Water Quality Indices: Challenges and Application Limits in the Literature

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Abstract: Since Horton in 1965, many authors have sought to aggregate different variables characterizing the state of water into a single value called Water Quality Index (*WQI*). This index is intended to facilitate the operational management of water resources and their allocation for different uses. Detailed and operational description of the main *WQI* calculations are here reviewed. The review contains: (1) an historical analysis of the evolution of *WQI* calculation methods by looking both at the choice of variables, the methods of weighting and aggregating these variables into a final single value; (2) an illustration of the contradictions observed in the final result when, on the same database, the *WQI* is calculated by different methods; (3) the significant progress possible via fuzzy logic to define a *WQI* adapted to specific water use.

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1. Introduction

The increasing population, the expansion of economic activities, and urban sprawl are leading to increased demand for water. The overuse of surface water and groundwater is jeopardizing numerous resources because of the reduction of the available quantities and the deterioration of their quality [1,2].

The deteriorated quality of surface water is becoming a serious issue in many countries [3] and water quality monitoring is among the highest priorities in resources protection policy [4]. Thus, recently developing countries have intensified efforts to evaluate the quality of rivers [5].

Due to spatial and temporal variations in water quality, which often are difficult to interpret, monitoring of the composition of waters is necessary [6].

The assessment of water quality is a prerequisite for the implementation of water protection policies and optimal allocation of different water sources according to their uses. Indeed, surface water has often been evaluated using norms [7]. However, sources of pollution are diverse: urban, industrial and agricultural pollution (diffuse or point source).

The frequency of monitoring and assessment of water quality helps to develop management strategies to control surface water pollution [8] facing to increasing urbanization and anthropogenic pressure on water resources.

As no unique variable can sufficiently describe water quality, it has been evaluated by measuring a series of physico-chemical intensive variables (e.g., the cation or anion concentrations, etc.).

Principal component analysis has been applied to assess the water quality in some studies [6,9], with the aim to identify the factor deteriorating water quality. Some other studies used the weighted score of each variable to propose a water quality index (*WQI*) [10–15]. The main idea in developing a *WQI* consists in encompassing a wide range of variables into a single numeric value.

The objective of the *WQI* is to classify the waters relative to biological, chemical and physical characteristics defining their possible uses and managing their allocations [16–18]. To this end, the analytical variables must be weighted and aggregated.

*WQI*s can be considered as models of water quality, i.e., a simplified representation of a complex reality, where variables are selected and methods for weighing and aggregating the variables are defined.

Alves et al. 2014 [19], in their statistical analysis review, found 554 articles dealing with the use of *WQI*'s between 1974 and 2011, of which only 38% are used in India and 9.5% in China.

Abbasi and Abbasi 2012 [20] published a book in which they reviewed water quality indices, and almost all indices existing in the literature were detailed.

In this paper, we propose a critical analysis of the *WQI* concept. We first present the historical evolution of the *WQI* concept, then discuss the limits of its application and stress the contradictions between results obtained. Eventually, we propose new perspectives for designing *WQI* adapted to specific water uses.

2. Historical Evolution of *WQI* (Water Quality Index) Concept

In all approaches of *WQI* calculation, four common steps are used [20]: (i) selection of variables, (ii) transformation, following a common scale, of these variables that have initially different dimensions, (iii) creation of subindices by assignment of a weighing factor to each transformed variable, and (iv) computation of a final index score using the aggregation of subindices.

*WQI*s can be classified according to these criteria (Table 1). The detailed principles of calculations for some *WQI* cited in this paper are summarized in the appendix.

Table 1. Structure, aggregation formula and number of variables in Water Quality Index (*WQI*) and some references using them.

WQI	No. of Variables	Structure	Aggregation	Example of Studies Using *WQI*	
				Ref.	Application Area
Horton	10	Formulas	Weighted geometrical average	[21] [22] [23] [24]	Pune, Maharashtra, India Suquia River, Argentina Río Lerma basin, Mexico Balikhlou River, Iran
NSFWQI *	9	Diagrams	Weighted geometrical average	[25] [26] [27] [28] [29] [30]	Cazenovia creek, USA Dakhla Oasis, Egypt Dourou River, Portugal Brazil Owo River, Nigeria Aydughmush Dam, Iran
Bhargava	According to the use	Formulas	Weighted product	[31]	Subernarekha, India
Dinius	12	Equations	Weighted geometrical average	[32,33]	
CCMEWQI **	Up to 47	Formulas	Harmonic Square Sum	[34] [35] [36] [7]	Atlantic region, Canada Mackenzie River basin, Canada Algeria Canada
Oregon	8	Equations	Unweighed harmonic Square Mean	[37]	
New *WQI* Said & al.	5	Formula	Logarithmic	[38]	

*: National Sanitation Foundation; ** Canadian Council of Ministers of the Environment.

Horton 1965 [10] initially proposed the *WQI* and since then many different methods of *WQI*'s calculation can be found in the literature. Even much earlier, previous studies in mid 1800s used the concept of categorizing waters according to their pollution degrees [39].

Horton fixed the steps to be followed in the development of an index: (i) Selection of quality characteristics on which the index is to be based; (ii) establishment of a rating scale of each characteristic and (iii) weighting of the several characteristics (see Appendix A.1 for calculation details).

Horton selected 10 variables, the most frequently measured to establish his index, which are: sewage treatment, i.e., the percentage of population upstream that is connected to a sanitation facility, dissolved oxygen (*DO*), pH, fecal coliforms (*FC*) count, specific conductance (*EC*), carbon chloroform extract, alkalinity, chloride, temperature and "obvious pollution". As indicated by Horton, "if additional refinements are desired, secondary indicators may be added".

Horton selected rating scales for each variable so that the subindex ranges from 0 to 100, where the highest quality is rated 100.

The weighting parameters range from 1 to 4. The final index score is composed of the weighted sum of the sub-indices, divided by the sum of weights and is multiplied by two coefficients m_1 and m_2, which depend on the temperature and the pollution level of water.

Horton's index is intended as a means for comparative evaluation of water quality conditions and pollution abatement programs. Thus, an index is basically a comparative tool to evaluate the efforts made to ameliorate the water quality and not really a tool to evaluate water quality absolutely.

Toxicity is explicitly excluded by Horton, on the basis that "under no circumstances should streams contain substances that are injurious to humans, animals and aquatic life. Water containing such substances, therefore, is considered not eligible for index rating."

Later, Brown et al. 1970 [11] established a new *WQI* and selected the nine following variables: *DO*, *FC*, pH, Biochemical Oxygen Demand (*BOD*), temperature, total phosphate and nitrate concentrations, turbidity, and total solid content. It is based on the professional opinion of a panel of 142 experts in water quality, who defined the weighting, *Q*, of each variable and established five classes for water quality: red (very poor), orange (poor), yellow (average), green (good) and blue (excellent). The first index proposed by Brown et al. took the arithmetic form (see Appendix A.2 for calculation details), later, Brown et al. 1973 [13] considered that a geometric aggregation (see Appendix B.1 for calculation details) was better than arithmetic aggregation, being more sensitive when a single variable exceeds the norm [40]. These works were supported by the National Sanitation Foundation, hence the appellation of their index NSFWQI [11,13].

Deininger and Landwehr 1971 [41] proposed their own *WQI* that is conceptually similar to Brown et al. [42]'s index, but it contains 12 variables for surface waters and 14 variables for groundwater. Variables used are: *DO*, *FC*, pH, *BOD*, the concentrations of nitrate, phosphate, phenol, dissolved solid, temperature, turbidity, colour and hardness for surface waters, and the same variables plus iron and fluoride concentrations for groundwater.

