Multi-scale statistical properties of disaggregated SMOS soil moisture products in Australia

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Highlights

- Fractal and multifractal properties were observed on remotely sensed soil moisture products acquired from SMOS satellite (Soil Moisture and Ocean Salinity), over space scales ranging from the kilometric field scale to the continental scale
- Two scaling regimes were noticed for the soil moisture data disaggregated with DisPATCh algorithm (Disaggregation based on Physical And Theoretical scale Change), with a scaling break observed at about ten kilometers
- Fractality and multifractality were also found on remotely sensed vegetation indices and surface temperature

Abstract

Soil moisture has a strong impact on climate, hydrology and agronomy at different space scales, from the continent global scale to the local watershed. Passive microwave sensors, like SMOS satellite (Soil Moisture and Ocean Salinity), allow a global study of soil moisture on the entire globe. To have access to kilometric variability, disaggregation algorithms have been developed, such as the Disaggregation based on Physical And Theoretical scale Change (DisPATCh). This method improves the space resolution of SMOS soil moisture from 40 km to 1 km. To do this, it combines coarse-scale (≈40 km) SMOS products with fine-scale (≈1 km) optical/thermal data. Validation studies on specific scales showed the potential of DisPATCh to enhance the spatio-temporal correlation of disaggregated SM with in-situ measurements, under low-vegetated semi-arid regions. Although the efficiency of the method was revealed in these regions, no studies fully explored its statistical behavior over a continuum of space scales. In this paper, we studied and compared the spatial multi-scale statistics of the different input and output datasets involved in DisPATCh downscaling. To do this, we applied spectral and multifractal analysis on the respective products for the region of southeastern Australia, from June to December 2010. Fractal and multifractal properties (in the framework of the Universal Multifractal model) were observed on inputs of DisPATCh (SMOS soil moisture, MODIS vegetation indices and surface temperature), which confirmed and completed

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some results reported in existing literature. For the output disaggregated soil moisture, two scaling regimes were observed, with a transition scale observed at about ten kilometers. Considering spectral analysis, at large scales (> 10 km), disaggregated soil moisture was found to have the same scaling as the original SMOS soil moisture. On finer scales (< 10 km), a different behavior was noticed, with a higher value of the slope of the power spectrum. The same scale break was detected on statistical moments, showing that both spectral and multifractal properties of DisPATCh soil moisture are characterized by this twofold scaling signature.

Keywords

Soil moisture; Multi-scale analysis; Multifractals; Disaggregation; SMOS; DisPATCh

1   Introduction

Soil moisture (SM) is a key component of the climate system and is strongly heterogeneous, at many time and space scales. Interactions between land surface and atmosphere, such as water, energy and carbon fluxes, are strongly related to SM (Ochsner et al., 2013). It has a significant role in the water cycle as it impacts runoff, infiltration and evaporation processes. Thus, SM is an important variable in several scientific fields such as hydrology (Western et al., 2004), meteorology (Dai et al., 2004), climatology (Anderson et al., 2007) and water resource management (Engman, 1991).

SM is heterogeneously distributed at different space scales, from few centimeters to several kilometers. This variability is due to environmental factors impacting directly SM at specific scale ranges (Brocca et al., 2007; Crow et al., 2012; Jana, 2010; Vereecken et al., 2014). For instance, we could mention here soil properties (texture and structure) acting at the field scale, topography features at the watershed scale, land cover (vegetation) and meteorological forcing at the regional and continental scales.

Many ground measurement techniques have been developed to acquire highly resolved SM data sets, down to centimeters in space and minutes in time (for more details see Dobriyal et al., 2012; Robinson et al., 2008; Robock et al., 2000). Although these methods are recognized as reliable and easy to implement, they are not adapted to represent spatial heterogeneity of SM at regional and continental scales (Collow et al., 2012; Crow et al., 2012).

Regional and global scale variability of SM may be acquired and studied with the help of remote sensing. Different active and passive microwave satellites allow daily measurement of surface soil moisture in the first 5 cm of the soil column (Petropoulos et al., 2015; Wigneron et al., 2003). These satellites acquire SM information thanks to the relationship between the soil dielectric constant and

water content. Active microwave sensors measure the energy reflected by the soil after sending a microwave pulse to the surface (backscatter): we find C-band Synthetic Aperture Radars (SAR) like Sentinel 1 satellite (S1) from European Space Agency (Wagner et al., 2009) and C-band scatterometers like the Advanced Scatterometer (ASCAT; Bartalis et al., 2007). These active sensors can provide space resolution from few meters (S1) to tens of kilometers (ASCAT). Their main drawback is their sensitivity to vegetation and surface roughness, which can alter useful information (SM) in the signal measured. Passive sensors, however, are less sensitive to scattering conditions. They measure the self-emission from the land surface (radiances). Good results were obtained by C- and X- band radiometers like the Advanced Microwave Scanning Radiometer-Earth observing system (AMSR-E; Njoku et al., 2003; Owe et al., 2001), or by L-band radiometers such as Soil Moisture and Ocean Salinity (SMOS; Kerr et al., 2010) and the recent Soil Moisture Active Passive (SMAP) mission (Entekhabi et al., 2010a).

L-band microwaves (1.4 GHz) have the benefit of little sensitivity to vegetation, providing optimal estimation of SM on a wider range of land cover conditions. L-band based satellite missions deliver SM products with a revisit time of 2 to 3 days. However, because of technological constraints, the spatial resolution is coarse (30-55 km), much coarser than the kilometer scale. This is a problem since hydro-agricultural applications need better resolved information, below kilometric space scales (Walker and Houser, 2004).

To address this issue, downscaling methods have been created to improve the low spatial resolution of satellite data. Downscaling algorithms are characterized by their input data (satellite products, auxiliary ground measurements, etc.) and by the type of method (physical or statistical). Peng et al. (2017) reviewed the several methods developed so far and proposed a three-group classification: satellite-based methods, methods using geo-information data and model-based methods.

The first group (I) gathers downscaling techniques which combines satellite passive microwave products with satellite highly resolved auxiliary data, such as radar or optical/thermal observations. This takes advantage of the assets of complementary remote sensing measurement techniques. Considering the fusion with high resolution radar, a change detection method was proposed by Njoku et al. (2002) to merge coarse-scale passive microwave soil moisture products and fine-scale active backscatter data. This technique consists in the linear relationship between soil moisture and backscatter data assuming the time-invariance of vegetation and surface roughness effects. The methodology was further tested in other experiments (Narayan et al., 2006; Piles et al., 2009) and proved its efficiency for improving the spatial details of soil moisture. Statistical tools were also used

95   to combine active and passive products, such as Bayesian merging method (Zhan et al., 2006) or

96   wavelet-based image enhancement method (Montzka et al., 2016). This kind of approach showed

97   the great potential of radar for improving soil moisture resolution, in particular for higher vegetation

98   water content and different land cover types (Akbar and Moghaddam, 2015). A possible limitation of

99   this approach is the time lag between active and passive data, due to the low revisit rate of high

100   resolution radar. Recently, SMAP satellite was launched to bypass this problem, embedding on board

101   one radiometer and one radar (Das et al., 2011). Unfortunately, the radar failed and no

102   active/passive combination could be performed. However, the previous studies made to prepare the

103   mission showed good capacity to improve spatial resolution of satellite products by merging active

104   and passive microwave data. Another type of satellite-based method is the combination of passive

105   microwave data with optical and thermal remote sensing data. The interest is to have the additional

106   information of high spatial resolution and short revisit time of the optical/thermal products. The

107   concept is to use highly resolved vegetation cover and surface temperature products to downscale

108   coarse-scale soil moisture product. Based on the surface temperature/vegetation index triangular

109   feature space proposed by Carlson (1994, 2007), Zhan et al. (2002) and later Chauhan et al. (2003)

110   developed and applied this method based on a polynomial function linking high resolution SM with

111   surface temperature, vegetation cover and surface albedo. At coarse resolution, scaling factors

112   (regression coefficients) are estimated from this polynomial function and then used at high

113   resolution in the same function to calculate the high resolution SM using NDVI (Normalized

114   Difference Vegetation Index) and LST (Land Surface Temperature) obtained from the LST/NDVI

115   feature space. An improved version was proposed by Piles et al. (2011) using brightness

116   temperatures instead of albedo, showing better results when comparing downscaled SM with in-situ

117   measurements. For instance, this downscaling technique was used to improve the resolution of

118   AMSR-E soil moisture merging it with optical/thermal data measured from MODIS (Moderate

119   resolution Imaging Spectroradiometer) (Choi and Hur, 2012) or MSG-SEVIRI (Meteosat Second

120   Generation Enhanced Visible and Infrared Imager) (Zhao and Li, 2013). The main problem in this

121   methodology is the non-conservativity of SM between fine-scale and coarse-scale SM. Based on the

122   same theory, other downscaling algorithms were proposed to relate the downscaled SM with coarse

123   observations of SM. An operationally implemented method is the downscaling algorithm DisPATCh

124   (Disaggregation based on Physical And Theoretical scale Change; Merlin et al., 2008a; Molero et al.,

125   2016). This algorithm is more physical because it uses soil evaporation processes to connect

126   optical/thermal and SM data. Different applications of DisPATCh were realized to increase the ≈40

127   km resolution of SMOS SM to 1 km and even 100 m respectively with MODIS (Merlin et al., 2012) and

128   Landsat-7 (Merlin et al., 2013) products. The originality of the method is the estimation of a SM proxy

129   called Soil Evaporative Efficiency (SEE; sections 4.2 and 6.2). The latter has the advantage, compared

130 to land surface temperature or evapotranspiration, to be more linked to SM and to be quite constant

131 during the day. Some improvements still need to be made about the modelling of SEE, especially on

132 elevation and illumination effects (Malbéteau et al., 2017) or soil properties and atmospheric

133 conditions (Merlin et al., 2016). Comparable evaporation-based methods were developed using

134 different proxies of SM such as the Soil Wetness Index (Kim and Hogue, 2012) or the Vegetation

135 Temperature Condition Index (Peng et al., 2016), both applied in the simple downscaling method

136 UCLA. We can also mention algorithms directly improving the resolution of brightness temperature

137 products (instead of retrieved SM), based on the relation between daily temperature change and

138 daily average SM (Song et al., 2014). Generally, these downscaling methods present a significant

139 asset considering the time coherence between the merged products, but some limitations exist.

140 Indeed, the cloud sensitivity of optical/thermal sensors makes the application of these methods

141 possible only under clear-sky conditions (Djamai et al., 2016).

142   Since SM is directly linked to geoinformation data such as topography, soil properties and

143 vegetation attributes (Werbylo and Niemann, 2014), a second group (II) of downscaling methods

144 were also proposed. These methods take advantage of highly resolved geoinformation data (giving

145 information on the local attributes of the zone studied), and could give access to very high spatial

146 resolution of SM. Topography for example was often used in downscaling approaches as an auxiliary

147 data (Busch et al., 2012; Pellenq et al., 2003). However, certain types of geoinformation data, like soil

148 properties, are usually provided by ground observations, which are really specific to the studied area.

149 Thus, the application is limited to local areas and it may not be suitable for global scale study of SM.

150   The third class (III) of methods concerns model-based downscaling techniques. There are two

151 types of models used here. On the one hand, there are hydrological (land surface) models. These

152 ones are more site-specific because they try to link coarse-scale remotely sensed SM and fine-scale

153 parameters obtained from local land surface models. The downscaling can be done through

154 optimization techniques (Ines et al., 2013), linear regressions (Loew and Mauser, 2008) or bivariate

155 relationships (Verhoest et al., 2015). On the other hand, there are models that analyze and describe

156 statistics across scales: they are more generic and try to preserve statistical properties across scales.

157 For example, Kaheil et al. (2008) proposed a wavelet-based downscaling method in order to model

158 spatial statistical properties of fine-scale SM thanks to coarse-scale airborne SM products. Other

159 approaches are based on the scaling (or fractal) properties of SM across spatial scales. Bindlish and

160 Barros (2002) proposed a fractal interpolation method applied on airborne SM products, measured

161 from Electronically Scanned Thinned Array Radiometer (ESTAR). They used power spectra to

162 represent the fractal behavior of SM, and could improve spatial resolution from 200 m to 40 m. A

163 few years later, Mascaro et al. (2010) applied Log-Poisson multifractal cascades on remote sensing

SM to generate simulations of fine-scale SM. The challenge here is to preserve non-stationarity from coarse to fine scales. Nevertheless, particular efforts are made to overpass this problem. For example, Kim and Barros (2002a) adapted the fractal interpolation method applying a sliding window on specific parts of the original field. They could simulate fractal variability while taking into account the local statistics of the field.

