take into account the uncertainties expressed by the experts.

2 *Methodological approaches for implementing expert judgement proposed for river levees*

The studies of risk analyses and evaluations of structural reliability relating to levees must include the evaluation of the probabilities of scenarios leading to failure [2]. The approaches proposed focus on the elicitation of expert judgement, and the calibration and aggregation of failure probabilities elicited relating to levee failure mechanisms that can lead to a breach of a levee.

2.1 Expert opinion elicitation approach

The implementation of the expert opinion elicitation approach starts with the identification of a panel of experts whose activities are directly involved with river levees. Levee risk analysis studies involve a wide range of expert opinions linked to different disciplines: geotechnics, river hydraulics, civil engineering, etc. Several experts from several disciplines with different levels of experience (junior, confirmed, senior) are liable to intervene in these levee risk analysis studies. Thus, we gathered an expert panel by taking into account the interdisciplinary dimension of the different experts and their diversity of opinions [21].

Once the panel has been formed, each expert is questioned individually on the failure probabilities relating to several profiles of the levee to be evaluated regarding different failure mechanisms. A form including a questionnaire and for collecting expert opinions in probabilistic format has been developed for this purpose. The form comprises three main elements for each variable to be elicited: *the question, the information and the demand for response*. In the example shown in figure 1, regarding the sliding mechanism, these items are set out in the following way:

- *The question* corresponds to the variable on which the experts are questioned. It contains a detail on the mechanism to be evaluated, the cross-section to be evaluated (downstream or upstream) and the flood for which the failure probability is demanded. This question on the sliding failure mechanism takes the following form: *"Given all the information available, what is the probability P_{fc} that the levee will fail due to sliding of the downstream slope in the case of a flood whose level reaches the crest of the levee?"*

- *The information* provides all the data required for the experts to formulate their responses and give their judgements. They correspond to the *objective cue information* presented by [26]. The items of information making up the form are: geometric data, hydraulic data, geotechnical data, the standard cross-section of the profile to be evaluated, the laws of probability of the geotechnical characteristics of the profile to be evaluated and the data resulting from a deterministic analysis represented by the safety factor SF of the levee regarding the slope sliding failure mechanism;

- *The demand for response* corresponds to the way the expert communicates his judgement on the failure probability (P_{fc}) of the cross-section to be evaluated. In view to taking into account the uncertainties linked to levees, the approach proposes expert elicitations in the form of uncertainty interval and a most probable value [1, 13, 27]. Thus the expert is first called on to elicit an uncertainty interval [q5%, q50%, q95%], then to specify the median value resulting from the interval elicited proposed by [18]. The choice of 5% and 95% quantiles is motivated by the practice of using statistical evaluations of resistance and stress parameters in civil engineering [8]. Also these are the quantiles generally used for the application of the classical calibration model [41].

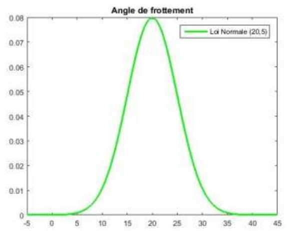
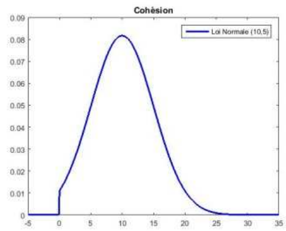
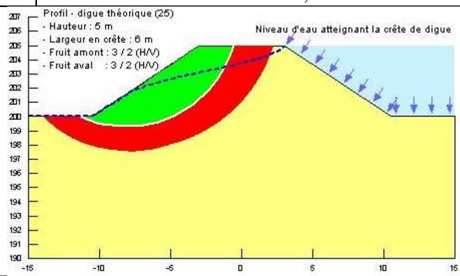
<table>
<tr><td>Form 01</td><td colspan="3">Cross-section – Theoretical levee 1</td></tr>
<tr><td colspan="4"><u>Question</u>
Given all the information available, what is the probability P_{fc} that the levee will fail due to sliding of the downstream slope in the case of a flood reaching the height of the levee crest?</td></tr>
<tr><td colspan="4"><u>Information</u></td></tr>
<tr><td>Geometric data
Cf. Figure below</td><td colspan="3">- Height = 5 m
- Altitude of crest = 205 m NGF
- Width of crest = 6 m
- Upstream batter = 3/2 (H/V)
- Downstream batter = 3/2(H/V)</td></tr>
<tr><td>Hydraulic data</td><td colspan="3">- Flood level reaching the levee crest
- Hydraulic gradient level calculated in steady state</td></tr>
<tr><td>Geotechnical data</td><td colspan="3">Body and foundation
- Sandy-silty material
- Permeability $K = 10^{-6}\ m/s$
- Unit weight: $\gamma_h = 18\ kN/m^3$</td></tr>
<tr><td rowspan="2">Probability laws of input data</td><td colspan="2">
</td><td>
</td></tr>
<tr><td colspan="2">Angle of friction
(body - foundation)
Probabilistic law: normal law
Mean: 20°
Standard deviation: 5°</td><td>Cohesion
(body - foundation)
Probabilistic law: normal law truncated at 0
Mean: 10 kPa
Standard deviation: 5 kPa, Min = 0 kPa</td></tr>
<tr><td>Deterministic analysis</td><td>Calculated SF
(with mean values)
Downstream slope
1.3</td><td colspan="2">
</td></tr>
<tr><td colspan="4"><u>Demand for response</u></td></tr>
<tr><td rowspan="3">Probabilistic analysis</td><td rowspan="3">Probability (P_{fc}) of sliding failure</td><td>P_{fc}: quantile q5%</td><td></td></tr>
<tr><td>P_{fc}: quantile q95%</td><td></td></tr>
<tr><td>P_{fc}: median value (q50%)</td><td></td></tr>
</table>

Figure 1. Example of a sheet for a levee profile resulting from the questionnaire form developed

The questionnaire is given individually to each of the experts. It contains an introduction briefly presenting the objectives of the study and a short manual for completing the form. Then, each expert freely completes the questionnaire according to his availability within a period of time agreed with the moderator. During this period, each expert may question the moderator if any elements of the questionnaire are unclear. In the same way, each expert may review his answers during this period. A training or dry run with feedback is desirable [19].

2.2 Approach for calibrating expert opinions

The approach to calibrating the panel experts' opinions consists in evaluating and weighting the panel experts' opinions in relation to calibration variables whose real values are known. The implementation of the approach for calibrating expert opinions is based on the model developed by Cooke [18]. This model is used to evaluate and weight the experts' opinions with the variables of interest and calibration. The variables of interest are those which the analyst seeks to know the values. The expert judgement is therefore used to build a subjective probabilistic distribution regarding the production of these variables. The calibration variables correspond to variables liable to inform the analyst on the capacity of the experts to provide an accurate and precise estimation. They are of the same nature as the variables of interest.

When applying them to levees, it is necessary to define pertinent calibration variables with respect to the variables of interest. Given that the variables of interest are the expert evaluations of risks and of the structural reliability of levees in a probabilistic framework, we adopted them as a calibration variable of the theoretical profiles of levee failure probability with respect to the sliding failure mechanism. This variable can be obtained quantitatively by carrying out a probabilistic reliability analysis, by considering the uncertainties on the input data (cohesion and angle of friction) and by Monte Carlo simulations [42]. The uncertainty of the input data (cohesion and angle of friction) is modelled by laws (see Figure 1). Random draws are made on these values from which stability calculations are made and an evaluation of the probability of failure by Monte Carlo simulation is obtained.

This choice of calibration variables has several advantages. The format of the chosen calibration variables is the same as that of the variables of interest: probability of failure of levee profiles. As the calibration variables correspond to the results of a numerical model of theoretical levee profiles, it is possible to construct a base with a large number of calibration variables (using different levee profiles with different geometries, material properties, etc.). It should be noted that the results of a numerical model depend on the assumptions adopted as input data. This allows the elicitation phase of the calibration variables to be easily framed by presenting the input data of the numerical model in the questionnaire.

The elicitations performed by the experts can then be compared with the results of the numerical model for the different calibration variables.