In Europe, Prati et al. 1971 [12] proposed another index based upon water quality standards (see Appendix A.3 for calculation details). Their idea consists of transforming concentrations of pollutants into levels of pollution. Variables used in this *WQI* are: pH, *BOD*, Chemical Oxygen Demand (*COD*), *DO*, concentrations of permanganate, ammonium, nitrate, chloride, iron, manganese, Alkyl Benzene sulphonates, Carbon Chloroform Extract and suspended solids (SS).

Nemerow 1971 [43] proposed three specific-use water quality indices, which, when added together, give a general water quality index. In their approach, they combined the average value and the maximum value of each variable. The method reduces the impact of one variable exceeding largely the permissible limits.

Dinius 1972 [32] proposed another *WQI*, based upon Horton's index, in order to calculate the costs of remediation of water pollution in Alabama (USA). This *WQI* defines a decreasing scale from 100 to 0, where the value 100 is assigned to the "perfect" quality water (see Appendix A.4 for calculation details).

Later, Dinius 1987 [33] developed another *WQI* using the method of sub-indices introduced by Dalkey 1967 [44] with some modifications [45] (see Appendix B.3 for calculation details).

In India, Bhargava 1983 [14]'s studies introduced a new *WQI* where the combination of variables highlights more specifically the pollution load. He defined later the variables to be introduced and specified the *WQI*'s formula according to the water use [46] (see Appendix B.2 for calculation details).

Tiwari and Mishra 1985 [47] proposed another *WQI* based on the same principles of those of Horton and Brown et al. but they modified the weighting method by introducing the normative values of the major variables of the water. Logarithm and antilogarithm have been used in their aggregation to keep harmonic the magnitude of sub-indices (see Appendix D.1 for calculation details).

House and Newsome 1989 [48] consider that using a quality index (*WQI*) allows for quantifying the "good" and "bad" water by reducing the number of data on a range of biological and physico-chemical variables and representing them in a single index simple, reproducible and objective.

In Canada, the Canadian Council of Ministers of the Environment (CCME) introduced a *WQI* developed by the "Water Quality Guidelines Task Group" in the mid 1990s, the idea was inspired from British Columbia Water Quality Index (BCWQI) [49]. The index proposed is non-linear. The concept of Canadian indices is based on the frequency of sampling and measurement, the frequency of values outside the required objectives and the deviation from recommended value of each variable (see Appendix C.1 for calculation details).

In 1996, the Lower Great Miami Watershed Enhancement Program (WEP) in Dayton, Ohio developed a water quality index WEPWQI, which encompasses the chemical, physical and biological variables and an index of the river, which includes water quality variables, flow measurements and water clarity (turbidity). Both indices are expressed on a scale: excellent, good, fair and poor. Pesticide and Polycyclic Aromatic Hydrocarbon contamination are included in the variables selected to calculate the *WQI*, which makes this index differ from the NSFWQI [38].

In the first decade of the 21st century, new indices appeared, simplifying further the existing formulas and better defining the field of the application of the index. For example, Overall Index of Pollution assesses a number of water quality variables based on the measurement and subsequent classification of each of them [50]; the Index of River Water Quality categorizes the variables following three aspects: "organic", "particulates" and "microorganisms", calculates a geometric average for each category before aggregation in a single index [17]; the Scatterscore index evaluates the water quality around mining sites in USA and identifies the changes of water quality with time and space [51].

Said et al. 2004 [38] developed a new *WQI*, using the logarithmic aggregation. Their idea was to reduce the number of variables to be introduced in the index and to change the aggregation method, while keeping its accuracy. Next, they used a random database to test their index and showed it gives similar results to NSFWQI and WEPWQI (see Appendix D.2 for calculation details).

The most recent method for calculating a *WQI* is based upon fuzzy logic and was introduced by Icaga 2007 [52], inspired from Silvert 2000's [53] work on environment assessment (see Appendix E for calculation details).

Fuzzy logic is a form of many-valued logic that expresses the partial truth between being false or true. It takes any real number between 0 and 1, conversely of the boolean logic where the truth values of variables may only be the integer values 0 or 1 [54].

Thus, subjective and non quantitative data can be used such as odours, which can be left out of the equations because they cannot be adequately measured. The concept of acceptability itself is considered as fuzzy [53].

The assessment by fuzzy logic generally is based on a numerical scale representing water quality. Thus, since the 1990s methods of aggregating water quality variables have been studied and used, particularly in the assessment of environmental quality of waters [55,56].

3. Evidence of Disagreement Between Water Quality Indices Results in Published Works

The initial objective of *WQI* was to classify waters into classes by aggregating and weighting different data. However, as shown above, the number of indices proposed in the literature increased so far that the present picture is not clear at all. Twenty indices were reviewed by Steinhart et al. 1982 [57], later Van Helmond and Breukel 1997 [58] demonstrated that at least 30 water quality indices are of common use around the world [45,59]. This is first apparent when using different *WQI*s to check a given database. Disagreements appear in three cases: (i) the same *WQI* is used, but the limits of classes differ; (ii) different *WQI*s are used, on the basis of the same variables, and lead to different classifications; (iii) different *WQI*s are used, on the basis of different types or numbers of variables.

4. Disagreement Using the Same *WQI*

In India, Sharma et al. [60] used two scales for the same index to evaluate surface waters quality in Ganges river at various locations in Allahabad (Table 2): the scale used by Yadav et al. [61] and the scale proposed by Ramakrishnaiah et al. [62], both to assess groundwater.

Table 2. Different Water Quality Scales based on the same *WQI* used by Sharma et al. [60].

Water Quality Class	WQI	
	Yadav et al.	**Ramakrishnaiah et al.**
Excellent	0–25	<50
Good	26–50	50–100
Poor	51–75	100–200
Very poor	76–100	200–300
Unsuitable	>100	>300

The results of Sharma et al. show that the waters of Ganges are of "good" water quality when Ramakrishnaiah et al.'s classification is used (Table 3). However, by reclassifying these waters according to Yadav et al.'s scale, we find that the quality index shifts towards the "poor" quality class, which fits better with the expert's judgement of the study case (Table 3).

Table 3. Water Quality Indices and Water Quality at different locations [60].

Locations	WQI	Water Quality	
		Yadav et al.	**Ramakrishnaiah et al.**
Ram Chaura Ghat	90.98	Very Poor	Good
Neeva	157.69	Unsuitable	Poor
Rasoolabad	95.43	Very Poor	Good
Daraganj	94.43	Very Poor	Good
Prior to Sangam	86.20	Very Poor	Good
Sangam	96.61	Very Poor	Good
Beyond Sangam	93.29	Very Poor	Good
Yamuna	115.16	Unsuitable	Poor

4.1. Different WQIs Lead to Contradiction

Fernández et al. 2004 [59] collected data from Columbian Health Ministry, and applied three methods of calculation of *WQI*, using Columbian recommended values and permissible limits of drinking water and wastewater (Table 4).

Table 4. Comparison between water quality indices using data from Colombia [59].

Index		NSFWQI	OWQI	Dinius *WQI*
Drinking water	Value	85.17	24.98	76.78
	Classification	Good	Very Poor	Polluted
Wastewater	Value	33.16	10.67	41.30
	Classification	Bad	Very Poor	Polluted

Despite the use of the same variables, the classification from each index differs, especially in the evaluation for drinking water. However, in the evaluation of wastewater, the values of indices and their classifications are closer. The difference in classification is interpreted by the fact that the indices and the limits of classes were designed for the USA and may not be applicable to Columbia due to different natural and anthropogenic activities [59].