The downscaling methods of the three groups presented above have their own advantages and disadvantages, with more or less efficiency according to specific surface or climate conditions. In this study, we focus on the evaluation of multi-scale variability of SM products generated by the method DisPATCh (Merlin et al., 2008a; Molero et al., 2016). Despite its limitations related to cloud cover, this semi-physical downscaling algorithm combines low sensitivity to vegetation of L-band microwaves, high spatial resolution of optical/thermal data and it is dispensed from estimation errors commonly generated by land surface models. Several studies have been realized so far to evaluate and validate this method (Malbéteau et al., 2016; Merlin et al., 2013, 2015; Molero et al., 2016). In general, the assessment of downscaling algorithms is made comparing fine-scale output products with ground measurements. Different performance metrics are used, such as correlation, root mean square error or bias (Albergel et al., 2013; Al Bitar et al., 2012; Entekhabi, 2010b). More recently, Merlin et al. (2015) proposed a new metric that estimates the gain given by the downscaling method in terms of representativeness of downscaled data compared to non-downscaled data. To take into account scale mismatch between downscaled and ground measurements, upscaling techniques have been developed in order to bring downscaled and ground data together at common space scales (Crow et al., 2012). For example, Merlin et al. (2013) applied the DisPATCh algorithm on SMOS data while using both MODIS (Moderate resolution Imaging Spectroradiometer) and Landsat-7 auxiliary data. Coarse-scale satellite data, downscaled data and aggregated ground measurements were compared at three different scales: 40 km, 3 km and 100 m. Good results confirmed the potential of DisPATCh to improve the spatio-temporal correlation of remotely sensed SM with in-situ measurements. However, the drawback of these validation techniques is that they are restricted to specific scales. Thus, the validation of disaggregated SM products over a continuum of space scales has not been fully explored yet. Investigation of the multi-scale statistics and of possible scaling properties of these products could provide relevant information on this aspect.

During the last thirty years, several studies were carried out to describe the statistical properties of SM across spatial scales (Famiglietti et al., 2008; Rodriguez-Iturbe et al., 1995). Different analytical methods were proposed. The most commonly used are spectral-wavelet analysis (Si, 2008) and multifractal analysis (Kim and Barros, 2002b; Mascaro et al., 2010; Oldak et al., 2002). In 1995, Rodriguez-Iturbe et al. highlighted for the first time the fractal behavior of SM from remote sensing:

the spatial variance of SM followed a power law decay as a function of aggregation scales ranging from 30 m to 1 km (Washita Experiment 1992, USA). Later studies showed that such a scaling behavior of SM variance could be extended to wider range of scales: up to regional scale (Hu et al., 1997) and even to continental scales (Rötzer et al., 2015). Similar research works demonstrated that increasing area extent (increasing size of the total area) induced the increase of SM variance according to a power law function (Famiglietti et al., 2008; Rötzer et al., 2015; Brocca et al., 2012). Moreover, in Oldak et al. (2002), the fractal scaling of SM was revealed to be multifractal: the power law was also applicable to the first six statistical moments of airborne SM products for scales ranging from hundreds of meters to tens of kilometers (Washita'92 Experiment and Southern Great Plains Experiment 1997, USA). Multifractal scaling was then detected in SM fields (Das and Mohanty, 2006; Kim and Barros, 2002b; Lovejoy et al., 2008; Mascaro et al., 2010). Since SM variability is directly related to the amount of soil wetness (Brocca et al., 2007; Famiglietti et al., 2008), it may be expected that scaling properties of SM may vary according to the state of SM. Indeed, when plotting SM variance power law in log-log coordinates, Rodriguez-Iturbe et al. (1995) and Manfreda et al. (2007) found that the corresponding slope of the curve was increased during drier periods, revealing seasonal variations of SM scaling (Rötzer et al., 2015). Moreover, it was observed that SM variability was not governed by a single scaling behavior, but by different scaling regimes depending on the range of scales. At the field scale, SM variability is mainly related to land surface characteristics such as soil properties or topography, whereas at larger scales it is impacted by meteorological quantities like rainfall or evapotranspiration (Cayan and Georgakakos, 1995; Entin et al. 2000). Studies based on semi-variograms (Ryu and Famiglietti, 2006; Korres et al., 2015) and spectral/moments analysis (Kim and Barros, 2002b) revealed the presence of scale breaks closed to this transition scale between land surface and meteorological regimes. Though, the aforementioned characteristics of SM highlight its complexity and its high degree of nonlinearity due to hydrometeorological processes acting at different space scales, attesting the necessity to better understand the scaling behavior of SM for applications such as data assimilation or downscaling (Rötzer et al., 2015).

In this paper, we propose an alternative and complementary method for verifying the multiscaling behavior of DisPATCh products. To do this, we studied and compared the statistical spatial properties across scales of the downscaled SM, the original SMOS SM and the MODIS auxiliary data, by applying spectral and multifractal analysis in the framework of the Universal Multifractal (UM) model (Schertzer and Lovejoy, 1987). The definition of multifractal formalism is given in section 2, with a particular attention paid to UM parametrization. The methodology followed for multifractal (and spectral) analysis is detailed in section 3. Section 4 describes the case study and the data set. Then, the different results obtained from spectral and multifractal analysis are presented in section

232 5. Finally, section 6 proposes explanations to the multiscaling behaviors of DisPATCh SM, and a

233 general conclusion of this study is given in section 7.

2   Theory of multifractals

235   During the last century, several studies showed that many geophysical processes could present

236 scale invariance properties. This was first anticipated by Richardson (1922) in the case of turbulence:

237 he described turbulent flows as cascade processes that transfer kinetic energy from large to small

238 scales. Based on this approach, statistical models of turbulence were proposed such as the famous

239 Kolmogorov law (1941) to describe velocity increments. Later research works generalized the study

240 to take into account the heterogeneity of the energy flux (Kolmogorov, 1962; Obukhov, 1962;

241 Yaglom, 1966). Multi-scale models such as multiplicative cascades were therefore proposed to

242 reproduce scale invariance properties through the use of fractal geometry. Later, scale invariance

243 was noticed in other geophysical fields: in his study of the coast of Britain, Mandelbrot (1967)

244 revealed the presence of fractal properties in topography.

2.1   From fractal sets to multifractal fields

246   The concept of fractal dimension has been used in many works related to multi-scale analysis and

247 geophysical modelling. Indeed, the term "fractal" refers to any entity (time series or 2D/3D random

248 field) in which each part presents similar properties, geometrically or statistically, to the ensemble. In

249 this manner, the structure of a fractal entity is characterized by scale invariance. Initially, the notion of

250 fractal was introduced in the late 19th century in geometry with the creation of sets, i.e. mathematical

251 objects, having unusual properties, especially a non-integer Hausdorff dimension, called later by

252 Mandelbrot as "fractal dimension" (Mandelbrot, 1967). Scale invariance, in the statistical sense, was

253 theoretically proposed by Kolmogorov in 1940 with the introduction of the fractional Brownian

254 motion. This model could generate random time series whose trajectories present fractal properties

255 in terms of statistical distribution. It illustrates the physical interest of fractal random processes, since

256 Brownian motions are somewhat ubiquitous in physics. Mandelbrot and Van Ness (1968) made it

257 famous by introducing it to more physical models. In particular, the first fractal stochastic models of

258 topography were developed based on this theory (Mandelbrot, 1975).

259   These stochastic models aim to represent the simple scaling (monofractal) behavior of

260 geophysical processes. In this context, the fractal dimension or scaling parameter is assumed to be

261 unique, restricting multi-scale modelling to a specific class of variability. However, most geophysical

262 processes are characterized by more complex statistics. In case of operational hydrology, rare and

263 extreme events, present in precipitation or soil moisture for example, correspond to high order

264 statistics and need to be detected (Hubert et al., 1993). Therefore, multifractal models, characterized

265 by an infinite spectrum of fractal dimensions, have been proposed to account for a more exhaustive

266 set of statistics. Schertzer and Lovejoy (1987), based on the findings of Parisi and Frisch (1985),

267 initially established the multifractal formalism through the fundamental equation:

268 $$Pr(\oplus_\lambda > \lambda^\gamma) \approx \lambda^{-c(\gamma)} \tag{1}$$

269 where $\oplus_\lambda$ is a positive normalized random scalar process, time series or random field defined on R^2

270 or R^3. The mean of the process is assumed to be statistically conserved across scales. λ is the

271 observation resolution, here defined as the inverse of the scale that can be seen as the sampling time

272 or pixel size for time and space domain processes respectively and $\approx$ indicates an equality within the

273 limits of slowly varying functions. Eq.1 expresses the fact that for a multifractal process, the

274 probability of exceeding a threshold varies as a power law of the resolution with exponent $c(\gamma)$. This

275 exponent is called as fractal codimension of the process, depending on the amplitude of thresholds.

276 The thresholds are defined by the following power law:

277 $$T_\lambda = \lambda^\gamma \tag{2}$$

278 with γ the notion of singularity, characterizing the amplitude of the process independently of the

279 scale. Each singularity is associated to a fractal codimension $c(\gamma)$, corresponding to a family of

280 thresholds of various amplitudes. From a more physical point of view, high singularities (detected by

281 high thresholds) are related to rare and extreme events, with high fractal codimensions and inversely

282 low (box-counting) fractal dimensions D_f (Mandelbrot, 1967). Indeed, the latter are related to the

283 dimension of space D through the relation $c(\gamma) = D - D_f$. Therefore, $c(\gamma)$ can be described as a

284 codimension function, increasing with γ, which completely characterizes the multi-scale statistical

285 properties of the field $\oplus_\lambda$. In general, if the field is multifractal, $c(\gamma)$ is found to be convex and

286 positive (with a fixed point C_1 imposed by the condition of canonical conservation), whereas

287 monofractality is associated to the trivial case $c(\gamma) = const$.

288   Since probability distributions and statistical moments are related by a Mellin transform,

289 Schertzer and Lovejoy (1987) proposed an equivalent equation to (1):

290 $$\langle \oplus_\lambda^q \rangle \approx \lambda^{K(q)} \tag{3}$$

291 where $\langle \cdot \rangle$ is the statistical averaging operator, q is the order of the moment ($q \geq 0$), and $K(q)$ is the

292 moment scaling function. Eq.3 expresses that, for any fixed moment order, statistical moments and

293 resolution are linked through a power law. Singularities and moment orders are directly linked, since

294 the moment scaling function $K(q)$ is the Legendre transform of the codimension function $c(\gamma)$.

295 Similarly to *c(γ)*, *K(q)* is a convex function (with the special case *K(1) = 0* related to the conservation of
296 the mean across scales), which entirely characterizes the multifractal field.

2.2   Multiplicative cascades

298   Multiplicative cascades are stochastic models that can be used to build multifractal fields.
299 Cascades are multiplicative processes because they are defined by an iterative multiplicative
300 construction: considering a two dimensional random signal (field), each pixel at resolution λ_{n+1} (with
301 *n* the construction level of the cascade) is the product of the embedding pixel at coarser resolution
302 (λ_n) multiplied by a random variable $\mu\varepsilon$. This is described by the following equation:

$$\bigcirc_{\lambda_{n+1}} = \mu\varepsilon \times \bigcirc_{\lambda_n} \tag{4}$$

304   In this manner, the statistical properties of the field $\bigcirc_{\lambda_{n+1}}$ are directly related to the statistical
305 properties of the coarser field $\bigcirc_{\lambda_n}$. If all the multiplicative random variables used for each step of
306 the iterative construction are independent and identically distributed, and distributed independently
307 of the scale, the final field presents scale invariant properties.

308   Several models of cascades have been developed so far. First models were built within the
309 framework of turbulence, such as the α-model (Schertzer and Lovejoy, 1984) which corresponds to
310 discrete construction of cascades: the multiplicative random variables are limited to two possible
311 fixed values, respectively leading to increasing or decreasing pixel value when the resolution is
312 refined. Later, more elaborated models were constructed generalizing the discrete case to
313 continuous cascades (Dubrulle, 1994; Schertzer and Lovejoy, 1987, 1991, 1997; She and Levêque,
314 1994). The latter are based on an infinite number of steps between any pair of resolutions, leading to
315 continuity in scale. The benefit of continuous cascades is twofold. First, they can represent possibly
316 more realistic structures by avoiding any arbitrary discretization of scales. Moreover, they often
317 converge toward random processes which are characterized by a small number of degrees of
318 freedom (special cases of log-infinitely divisible distributions). This is interesting considering that
319 multifractal fields built by multiplicative cascade processes would otherwise need an infinite number
320 of scaling parameters (one for each fractal dimension). For example, She and Levêque (1994)
321 proposed a continuous cascade model based on Log-Poisson statistics, and Schertzer and Lovejoy
322 (1987) used Log-stable random variables to build the Universal Multifractal model. In both models,
323 only two fundamental parameters are needed to fully define multifractality.