Implementing Cooke's [18] model permits calculating the individual *performance weight* w_e of each expert, which is also expressed as the *relative calibration weight* w'_e in relation to all the experts of the panel. The calibration weight is determined from the *calibration score* C_e and the *information score* K_e as a function of the following relations [18]:

$$w_e = C_e . K_e \quad [1]$$

$$w'_e = C_e . K_e \,/\, \sum_{e=1}^{z} C_e . K_e \quad [2]$$

where:

w_e	:	weight of the expert's calibration *e*
w'_e	:	weight of the relative calibration of expert *e* in the panel of experts
C_e	:	calibration score of expert *e*
K_e	:	information score of expert *e*
Z	:	number of experts

Calibration score C_e permits evaluating the quality (statistical accuracy) of the information given by the expert. Calibration measures the statistical likelihood that a set of experimental results corresponds, in a statistical sense, with the experts' assessments. Loosely, the calibration score is the probability that the divergence between the expert's probabilities and the observed values of the seed variables might have arisen by chance. A low score (near zero) means that it is likely, in a statistical sense, that the expert's probabilities are 'wrong' [19].The calibration score is evaluated by the following relation 3 below [18]. It is determined by the addition of an error probability $P(\mathcal{X}^2)$ obtained by the $\mathcal{X}^2$ distribution for large samples size (see [41] for more details). We used Kullback-Laibler (KL) divergence measure which measures the difference between two probability distributions (expected and elicited by expert), noted $I_e(c_i,p_i)$ within formula 3 and 4 below.

$$C_e = 1 - P\big(2 * n * I_e(c,p)\big) = 1 - \mathcal{X}^2\big(2 * n * I_e(c,p)\big) \quad [3]$$

where :

P( )	:	probability of a random variable according to a $\mathcal{X}^2$ law
$\mathcal{X}^2($ $)$	:	distribution function of a random variable according to the $\mathcal{X}^2$law
n	:	number of calibration variables
c	:	calibration vector representing the portion of true values in each interquartile interval $c = \{c_1, c_2, \dots, c_j\}$
j	:	number of interquartile intervals, in this study j=4
p	:	theoretical probability vector representing the theoretical probabilities of each interquartile interval $p = \{p_1, p_2, \dots, p_j\}$
$I_e(c_i, p_i)$	:	relative information between the theoretical probability vector p and the calibration vector c . It is calculated as follows:

$$I_e(c,p) = \sum_{i=1}^{j} c_i \ln(c_i/p_i) \quad [4]$$

The *information score* K_e permits measuring the quantity of information contained in the probabilistic distributions given by the experts. It relies on a measure of distances between the vector of subjective probabilities s and the vector of theoretical probabilities p. The information score is evaluated by the following relation:

$$K_e = (1/\mathrm{n}) \sum_{i=1}^{n} I_{e,i}(p,s) \qquad [5]$$

where:

s : vector of subjective probability representing the subjective probabilities of each interquartile interval $s = \{s_1, s_2, \dots, s_j\}$. It is used for measuring the distance between the expert's distribution and a non-informative uniform distribution;

$I_e(p,s)$: the relative information between the vector of subjective probabilities s and the vector of theoretical probabilities p.

The vector of subjective probability s and the relative information $I_e(p,s)$ are calculated for each calibration variable by the following formulas:

$$s = \left\{\frac{q_5 - q_0}{q_{100} - q_0}, \frac{q_{50} - q_5}{q_{100} - q_0}, \frac{q_{95} - q_{50}}{q_{100} - q_0}, \frac{q_{100} - q_{95}}{q_{100} - q_0}\right\} \qquad [6]$$

$$I_e(\mathrm{p},\mathrm{s}) = \sum_{i=1}^{j} p_i \ln(p_i/s_i) \qquad [7]$$

2.3 Expert opinion aggregation approach

Several approaches have been suggested and/or implemented on case studies for the aggregation of expert opinions, concerning unweighted simple average aggregations: elicited probability distributions [14] or elicited quantiles [39], or weighted averages of probabilities distributions constructed from the elicited quantiles via the respective performances of the experts on the calibration variables [14]. An interesting discussion of the comparison of the results of these methods can be found in [29]. The expert opinion aggregation approach is implemented in this study with performance weighted quantiles [39]. The objective is to conserve the initial form of the expert elicitations given in probabilistic format with a value considered as the median and an uncertainty interval. This approach corresponds to a weighted sum of the calibrated quantiles of expert opinions. The aggregated quantiles ($q_{a5\%}, q_{a50\%}, q_{a95\%}$) are determined by the following relations:

$$q_{a5\%} = \sum w'_e \cdot q_{5\%(e)} \quad [8]$$

$$q_{a50\%} = \sum w'_e \cdot q_{50\%(e)} \quad [9]$$

$$q_{a95\%} = \sum w'_e \cdot q_{95\%(e)} \quad [10]$$

where :

$q_{a5\%}$: aggregated quantile corresponding to a probability of 5%;

$q_{a50\%}$: aggregated quantile corresponding to a probability of 50%;

$q_{a95\%}$: aggregated quantile corresponding to a probability of 95%.

3 Application of the approach – Results

3.1 Presentation of the case study

The levee studied is an earth-fill levee 5500 meters long intended to protect a French city against floods. The height of the levee in the study area varies from 1 m to 6 m.

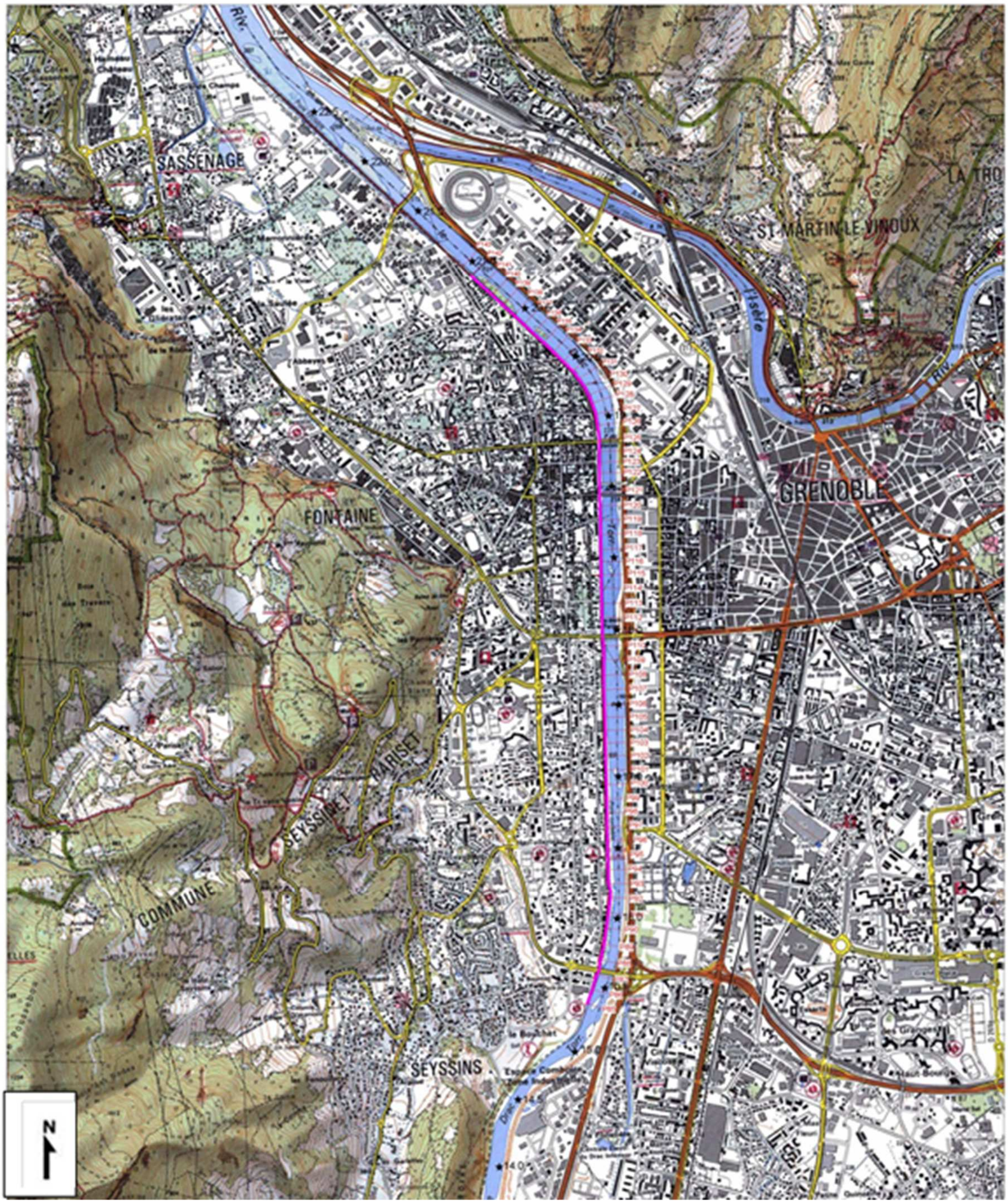


Figure 2. Aerial view of the levee studied (scale: 1/25 000 in A3 format)