Akkoyunlu and Akiner 2012 [63] used CCMEWQI, OWQI (Oregon WQI, see Appendix C.2 for calculation details) and NSFWQI to evaluate water quality in eight rivers in Turkey; they show a significant difference between the classes of the water quality in the same site but with different indices (Table 5).

Table 5. Comparison of different Water Quality Indices for the rivers of Sapanca Lake Basin [63].

River	Median *WQI* Score and Quality					
	CCMEWQI		OWQI		NSFWQI	
Arifiye	34	Poor	13	Very Poor	52	Medium
Balikhane	45	Marginal	24	Very Poor	70	Medium
Istanbul	49	Marginal	26	Very Poor	74	Medium
Karacay	73	Fair	60	Poor	78	Good
Keci	40	Poor	19	Very Poor	69	Medium
Kurucay	58	Marginal	60	Poor	77	Good
Mahmudiye	70	Fair	60	Poor	77	Good
Sarp	35	Poor	13	Very Poor	57	Medium

This work, referring to the water quality classes defined in the Water Pollution Control Regulation of Turkey for inland surface waters, conclude that CCMEWQI and OWQI cannot be used for evaluation of rivers of Sapanca lake Basin in Turkey. In this particular case, the authors seek to measure the impact of water quality on eutrophication by classifying their waters according to this risk. The CCMEWQI and OWQI indices, having been constructed for a broader pollution assessment, are therefore also sensitive to variables such as *FC*, a variable which, in principle, does not affect eutrophication but which contributes to degrading the position of water in the water quality classification. Thus, if *FC* values are strongly high (range 10,000–1,000,000 [63]) and phosphate values are larger than acceptable (up to 1), CCMEWQI and OWQI classify the waters directly in class IV, i.e., "poor quality", whereas NSFWQI classifies in the "medium" class.

Finally, they selected a subset of variables, linked *a priori* with the eutrophication risk, including o-phosphate and five others but excluding *FC*, to define WQI_{eut}, aimed to specifically assess the eutrophication risk.

4.2. Modification of the Aggregation Method and of the Nature of Variables

Said et al. [38] proposed a new *WQI*, called NewWQI, requiring fewer variables than NSFWQI and WEPWQI and compared these three indices (for the detail of calculation see Appendix D.2). Although the limits of classes of water are different, they observe that a given water is found in the same quality class. For example, a water with an NSFWQI of 77.9 or a WEPWQI of 54, has a New *WQI* of 2.22 and in all three cases is classified as "good quality water".

Lumb et al. [40] compared the US indices with the Canadian index (i.e., CWQI). The selected American indices are: the two indexes AWQI [11], MWQI [13] and Oregon *WQI* [37] (i.e., OWQI). They computed them on two sets of data based on different numbers of variables, respectively 7 and 10, and raw data from a sampling of 144 stations per year.

The results obtained from their comparison (Table 6) show that the agreement between American *WQI*s and CWQI taken as reference, e.g., OWQI is in agreement with CWQI for 77% of stations sampled and 65% of years studied.

Table 6. Measures for comparison between Canadian (CWQI) and US-based *WQI*s [40].

Measure for Comparison	CWQI-OWQI		CWQI-AWQI		CWQI-MWQI	
	7 Variables	10 Variables	7 Variables	10 Variables	7 Variables	10 Variables
Stations matched (%)	16	77	7	0	0	3
Station year matched (%)	15	65	7	0	0	4

It appears clearly that only the comparison between CWQI and OWQI with 10 variables is acceptable. However, between CWQI and the two indices AWQI [11] and MWQI [13], the percentage of similarity is only between 0 and 7%. Concerning the number of variables introduced in calculations, the higher the number of variables in OWQI and MWQI, the more similar the results are to CWQI. However, the result is the reverse for the AWQI. They explained that the differences of classification of waters depended on the four indices by the mode of aggregation of variables and parameters used in calculation of each index. In fact, OWQI and CWQI are indices obtained by non-linear aggregation, while AWQI and MWQI are obtained by linear aggregation.

4.3. Test of Four WQIs Using the Same Dataset

In order to highlight the spatio-temporal contradiction of the water classification using different WQI, four Water Quality Indices were tested. For this context, the logarithmic and arithmetic index proposed by Tiwari and Mishra (TMWQI) [47] (for calculation details, see Appendix D.1), the arithmetic index proposed by Ramakrishnaiah et al. (RWQI) [62] (for calculation details, see Appendix A.5), the harmonic square average index proposed by CCME (CCMEWQI) [64] (for calculation details, see Appendix C.1) and the fuzzy logic WQI proposed by icaga (FWQI) [52] (for calculation details, see Appendix E) were compared using the same dataset.

Data were collected from two different locations in Mejerda watershed in Tunisia and in three different periods: summer 2015, winter 2015 and spring 2016. The 13 variables measured are: pH, EC, TDS, DO, COD, calcium, magnesium, sodium, potassium, bicarbonate, sulphate, chloride and nitrate (Table 7).

The results described in Table 8, confirm the contradiction detected with the application of different WQIs. In the majority of cases, the indices qualify the samples in adjacent water quality classes. However, it is possible to find that the indices characterize them conversely as it was obtained through the analysis of spring sample of station 2.

In fact, these results can be explained by the way of the aggregation of the indices.

Among the four WQIs, only the RWQI depends on the expert opinion. For this reason, it was necessary to check the difference of classification between RWQI and other methods based on standards.

In addition, RWQI and FWQI methods may conduct to stable water quality classification comparing to the other approaches. For the case of CCMEWQI and FWQI, all the calculated subindices are normalized into a common scale, which reduces the effect of the ranges of the measurable variables (i.e., pH ranges from 0 to 14 in any case; however, the CE ranges from 0 to 1559 μS/cm in our case). The depth of analysis of each *WQI* helped to detect that the FWQI approach classifies the variables in the first steps of the calculation but the other methods use all variables as if they belong to the same class. This method of calculation may lead to more accurate results with the application of FWQI.

Table 7. Analytical results of the two samples.

Variable	Station					
	Station 1			Station 2		
	Summer	Winter	Spring	Summer	Winter	Spring
pH (pH-unit)	7.79	7.53	7.90	7.47	7.29	8.30
CE (µS/cm)	1559.00	1025.00	1170.00	1552.00	1110.00	510.00
TDS (mg/L)	1527.00	1000.00	680.00	1519.00	1065.00	289.00
DO (mg/L)	4.66	4.25	2.70	0.70	3.20	0.70
Calcium (mg/L)	134.30	69.80	125.15	154.05	192.65	148.80
Magnesium (mg/L)	36.10	54.50	22.80	25.30	33.80	19.90
Sodium (mg/L)	118.38	379.22	601.20	151.04	136.45	150.30
Potassium (mg/L)	1.83	8.22	9.15	2.87	3.04	2.13
Bicarbonates (mg/L)	169.58	256.81	316.59	507.22	549.06	597.80
Sulphates (mg/L)	158.57	158.57	170.78	83.69	83.69	97.15
Chloride (mg/L)	234.80	469.60	512.00	237.16	337.16	220.15
Nitrates (mg/L)	12.35	3.91	5.40	0.22	3.19	24.00
COD (mgO$_2$/L)	19.70	30.00	16.00	35.40	120.00	15.75

Table 8. WQI values and classifications.