2.3   Universal Multifractals

325   Physically, multifractal fields built by Log-Poisson or Log-stable cascades have a high degree of
326 generality in geophysics. Log-Poisson model has been successfully applied to different geophysical

327 variables such as rain (Deidda, 2000), or even soil moisture (Mascaro et al., 2010). However, this
328 model can have disadvantages of representing a restricted range of variabilities, which may make it
329 unsuitable for modeling processes with unbounded singularities. On the other hand, by assuming the
330 stability of the random variables and suitable renormalization, UM model is likely adapted for
331 characterizing a wide range of processes: topography (Lavallée et al., 1993), rain and clouds (Tessier
332 et al., 1993) and more recently soil moisture and vegetation optical indexes (Lovejoy et al., 2008).
333 Moreover, a possibly more immediate physical interpretation of the parameters is found in this
334 model. For mathematical and physical arguments supporting the universality of UM model, see
335 Schertzer and Lovejoy (1997); see also Gupta and Waymire (1997) for discussion about its generality.
336 UM model defines the moment scaling function using two "universal" parameters, through the
337 following equation (Schertzer and Lovejoy, 1987):

338 $$K(q) = \frac{C_1}{(\alpha-1)}(q^\alpha - q) \qquad\qquad (5)$$

339 where α is the degree of multifractality of the field. It varies between 0 (monofractality) and 2 (log-
340 normality) and expresses how fast the codimension evolves as a function of the singularity. The
341 second parameter C_1 is the codimension giving the dominant contribution to the mean value of the
342 field (related to moment of order 1): $C_1 = K'(1)$. Physically, it indicates inhomogeneity (dispersion)
343 of the field: it varies from 0 (homogeneous field) to the dimension D of the embedding space (very
344 intermittent field). Because of Legendre transform, $c(C_1) = K'(1)$ is also defined as the fixed point
345 of the codimension function.

346 *2.4 FIF model*

347 Generally, most of the geophysical fields are non-conservative, i.e. integrated processes defined
348 by a certain degree of fractional integration. This appellation comes from multifractal cascade
349 models: see Gagnon et al., 2006 for detailed explanations on this formalism. Thus, to account for a
350 wider range of processes, an extension of the UM model to non-conservative fields has been
351 proposed (Schertzer and Lovejoy, 1991): the Fractionally Integrated Flux (FIF) model. It expresses the
352 degree of fractional integration of the UM field, using a third parameter *H*. The latter is called the
353 order of integration and defines the non-conservativity of the field: in plain words, the larger is *H*, the
354 smoother is the field. The integrated flux is noted R_λ and is characterized by a power law variation of
355 its stationary increments:

356 $$\Delta R_\lambda \approx \oint_\lambda \Delta x^H \qquad\qquad (6)$$

357 where ΔR_λ are the increments (fluctuations of the flux) estimated over a varying window Δx, which
358 is equivalent to the space scale *l*. Note that when *H = 0*, the equation corresponds to the

359   conservative case $\oint_\lambda$. Additionally, in the case of two dimensional fluxes, Eq.6 also applies for other

360   directions (i.e. Δy increments), with the same exponent *H* if the process is isotropic.

361   Hereafter in this article, the appellation proposed in Lovejoy and Schertzer (2010) will be

362   followed: non-integrated cascades will be called conservative fluxes, due to the conservation of the

363   mean, and fractionally integrated "non-conservative" processes will be called "random fields" or

364   simply "fields".

365   **3   Multifractal analysis methodology**

366   The different techniques used to analyze the multi-scale properties of DisPATCh related products

367   are detailed in this section. The methodology is based on the multifractal theory presented in the

368   precedent section. Because our study treats only satellite images, we will focus on the two

369   dimensional versions of these techniques.

370   *3.1   Power spectrum: preliminary evidence of scaling*

371   Spectral analysis is a methodology often used in geophysics to characterize, in an easy and rapid

372   way, some scaling properties of fields over different space scales (Lovejoy et al., 2008). Thanks to its

373   high sensitivity to scale breaks, scaling regimes can be easily identified. In a first step, the two-

374   dimensional power spectral density $P(k_x, k_y)$ of the data under analysis, X, is estimated:

375    $$P(k_x, k_y) = |fft(X)|^2 \tag{7}$$

376   with *P* the power spectral density defined on both vertical and horizontal image axis, corresponding

377   respectively to k_x and k_y wavenumbers (spatial frequencies). Here, the estimation of the PSD is done

378   through a two-dimensional *fft* or Fast Fourier Transform. Then, the one dimensional isotropic angle-

379   integrated power spectrum $E(k)$ is obtained (Lovejoy et al., 2008; §8):

380    $$E(k) = \int_{\|\vec{k}\|=k} P(\vec{k}) d\vec{k} \tag{8}$$

381   where *k* is the modulus of the wavenumber and $\|.\|$ is the Euclidean norm. Since it expresses space

382   frequencies, *k* is directly related to the space resolution λ. If the process presents scaling properties,

383   the spectrum should follow a power law, where β is the negative slope of $E(k)$ on a log-log graph:

384    $$E(k) \approx k^{-\beta} \tag{9}$$

385   β is called the spectral exponent and is directly related to the FIF parameters through the equation:

386    $$\beta = 1 + 2H - K(2) \tag{10}$$

387 In this manner, β also gives first indications about the possible conservative nature of the field, since

388 integrated flux ($H > 0$) should correspond to spectral exponent greater than 1. Note that power

389 spectrum is a second-order statistic, hence the term $K(2)$.

390 *3.2 Statistical moments: multifractal properties*

391 To test the presence of multifractal properties in the data (Eq.3), statistical moments and moment

392 scaling function need to be estimated. To do this, different steps must be followed. First, the

393 underlying conservative field $\bigcirc_{\lambda_{max}}$ has to be reconstructed from the data, at the maximum

394 observation resolution λ_{max}. Because the possible existence of a fractional integration of order H

395 (Eq.6), a fractional derivative of the same order should be done. In this study, the modulus of the

396 gradient was applied to the data. Indeed, this operator provides a simple and good numerical

397 approximation of the fractional derivation without prior knowledge of *H* order (Lavallée et al., 1993):

398 $$\bigcirc_{\lambda_{max}} = \sqrt{\left(\frac{\partial R_{\lambda_{max}}}{\partial x}\right)^2 + \left(\frac{\partial R_{\lambda_{max}}}{\partial y}\right)^2} \tag{11}$$

399 Once the conservative field is retrieved, $\bigcirc_{\lambda_{max}}$ is normalized by its mean.

400 The second step involves the degradation of the field at lower resolutions $\lambda < \lambda_{max}$. It aims to

401 approximate the inversion of the stochastic multiplicative cascade by iteratively averaging the field at

402 coarser scales: each coarse pixel (level *n* of the cascade*)* is obtained by a simple average of

403 neighboring finer pixels (level *n+1*). Note that each roughened pixel size is a power of two multiplied

404 greater than the observation scale l_{min} (= λ_{max}^{-1}).

405 Finally, empirical moments (i.e. computing *q*-th order moments in Eq.3 while replacing statistical

406 averages by empirical averages) are then computed for various orders and resolutions. On a log-log

407 graph, the different moments are plotted as a function of the resolution. If linearity is observed for

408 each moment curve, at least over a significant range of resolutions, Eq.3 is therefore verified, which

409 is the signature of multifractality. The empirical moment scaling function can be estimated, with $K(q)$

410 corresponding to each linear fit of *q*th order moment. Afterwards, the universal parameters α and C_1

411 may be obtained by optimization according to the UM model form of $K(q)$ (Eq.5).

412 *3.3 Structure functions: some evidence of non-conservativity*

413 A convenient way to reveal the non-conservative/fractionally integrated nature of the integrated

414 flux R_λ (Eq.6), is to compute its first order structure function:

415 $$\Delta R_\lambda(\Delta x) = \langle |R_\lambda(x + \Delta x) - R_\lambda(x)| \rangle \tag{12}$$

416 If the flux is indeed non-conservative, the order of integration H should be the slope of the

417 increments ΔR_λ, plotted in a log-log graph as a function of space scale Δx. This technique will be

418 used in this study to estimate the H parameter.

4 Case study and data

4.1 C4DIS processor and satellites products

421   The different products analyzed in this study are input and output data of C4DIS (CATDS level-4

422 DISaggregation) processor (Molero et al., 2016). This processor includes the first operational version

423 of the DisPATCh algorithm, taking into account the best configurations according to the latest studies

424 (Merlin et al., 2010a, 2010b, 2013). Because the algorithm is still evolving, C4DIS products are called

425 as "scientific" (Molero et al., 2016): users can have access on demand to the products over specific

426 areas of the world.

427   As presented earlier, the downscaling method combines SMOS microwave data and MODIS

428 optical/thermal data. The SM data is given by the SMOS Level-3 daily global SM product (reference:

429 MIR CLF31A/D). This product is provided by the Centre Aval de Traitement des Données SMOS

430 (CATDS), which is the French ground segment for SMOS Level-3 and Level-4 products. The SM data is

431 acquired every day at a radiometric resolution that varies between 35 and 55 km, 40 km in average,

432 from L-band brightness temperature measurements (Kerr et al., 2012; Wigneron et al., 2007). SMOS

433 Level-3 products are delivered on the EASE (Equal Area Scalable Earth) grid, with a grid spacing of 25

434 km × 25 km.

435   The optical/thermal products come from the MODIS sensor, embedded on both Aqua and Terra

436 satellites. Two types of auxiliary data are used in DisPATCh. First, there is Land Surface Temperature

437 (LST). It is extracted from the MODIS Level-3 daily products: MYD11A1 (Aqua) and MOD11A1 (Terra).

438 These temperature products are estimated from thermal infrared radiances emitted from the surface

439 (3-15 μm). Then, the second auxiliary data is Normalized Difference Vegetation Index (NDVI), given

440 by the Level-3 16-day Terra product (MOD13A2). The vegetation index is computed from surface

441 reflectances in red (0.7 μm) and near infrared (0.8 μm) wavelengths. Both LST and NDVI products are

442 provided at 1 km resolution by the NASA Land Processes Distributed Active Archive Center (LP DAAC).

443 They are presented on a sinusoidal grid, with a grid spacing slightly smaller than kilometer: 0.93 km ×

444 0.93 km (Solano, 2010; Wan, 2006). We may notice that LST products have daily time resolution,

445 whereas NDVI products are representative of a period of 16 days.

446   Output DisPATCh products are generated every day by the C4DIS processor. Their resolution is

447 that of MODIS products (1 km), and they are presented on an equal-spaced lat-lon WGS84 grid, with

448 a grid spacing of 0.01° (≈1.12 km). For simplicity, in the following we'll make the approximation 0.01°

449 = 1 km. One single downscaled image is the result of the combination of four downsampled SMOS

450 SM images, one MODIS NDVI image, and up to six MODIS LST images corresponding to 3 consecutive

451 days of Aqua and Terra acquisitions (for more details on the combination methodology see

452 Malbéteau et al., 2016; Merlin et al., 2012; Molero et al., 2016). In other words, in the final product,

453 each high resolution output pixel comes from the average of 24 possible disaggregated pixels (up to

454 24 SM-LST possible pairs). The advantage of this composition is that uncertainty in downscaled SM

455 can be potentially reduced and estimated, and time-coverage is improved (Malbéteau et al., 2016).

456 *4.2 DisPATCh algorithm*

457 DisPATCh is based on a semi-empirical model that estimates the Soil Evaporative Efficiency (SEE)

458 from high resolution (HR = 1 km) LST and NDVI products. The method is based on the separation of

459 MODIS LST into its soil and vegetation components, respectively referred in this study as $T_{s,HR}$ and

460 $T_{v,HR}$. To do this, the approach relies on a variant of the trapezoid method from Moran et al. (1994)

461 which interprets the feature space defined by MODIS LST and NDVI-derived fractional vegetation

462 cover $f_{v,HR}$. The purpose here is to extract the soil temperature $T_{s,HR}$ according to the following

463 equations:

464 $$T_{s,HR} = (LST - f_{v,HR} * T_{v,HR})/(1 - f_{v,HR}) \qquad (13)$$

465 $$\text{with } f_{v,HR} = (NDVI - NDVI_{soil})/(NDVI_{veget} - NDVI_{soil}) \qquad (14)$$

466 In equation 13, the vegetation temperature $T_{v,HR}$ is calculated according to Moran et al. (1994).

467 $NDVI_{soil}$ and $NDVI_{veget}$ in (14) are respectively the NDVI obtained from bare soil (set to 0.15) and

468 from full-cover vegetation (set to 0.90).