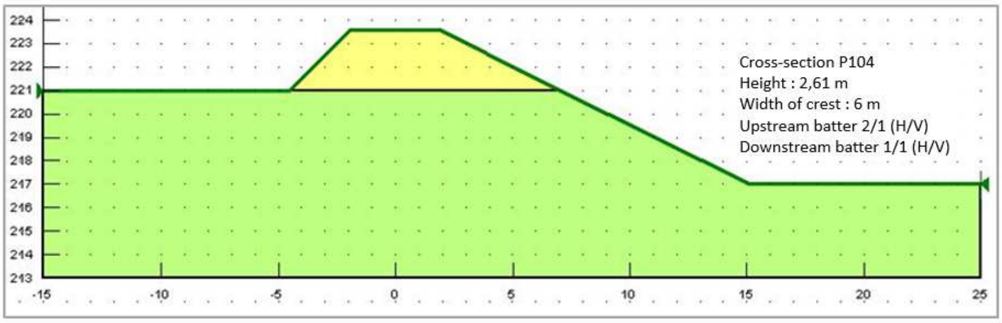


Figure 3. Standard cross-section of the levee studied

The approach developed was implemented by a panel of 6 expert, all civil engineers with different specialities (geotechnics, river hydraulics, civil engineering structures, risk analysis), all levee safety experts working for the French Ministry of Ecology, and all volunteers. In order to realize the experience in the best conditions, the elicitation was done anonymously. The questionnaire used in this case study contains a brief introduction (including a notice for completing the form), a section for calibration variables and a section for variables of interest. This questionnaire was given individually to each of the experts. They provided their judgements on the calibration variables and the variables of interest corresponding to the failure probabilities of the levees with respect to the different failure mechanisms. The experts completed the form within one week. Previously, there was a dry run with a limited number of variables, which improved the clarity of the form. Other than this dry run, there was no specific training with feedback on the true values of the calibration variables. This choice was adopted specifically for this case study for further improvements concerning the experts' biases.

In this study, the calibration variables correspond to the probabilities of sliding failure for 30 theoretical levee cross-sections (cf. section 2.2). This number of calibration variables permits a robust statistical analysis [43].

The study is applied to 40 variables of interest corresponding to 10 levee cross-sections studied with 4 failure mechanisms per cross-section. The following table summarises the type of calibration variable and the variables of interest used in this study:

Table 1. Table summarising the calibration variables and the variables of interest in the application of the approach.

Failure mechanism	Calibration variables (theoretical levee profiles)		Variables of interest (profiles of the levee studied)	
	Number of variables	no. of variable	Number of variables	no. of variable
Sliding	30	no.1 – no.30	10	no.31 – no.40
Internal erosion	-	-	10	no.41 – no.50
Scouring	-	-	10	no.51 – no.60

Overtopping	-	-	10	no.61 – no.70

3.2 Results of the elicitation phase

The following graph presents the elicitations of expert no.1 regarding the failure probabilities (P_{fc}) of 4 failure mechanisms. The black squares correspond to the quantiles (5%, 50% and 95%) elicited by the expert, while the grey circles correspond to the "true value" of the calibration variable (evaluated through a reliability analysis).

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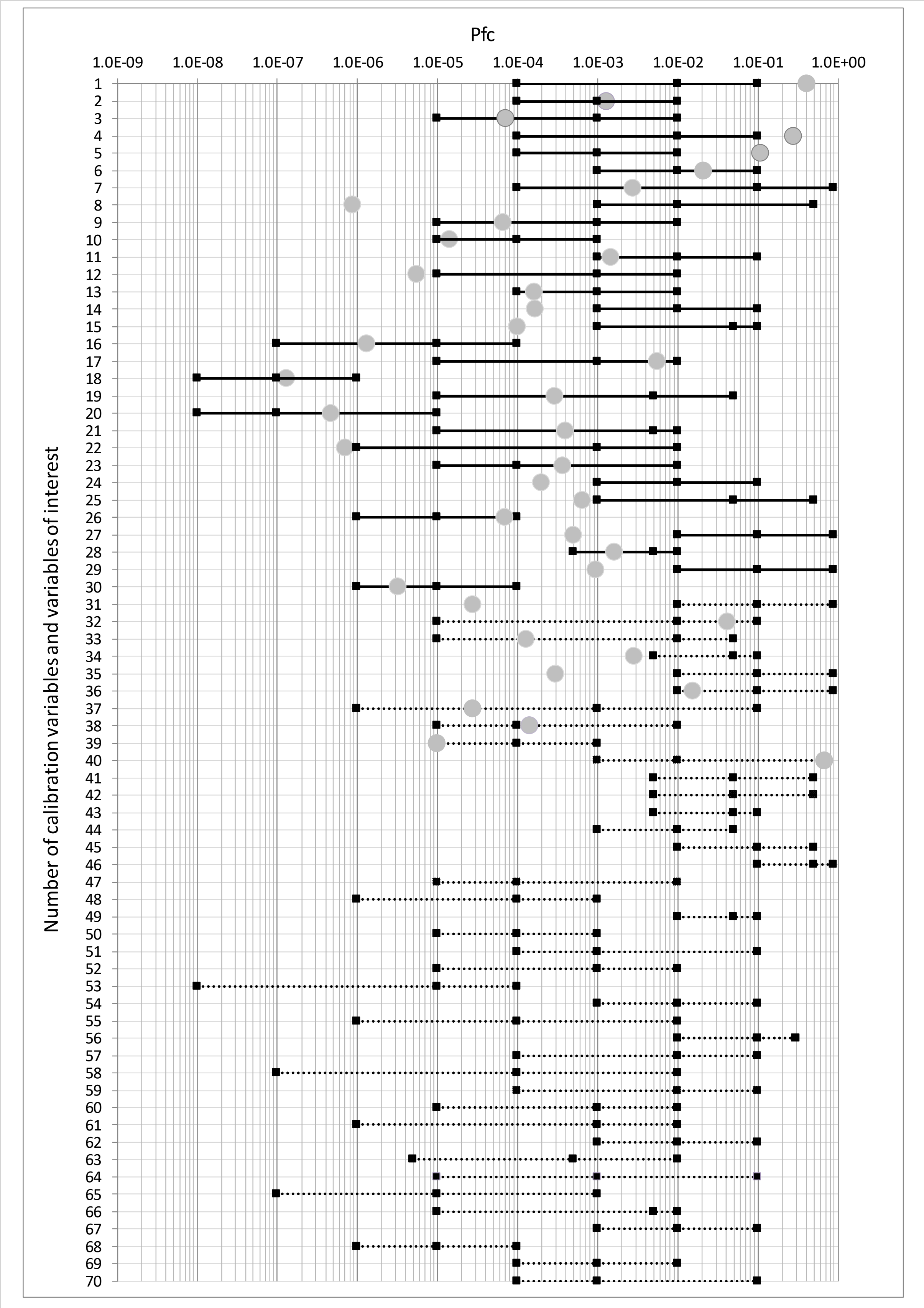

Pfc
1.0E-09
1.0E-08
1.0E-07
1.0E-06
1.0E-05
1.0E-04
1.0E-03
1.0E-02
1.0E-01
1.0E+00
Number of calibration variables and variables of interest
1
2
3
4
5
6
7
8
9
10
11
12
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Figure 4. Raw elicitations of expert no.1 relating to calibration variables no.1 to no.30 and to variables of interest no.31 to no.40 – sliding failure mechanism, no.41 to no.50 – internal erosion failure mechanism, no.51 to no.60 – scouring failure mechanism), no.61to no.70 – Overtopping failure mechanism.