WQI	Season	Station			
		1		2	
		Value	Classification	Value	Classification
TMWQI	Summer	59.68	moderate	56.52	moderate
	Winter	75.41	poor	59.02	moderate
	Spring	85.36	poor	75.47	poor
RWQI	Summer	109.01	poor	121.23	poor
	Winter	109.72	poor	128.65	poor
	Spring	114.26	poor	89.17	good
CCMEWQI	Summer	66.56	Fair	53.96	marginal
	Winter	47.90	marginal	45.18	marginal
	Spring	52.98	marginal	70.25	Fair
FWQI	Summer	52.27	moderate	53.24	moderate
	Winter	56.35	moderate	86.32	poor
	Spring	53.74	moderate	53.71	moderate

5. Discussion

The main idea of *WQI* is to transform a number of selected variables, which are quantitative and intensive, into a single variable which is qualitative, ordinal and intensive. However, the considered variables have different units and ranges of values. Thus, in the aggregation process, all variables must be turned into subindices expressed on a single scale. In the principle of *WQI* calculation, expert opinions are required for the selection of the variables and in the choice of their weights in the aggregation. A weighting coefficient is assigned to each variable based on their potential impacts on water quality. The Figure 1 describes the steps followed generally in the elaboration of a *WQI*.

In the literature, the development of water quality indices has undergone two types of evolutions. The first is the consideration of normative progress (e.g., WHO guidelines and standards) on the quality of water that has affected the weighting methods. The second is related to the progression of digital processing that has impacted aggregation methods.
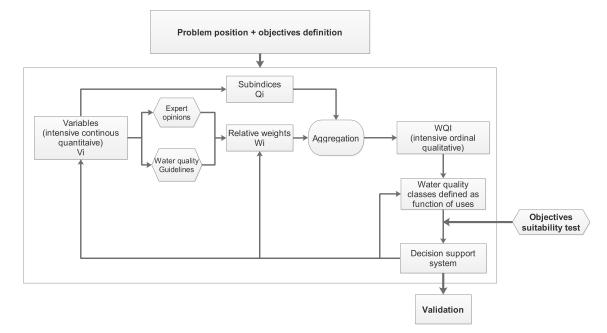
Figure 1. Common steps on *WQI* conception.

5.1. Choices of Variables

The quality of the water varies according to the spatio-temporal dimensions of its course during its cycle and according to its allocations and uses. The latter determine the choice of water quality variables, the analytical method and the sampling period. The acquisition of water quality data must follow harmonized and well defined protocols both in field sampling methods, monitoring of non-conservative variables such as temperature, pH, oxido-reduction potential... and in the laboratory analyses (*COD*, *BOD*5, anions, cations...). Water quality is not defined by a single specific variable, whereas the *WQI*, by combining many variables, expresses this quality in a single relevant value, in relationship with the water management objectives, on a scale of value. The Delphi method [44] can be used in the selection of these variables, but it remains dependent on the expert opinion. The consequence is that the final *WQI* can be highly variable depending on the panels of experts solicited.

However, the introduction of statistics and multivariate analysis over the last decade now permits a selection of variables while erasing this "opinion" effect, and this increases the robustness of the final result. Bhargava [65], like Icaga [52], proposes that the selection of variables be made according to the water management objectives.

Since the first water quality index [10], the number and nature of the variables introduced into the different calculation proposals range from 5 [38] to 78 [66]. This raises from the outset difficulties in the acquisition of data and of analytical loads of laboratories when regular monitoring of water quality is carried out. Thus, the variables must be carefully selected according to the location of the waters studied and the sampling periodicity [39]. In addition, the number of variables introduced in *WQI* affects the final classification of water qualities [40]. One must ask: if few variables are considered in the calculation of *WQI* are they the most relevant to define water quality? Furthermore, if other pollutants, limiting this quality, are present in the water, such as heavy metals or xenobiotics, is the conventionally calculated *WQI* still relevant? In principle, this is inconsistent with the original concept of *WQI* by Horton: "Water containing such substances is considered not eligible for index rating" (see above).

If on the contrary, a large number of variables is considered, it is likely that they are not independent, so that fewer variables can be used to detect any deterioration of water quality.

5.2. The Weighting Methods

Two methods of weighting are generally used. The first is based on the experts' opinions, either to choose the variables or to assign weights to the variables (e.g., [10,11,13], etc.). The second is based upon water quality guidelines, and the weights are defined as functions of the standards proposed in these guidelines (e.g., [37,47], etc.). In both of the methods, the weighting highly affects the final index obtained and can change significantly when changing the experts panel or when changing the guidelines used.

5.3. The Aggregation Methods

Different ways to aggregate variables have been used. Mainly, they are: the weighted arithmetic average, the weighted geometric average, weighted and unweighted harmonic square average, and more recently, aggregation by using logarithmic functions or founded on fuzzy logic. The first *WQI*s were arithmetic averages, then they took a geometric form. The disadvantage of the geometric average with respect to the arithmetic average is that if the value of one of the variables is close to zero, whatever the weighting of the variables, the *WQI* will tend to 0. However, a *WQI* calculated with a geometric average is less affected by the extreme values of variables than a *WQI* calculated by an arithmetic average [67]. Then, in the beginning of the 1990s, the unweighted harmonic square average method was considered an improvement over the weighted arithmetic average and the weighted geometric average. In particular, under these conditions the influence of the values of the variables had a direct impact on the *WQI* value [68]. Thus, strong anomalies that would be measured on certain variables during monitoring the water quality, will be observable in the value of the final *WQI*. Later, at the end of the 20th century, with the generalisation of computer use, the aggregations took other forms, such as logarithmic functions. However, even in recent literature most of the proposed *WQI*s remain based on arithmetic or geometric aggregations.

5.4. Fuzzy Logic

The concept of "acceptability" is seen by some people as fuzzy [53]. It consists of defining sets of acceptable conditions in which quantitative variables are transformed through fuzzy membership functions representing a degree of acceptability (Figure 2). The membership functions allow for defining acceptable upper limit (mf1, e.g., nitrate, FC), lower limit (mf4, e.g., DO), inside interval (mf2 and mf3, e.g., temperature and pH) or outside interval (mf5, e.g., temperature unfavourable to pathogens). Another significant difference between the Icaga index and the others is that it allows for the consideration of subjective data, such as smell or color.

For each variable, the membership function takes a value between 0 and 1 and this value is affected to the corresponding subindex. Then these subindices must be combined. The paper by Icaga is however not explicit on this point: this author uses OR logic operator in a series of tests. This operator ensures the maximum membership to a water quality class. Implicitly, it seems that for assessing the water quality, a hierarchy of variables is considered, and weighting factors, not explicitly given, are used. However, the logic can propose also a minimum membership to a water quality class when using the AND operator [69].

This index is thus innovative because it is defined by critical variables allowed by fuzzy logic. Indeed, once one of them exceeds the allowable limits, the water quality is automatically downgraded.