469 Then, MODIS-derived soil temperature $T_{s,HR}$ allows to estimate SEE_{HR} at 1 km resolution

470 following the methodology proposed by Merlin et al. (2012):

471 $$SEE_{HR} = (T_{s,max} - T_{s,HR})/(T_{s,max} - T_{s,min}) \qquad (15)$$

472 where $T_{s,max}$ and $T_{s,min}$ are endmembers estimated from the approximations of Merlin et al. (2013)

473 considering the relations between the minimum/maximum of LST and the associated $f_{v,HR}$ (more

474 details can be found on these estimates in Molero et al., 2016; p.4).

475 SEE is used to describe the spatial variability of SM within the low resolution (LR = 40 km) pixel

476 given by SMOS product. High resolution SM (SM_{HR}) is linked to high resolution SEE (SEE_{HR}) through

477 the linear model proposed by Budyko (1956) and Manabe (1969):

478 $$SEE_{HR} = \frac{SM_{HR}}{SM_p} \qquad (16)$$

479 where SM_p is a LR parameter depending on atmospheric conditions and soil properties. In the C4DIS

480 processor, this parameter is computed at low resolution at each execution from daily SMOS SM

481 (SM_{LR}) and SEE averaged inside the LR pixel (SEE_{LR}):

482 $$SM_p = \frac{SM_{LR}}{SEE_{LR}} \qquad (17)$$

483 The disaggregation is finally realized by applying a first order Taylor expansion to the SEE and SM

484 dataset. The downscaling relationship is written as:

485 $$SM_{HR} = SM_{LR} + SM'(SEE_{LR}) \times (SEE_{HR} - SEE_{LR}) \qquad (18)$$

486 with $SM'(SEE_{LR})$ the partial derivative of SM relative to SEE computed at low resolution. Here, this

487 derivative simply equals the SM_p parameter estimated according to (17).

488 *4.3   Study area*

489 Australia is a wide country, with an area of almost 8 million km² and characterized by various

490 surface and climate conditions. Thus, it is a suitable area to study spatial variations of soil moisture

491 over a wide range of scales. Many studies on SM have been carried out in Australia in order to

492 monitor SM variability using ground, airborne and satellites data (Smith et al., 2012). Among others,

493 we can mention the National Airborne Field Experiment 2006 (NAFE'06; Merlin et al.., 2008b) and the

494 Australian Airborne Calibration/validation Experiments for SMOS (AACES; Peischl et al., 2012). These

495 experiments were realized over the Murrumbidgee catchment (82 000 km², Fig.1), located at the

496 southeastern part of Australia. Because of its variable climatic conditions (humid in the east, semi-

497 arid in the west), this region was used for validating satellites missions such as SMAP (Panciera et al.,

498 2014) or SMOS. SM products delivered by SMOS were assessed during the AACES experiments, which

499 took place in 2010 over two periods: January-February (AACES-1) and September (AACES-2). Wide

500 spatially distributed networks of in-situ measurements (OzNet hydrological monitoring network;

501 Smith et al., 2012) and transect flights (Polarimetric L-band Multibeam Radiometer; Peischl et al.,

502 2012) were used to validate SMOS data. In this context, benefiting from a dense SM dataset at

503 different space scales, some of the first applications of DisPATCh algorithm were realized during the

504 AACES experiments (Merlin et al., 2012). These works showed the efficiency of DisPATCh under low-

505 vegetated semi-arid areas, and its potential to evaluate coarse-scale SMOS products. Later studies

506 (Malbéteau et al., 2016; Molero et al., 2016) continued the evaluation and improvement of DisPATCh

507 algorithm over the Murrumbidgee catchment.

508   In this paper, DisPATCh analysis is made during the 7-month period from June to December 2010,
509   taking advantage of previous DisPATCh studies over this period. We choose to extend the study area,
510   from the Murrumbidgee catchment to the Murray Darling Basin (MDB, 1 million km², Fig.1). The first
511   reason of this choice is related to the main objective of the study, which is the analysis of DisPATCh
512   related products over different space scales. Though, our study covers a large range of scales, from
513   the pixel size (kilometer scale) to the full basin extent (1300 $\times$1400 km²), giving a new point of view
514   considering DisPATCh validation. Moreover, spectral and multifractal tools presented in section 3
515   cannot be properly applied if the data size is not sufficient enough. Because of its low resolution, it
516   would be inappropriate to do multiscale analysis of SMOS SM over the Murrumbidgee catchment
517   ("images" would be smaller than 5 $\times$ 5 pixels).

518   MDB is located in southeastern Australia and contains more than 20 catchments such as
519   Murrumbidgee in its south part (Fig.1) The climate is sub-tropical in the North-East (average annual
520   precipitation up to 1500 mm), semi-arid in the West (average annual precipitation less than 300 mm)
521   and mostly temperate in the South (snowfall during winter on the peaks of the Great Dividing
522   Range). Regarding to land use, West is made of wide plains essentially composed of saltbush
523   shrublands and mulga lands. From South to North-East, there are the mountains of the Great
524   Dividing Range reaching 2 300 m in altitude. Irrigation, dry land cropping and pastures are spread
525   over the basin, but most of the irrigated areas are located in the South (like Murrumbidgee region).
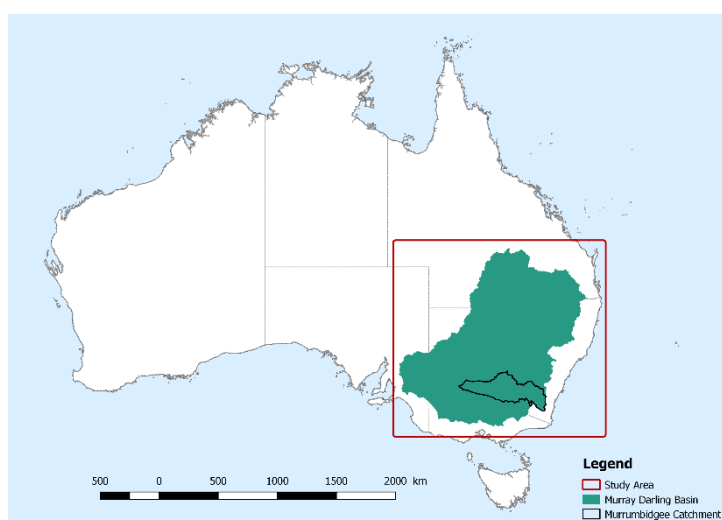
526

Figure 1. The study area includes the Murray Darling Basin (1 million km²), southeastern Australia.

527

4.4   Data preprocessing

Before applying the multi-scale analysis, preprocessing must be done on the different satellite products. The first preprocessing step is to handle the missing values. Because of technology or acquisition conditions, all satellite sensors provide products that present more or less missing values. These can be caused by failures in the data acquisition or delivering, or even voluntarily generated by the production center when discarding incorrect values. In our case, SMOS products can be affected by unauthorized emissions that cause radio frequency interference (RFI). SMOS SM used in this study are pre-filtered by CATDS in order to remove pixels with more than 10% RFI probability (Kerr et al., 2013; Olivia et al., 2012). Considering MODIS products, cloud pixels are also removed to avoid the impact of atmosphere on downscaled data. Though, missing values in output DisPATCh products are mainly caused by the accumulation of missing values coming from inputs. Thanks to the 24 averaged HR outputs combination implemented in C4DIS processor (section 4.1), the probability to get missing values in the final averaged downscaled product is reduced. In our study, we applied bilinear interpolation in each satellite image to fill in missing data (noted NaN). To do this properly, some conditions were established. To minimize the impact of data interpolation on spectral and multifractal analysis, each image with more than 40% of NaN were discarded. Moreover, in order to treat separately land-surface NaN values from sea areas located outside the continent, the latter were filled with zeros. Previous studies showed that biased multifractal parameters could be obtained from data containing significant proportion of zeros (De Montera et al., 2009; Verrier et al., 2010, 2011). Thus, we made sure to select images whose ground area contains a minimum of sea pixels (less than 10 %).

In a second stage, sub-images of $2^n \times 2^n$ pixels need to be selected over the MDB area. To estimate statistical moments over different spatial resolutions, images must indeed be square, with a number of pixels equal to a power of two along each dimension (section 3). Because of different satellites projection grids and spatial resolutions, selected sub-images from different satellites do not cover exactly the same area and they do not completely match to the original MDB area. Figure 2 presents examples of sub-images obtained for DisPATCh SM, SMOS SM and MODIS NDVI, whose size is respectively 1024 × 1024, 64 × 64 and 1024 × 1024 pixels (for readability, during preprocessing all MODIS products were projected from sinusoidal to orthogonal lat/lon coordinates; Sohrabinia, 2012). Considering the different grid spacing of the products and sub-images size condition, the sub-image selected for SMOS SM covers the entire MDB (1600 × 1600 km²), whereas the sub-images selected for DisPATCh and MODIS products are smaller (around 1000 × 1000 km²). It is important to notice that, while they have similar spatial resolution and a same number of pixels, DisPATCh SM and MODIS images do not exactly correspond to the same ground area. This is caused by slightly different

562 grid spacing for the two products, 1 km for DisPATCh and 0.93 km for MODIS (Solano, 2010; Wan,

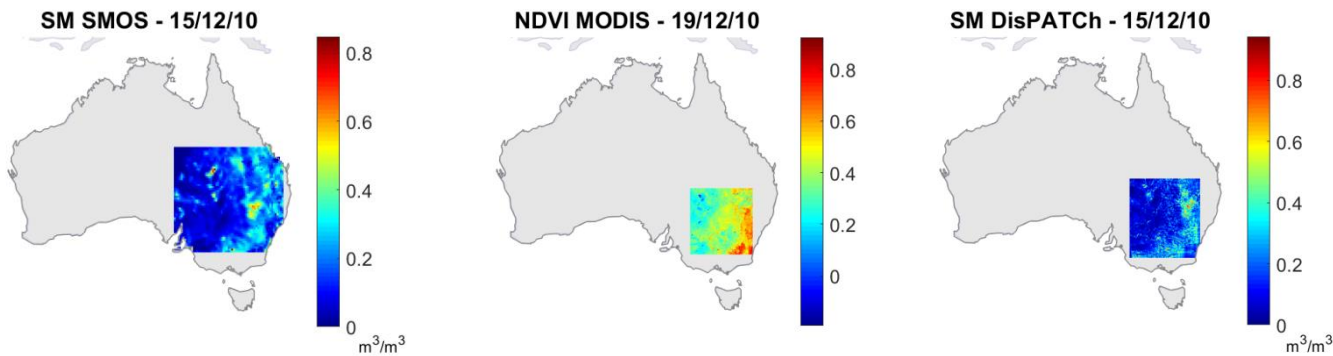563 2006). For simplicity, we'll consider in the following that both DisPATCh and MODIS products present

Figure 2. Sub-images selected for each satellite product over the Murray Darling Basin.

564 a grid spacing of around 1 km.

565

566 Table.1 summarizes the main characteristics of our preprocessed satellite dataset. Two

567 important observations should be highlighted. First, considering their daily revisit time, few DisPATCh

568 SM and MODIS LST images are retained over the full June-December period: only 12 maps for

569 DisPATCh and around 70 maps for MODIS LST. This is directly related to the significant number of

570 missing values that is in average 30 % in these two types of products. Therefore, missing values in

571 downscaled SM seem to be mostly generated by those in LST products, probably due to the presence

572 of clouds in the data. Then, another point concerns the different surface areas of the preprocessed

573 products. Because they do not fully overlap, SMOS and DisPATCh sub-images may capture different

574 SM dynamics. Extreme events occurring in northern MDB are observed in SMOS data whereas it may

575 not be taken into account in DisPATCh data. However, we ensured that all products did have the

576 widest area in common, focusing on irrigated regions in the middle-south part of the basin (like

577 Murrumbidgee).

578

PRODUCTS	Revisit (day)	Effective resolution (km)	Grid spacing (km)	Surface area (km²)	Number of images (with NaN% < 40%)	Average NaN rate (%)
SM DISPATCH	1	1	1	1024 x 1024	12	32
SM SMOS	1	40	25	1600 x 1600	203	3
LST Aqua-MODIS	1	1	≈ 1	950 x 950	60	28
LST Terra-MODIS	1	1	≈ 1	950 x 950	82	26
NDVI Terra- MODIS	16	1	≈ 1	950 x 950	13	10

19

Table 1. Main characteristics of satellites products analyzed in this study. We also mentioned the surface area, the number of images conserved, and the average rate of missing values (without sea areas) of the dataset after preprocessing.