Table 2 gives the distribution of the true values (in number and in percentage) in the interquartile intervals elicited by the experts for theoretical levee cross-sections no.1 to no.30:

Table 2. Distribution of true values of the calibration variables (no.1-no.30) in the interquartile intervals elicited by the experts and the mean of elicitations.

Calibration variable (no.1 to no.30)					
Interquartile interval		$[0\%, 5\%]$	$[5\%, 50\%]$	$[50\%, 95\%]$	$[95\%, 100\%]$
Expert no.1					
$(\mathbf{P_{fc}})$ (no.1 to no.30)	n	9	11	7	3
	%	30%	37%	23%	10%
Expert no.2					
$(\mathbf{P_{fc}})$ (no.1 to no.30)	n	9	10	10	1
	%	30%	33%	33%	3%
Expert no.3					
$(\mathbf{P_{fc}})$ (no.1 to no.30)	n	10	5	7	8
	%	33%	17%	23%	27%
Expert no.4					
$(\mathbf{P_{fc}})$ (no.1 to no.30)	n	10	8	8	4
	%	33%	27%	27%	13%
Expert no.5					
$(\mathbf{P_{fc}})$ (no.1 to no.30)	n	0	4	15	11
	%	0%	13%	50%	37%
Expert no.6					
$(\mathbf{P_{fc}})$ (no.1 to no.30)	n	8	8	6	8
	%	27%	27%	20%	27%
Means of elicitations (before calibration)					
$(\mathbf{P_{fc}})$ (no.1 to no.30)	n	7.67	7.67	8.83	5.83
	%	26%	26%	29%	19%

The results show that the percentage of true values in the interval [5%, 95%] varies from 40% (17%+23%) for expert no.3to 66% (33%+33%) for expert no.2. More exactly for expert 3, 33% in the interval [0%, 5%] instead of 5%, 17% in the interval [5%; 50%] instead of 45%, 23% in the interval [50%, 95%] instead of 45, and 27% in the interval [95%, 100%] instead of 5%, and for expert 2 : 30% in the interval [0%, 5%] instead of 5%, 33% in the interval [5%; 50%] instead of 45%, 33% in the interval [50%, 95%] instead of 45, and 3% in the interval [95%, 100%] instead of 5%. For experts nos. 1, 4 and 6, the true values are concentrated in the interval [0%, 50%], expressing a trend towards overestimating the failure probabilities. On the contrary, for expert no.5, the true values are concentrated in the interval [50%, 100%], expressing a trend towards underestimating the failure probabilities.

3.3 Results of the calibration phase

The calibration phase is performed using 30 calibration variables (n=30). The result of the calibrating the expert opinions is presented as a calibration score, information score and relative calibration weight.

Since elicitation is performed on the $q_{5\%}$, $q_{50\%}$ and $q_{95\%}$ quartiles, the theoretical probability vector p associated with the interquartile intervals is p={0.05, 0.45, 0.45, 0.05}. The calibration vectors c can be obtained directly from table 2 (for expert 1, c={0.30, 0.37, 0.23, 0.10}). Then, the relative information between c and p vectors, $I_e(c,p)$ is calculated according Eq.4 (for expert 1, $I_e(c,p)$=0.38), which finally gives the calibration score C_e presented in Table 3 by using Eq.5.

For the calculation of the information score, the elicited failure probability values can be very low, expressed in the format (10^{-x}). We change the variable in order to conserve a constant difference in absolute value between (10^{-x}) and $\left(10^{-(x-1)}\right)$, which amounts to applying the common logarithm to the elicited probabilities ($\log_{10}(P_{fc})$). This change of variable permits a more pertinent interpretation of the information score. The subjective probability vector s is then evaluated for each expert and for each of the calibration variables. For example, expert 1 elicits the probability values of 10^{-4}, 10^{-2} and 10^{-1} for the quartiles 5%, 50% and 95% respectively for the first calibration variable. With the variable change to base $\log_{10}$, we thus obtain $q_{5\%}$=-4, $q_{50\%}$=-2 and $q_{95\%}$=-1, which makes it possible to evaluate the vector s_1={0.24, 0.39, 0.19, 0.18} with Eq.6 and the relative information $I_{e,1}(p,s)$=0.31 with Eq.7. The information score K_e is finally evaluated with Eq.5, and corresponds to the average of the relative information $I_{e,i}(p,s)$ evaluated for the 30 calibration variables.

Once the calibration score C_e and information score K_e have been evaluated, the individual weight w_e and the relative weight of calibration w'_e are directly evaluated by Eq.1 and Eq.2 respectively. The results of the calibration score C_e, information score K_e, individual weight w_e, and relative weight of calibration w'_e are presented in the following table for each expert. The calculations to obtain these results were carried out in an Excel file.

Table 3. Results of the expert opinion calibration phase in relation to the calibration variables.

	Calibration score	Information score	Weight of individual	Relative weight
No. expert	C_e	K_e	w_e	w'_e
1	0.79	0.74	0.59	0.28
2	0.91	0.91	0.83	0.40
3	0.03	0.79	0.02	0.01
4	0.46	1.04	0.48	0.23
5	0.14	0.44	0.06	0.03
6	0.19	0.52	0.10	0.05
			Total	1

The results obtained from the calibration of expert opinions showed that the values of the calibration scores (C_e) varied from 3% to 91%. The highest value was obtained by expert no. 2, meaning that their elicitations contained more true calibration values in comparison to the elicitations of the other panel experts. On the contrary, the lowest calibration score was obtained by expert no. 3, meaning that their elicitations contained the fewest true calibration values.

Regarding the information score, the values varied from 0.44 to 1.04. The highest value was obtained by expert no.4, meaning that the uncertainty intervals elicited by this expert were narrower than those given by the other panel experts. On the contrary, the lowest value of this score was obtained by expert no.5, meaning that the uncertainty intervals elicited by this expert were wider than those given by the other panel experts.

We observed that the calibration score in our study had a more significant influence than the information score on the final result of the expert opinion calibration phase. Indeed, the range of the final calibration score was wider than that of the information score. Thus, expert no.2 with the highest calibration score obtained the best relative calibration weight ($w_2' = 40\%$) whereas expert no.3 with the lowest calibration score obtained the lowest relative calibration weight ($w_3' = 1\%$). The question of not taking into account his opinion may then arise. In the context of this study, we preferred to keep all the opinions, taking into account the fact that that of some experts does not significantly influence the final results.

3.4 Results of the aggregation phase

The result of the expert opinion aggregation phase is shown by one probabilistic distribution relative to each levee cross-section evaluated by the experts. We operated results exposed in table 3 to combine expert assessments, using quantiles combination [39] to produce results exposed in figure 5 and table 4. The following graph shows the result of the aggregation of the calibrated expert opinions for the 30 calibration variables related to the theoretical levees (continuous lines) where the true values are represented by a circle. The calculations to obtain these results were carried out in an Excel file.