Bhargava's index uses implicitly the fuzzy logic, since he defined a constant calculated in function of permissible limits. The subindex used in the aggregation decreases exponentially, which leads to a downgrade of the water quality into "unsuitable" class. This makes it possible to generate numerous intermediate classes of quality between the classes "good" and "bad", which cannot be defined by any isolated variable. This increases the sensitivity of the method and gives a rigorous framework to evaluation.
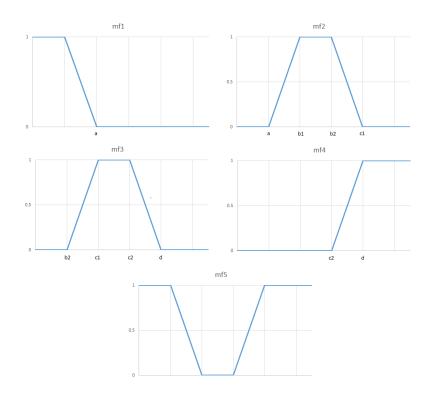
Figure 2. Membership functions in the fuzzy logic.

In the indices founded on fuzzy logic, experts intervene only at two levels. In the first step, when classes and membership variables are defined, and in the last step, in the defuzzyfication to obtain the output index. In his paper, Icaga did not define the output function, but we propose a simpler way to combine the subindices.

According to the water management objective defined *a priori*, i.e., drinking water, irrigation etc., we can define weighting factors and consider some quality variables. Then, we can combine the subindices in the following form 1:

$$Output = \sum_i a_i mf_i \tag{1}$$

where:

a_i, are coefficients of degree of impact of each membership function mf_i on the water quality with the constraint ($\sum_i a_i = 1$).

6. Conclusions

Since the development of the first WQI by Horton (1965), several complex approaches were used to establish more accurate WQI. Significant changes were noticed with a transformation of continuous quantitative intensive variables into an ordinal qualitative intensive value through either a weighted or unweighed aggregation. Our review of several WQI developed by experts and scientists, shows that the context is rarely considered, the water allocation objectives, and the way in which it affects weighting factors are poorly determined. Therefore, the weighting should be decided according to the use of water. Accordingly, a Universal WQI cannot be defined.

In addition, the use of fuzzy logic seems the clearest innovation in the last decade and it is appropriate for an accurate WQI. This approach allows to evaluate the impact of each variable in the final index of the quality of the water. However, it remains to establish weighting factors for specific water use. These weighting factors must be locally determined.

Appendix A. Weighted Arithmetic Average

Appendix A.1. Horton's Index

The *WQI* calculated by the method described by Horton, is given by the following formula:

$$WQI = \left(\frac{\Sigma S_n \times w_n}{\Sigma w_n}\right) \times m_1 \times m_2, \tag{A1}$$

where:

S_n is the subindex assigned to the nth variable;

w_n is the relative weight of the nth variable;

m_1 is a temperature correction factor (0.5 if the temperature is below 34 °C, otherwise 1);

m_2 is a correction pollution factor (0.5 or 1).

Rating scales (0 to 100 for each variable) were assigned and each variable was then weighed from 1 to 4 according to its relative impact. The more significant variable was given the higher weight [39].

Appendix A.2. First National Sanitation Foundation Water Quality Index (NSFWQI) in 1971

One of the challenges in the concept of Horton was selecting the right variables to include in the *WQI* [39].

Brown et al. developed another version of the *WQI* with the support of the National Sanitation Foundation hence the name of this index NSFWQI.

Brown's method is based on the method of Dalkey (Rand Corporation's Delphi technique) and was done by selecting variables rigorously, the development of a common scale and assigning weights to the variables.

A panel of 142 experts was asked to compare overall water quality using a scale of 1 (highest) to 5 (lowest) for the selected variables.

Variables to be involved in the formula are : *DO*, Fecal Coliforms, pH, *BOD*, Nitrates, Phosphates, Temperature, Turbidity and Dissolved Solids. Each variable got a rating calculated using arithmetic mean, then rates have been converted into temporary weights. Next, each temporary weight was then divided by the sum of all the temporary weights to arrive at the final weight.

Table A1 expresses the weight of each variable introduced in the the NSFWQI:

The NSFWQI next is given by the following formula:

$$WQI = \sum_{i=1}^{n} q_i W_i \tag{A2}$$

where

q_i is The quality class for the nth variable;

W_i is the relative weight for the nth variable ($\sum W_i = 1$).

The index works well if all variables are independent of each other. Special procedures have been proposed to "pesticides" and "toxic compounds", if any of them exceeds its assigned upper limit (e.g., 0.1 mg/L for pesticides), the *WQI* is automatically recorded as 0.

Brown et al. considered the possibility of using a color spectrum to illustrate the scale of water quality in each region, with the dark red denoting a very poor water quality (*WQI* = 0–10), a strip

narrow yellow representing average quality (50 *WQI*) and dark blue (*WQI* = 90–100) representing excellent water quality.

Table A1. Weights for variables included in National Sanitation Foundation Water Quality Index (NSFWQI).

Variable	Weight
DO	0.17
Faecal Coliforms	0.16
pH	0.11
BOD	0.11
Nitrates	0.10
Phosphates	0.10
Temperature	0.10
Turbidity	0.08
Dissolved Solids	0.07
Total	**1.00**

Appendix A.3. Prati's Pollution Index

As it is explained in the history of *WQI*, the idea of Prati et al. was transforming concentrations of pollutants into levels of pollution.

In the first step, water quality was classified vis a vis all the variables based on water quality standards, the classification is explained in Table A2, next, in the second step, one pollutant was taken as reference and its actual value was considered directly as reference index. The third step consists of transforming these values into sub-indices using mathematical equations. This transformation took into account the polluting capacity of the variables related to a selected reference variable [20].

Next, the index was computed as the arithmetic mean of the 13 subindices for groundwater with the following formula:

$$I = \frac{1}{13} \sum_{i=1}^{13} I_i \tag{A3}$$

For the surface water, only the first 11 variables are used so the sum is from 1 to 11.

The index ranges from 0 to 14, and was applied by Prati et al. on surface water in Italy.

Table A2. Classification of water quality and subindex functions for the Development of Prati's Index.

Variable	Classification of Water Quality					Subindex
	Excellent	Acceptable	Slightly Polluted	Polluted	Heavily Polluted	
pH (units)	6.5–8.0	6.0–8.4	5.0–9.0	3.9–10.1	<3.9->10.1	$I_i = -0.4x^2 + 14$, $0 \le x < 5$, $I_i = -2x + 14$, $5 \le x < 7$, $I_i = x^2 - 14x + 49$, $7 \le x < 9$, $I_i = -0.4x^2 + 11.2x - 64.4$, $9 \le x < 14$,
DO (% Sat)	88–112	75–125	50–150	20–200	<20 ->200	$I_i = -0.08x + 8$, $50 \le x < 100$, $I_i = -0.08x - 8$, $100 \le x$,
BOD (ppm)	1.5	3.0	6.0	12.0	>12.0	$I_i = 0.66666x$, (x is in mg/L)
COD (ppm)	10	20	40	80	>80	$I_i = 0.10x$, (x is in mg/L)
Permanganate (mg/L)	2.5	5.0	10.0	20.0	>20.0	$I_i = 0.04x$,
Suspended solids (ppm)	20	40	100	278	>278	$I_i = 2^{[2.1log(0.1x-1)]}$, (x is in mg/L)
Amonia (ppm)	0.1	0.3	0.9	2.7	>2.7	$I_i = 2^{[2.1log(10x)]}$, (x is in mg/L)
Nitrates (ppm)	4	12	36	108	>108	$I_i = 2^{[2.1log(0.25x)]}$, (x is in mg/L)
Chlorides (ppm)	50	150	300	620	>620	$I_i = 0.000228x^2 + 0.0314x$, $0 \le x < 50$, (x is in mg/L) $I_i = 0.0000132x^2 + 0.0074x + 0.6$, $50 \le x < 300$, (x is in mg/L) $I_i = 3.75(0.02x - 5.2)^{0.5}$, $300 \le x$, (x is in mg/L)
Iron (ppm)	0.1	0.3	0.9	2.7	>2.7	$I_i = 2^{[2.1log(10x)]}$ (x is in mg/L)
Manganese (ppm)	0.05	0.17	0.5	1.0	>1.0	$I_i = 2.5x + 3.9x^{0.5}$, $0 \le x < 0.5$, (x is in mg/L) $I_i = 5.25x^2 + 2.75$, $0.5 \le x$, (x is in mg/L)
Alkyl Benzene sulphonates (ppm)	0.09	1.0	3.5	8.5	>8.5	$I_i = -1.2x + 3.2x^{0.5}$, $0 \le x < 1$, (x is in mg/L) $I_i = 0.8x + 1.2$, $1 \le x$, (x is in mg/L)
Carbon Chloroform Exact (ppm)	1.0	2.0	4.0	8.0	>8.0	$I_i = x$ (x is in mg/L)