5   Results

580 *5.1   Spatial power spectra*

581   Figure 3 shows the mean power spectra estimated over the full period (June-December 2010) of

582 the different input and output products involved in DisPATCh (it represents an average spectrum

583 based on individual spectra obtained within the period). Each spectrum is plotted in log-log

584 coordinates, with horizontal axis converted into space scale l ($= k^{-1}$), expressed in kilometers.

585 Considering SMOS SM and MODIS products, the mean spectra are found to be scaling over the entire

586 range of scales. This is observed by a linear evolution of $log(E(k))$ (Eq.9), with coefficients of

587 determination R² greater than 0.9 for each spectrum (Table.2). Note that R² is used as a measure of

588 the goodness-of-scaling, estimated from the linear regression between $log(E(k))$ and $log(l)$.

589 However, a different behavior is noticed for the disaggregated SM spectrum. Two scale ranges seem

590 to appear, with an increasing slope on scales lower than about ten kilometers. A segmentation

591 algorithm was applied on this spectrum (D'Errico, 2017), which confirmed a scale break at $l \approx$ 10 km.

592 According to the different values of spectral slopes obtained (Table.2), a three-group classification

593 was proposed:

594 ● β ≈ 1: *SMOS SM, MODIS vegetation index and disaggregated SM (l > 10 km)*

595   For these three products, the negative slope is found to be close to one. Though, according to

596   Eq.10, this may reveal the conservative nature of the fields ($H \approx$ 0). Moreover, these values are

597   quite similar to the estimates proposed in literature: Lovejoy et al. (2008) found β = 1.2 for both

598   vegetation and soil moisture indexes (from MODIS products, Guadalajara, central Spain, July

599   2006). Previous studies on topography, especially on volcanic surfaces (Laferrière and Gaonac'h,

600   1999), found comparable results with quite low degree of fractional integration. Since

601   topography can affect the spatial distribution of SM and vegetation (Kim and Barros, 2002b), it is

602   not surprising to observe similar scaling behavior between these fields.

603 ● 1 < β < 2: *MODIS surface temperature (from both Aqua and Terra satellites)*

604   LST spectra have β values greater than 1. Here, surface temperature seems to correspond to a

605   non-conservative field (H > 0). These spectral slopes may be comparable to those obtained in

606   literature on precipitation fields (Lovejoy and Schertzer, 2008), showing possible connections

607   between the spatial distribution of surface temperature and that of rainfall, and therefore with

608   the underlying (turbulent) atmospheric dynamic (Schmitt et al., 1993).

609 ● β > 2: *disaggregated SM (l < 10 km)*

610   On small scales, DisPATCh SM spectrum presents a relatively large slope, reflecting a high degree

611   of fractional integration (H > 0.5). To our knowledge, such high value of spectral exponent has

never been observed in previous studies on SM fields. However, comparable scaling was

613 obtained on SM time series, revealing spectral slopes greater than 2 (Katul et al., 2007).
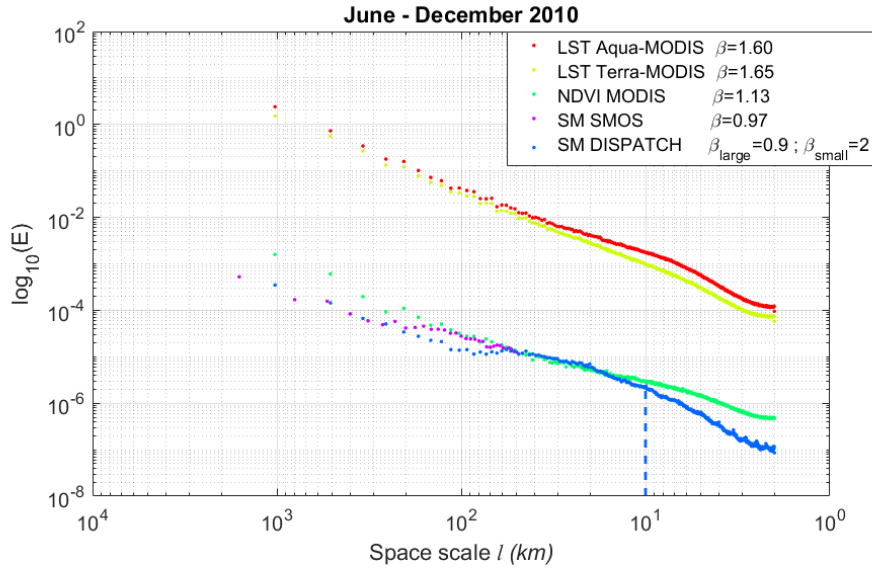

Figure 3. Mean angle-integrated power spectra of DisPATCh related products (over the full June-December period). β_{large} and β_{small} refer to disaggregated SM spectral exponents obtained respectively from scales $l > 10$ km and scales $l < 10$ km.

614 From these spectral observations, a similar scaling seems to appear between the original SMOS SM

615 and the disaggregated SM on scales greater than 10 km, but this behavior is found to change for

616 scales lower than about ten kilometers. A comment may also be made on LST power spectra and

617 their linear regressions : although R^2 coefficients present good values on the entire range of scales (>

618 0.9), a scale break may be observed at about the same spatial scale found for DisPATCh spectrum (l

619 ≈ 10 km). The scale break seems less pronounced but it could be related to that of DisPATCh. This

620 point will be discussed in section 6.2.

621 This twofold scaling regime of DisPATCh SM can be also observed on each specific date of the

622 study period (with R^2 coefficients greater than 0.9 on almost all images and on both scale ranges).

623 Figure 4a shows the time series of the individual spectral exponents estimated for all products (i.e.

624 spectra computed for each image). From June to December, a significant difference of β values is

625 observed between the two scale ranges of disaggregated SM. For example, on July 9 (Fig.4b), power

626 spectra are found to be similar as mean ones presented above (Fig.3). In particular, the same scale

627 break is still observed for disaggregated SM at about ten kilometers. Another remark concerns the

628 amplitude of the scale break according to seasons. Figure 4a shows that, for disaggregated SM, the

629 difference between the spectral exponents of small scales and large scales (respectively blue triangle

630 and blue star symbols) is more important during the last three months of the period. At small scales,

631 the spectral slope suffers a drastic change from around 1.9 (Jun-Jul-Aug-Sept) to 2.3 (Oct-Nov-Dec,

632 i.e. spring and early summer in Australia). The amplitude of the scale break observed in DisPATCh SM

633 could be related to the seasonal conditions of the study area. This will be discussed in section 6.1.
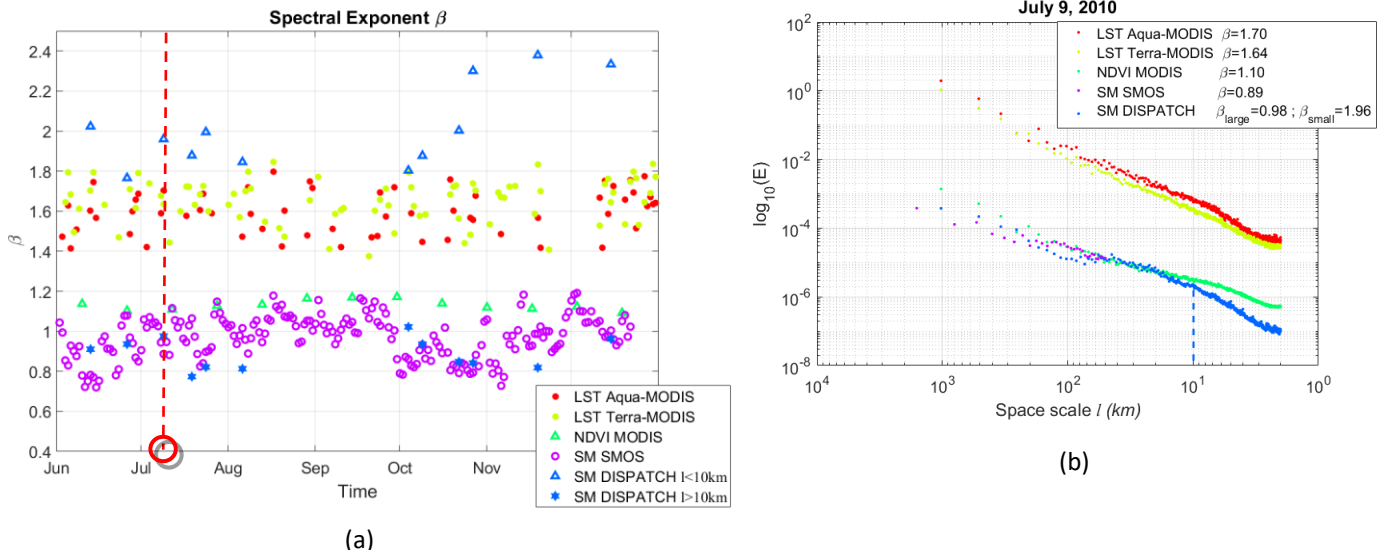

Figure 4. (a) Time series of spectral exponents over the period June-December 2010, (b) angle-integrated power spectra obtained on July 9, 2010.

634

5.2   Multifractal analysis

636   The moments of the normalized absolute gradients were estimated at all accessible resolutions.

637 Since divergence for q greater than $q_D \approx 3$ was reported in most of the literature (Hubert et al.,

638 2007), and because of sample size limitations, in this study moments were computed for orders set

639 from 0 to 3, in steps of 0.1. Figure 5 shows the mean moments over the 7-month period, plotted in

640 log-log coordinates as a function of the space scale l ($= \lambda^{-1}$). For each product, multifractal regimes

641 are identified on specific scale ranges. The power-law described by Eq.3 is well verified over these

642 spatial scales, corresponding to a linear variation of $log(M_l^q)$ for all orders of moments $\langle \diamondsuit_\lambda^q \rangle = M_l^q$.

643 This behavior means that a multifractal model is well adapted on the corresponding scale ranges.

644   Considering vegetation and temperature MODIS products, a scaling regime is found on scales

645 greater than 8 km (Fig.5a-c). On these scales, moments curves were fitted by linear regression (red fit

646 lines on Fig.5), and the corresponding scaling functions $K(q)$ were computed (red, yellow and green

647 curves Fig.5e). UM parameters were then estimated applying (derivative-free) minimization method

648 between empirical scaling function $K(q)$ and the model form of $K(q)$ described in Eq.5. For the

649 vegetation, parameters values are found to be α = 1.74 and C_1 = 0.03 (Table.2). They are quite close

650 to those estimated by Lovejoy et al. (2008) on similar NDVI MODIS products (α = 2 and C_1 = 0.06). For

651 Aqua surface temperature, we found the same parameter values as the vegetation ones (α = 1.7 and

652   C_1 = 0.03), which is related to the very similar $K(q)$ functions for all orders q. Slightly different

653   parameters are found for Terra products (α = 1.91 and C_1 = 0.04). This difference could be due to the

654   different acquisition time of the two satellites (10:30 for Terra and 13:30 for Aqua). This may have

655   some effect on the multiscaling behavior of surface temperature. Another reason to this difference

656   could be the larger scaling regime considered for Terra: a multifractal behavior is observed on scales

657   ranging from 8 km to 1024 km, against 8 km to 300 km for Aqua and NDVI products. Anyway, these

658   results confirm (NDVI) and reveal (LST, not yet studied at this time) the multifractal properties of the

659   considered MODIS products. In both cases, they are characterized by a high degree of multifractality

660   (α is close to 2, value corresponding to the log-normal case) and by a low dispersion of the field (C_1 <

661   0.1).

662   SMOS SM products show good multifractal behavior too: moments are found to be well

663   fitted (R^2 = 0.99, cf. Table.2), on most of the aggregation scales (apart from the 2 greatest scales,

664   1600 km and 800 km). Scaling function was computed over spatial scales going from the 25 km

665   observation scale to 400 km (purple curve in $K(q)$ graph, Fig.5). Compared to MODIS products, a

666   growing divergence is noticed between SMOS and NDVI/LST scaling functions, especially for orders q

667   greater than 1. This scaling behavior is confirmed by different UM parameters: α = 1.46 and C_1 = 0.16.