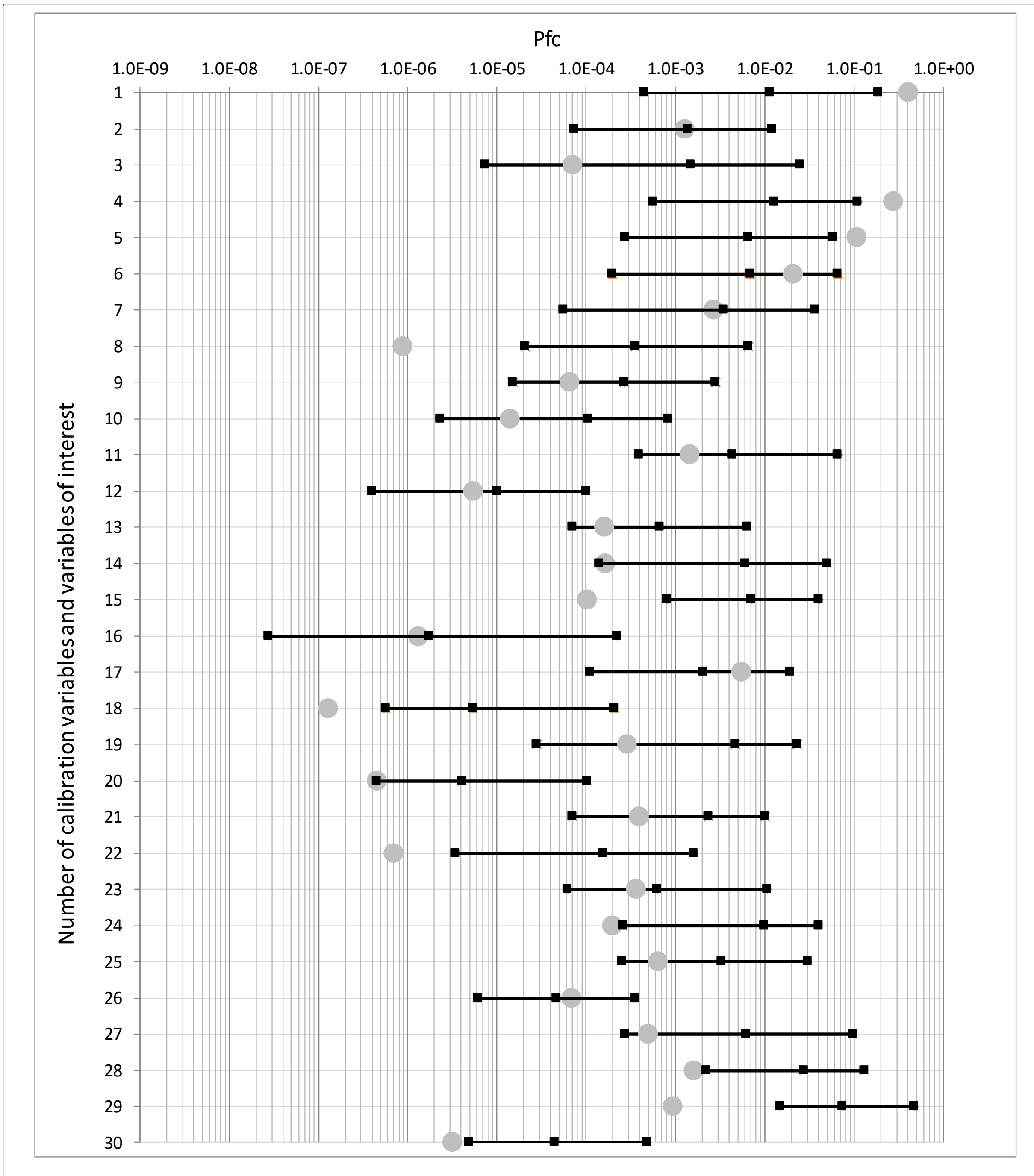

Figure 5. Calibration variables calibrated and aggregated and their uncertainty intervals

Figure 5 can be interpreted using Table 4 below which presents the true calibration variables in the interquartile intervals at the end of the expert opinion aggregation phase:

Table 4. Distribution of the true calibration variable values (no.1-no.30) in the interquartile intervals.

Calibrated and aggregated elicitations of calibration variables (no.1 to no.30)					
Interquartile interval		$[0\%, 5\%]$	$[5\%, 50\%]$	$[50\%, 95\%]$	$[95\%, 100\%]$
$(\mathbf{P_{fc}})$ (no.1 to no.30)	n	9	15	3	3
	%	30%	50%	10%	10%

At the end of the aggregation phase we observed that the final distributions of the calibrated and aggregated expert opinions contained true values in the uncertainty interval for 18 calibration variables out of a total of 30 variables, i.e. 60% of the true values belonging to the uncertainty interval [5%, 95%]. Thus, the aggregation of the calibrated expert opinions leads to an uncertainty interval [5%, 95%] containing more true values in comparison to the raw expert uncertainty intervals.

3.5 Application to variables of interest

On the basis of the expert calibrations, the approach was applied to the variables of interest. Figure 6 presents the results for 40 variables of interest corresponding to 10 levee cross-sections studied for 4 failure mechanisms:

- variables no.31 to no.40 corresponding to the sliding failure mechanism,
- variables no.41 to no.50 corresponding to the internal erosion failure mechanism,
- variables no.51 to no.60 corresponding to the scouring failure mechanism,
- variables no.61 to no.70 corresponding to the overtopping failure mechanism.

Table 5 shows the distribution of true values (in number and in percentage) in the raw interquartile intervals (before calibration and aggregation) elicited by the experts for the sliding failure mechanism for levee cross-sections studied no.31 to no. 40:

Table 5. Distribution of true values of variables of interest (no.31-no.40) in the interquartile intervals elicited by the experts for the sliding failure mechanism.

Variable of interest (no.31 to no.40) for the sliding failure mechanism					
Interquartile interval		$[0\%, 5\%]$	$[5\%, 50\%]$	$[50\%, 95\%]$	$[95\%, 100\%]$
Expert no.1					
$(\mathbf{P_{fc}})$ (no.31 to no.40)	n	3	3	4	0
	%	30%	30%	40%	0%
Expert no.2					
$(\mathbf{P_{fc}})$ (no.31 to no.40)	n	4	3	3	0
	%	40%	30%	30%	0%

Expert no.3					
(P_{fc}) (no.31 to no.40)	n	3	2	4	1
	%	30%	20%	40%	10%
Expert no.4					
(P_{fc}) (no.31 to no.40)	n	3	5	2	0
	%	30%	50%	20%	0%
Expert no.5					
(P_{fc}) (no.31 to no.40))	n	0	0	6	4
	%	0%	0%	60%	40%
Expert no.6					
(P_{fc}) (no.31 to no.40)	n	0	1	5	4
	%	0%	10%	50%	40%
Mean of elicitations (before calibration)					
(P_{fc}) (no.31 to no.40)		2.17	2.33	4	1.5
		21.7%	23.3%	40%	15%