Appendix A.4. First Dinius Water Quality Index (DWQI) in 1972

In the 1970s, many indices were developed, so in 1972, Dinius developed his first *WQI* from a review of published scientific literature, so his index was calculated as the weighted sum of the subindices, like Horton's index, and the additive version of the NSFWQI, he chose 11 variables to be introduced into his index (Table A3).

Table A3. Subindex Functions of Dinius' Index.

Subindex Number	Variable	Subindex
1	*DO* (% Sat)	$I_1 = x$
2	BOD (mg/L)	$I_2 = 107x^{-0.642}$
3	Total Coliforms (MPN/100 mL)	$I_3 = 100x^{-0.3}$
4	*FC* (MPN/100 mL)	$I_4 = 100(5x)^{-0.3}$
5	Specific Conductance (µS/cm)	$I_5 = 535x^{-0.3565}$
6	Chlorides (mg/L)	$I_6 = 125.8x^{-0.207}$
7	Hardness ($CaCO_3$, ppm)	$I_7 = 10^{1.974-0.00132x}$
8	Alkalinity ($CaCO_3$, ppm)	$I_8 = 108x^{-0.178}$
9	pH (units)	$I_9 = 10^{0.2335+0.44x}$, $x < 6.7$ $I_{10} = 100$, $6.7 \leq x \leq 7.58$ $I_{11} = 10^{4.22-0.293x}$, $x > 7.58$
10	temperature/°C	$I_{12} = -4(t_a - t_s) + 112$, t_a = actual temp, t_s = standard temp
11	Colour (C units)	$I_{13} = 128x^{-0.288}$,

Next the *WQI* is computed as:

$$WQI = \frac{1}{21} \sum_{i=1}^{11} I_i^{w_i} \tag{A4}$$

where

I_i is the subindex function of the pollutant variable;

W_i is the unit weight of the pollutant variable whose value ranges from 0 to 1;

The weights ranged from 0.5 to 5 on a basic scale of importance and the sum of these weights was 21, which is the denominator in Dinius formula.

Appendix A.5. Method of Ramakrishaniah [62] (RWQI)

RWQI is an arithmetic average approach proposed by [62], where the weighting is calculated according to experts' opinions and depending on the importance of the parameter.

The RWQI is given by the following formula:

$$RWQI = \Sigma w_i q_i \tag{A5}$$

where:

q_i is the quality class for the nth variable;

w_i is the relative weight of the nth variable;

In the first step, q_n is calculated according to the following formula:

$$q_n = \frac{V_n - V_{ideal}}{S_n - V_{ideal}} \times 100 \quad \text{for pH and } DO \tag{A6}$$

$$q_n = \frac{V_n}{S_n} \times 100 \quad \text{for other variables} \tag{A7}$$

In the second step, the relative weight (W_i) is calculated using the following equation:

$$W_i = \frac{w_i}{\Sigma wi} \tag{A8}$$

where

W_i is a temporary weight of each variable.

From RWQI values, waters are classified into five categories (Table A4).

Table A4. The RWQI classification.

RWQI	Quality Class
<50	Excellent
50–100	Good
100–200	Poor
200–300	Very poor
>100	Unsuitable

Appendix B. Weighted Geometric Average

Appendix B.1. Second National Sanitation Foundation Water Quality Index (NSFWQI) in 1973

Brown et al. proposed another formula for calculating the WQI after finding that the multiplicative formula is better suited with the opinions of experts, this formula is given by:

$$WQI = \prod_{i=1}^{n} q_i^{W_i}, \tag{A9}$$

where

q_i is the quality class for the nth variable; W_i is the relative weight for the nth variable($\sum W_i = 1$).
Brown et al. kept the same scale of classification of water quality.

Appendix B.2. Bhargava Method

Bhargava used the concept of WQI evaluation cited by Brown et al. to classify the quality of water for drinking purposes.

Bhargava identified four groups of variables. Each group contained sets of one type settings [14].

Coliforms were included in the first group, which represents the bacteriological quality of drinking water. Heavy metals and toxic substances were included in the second group. The third group includes variables that cause physical effects such as odour, color and turbidity. Organic and inorganic substances such as sulphate and chloride, etc. have been included in the fourth group [70].

The simplified model for WQI Bhargava is given by:

$$WQI = [\prod_{i=1}^{n} f_i]^{\frac{1}{n}} \times 100 \tag{A10}$$

where f_i is the value of the sensitivity function of the i-th variable includes the effect of the concentration and weight of the variable i in use varies from 0 to 1 and n is the number of variables taken into account.

Bhargava has established a maximum level of allowable contaminants (C_{MCL}) for each variable used in his formula by referring to the US Environmental Protection Agency [20].

The C_{MCL} is used later to define a parameter C to calculate the sensitivity function f_i.

If the concentration of the i-th variable is inferior or equal $C_{MCL,i}$, f_i takes automatically the value 1. Otherwise, f_i is given by the formulas expressed in Table A5.

Table A5. Subindex Functions of Bhargava's Index.

Variables	Subindex Function
Group I : Coliform organisms	$f_1 = exp^{[-16(C-1)]}$
Group II : Heavy metals, other toxicant, etc..	$f_2 = exp^{[-4(C-1)]}$
Group III: Physical variables	$f_3 = exp^{[-2(C-1)]}$
Group IV : Organic and inorganic intoxicant substances	$f_4 = exp^{[-2(C-1)]}$

The method of Bhargava is only derived from the method of Brown and his index classify the waters into a scale ranging from 0 for extremely polluted to 100 for absolutely unpolluted water.

Appendix B.3. Second Dinius Water Quality Index

In 1987, Dinius developed a multiplicative DWQI using Delphi method in the choice and the range of variable to be included in his new formula.

The DWQI was developed using Delphi method, and it was designed for particular uses of the water which are: recreation, fish, shellfish, public supply, agriculture and industry. Twelve variables were used in this index: *DO*, 5-day *BOD*, coliform count, E-coli count, pH, alkalinity, hardness, chloride, specific conductivity, temperature, color and nitrate [33].

Variables' weights were assigned using Delphi method and subindices' functions were combined using additive aggregation function expressed in Table A6.