668   To our knowledge, no application of the UM model has already been made on remotely sensed SM

669   from passive microwaves. Therefore, it is difficult to compare these results with literature. However,

670   although they didn't use the UM model, Kim and Barros (2002b) studied spatial scaling properties of

671   passive microwave SM, estimated from airborne L-band radiometer (Southern Great Plains

672   Experiment 1997, USA). They observed a multifractal scaling on a similar scale range (1.6 km to 250

673   km), which is coherent with our results. Lovejoy et al. (2008) indeed applied the UM model, but on

674   an optical SM index, estimated from MODIS reflectances (Lampkin and Yool, 2004). They found α = 2

675   and C_1 = 0.05 over lower spatial scales (0.5 km to 25 km). These parameter values are quite different

676   from ours. The different scale range and the different study area (Guadalajara, central Spain, in

677   Lovejoy et al., 2008) between their work and ours could be a possible explanation to this result.

678   Another reason might be linked to the nature of the signal studied. Optical-estimated indexes, like

679   MODIS SM index, are more sensitive to land cover such as vegetation (Fabre et al., 2015; Haubrock et

680   al., 2008), then "polluting" the scaling properties of SM.
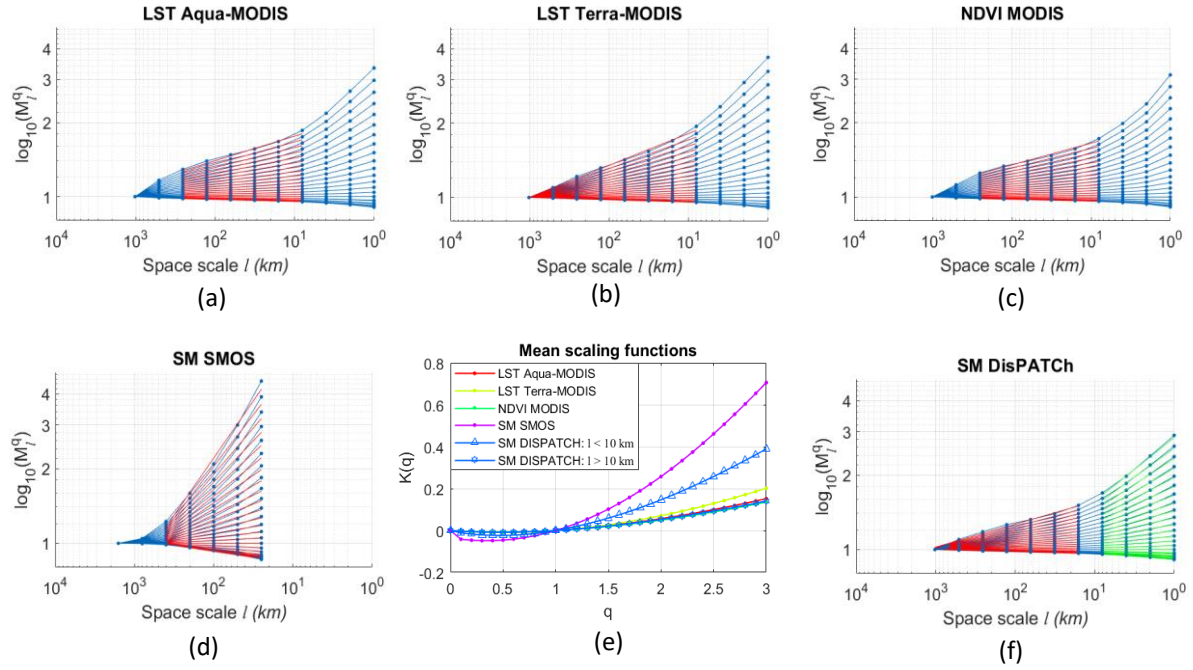
Figure 5. Mean moments as a function of space scale l in km (a-d; f) and mean scaling functions (e) of DisPATCh related products (over the full June-December period). Scaling regimes are distinguished and fitted with the straight lines (linear regressions on moments graphs).

681

682   Focusing now on disaggregated SM products, a change of slope is noticed for each of the
683   statistical moments. The same segmentation algorithm was applied on all moment curves, revealing
684   a scale break at about ten kilometers. Two multifractal scaling regimes may be observed here,
685   confirming the twofold scaling behavior found in the power spectra. Considering larger scales ($l > 10$
686   km, red fit lines on Fig.5), estimated UM parameters are: α = 1.64 and C_1 = 0.03. They are close to the
687   parameters found for our MODIS products (NDVI and LST), with a high degree of multifractality and a
688   low dispersion of the field. For smaller scales ($l < 10$ km, green fit lines on Fig.5), the degree of
689   multifractality is almost unchanged (α = 1.59) compared to the large scales regime. However, the
690   dispersion parameter is increased (C_1 = 0.09), which is three times the value obtained on greater
691   scales. Though, the difference between the two multifractal scaling regimes seems to be mainly
692   linked to the dispersion of SM through scales. If we refer to the multifractal analysis of MODIS SM
693   index made by Lovejoy et al. (2008), our estimates are coherent considering α (for both ranges of
694   scales) and C_1 (on large scales). Lovejoy et al. (2008) didn't notice any scale break, therefore it is
695   difficult to comment our estimate of C_1 at small scales. Nevertheless, Kim and Barros (2002b)
696   observed a similar scale break (at about the same 10 km scale) on passive microwave SM. Indeed,
697   they noticed two scaling regimes from variance, spectra and moments graphs. The twofold scaling

698 behavior of DisPATCh SM products looks consistent with the scale break identified first by Kim and

699 Barros (2002b).

PRODUCTS	Grid spacing (km)	Scale range (km)	β	R^2_{β}	α	C_1	$R^2_{K(q)}$	H	R^2_H
SM DISPATCH	1	$[1-10]$	2.01	0.97	1.59	0.09	0.97	0.45	0.99
		$[10-1024]$	0.89	0.92	1.64	0.03	0.96	0.15	0.98
SM SMOS	25	$[25-1600]$	0.97	0.94	1.46	0.16	0.99	0.29	0.98
LST Aqua-MODIS	≈ 1	≈ $[1-1024]$	1.60	0.98	1.7	0.03	0.96	0.26	0.98
LST Terra-MODIS	≈ 1	≈ $[1-1024]$	1.65	0.99	1.91	0.04	0.95	0.31	0.99
NDVI Terra- MODIS	≈ 1	≈ $[1-1024]$	1.13	0.98	1.74	0.03	0.96	0.15	0.99

Table 2. Scaling parameters obtained from multifractal analysis over the period June-December 2010. R² coefficients were estimated from linear regressions on the specified scale range. Note that $R^2_{K(q)}$ is the average of the coefficients obtained on every moment curves.

6   Discussion

6.1   A physically-explained twofold scaling behavior of soil moisture?

702 Since SM variability is impacted by several environmental factors (Brocca et al., 2007; Crown et

703 al., 2012), the scale break observed on disaggregated SM could be the result of processes acting at

704 different space scales. At finer scales ($l < 10\ km$), spatial structure of SM is governed by infiltration

705 or runoff, which are mainly related to the soil properties (texture, structure) (Hawley et al., 1983;

706 Famiglietti et al., 1998). On the other hand, at larger scales ($l > 10\ km$), SM variability is more

707 affected by evapotranspiration processes (Mohanty and Skaggs, 2001) or precipitation (Jackson et al.,

708 1999).

709 A similar scale break at ~ 10 km was also noted by Kim and Barros (2002b) based on power

710 spectra and statistical moments of SM, estimated from airborne L-band radiometer (Southern Great

711 Plains Experiment 1997, USA). SM retrievals were obtained at 1 km nominal resolution from the $\mathcal{T}\text{-}\omega$

712 model (Jackson and Schmugge, 1991) which depends on Vegetation Water Content (VWC) estimates

713 based on NDVI. They observed that the relationship between the spatial structure of SM and

714 landscape characteristics was strongly modulated by the wetness of the soil. Indeed, they applied an

715 EOF analysis (Empirical Orthogonal Function) between SM and auxiliary data which are topography,

716 VWC and soil content. This revealed that SM was much correlated to topography during rain events,

717   whereas stronger correlation with vegetation (water content) was noticed during drier periods

718   (mainly   governed   by   evapotranspiration   processes).

719   These results are interesting since other research studies also observed similar scale break in the

720   case of precipitation products obtained from radar at 1 km resolution (southeastern France, Gires et

721   al., 2011). Indeed, a transition in spectra and moments was noticed at about twenty kilometers (not

722   far from our 10 km scale break). However, some limitations relative to radar data acquisition must be

723   taken into account considering these results. Indeed, constraints due to algorithmic processing

724   (change from polar to Cartesian coordinates, impact of missing data, temporal integration…) and to

725   physics (attenuation by rainfall, etc.) may impact the scaling properties of precipitation radar images.

726   Moreover, the Z-R relationship between radar reflectivity and rain rate (Marshall and Palmer, 1948)

727   remains somehow controversial, with a non-robust parameterization from a multi-scale point of view

728   (Verrier et al., 2013). Thus, in this context, the scale break detected by Gires et al. (2011) may not be

729   as relevant as it could be. However, they also analyzed the multifractal behavior of simulated

730   precipitations generated on the same area, at ~ 2 km resolution, from the Meso-NH atmospheric

731   model (Lafore et al., 1997). The analysis revealed the presence of a comparable scale break at about

732   30 km, which tends to show that this transition scale in precipitation data is not an artifact. Since

733   rainfall is an important forcing of SM, it may be thought that a break in the rainfall spectra would

734   affect the SM, in a more significant way when the rain event is important. Moreover, a theoretical

735   model of SM in the time domain was proposed by Katul et al. (2007) to relate the scaling of

736   precipitation to that of SM. The spectral exponents of these two variables were found to be

737   connected over time scales finer than 7 days, through the simple equation: $\beta_{SM} = \beta_P + 2$ (with β_{SM}

738   and β_P the negative spectral slopes of respectively soil moisture and precipitation time series).

739   Despite these results were observed on time series, it may corroborate the possible dependence

740   between the SM variability and that of heavy rainfall, even in the space domain.

741   Considering seasonal variations, the power spectra of disaggregated SM seem to reveal a

742   pronounced twofold scaling behavior especially during spring and early summer (October to

743   December period). Since DisPATCh images are mainly located over the middle-south part of the

744   Murray Darling Basin, climate is then mostly temperate. Therefore, the last months of the study

745   period correspond to a drier landscape. Thus, the two scaling regimes seem to be even more distinct

746   when the soil is drier. To demonstrate this effect, the spatial mean of DisPATCh SM ($\mu(SM)$) and the

747   absolute difference $\left|\beta_{large} - \beta_{small}\right|$ were computed for each disaggregated image. In Figure 6, the

748   normalized anomalies of these two variables are in line with this hypothesis (blue and red circle

749   symbols): a more pronounced twofold scaling behavior seems to be found on the driest days (Oct-

750   Nov-Dec). Kim and Barros (2002b) noticed a similar behavior, with lower scaling differences during

rain events (observed on both spectra and moments of SM). In certain dates, they even noticed that there was no scale break at all, corresponding to very high wetness conditions of the soil. Moreover, we estimated the position of the scale breaks on each power spectrum during the period (corresponding normalized anomalies plotted in gray star symbols, Fig.6). Although it is positioned on average around 10 km (not shown here but the mean value over the full period was estimated at ~ 13 km), the transition scale between the two scaling regimes seems to follow a decreasing trend as
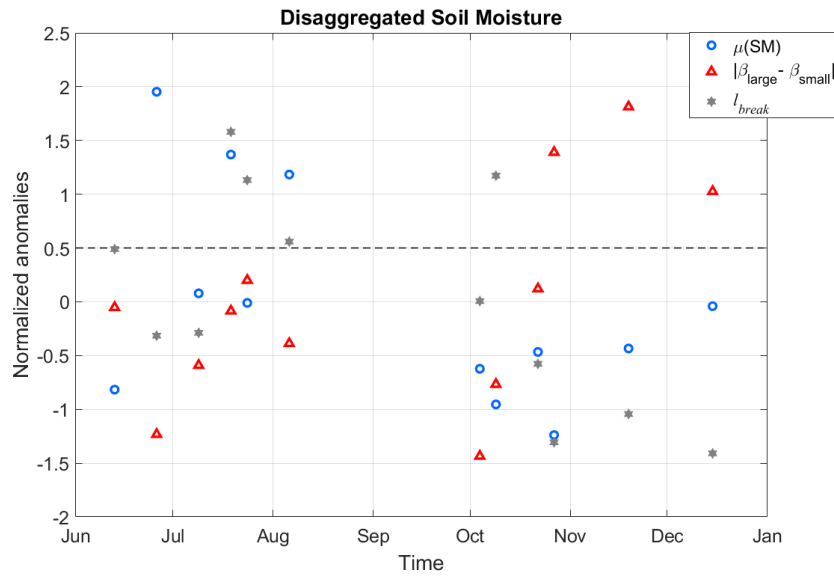
Figure 6. Time series over the period June-December 2010 of the normalized anomalies of the three following variables: the spatial mean of each DisPATCh SM product; the absolute difference between the two spectral exponents β_{large} and β_{small} (estimated respectively for $l > 10\ km$ and $l < 10\ km$); the position of the scale breaks estimated on each spectrum by the segmentation algorithm. The dotted line differentiates the two regimes: wetter trend from June to September and drier trend from October to December.

the soil is drying, with estimated scale breaks ranging from ~ 15 km in wet period to ~ 12 km in dry period. A comparable behavior was observed by Kim and Barros (2002b), showing that the position and the amplitude of the scale break in the scaling behavior of SM is dependent on the state of SM, and thus on the hydrometeorological conditions like rain, evapotranspiration and infiltration processes.