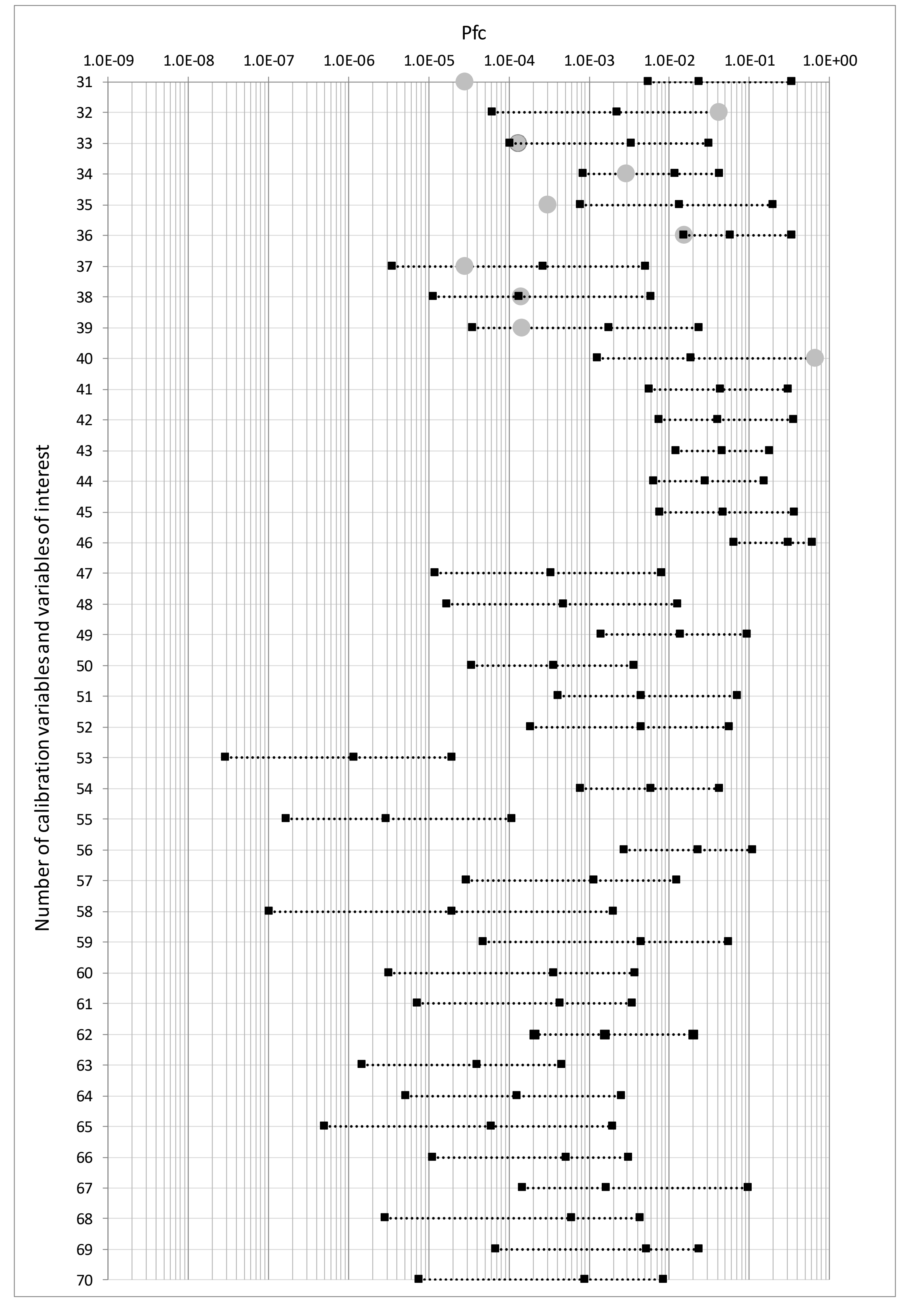

Figure 6. Calibrated and aggregated variables of interest and their uncertainty intervals: **no.31 to no.40 – sliding failure mechanism, no.41 to no.50 – internal erosion failure mechanism, no.51 to no.60 – scouring failure mechanism, no.61 to no.70 – overtopping failure mechanism.**

Regarding the different variables of interest considered in this study, it is possible to perform a quantitative evaluation by analysing the failure probabilities for variables no.31 to no.40 of the sliding failure mechanism, which correspond to the sliding failure probabilities of six cross-sections of the levee studied. These variables allow comparing the results of the calibration and aggregation approach applied to the expert opinions with the results of a quantitative analysis. Table 6 below presents the distribution of true values in the interquartile intervals for the variables of interest no.31 to no.40 at the end of the expert opinion aggregation phase and using the calibration weights determined during the expert opinion calibration phase for the cross-sections of the theoretical levees:

Table 6. Distribution of true values of the variables of interest (no.31-no.40) in the interquartile intervals.

Calibrated and aggregated elicitation pf variables of interest (variables no.25-no.30)					
Interquartile interval		$[\mathbf{0\%, 5\%}]$	$[\mathbf{5\%, 50\%}]$	$[\mathbf{50\%, 95\%}]$	$[\mathbf{95\%, 100\%}]$
$(\mathbf{P_{fc}})$ (no.31-no.40)	n	2	5	2	1
	%	20%	50%	20%	10%

The final distributions of the aggregated expert opinions of variables no.31 to no.40 contain the true value in the uncertainty interval [5%, 95%] seven times, giving a percentage of 70% of the true values of the uncertainty values given by the experts. The evaluation of variables 31 and 40 is similar to a cross-validation of the results obtained for the calibration variables (1 to 30). We find that we get similar results. We propose to discuss this in paragraph 3 of section 4.2.

Regarding the other failure mechanisms (internal erosion, overtopping and scouring), the structural limit states were not quantifiable since it was not possible to quantitatively evaluate the failure probability of these failure mechanisms. The true values were therefore unknown for these mechanisms and the approach proposed therefore made it possible to implement the expert judgement to evaluate these failure probabilities in the framework of a levee risk analysis.

4 Discussion

The approach developed comprises a protocol for using expert judgement in the area of river levees to evaluate their structural reliability regarding the failure probabilities of several failure mechanisms. Our discussion focuses on the approach developed and on the results obtained.

4.1 Discussion on the approach developed

The approach proposed answers the issue of evaluating the reliability of levees as raised by French regulations relating to hazard studies and determining the probability of failure on the basis of expert evaluations. The approach permits eliciting, calibrating and aggregating the evaluations of experts in different disciplines. The treatments of the expert elicitations allowed us to obtain an evaluation of failure probability and an evaluation of the uncertainty on this probability by way of an uncertainty interval. The application of this methodological approach led to two significant improvements: i) it combined additional expert opinions from the calibration and information scores to give more weight to the most relevant experts; ii) it used an uncertainty interval to provide quantitative information on the uncertainty on the final failure probability. In addition, the analysis of the uncertainty interval can be completed with the information and calibration scores obtained for the different experts, or, more intuitively, by visualising the quantities of calibration variables falling outside the uncertainty intervals given by the experts.

The major difficulty for implementing the model concerns the construction of the calibration variables. In our work, we chose the sliding failure scenario as the calibration variable of a levee failure. The calibration variables were obtained by analysing probabilistic reliability by taking into account the uncertainties on the input data. However, as we mentioned in introduction within our research study, there was no calibration variable directly involving the failure probability of a levee for internal erosion, overtopping and scouring failure scenarios. Also, the use of calibration variables focused on the sliding mechanism to assign calibration weights to the other mechanisms (internal erosion, overtopping and scouring) could be discussed on the pertinence of the process.

However, we consider that our approach is relevant because the parameters involved in the mechanics of internal erosion, overtopping and scouring are the geotechnical parameters of the soils and of the levee also involved in the sliding mechanism: the density of the materials, cohesion, angle of friction, geometry of the levee, etc. Thus, calibrating the experts on the sliding mechanism amounts to calibrate the experts on the geotechnical parameters involved in the sliding mechanism, and consequently on the other mechanisms of failure. Therefore this approach of calibration of the variables of interest seems well founded and justified.

One perspective for improving the calibration variables of the latter scenarios would be to break these failure scenarios down into elementary functional events [17, 44], for which it would be easier to obtain calibration variables. It would then be possible to question the experts on the calibration variables linked to elementary events and obtain the failure probability for these scenarios by multiplying the probabilities elicited for the elementary variables.

4.2 Discussion on the results

It is considered that there is an overconfidence bias when the intervals estimated by the experts are too narrow [45, 25]. We propose in this study to consider two sub-components of the over-confidence bias, excess of optimism and excess of caution. Situations of bias on excess of optimism and excess of caution can be identified when the true values of the calibration variables fall within the intervals [95%; 100%] and [0%; 5%] in more than 10% of the expert's or the experts' responses. If the value of the calibration variable is in the [0%; 5%] interval, this means that the expert has given a higher failure probability than that of the true value of the calibration variable, in which case the evaluation can be deemed as over-cautious, possibly leading to costly structural reinforcement works. On the contrary, if the true value of the calibration variable falls within the [95%; 100%] interval, this means that the expert was overconfident, since they gave a lower failure probability and uncertainty interval than the real failure probability of the structure. This situation is dangerous because it may lead to a poor evaluation of the structure's failure probability liable to place populations in danger.

The results obtained concerning the elicitations of experts before treatment confirm previous observations of the probabilistic evaluation based on expert opinion and allow distinguishing biases of excess of caution and excess of optimism. We observed that the experts tended to be overcautious regarding the evaluation of failure probabilities and to be overconfident regarding the evaluation of uncertainties. Indeed, the results presented in table 2 show that on average 26% of the expert evaluations were cautious, instead of the 5% that should have been obtained if the distribution of the experts were perfectly calibrated. As mentioned above, this bias of caution points towards safety in comparison to a bias of excess of optimism. The bias of excess of optimism obtained reached 19%, which is high for structures critical for the safety of people and property. It is noteworthy that the literature concerning heuristic biases agrees in saying that they can be reduced by implementing several procedures: in the calibration phase as presented in this article, or in the elicitation phase by making the experts aware of biases and their explicit description [12], or in the framework of calibration exercises prior to elicitation. This provides an interesting perspective of improvement for this study.