Table A6. Subindex Functions of the 1987 Dinius' Index.

variable	Weight	Function
DO (% Sat)	0.109	$0.82DO + 10.56$
BOD_5 (mg/L at 20 °C)	0.097	$108(BOD)^{-0.3494}$
Coliforms (MPN-Coli/100 mL)	0.090	$136(COLI)^{-0.1311}$
E.coli (E-coli/100 mL)	0.116	$106(E-COLI)^{-0.1286}$
Alkalinity (ppm $CaCO_3$)	0.063	$110(ALK)^{-0.1342}$
Hardness (ppm $CaCO_3$)	0.065	$552(H)^{-04488}$
Chloride (mg/L, fresh water)	0.074	$391(CL)^{-0.3480}$
Specific Conductance (μhos/cm at 20 °C)	0.079	$506(SC)^{-0.3315}$
pH (units)	0.77	$10^{(0.6803+0.1856pH)}$ 1 $10^{(3.65-0.2216pH)}$
Nitrates (mg/L)	0.090	$125(N)^{-0.2718}$
Temperature/°C)	0.077	$10^{(2.004-0.0382(t_a-t_s))}$
Colour	0.063	$127(C)^{-0.2394}$

The new index of Dinius had the following form:

$$WQI = \prod_{i=1}^{12} I_i^{w_i} \tag{A11}$$

where I_i is the subindex of pollutant variable (between 0 and 100), w_i is the unit weight of pollutant variable (between 0 and 1), n is the number of pollutant variables.

As other water quality indexes, the DWQI value ranges from 0 to 100.

Appendix C. Weighted and Unweighted Harmonic Square Average

Appendix C.1. Canadian Council of Ministers of the Environment Water Quality Index (CCMEWQI)

The index of the quality of Canadian waters is taken from the formula of British Columbia Water quality index (BCWQI) found in 1990 [49,71] and was used as the basis for identifying the public priority actions of the Manitoba department of Environment [72].

Conceptually, CCME *WQI* is based on a combination of three factors determined in CCME.

Factor 1 (F1) represents the scope, which assesses the extent and non-compliance with water quality directive in the interest period.

Factor 2 (F2) represents the mean frequency and number of times tested or observed value was out of acceptable limits or standards.

Factor 3 (F3) represents the magnitude of the deviation or values whose objectives are not achieved.

$$F_1 = (\frac{\text{Number of failed variables}}{\text{Total number of variables}}) \times 100 \tag{A12}$$

$$F_2 = (\frac{\text{Number of failed tests}}{\text{Total number of tests}}) \times 100 \tag{A13}$$

$$F_3 = (\frac{nse}{0.01nse + 0.01}) \tag{A14}$$

The *nse* variable is expressed as:

$$nse = \frac{\sum_{i=1}^{n} departure_i}{\text{Number of tests}} \tag{A15}$$

The collective amount by which individual tests are out of compliance is calculated by summing the departures of individual tests from their objectives and dividing by the total number of tests. For the cases in which the test value must not exceed the objective:

$$departure_i = (\frac{\text{Failed Test}_i}{Objective_j}) - 1 \tag{A16}$$

For the cases in which the test value must not fall below the objective:

$$departure_i = (\frac{Objective_j}{\text{Failed Test}_i}) - 1 \tag{A17}$$

For the cases in which the objective is zero:

$$departure_i = \text{Failed Test}_i \tag{A18}$$

The value of the index is calculated using the following formulation:

$$CCMEWQI = 100 - \frac{\sqrt{F_1^2 + F_2^2 + F_3^2}}{1.732} \tag{A19}$$

The vector length can reach $\sqrt{100^2 + 100^2 + 100^2} = \sqrt{30000} = 173.2$, so division by the factor 1.732 is only to adjust CCMEWQI into 0 to 100 scale. The above formula produces a value of CCMEWQI between 0 and 100 and provides a digital value to the state of the water quality. Zero means very poor quality of water, while a value close to 100 means excellent water quality. The assignment of CCME *WQI* values to different categories is a somewhat subjective process and also requires expert judgment and expectations of public water quality. The water quality is classified into five categories with descriptions as shown in Table A7.

Table A7. Classification of water quality according to Canadian Council of Ministers of the Environment (CCME) method [64].

WQI	**Quality Class**	**Description**
<44	Poor	Water quality is almost always threatened or impaired; conditions usually.
45–64	Bad	Water quality is usually protected but occasionally threatened or impaired; conditions sometimes depart from natural or desirable levels.
65–79	Marginal	Water quality is usually protected but occasionally threatened or impaired; conditions sometimes depart from natural or desirable levels.
80–94	Good	Water quality is protected with only a minor degree of threat or impairment; conditions rarely depart from natural or desirable levels.
95–100	Excellent	Water quality is protected with a virtual absence of threat or impairment; conditions very close to natural or pristine levels

Appendix C.2. Oregon Water Quality Index (OWQI)

The *WQI* of Oregon, developed by the Oregon Department of Environmental Quality (ODEQ) in the late 1970s and updated several times since then, is another frequently used *WQI* in the public domain [37].

However, the original OWQI was abandoned in 1983 because of the tremendous data and resources needed for the calculation and presentation of results.

With the advances in computer technology, improved tools display and data visualization, allowing a better understanding of water quality, the OWQI was updated in 1994 by refining the original sub-indices adding temperature and a total phosphorus subindex thus improving aggregation calculation.

The original OWQI was modeled after the NSFWQI where Delphi method was used for the selection of variables [73].

Most of the indicators of the quality of water used in the United States are based on the method of Brown, so the OWQI, except in the concept of this index, incorporates the measures of the eight variables of water quality (temperature, *DO*, *BOD*, pH, ammonia + nitrate nitrogen, total phosphate, total solids, and *FC*).

The water-quality variables were classified according to the impairment categories, i.e., oxygen depletion, eutrophication or potential for excess biological growth, dissolved substances and health hazards [20].

In 1994, OWQI took another shape after the work of Dojlido et al. and the calculation formula became:

$$WQI = [\frac{1}{n} \sum_{i=1}^{n} q_i^{-2}]^{-0.5} \tag{A20}$$

where q_i is the quality class for the nth variable.

Water quality classes' limits in terms of OWQI scores are different from those based on NSFWQI, the new classes are: excellent: 91–100; good: 85–90; just: 80–84; poor: 60–70; and very poor: 0–59.

Appendix D. Logarithmic Aggregations

Appendix D.1. The WQI Proposed by Tiwari and Mishra (TMWQI)

The *WQI* calculated by the method described by Tiwari and Mishra is similar to the basic method of Horton. Except in this method, the logarithm and antilogarithm is used, and it is only for mathematical purposes, indeed the logarithm reduces the magnitude of the chemical variables and the

antilogarithm is used after, to broaden and make visible the classification scale. The WQI is calculated by the following formula:

$$TMWQI = Antilog \Sigma w_n log q_n \tag{A21}$$

where:

q_n is the quality class for the nth variable;

W_n is the relative weight of the nth variable;

In the first step, q_n is calculated according to the following formula [74]:

$$q_n = \frac{V_n - V_{ideal}}{S_n - V_{ideal}} \times 100 \quad \text{for pH and } DO \tag{A22}$$

$$q_n = \frac{V_n}{S_n} \times 100 \quad \text{for other variables} \tag{A23}$$

where

V_n is the value of variable in sample n;

S_n = value of variable recommended by guidelines;

V_{ideal} = the ideal value which is considered by some researchers, who applied this method like Tripathy and Sahu, 7.0 for pH and 14.6 for DO .