To go further on the dependences between seasons and SM scaling, Kim and Barros (2002b) observed that multifractality was almost always involved on scales smaller than 10 km, whatever the dryness of the soil. However, on scales greater that 10 km, multifractality was found to become monofractality, especially during drier conditions. At large scales (between 25 km and 400 km), a comparable effect was noticed on our SMOS SM products (purple circles, Fig.7): the multifractality

768   index α is decreased from around 1.6 (June) to 1.3 (December), which may reflect a moderate
769   decrease of multifractality during the study period. Therefore, multifractal properties of SM at large
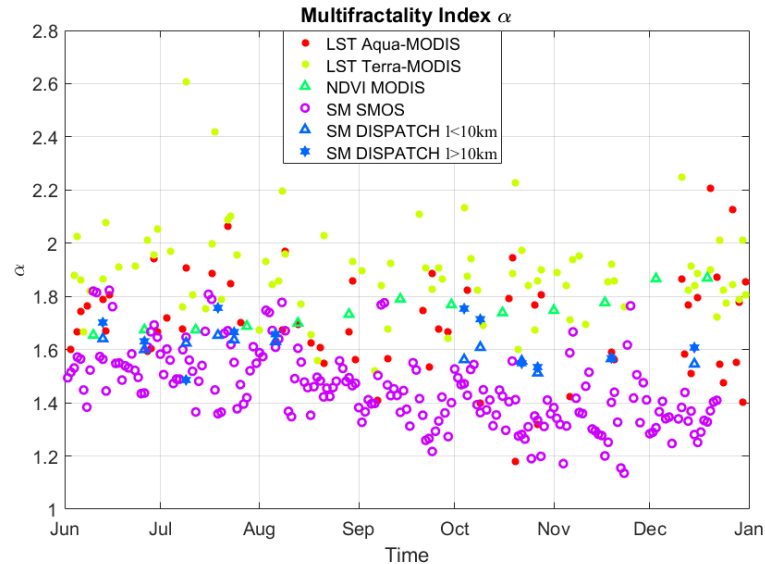770   scales seem to be related to the soil dryness. This may give complementary explanations to the

Figure 7. Time series of the multifractality index α, over the period June-December 2010.

771   twofold scaling behavior of SM. On the other hand, considering DisPATCh SM, a rather constant
772   evolution of α is noticed on both small scales (blue triangle symbols) and large scales (blue star
773   symbols). The first case confirms the idea that multifractality is not dryness-dependent on smaller
774   scales, whereas the second is in contradiction with this assumption. Thus, the latter should be
775   considered cautiously to explain the scaling properties of SM.

776

6.2   A model-induced twofold scaling behavior of soil moisture?

778   In relatively recent works (Mascaro et al., 2010; Mascaro and Vivoni, 2012), scale invariance and
779   multifractality were noticed from SM products measured from airborne L-band radiometers (Southern
780   Great Plains 1997 and 1999 Experiments, USA). In these studies, a Log-Poisson multifractal model was
781   applied (She and Levêque, 1994), and a single scaling regime was observed on statistical moments,
782   from 0.8 km to 25.6 km scales. Although this result confirms the multifractal properties of SM on
783   space scales similar to ours, it refutes the existence of two scaling regimes. No scale break was
784   observed at about ten kilometers. Since the Log-Poisson model is based on a similar universal theory
785   as the UM model (continuous cascades), it is somewhat unexpected not to detect the same transition
786   on comparable SM products (same technology and same scale range).

787   To investigate if this difference could be related to the case study (different areas or periods), we

788   compared our DisPATCh products to fine scale airborne data acquired during the AACES-2 mission

789   (Peischl et al., 2012). This mission was performed in September 2010, during which transect flights

790   were carried out over the Murrumbidgee catchment. Brightness Temperatures (BT) were acquired

791   from L-band radiometer (on both H- and V-polarizations), at a nominal 1 km spatial resolution. The

792   study area was divided in 5 patches of 50×100 km², each corresponding to a single flight day (13, 16,

793   19, 21, 22 September). We gathered these patches into one single BT image, and we selected a sub-

794   image of 128×128 km². To verify the presence of two scaling regimes in the data, we applied spectral

795   analysis on both H-polarized and V-polarized BT sub-images. In Figure 8, the power spectrum of H-

796   polarized BT was compared to the power spectra of DisPATCh related products on equivalent period.

797   The spectra of each satellite product available between the 13 and 22 of September were averaged

798   together. Since no DisPATCh SM products were pre-selected on this period (because of too many

799   NaN), we chose the nearest available product, which corresponds to 4 October. In Figure 8, one single

800   linear fit is observed on BT power spectrum, over the entire scale range (from 1 km to 128 km) and

801   with a spectral slope equivalent to that of SMOS SM spectrum (β ≈ 1). Note that V-polarized spectrum

802   was not plotted here, but it was found to be very similar to the H-polarized one.
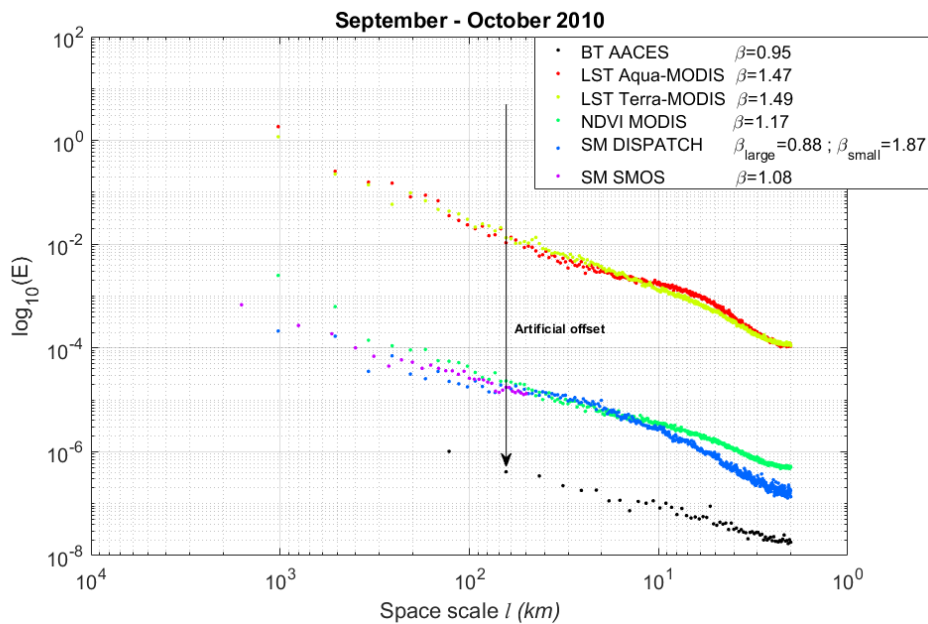

Figure 8. Angle-integrated power spectra of Brightness Temperature (BT) and DisPATCh related products, obtained on the period September-October 2010. For better visualization and comparison, BT power spectrum was shifted down (black arrow on the graph).

803   Different scaling behaviors were noticed for AACES BT and DisPATCh SM, on similar area and

804   similar period. This may sustain the idea that the scale break observed at 10 km could be caused by

805   the DisPATCh model and, specifically, by the way in which the multi-scale properties of each product

806   are mixed in the algorithm. To verify this hypothesis, *a simplified version of the C4DIS processor* was

807   implemented in order to study the multi-scale behavior of the different variables combined and

808   generated through the algorithm. To do this, the method proposed by Molero et al. (2016) was

809   followed, which includes the two main steps described in section 4.2: (1) the estimation of SEE_{HR}

810   variable (Soil Evaporative Efficiency) from MODIS products (Eq.13-15) and (2) the proper

811   disaggregation process of SM from SMOS products, SEE_{HR} and SM_p parameter (Eq.17-18).

812   According to this method, our algorithm was applied on SMOS and MODIS products acquired on

813   November 19, 2010. A sub-area was selected (≈ 700 x 700 km²) in order to have a smaller number of

814   missing data, and thus to get the minimum impact of gap-filling on the studied products. Figure 9

815   shows the power spectra obtained from the input products of DisPATCh (LST, NDVI), intermediate

816   products (T$_s$, SEE) and output product (SM MEAN). The latter product is the average of the 6

817   disaggregated SM images obtained from the 6 SM-LST combinations (see section 4.1). Here, just one

818   SMOS image of ≈25 km grid spacing was combined with the MODIS products. Indeed, both cases with

819   one SMOS image and four downsampled ones were implemented (section 4.1 and Molero et al.,

820   2016), and no significant differences were observed between the final products and between their

821   power spectra. Therefore, for simplicity of implementation, only the case of one SMOS image was

822   considered here. For comparison, the power spectrum of C4DIS SM product acquired on the same

823   date and on the same sub-area was also plotted here. The segmentation algorithm used in section 5

824   was applied on each power spectrum. A geometric mean was estimated from the different scale

825   breaks obtained, revealing two averaged scale breaks which are nearly common to all spectra: the

826   first at almost ten kilometers (l = 9 km) and the second at about thirty kilometers (l = 33 km). To

827   evaluate the link between the multi-scale behavior of each product, spectral exponents were

828   estimated on the two following scale ranges: from 33 km to 9 km (*large scales*) and from 9 km to 1

829   km (*small scales*). Comparing our SM MEAN product with SM C4DIS product (Table.3), a very similar

830   scaling is observed on *large scales* (β_{large} ≈ 1.3). On *small scales*, high spectral exponents are found,

831   with β_{small} ≈ 2 for SM MEAN and β_{small} = 2.86 for SM C4DIS. These different spectral slopes on finer

832   scales could be related to the non-implementation of some filtering steps in our algorithm which are

833   indeed coded in C4DIS processor: corrections of topography effects, filtering LST data with low
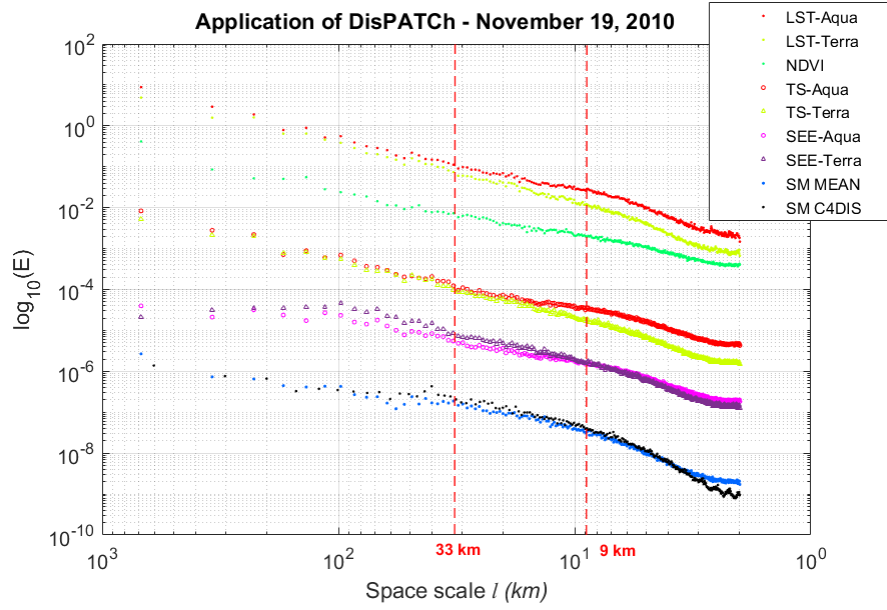
834   quality, etc. (Molero et al., 2016).