The results obtained by applying our calibration approach to calibration variables (nos.1 to 30) showed a marked reduction of variables in the excess of optimism interval [95%; 100 %], thus falling from 19% (mean of individual expert elicitations) to 10% (table 4), which is a considerable improvement that goes towards ameliorating the safety of the structures concerned. There is still room for progress to reach the target value of 5% in the excess of optimism interval [95%; 100%]. The results obtained for the variables of interest (no.31 and 40) are analogous with a substantial reduction of variables in the excess of optimism interval [95%; 100 %] reduced from 15% to 10% (cf. table 5 and table 6). To improve this result, it would be interesting to implement specific mathematical procedures to correct these types of bias using inflation factors [46].

5 *Conclusion*

Studying the structural reliability of levees with quantitative approaches is complex and often impossible due to the lack of statistical data or the lack of limit state equations for certain mechanisms. Thus this article presented an approach for evaluating the failure probabilities of levees based on the calibration of expert judgement. The aim of the procedure was to determine a calibration weight that can be used for other levee risk analyses.

The approach developed is pertinent for using expert judgement in the area of river levees. It can be used to evaluate the reliability of structures in terms of failure probability based on this judgement. The approach first proposes eliciting expert opinions in probabilistic format then identifying a calibration weight for the best expert elicitation in view to aggregating them. At the end of an analysis of the accuracy and informativeness of expert opinions, it assigns a high calibration weight to the most accurate expert elicitations and a low calibration weight to inaccurate and thus less informative elicitations.

The results obtained from the application of the approach developed in the area of levees showed the advantage of calibrating expert judgement. Finally, the approach can be used to correct through aggregation expert evaluations of levee reliability and obtain a more informative aggregated evaluation.

The calibration variables used in the case study correspond to probabilities of sliding failure evaluated by a reliability analysis. The accuracy of the expert judgments calibration can be assessed for the sliding mechanism by comparison with the results of a reliability analysis. However, the accuracy of the calibration cannot be evaluated for the other failure mechanisms that have not been assessed by a reliability analysis in this case study. Some research works are available in the literature concerning the reliability analysis of levees with respect to other failure mechanisms such as internal erosion or piping [5]. As these works have not been considered herein, one perspective should be to decline the methodology of this paper by also using calibration variables corresponding to other levee failure mechanisms (such as the probability of failure due to internal erosion assessed by a reliability analysis). This would allow the results to be compared in order to further explore the validation/verification of calibration of experts across different failure modes.

Besides, the approach has limitations regarding the quantitative correction of the trends of overconfidence followed by expert opinions. Another perspective for this work would be to improve correction with correction protocols including the treatment of expert biases, such as overconfidence which is one of the most common (and potentially severe) problem in expert judgment [45]. In this study, each expert is questioned individually, without allowing discussion or consensus among experts (as in the case of collective elicitation by a panel of experts). Future work could study and compare these approaches (individual and collective elicitation) for application to flood protection levees.

This approach developed for river levees could be adapted and applied in other areas where recourse to expert judgement is the only means of obtaining information usable for carrying out a reliability analysis. The approach will require adjustments specific to each category of civil engineering structure before its implementation.

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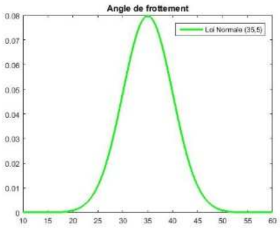
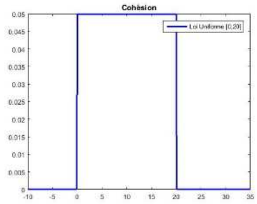
7 Appendix A: Variables of interest used in the case study

The following four figures present an example questionnaire sheet for each failure mode considered as a variable of interest.

<table>
<tr><td>Form 14</td><td colspan="3">Cross-section – P 108</td></tr>
<tr><td colspan="4">Given all the information available, what is the probability P_{fc} that the levee will fail due to sliding of the downstream slope in the case of a flood 500 (annual P_{flood} = 1/500=2 $*$ 10^{-3})?
$P_{Total} = P_{fc} * P_{flood}$</td></tr>
<tr><td>Geometric data
Cf. Figure below</td><td colspan="3">- Height = 4.61 m
- Altitude of crest = 222.69 m NGF
- Width of crest = 2.94 m
- Upstream batter = 1/1 (H/V)
- Downstream batter = 3/2 (H/V)</td></tr>
<tr><td>Hydraulic data</td><td colspan="3">- Flood level Q500 = 220.91 m NGF (1.78 m under the crest)
- Hydraulic gradient level calculated in steady state</td></tr>
<tr><td>Geotechnical data</td><td colspan="3">Body and foundation
- Body : Sandy-gravel
- Permeability $K = 10^{-3}\ m/s$
- Foundation Sandy-gravel, lenses of more or less compact materials.
- Permeability $K = 10^{-6}$ à $10^{-5}\ m/s$
- Unit weight: $\gamma_h = 21\ kN/m^3$</td></tr>
<tr><td rowspan="2">Probability laws of input data</td><td colspan="2">
</td><td>
</td></tr>
<tr><td colspan="2">Angle of friction
(body - foundation)

Probabilistic law : normal law
Mean : 35°
Standard deviation : 5°</td><td>Cohesion
(body - foundation)

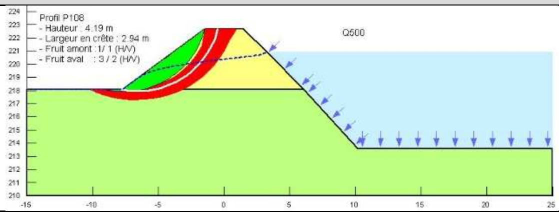
Probabilistic law : loi uniforme
Min = 0 kPa
Max = 20 kPa</td></tr>
<tr><td colspan="4">Flood : Q500</td></tr>
<tr><td>Deterministic analysis</td><td>Calculated SF (with mean values) Downstream slope
2,01</td><td colspan="2">
</td></tr>
<tr><td colspan="4"><u>Demand for response</u></td></tr>
<tr><td rowspan="3">Probabilistic analysis</td><td rowspan="3">Probability (P_{fc}) of sliding failure</td><td>P_{fc}: quantile q5%</td><td></td></tr>
<tr><td>P_{fc}: quantile q95%</td><td></td></tr>
<tr><td>P_{fc}: median value (q50%)</td><td></td></tr>
</table>