In the second step, the relative weight (W_n) is calculated using the following equation:,

$$W_n = \frac{K}{S_n} \tag{A24}$$

where

K is a proportionality constant given by :

$$K = \frac{1}{\sum_{i=1}^{n} \frac{1}{S_i}}. \tag{A25}$$

From $TMWQI$ values, waters are classified into five categories (Table A8).

Table A8. Classification of water according to Tiwari and Mishra.

$TMWQI$	Quality Class
<26	Excellent
26–50	Good
51–75	Medium
76–100	Poor
>100	Unsuitable

Appendix D.2. New Water Quality Index Proposed by Said et al.

The idea of Said et al. was to calculate the WQI into two steps, in the first one, water quality variables were ranked according to their significance. In the second step, many forms were tested to give to DO the highest weight followed by fecal coliform and total phosphorus, also to keep the index in a simple equation. Finally, the logarithm was used to give small numbers that are easily used by the management decision-makers, the stake-holders, and the general public.

The variables included on the new WQI are: DO, total phosphates, fecal coliform, turbidity, and specific conductivity, and were chosen according their powers on the effect on water conditions.

Said et al. felt that turbidity and specific conductance have linear effects, which have lesser influence on the values of the variables, in the index formula [20].

The subindices were eliminated in the formula of the new WQI, and there is no need to standardize the variable, which made the calculation more simplified. The formula of the New WQI is given by:

$$NewWQI = log\left[\frac{(DO)^{1.5}}{(3.8)^{TP} \times (TURB)^{0.15} \times (15)^{\frac{FC}{10000}} + 0.14 \times (SC)^{0.5}}\right] \tag{A26}$$

where

DO is the Dissolved Oxygen (% oxygen saturation);

$TURB$ is the Turbidity (Nephelometric turbidity units [NTU]);

TP is the total phosphates (mg/L);

FC is the fecal coliform bacteria (counts/100 mL);

SC is the specific conductivity in (MS/cm at 25 °C);

The index was designed to range from 0 to 3. The maximum or ideal value of this index is 3. In very good waters the value of this index will be 3. From 3 to 2, the water is acceptable, and less than 2 is marginal and remediation, if one or two variables have deteriorated, the value of this index will be less than 2.

If most of the variables have deteriorated, the index is less than 1, which means that water quality is poor.

Appendix E. Fuzzy Logics (FWQI)

According to Icaga six steps are needed to develop a fuzzy logics index, for a conventional classification containing four classes, as follows:

1. determination of the quality classes for the measured variables;
2. arrangement of the variables according to their classes into the four groups;
3. application of membership functions (m_f) to standardize the natural measurement scales of the quality variable into a measurement of the quality degree (membership grade). In this step, four membership functions are used.

mf_i is the membership function of the observed value i depending on the limits given in Tables A9 and A10. The reviewed membership functions from Icaga's paper are defined as below:

$$mf_1 = \begin{cases} 1 & \text{for } x < a \\ 1 - \left[\frac{x-a}{b_1-a}\right] & \text{for } a \leq x \leq b_1 \\ 0 & \text{for otherwise} \end{cases}$$

$$mf_2 = \begin{cases} \frac{x-a}{b_1-a} & \text{for } a \leq x \leq b_1 \\ 1 & \text{for } b_1 \leq x \leq b_2 \\ 1 - \left[\frac{x-b_2}{c_1-b_2}\right] & \text{for } b_2 \leq x \leq c_1 \\ 0 & \text{for otherwise} \end{cases}$$

$$mf_3 = \begin{cases} \frac{x-b_2}{c_1-b_2} & \text{for } b_2 \leq x \leq c_1 \\ 1 & \text{for } c_1 \leq x \leq c_2 \\ 1 - \left[\frac{x-c_2}{d-c_2}\right] & \text{for } c_2 \leq x \leq d \\ 0 & \text{for otherwise} \end{cases}$$

$$mf_4 = \begin{cases} \frac{x-c_2}{d-c_2} & \text{for } c_2 \leq x \leq d \\ 1 & \text{for } d < x \\ 0 & \text{for otherwise} \end{cases}$$

4. four rule bases are successively used:

> **if** $QV_{i1} = IV$ **or** $QV_{i2} = IV$ **or** ... **or** $QV_{in} = I$ **then** *Output* $= I$
>
> **if** $QV_{j1} = III$ **or** $QV_{j2} = III$ **or** ... **or** $QV_{jn} = II$ **then** *Output* $= II$
>
> **if** $QV_k = II$ **or** $QV_{k2} = II$ **or** ... **or** $QV_{kn} = III$ **then** *Output* $= III$
>
> **if** $QV_{l1} = I$ **or** $QV_{l2} = I$ **or** ... **or** $QV_{ln} = IV$ **then** *Output* $= IV$

where QV_i is the quality variable; *I, II, III, IV* are the quality classes in conventional classification; N is the number of quality variables. In the rule bases the "or" operators are used to obtain maximum values.

5. Using the fuzzy algorithm: In fuzzy algorithm, the Mamdani [75] approach is used. Fuzzy inferences of the groups are determined using grades of membership functions of the variables;

6. Defuzzification of the inferences to obtain an index whose value ranges (0;100) interval using Centroid methods, which calculate the center of gravity of the output function.

Table A9. Quality classes for physical and inorganic chemical variables of inside water resources [52].

Variable	Limits Of Water Quality Classes			
	I	II	III	IV
temperature (t) (°C)	25	25	30	> 30
pH	6.5–8.5	6.5–8.5	6–9	<6 or >9
DO $(g\,m^{-3})$	8	6	3	<3
Oxygen Saturation (OS) (%)	90	70	40	<40
Chloride (Cl^-) $(g\,m^{-3})$	25	200	400	>400
Sulphate (SO_4^{2-}) $(g\,m^{-3})$	200	200	400	>400
Ammonia $(NH_3)(g\,m^{-3})$	0.2	1	2	>2
Nitrite (NO_2^-) $(g\,m^{-3})$	0.002	0.01	0.05	>0.05
Nitrate (NO_3^-) $(g\,m^{-3})$	5	10	20	>20
Total phosphorus $(g\,m^{-3})$	0.02	0.16	0.65	>0.65
TDS $(g\,m^{-3})$	500	1,500	5,000	<5000
Color (P_t-co unit)	5	50	300	>300
Sodium (Na^+) $(g\,m^{-3})$	125	125	250	>250

Table A10. The limits of the membership functions [52].

	The Variables of the Membership Functions					
	a	b		c		d
		b_1^a	b_2^b	c_1^a	c_2^b	
T (C)	17.5	22.5		27.5		32.5
pH > 7.5	7.5	7.75		8.75		9.25
pH < 7.5	5.75	6.25		6.75		7.5
DO $(g\,m^{-3})^c$	9	7		4.5		1.5
Chloride $(g\,m^{-3})$	0	50	100	300		500
Sulphates $(g\,m^{-3})$	50	150		250	350	450
Ammonia $(g\,m^{-3})$	0	0.4		1.5		2.5
Nitrite $(g\,m^{-3})$	0	0.004		0.03		0.07
Nitrate $(g\,m^{-3})$	2.5	7.5		15		25
TDS $(g\,m^{-3})$	0	1000		3250		6250
Color (Pt-co unit)	0	27.5		175		425
Sodium $(g\,m^{-3})$	31.25	93.75		156.3	218.8	281.3
Output membership function	12.5	37.5		62.5		87.5

[a] First upper corner of the trapezoidal membership function; [b] Second upper corner of the trapezoidal membership function; [c] The number membership functions in reverse order.

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