Figure 7: example questionnaire sheet for sliding

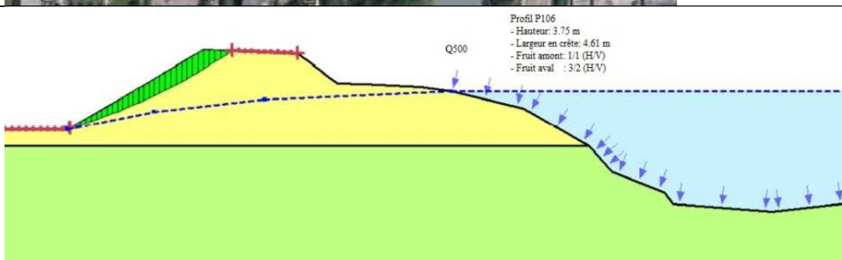
<table>
<tr><td>Form 17</td><td colspan="3">Cross-section P 106</td></tr>
<tr><td colspan="4">Given all the information available, what is the probability P_{fc} that the levee will fail due to internal erosion in the case of a flood 500 (annual P_{flood} = 1/500=2 $*$ 10^{-3})?
$P_{Total} = P_{fc} * P_{flood}$</td></tr>
<tr><td>Aerial view of the section</td><td colspan="3">
</td></tr>
<tr><td>Cross-section</td><td colspan="3">
</td></tr>
<tr><td colspan="4">Evaluation criteria presenting technical data on the entire section</td></tr>
<tr><td>$C_{1,IE}$ Levee body permeability</td><td colspan="3">- Body permeability : $K = 10^{-3}\ m/s$
- Foundation permeability : $K = 10^{-6}\ to\ 10^{-5}\ m/s$
- Geometric Data : Height 3,24 m – Width of crest 4,61 m – Upstream batter 3H/2V – Downstream batter 1H/1V – flood level is 1.78 m under the crest for Q500
- Water leak : undetected
- Sealing of the upstream protection: rip rap of the 50s</td></tr>
<tr><td>$C_{2,IE}$ Levee body resistance to internal erosion</td><td colspan="3">- Materials constituting the levee body: sandy-gravelly, seem locally not very compact.
- Materials constituting the foundation of the levee: sand-gravel, seem to present lenses of more or less compact materials.
- Cohesion $c = 0\ kPa$
- Angle of friction $\emptyset = 35$ °
- Discontinuities of the embankment: not identified by the observation of an in situ geotechnician
- The critical hydraulic gradient of materials is i_{cr} = 0,1
- The overall hydraulic gradient of this profile (P 106) is of i_{glob} = 0.09 (steady state) The duration of floods 2 days</td></tr>
<tr><td>$C_{3,IE}$ Irregularities in levee body</td><td colspan="3">- Trees and roots: significant presence of trees along the entire length of the dyke on the embankment side of the river.
- Terriers: no significant burrows detected.
- Crossing structures: no crossing works identified</td></tr>
<tr><td rowspan="3">Probabilistic analysis</td><td rowspan="3">Probability (P_{fc}) of internal erosion failure</td><td>P_{fc}: quantile q5%</td><td></td></tr>
<tr><td>P_{fc}: quantile q95%</td><td></td></tr>
<tr><td>P_{fc}: median value (q50%)</td><td></td></tr>
</table>

Figure 8: example questionnaire sheet for internal erosion

<table>
<tr><td>Form 22</td><td>Cross-section – P 140</td></tr>
<tr><td colspan="2">Given all the information available, what is the probability P_{fc} that the levee will fail due to overtopping in the case of a flood 500 (annual P_{flood} = 1/500=2 $*$ 10^{-3})?
$P_{Total} = P_{fc} * P_{flood}$</td></tr>
</table>

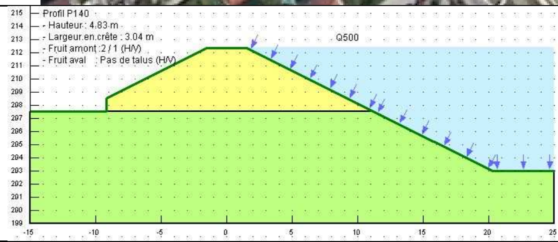
<table>
<tr><td>Aerial view of the section</td><td colspan="3">
</td></tr>
<tr><td>Cross-section</td><td colspan="3">
</td></tr>
<tr><td colspan="4">Evaluation criteria presenting technical data on the entire section</td></tr>
<tr><td>$C_{1,0}$
Levee crest elevation</td><td colspan="3">- The embankment height is 4.83 m
- The water is above the crest of 5 cm for a Q500 flood</td></tr>
<tr><td>$C_{2,0}$
Flow obstruction factors</td><td colspan="3">- No obstruction factors identified</td></tr>
<tr><td>$C_{3,0}$
Presence of low elevation points in crest</td><td colspan="3">- No low elevation point in crest identified</td></tr>
<tr><td>$C_{4,0}$
Operation of spillway(s)</td><td colspan="3">- No spillway of security along section</td></tr>
<tr><td>$C_{5,0}$ & $C_{6,0}$
Resistance to overtopping of crest protection, landslide levee slope protection and waterside levee toe
&
Levee body resistance to overtopping</td><td colspan="3">- The duration of flood 2 days with overtopping for 3 hours
- Height of downstream slope 3.04 m
- Materials constituting the body of the dam: sand-gravel, seem locally not very compact.
- Materials constituting the foundation of the levee: sand-gravelly, seem to present lenses of more or less compact materials. Permeability.
- Cohesion of the body and the foundation $c = 0\ kPa$
- Angle of friction of the levee body Ø=35°
- Angle of friction of the levee foundation Ø=32°
- Etanchéité de la protection aval : terre végétale
- Geometric irregularities: unidentified
- Protection of downstream protection: topsoil
- Geometric irregularities: unidentified</td></tr>
<tr><td rowspan="3">Probabilistic analysis</td><td rowspan="3">Probability (P_{fc}) of overtopping failure</td><td>P_{fc}: quantile q5%</td><td></td></tr>
<tr><td>P_{fc}: quantile q95%</td><td></td></tr>
<tr><td>P_{fc}: median value (q50%)</td><td></td></tr>
</table>

Figure 9: example questionnaire sheet for overtopping

Form 19	Cross-section P 106
Given all the information available, what is the probability P_{fc} that the levee will fail due to **Scouring** in the case of a flood 500 (annual P_{flood} = 1/500=2 $*$ 10^{-3})? $P_{Total} = P_{fc} * P_{flood}$	

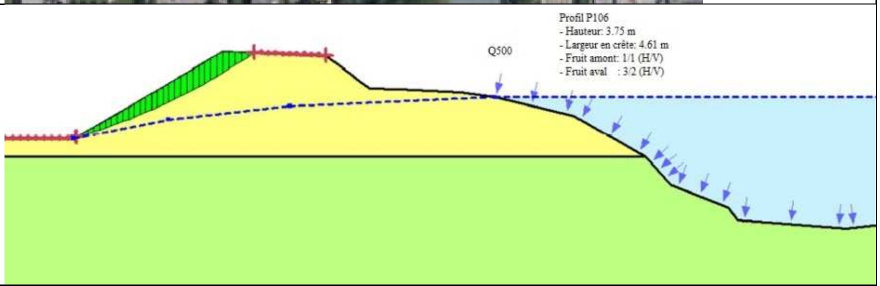
<table>
<tr><td>Aerial view of the section</td><td colspan="3">
</td></tr>
<tr><td>Cross-section</td><td colspan="3">
</td></tr>
<tr><td colspan="4">Evaluation criteria presenting technical data on the entire section</td></tr>
<tr><td>$C_{1,sc}$ & $C_{2,sc}$
Levee body protection resistance to external erosion on river side
&
Levee body resistance to external erosion</td><td colspan="3">- Sealing of the upstream protection: pearl of the 50s
- Height 3,75 m
- Flood level Q500 1.25 m under the crest
- The duration of floods 2 days
- - The average flow velocity of this profile (P 106) for a Q500 flood is 3.42 m / s
- En générale, les vitesses d'écoulement dans le lit du Drac sont fortes, supérieures à 3,5 m/s pour le tronçon 1
- Body permeability : $K = 10^{-3}\ m/s$
- Materials constituting the levee body: sandy-gravelly, seem locally not very compact.
- Cohesion $c = 0\ kPa$
- Angle of friction $\emptyset = 35\ °$
-</td></tr>
<tr><td>$C_{3,sc}$ & $C_{4,sc}$
Foundation protection resistance to external erosion
&
Foundation resistance to external erosion</td><td colspan="3">- Sealing of the upstream protection: pearl of the 50s
- Permeability of the foundation : $K = 10^{-6}$ à $10^{-5}\ m/s$
- Materials constituting the foundation of the levee: sandy-gravelly, seem to present lenses of more or less compact materials.
- Cohesion $c = 0\ kPa$
- Angle of friction $\emptyset = 35\ °$</td></tr>
<tr><td colspan="4"><u>Demand for response</u></td></tr>
<tr><td rowspan="3">Probabilistic analysis</td><td rowspan="3">Probability (P_{fc}) of scouring failure</td><td>P_{fc}: quantile q5%</td><td></td></tr>
<tr><td>P_{fc}: quantile q95%</td><td></td></tr>
<tr><td>P_{fc}: median value (q50%)</td><td></td></tr>
</table>

Figure 10: example questionnaire sheet for scouring