Figure 9. Angle-integrated power spectra of some of the input, intermediate and output products obtained from our implementation of DisPATCh on November 19, 2010. Power spectrum of the final C4DIS product is also plotted here. For better visualization and comparison, the positions of power spectra according to Y axis were modified.

835

836   Despite these small differences between SM MEAN and SM C4DIS spectra, the scale break
837   remains noticeable, as it seems to be on the other products of the algorithm. To demonstrate this,
838   the absolute difference $\Delta\beta = |\beta_{large} - \beta_{small}|$ was computed as an indicator of the amplitude of
839   the scale break. Values greater than 0.6 were found for LST (Aqua and Terra), T_s (Aqua) and SEE
840   (Aqua) products. These results seem to reveal that MODIS LST products would be the cause of the
841   scale break located at about ten kilometers in the disaggregated SM product. This scale break would
842   propagate in the algorithm through the estimation of T_s and SEE. A possible explanation to this scale
843   break in LST products may be related to the physical nature of the signal used. Indeed,
844   optical/thermal sensors can be characterized by modified spectral slopes near the satellite
845   resolution. Moreover, this effect seems more important on Aqua LST ($\Delta\beta$ = 0.83) than on Terra LST
846   ($\Delta\beta$ = 0.62). Similar differences can be observed between the mean power spectra of Aqua and Terra
847   LST over the full period (Fig.3 in section 5.1). Thus, it may be thought that the amplitude of the scale
848   break could be related to the diurnal cycle of surface temperature. Since surface temperatures
849   measured from Aqua are acquired at the hottest hours of the day (13:30), there might be a
850   correlation between the amplitude of scale break and the level of surface temperature.

Scale range (km)	[33 - 9]		[9 - 1]		Δβ
Spectral exponent	β	$R^2{}_\beta$	β	$R^2{}_\beta$	
LST - *Aqua*	1.08	0.97	1.91	0.98	0.83
LST - *Terra*	1.39	0.97	2.01	0.98	0.62
NDVI	0.93	0.94	1.22	0.98	0.29
TS - *Aqua*	0.90	0.95	1.55	0.98	0.65
TS - *Terra*	1.33	0.98	1.79	0.98	0.46
SEE - *Aqua*	0.88	0.96	1.61	0.98	0.73
SEE - *Terra*	1.25	0.97	1.83	0.98	0.58
SM MEAN	1.22	0.96	2.08	0.97	0.86
SM C4DIS	1.29	0.96	2.86	0.98	1.57

Table 3. Spectral exponents obtained from spectral analysis of DisPATCh related products on November 19, 2010 (Fig.9). R² coefficients were estimated from linear regressions on the specified scale range. $\Delta\beta = |\beta_{large} - \beta_{small}|$ is used as an indicator of the amplitude of the scale break, with *large* and *small* referring respectively to $[33 - 9]$ km and $[9 - 1]$ km scale ranges.

851

852   Considering the scale break observed at about 30 km, this one may not be related to the multi-
853   scale properties of MODIS products but possibly to the combination of different products defined on
854   different grid spacings. Indeed, DisPATCh algorithm combines and creates products which have
855   either the grid spacing of MODIS data (≈1 km) or the grid spacing of SMOS data (≈25 km). For
856   example, the estimation of SEE (Eq.15) combines end-members ($T_{s,max}$ and $T_{s,min}$) defined on the
857   SMOS grid, with another product (T_s) defined on the MODIS grid. As seen on Figure 10, a footprint of
858   SMOS pixels is then observable on the resulting image of SEE. This property is due to the resampling
859   strategy of SMOS data and to the end-members that are defined on the SMOS grid. This systematic
860   footprint is visible in the real domain but can also have an impact in the Fourier domain. Indeed,
861   sharp transitions at the SMOS pixels limits may create spurious convolutions by cardinal sine-like
862   functions which may affect the spectrum. On the disaggregated SM, this effect can generate an
863   imperfect transition between the part of the spectrum related to SMOS SM ($l > 25$ km) and the part
864   related to MODIS products ($l < 25$ km). Regarding the behavior of SEE power spectra on scales
865   greater than thirty kilometers (Fig.9), a lower spectral slope is observed (β ≈ 0.5) comparing to that
866   obtained on finer scales (β ≈ 1 for $[33 - 9]\ km$). This could be related to the oversampling of SMOS

867 data, generating harmonics on fine scales and therefore not including variability on large scales. In
868 this manner, not only large scales but even fine scales could be affected by this effect. The latter may
869 also contribute, in a way, to the accentuation of the spectral drop observed at finer scales on the
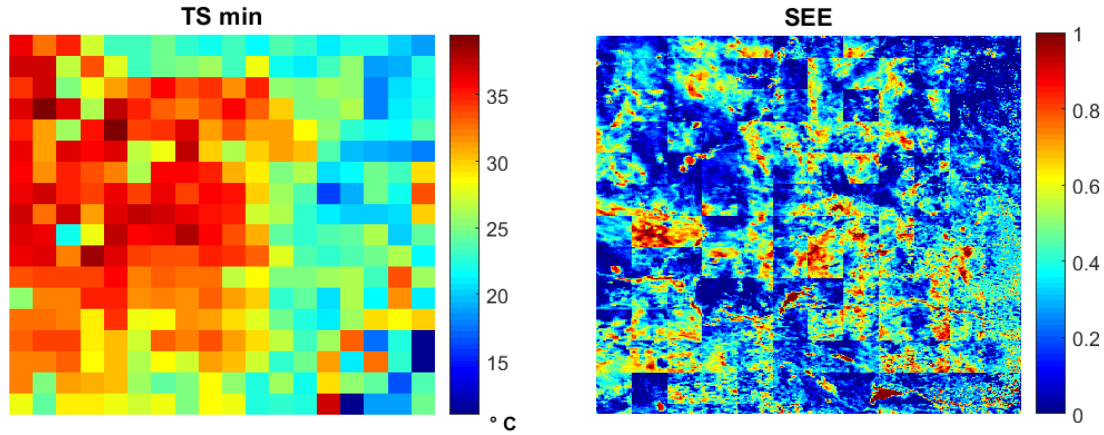870 final disaggregated SM product.

Figure 10. Images corresponding to $T_{s,min}$ and SEE_{HR} products obtained from our implementation of DisPATCh on November 19, 2010.

871

872

873 ## 7   Conclusion

874 During the last century, several studies were carried out to investigate the scaling properties of
875 SM. Very diversified technologies were used to access and study the spatial structure of SM: airborne
876 microwaves products, satellite optical indices, etc. Moreover, different approaches have been
877 considered, such as power spectra, statistical moments, fractal dimensions, and even different types
878 of cascade models (Log-Poisson, Universal Multifractal, and even no explicit parameterization in
879 some cases…). In this study, we analyzed the multifractal behavior of remotely sensed SM products
880 over space scales ranging from the kilometric field scale to the continental scale. Universal
881 Multifractal model was applied for the first time on SMOS SM data, giving access to large scale
882 variabilities of SM, over the Australian landscape. Fractal and multifractal properties were observed,
883 which confirmed and completed some results reported in existing literature.

884 The relevant aspect of the present work may be the multi-scale analysis of the outputs of the
885 disaggregation algorithm DisPATCh (Merlin et al., 2008a; Molero et al., 2016). This deterministic
886 algorithm improves the space resolution of SMOS SM products from 40 km to 1 km. To do this, it

combines coarse-scale SMOS SM with fine-scale (≈1 km) MODIS optical/thermal data. Although several validation studies have been realized on this downscaling method (Malbéteau et al., 2016; Merlin et al., 2012, 2013, 2015; Molero et al., 2016), none fully explored its statistical behavior over a continuum of space scales. In this context, we applied fractal and multifractal analysis on the different products involved in DisPATCh algorithm, including disaggregated (and original) SM products, and MODIS auxiliary data which are vegetation indices (NDVI) and surface temperatures (LST).

Input products of DisPATCh revealed relatively good scaling properties over the considered scale ranges. Indeed, NDVI, LST and original SM were characterized by a power law evolution of their power spectra and statistical moments, meaning respectively fractality and multifractality. However, a specific scaling behavior was noticed for the output disaggregated SM. Two scaling regimes were obtained, with a transition scale observed at about ten kilometers, on both spectra and moments. Considering spectral analysis, on large scales ($l > 10$ km), disaggregated SM was found to have the same scaling as the original SM measured from satellite. On finer scales ($l < 10$ km), a different behavior was noticed, with an increasing slope of the power spectrum. Similar scale break was detected on statistical moments, showing that both spectral and multifractal properties of DisPATCh SM are characterized by this twofold scaling signature.

Two possible arguments were given to explain the specific scaling of the disaggregated SM. First, a more physical interpretation may indicate that this twofold scaling behavior would be related to the real properties of SM. As it was previously observed by Kim and Barros (2002b), such scale break would be reflective of nonlinear hydrometeorological processes (rainfall, infiltration, evapotranspiration) acting at different space scales and modulated by terrain, soils and vegetation distributions. The spatial structure of SM may be more impacted by infiltration or runoff at the field scale, whereas it would be mainly controlled by evapotranspiration or precipitation at the regional/continental scale. A more significant scaling transition was observed on the driest days (early summer), which may support the link between SM and external forcing agents such as precipitation.

A second explanation would be more algorithmic and directly related to the processing of the different products used and created within the algorithm. The model used in DisPATCh would generate SM whose statistics are not properly distributed across scales. This may occur at two levels in the algorithm. First, some MODIS products properties (such as breaks in the scaling) may be retrieved in the final DisPATCh products. Indeed, a spectral drop at about the same ten kilometers scale was detected on LST power spectra. Although it is less pronounced than on disaggregated SM,

920 this scale break may be introduced by MODIS LST and amplified by the disaggregation model. Since
921 one single scaling regime was noticed on Brightness Temperature (BT) products acquired in the L-
922 band over the same area and the same period, these observations suggest that the unexpected
923 scaling in MODIS products would be caused by the technology specific to optical/thermal sensors.
924 Then, another impact of the algorithm on the multi-scale properties of SM may be related to signal
925 processing artifacts occurring with the combination of several products defined with different grid
926 spacings. This combination is required to permit conservativity between input and output SM
927 products. However, from a signal processing point of view, this could create systematic footprints on
928 the final image (i.e., visible SMOS pixels in the downscaled products) and therefore affect the power
929 spectrum (convolutions by cardinal sine-like functions).

930   At this point, it is difficult to determine which of the physical or algorithmic factors would be at
931 the origin of this twofold scaling behavior. Though, a plausible hypothesis may be that both factors
932 could affect the scaling of disaggregated SM. Indeed, a scale break at about the SMOS SM resolution
933 could be initially produced by combination artifacts, which would be more or less amplified in the
934 algorithm according to seasonal conditions, resulting in moving the scale break during the period to
935 finer scales.

936   Further work need to be addressed to fully explain these results, in particular to determine to
937 what extent each of the two factors impacts the scaling of DisPATCh SM. Complementary auxiliary
938 data should be compared to our products. Indeed, an EOF or comparable analysis made on DisPATCh
939 SM and topography, vegetation water content or soil content would provide relevant information
940 about the connection between the spatial variability of these products and help with interpretation.
941 Moreover, it would be interesting to verify if precipitation products can be characterized by a similar
942 scale break on equivalent space scales and over the same area (Murray Darling Basin). However, it
943 must be considered that such a comparison might be complex to interpret since, to our knowledge,
944 no theoretical model has been proposed yet to relate the spatial scaling properties of SM and that of
945 rainfall (as it was already done in the time domain by Katul et al., 2007). In the same way, the
946 comparison between DisPATCh SM and airborne BT is not that trivial, because relatively complex
947 operations are involved to get inverted SM from BT. To illustrate this, Mascaro and Vivoni (2012)
948 noticed monofractality from BT data, whereas multifractality was observed from the corresponding
949 inverted SM data. The scaling properties of BT could be affected during the inversion process,
950 explaining why the single scaling we observed on BT does not imply single scaling of DisPATCh SM.
951 Therefore, multifractal analysis of proper fine scale SM products may clarify this idea and help
952 validating DisPATCh SM variability.

953   In the hypothesis of a model-induced scale break, current work is engaged to quantify the effect

954   of MODIS products and the trace of pixel SMOS on different dates and on proper operational

955   conditions (analysis of products used and generated within the C4DIS processor). An application of

956   DisPATCh using Landat-7 auxiliary data instead of MODIS products was realized by Merlin et al.

957   (2013), allowing a disaggregation process at sub-kilometric scales (100 m). Since Landsat-7 provides

958   optical/thermal data with higher resolution than MODIS, it could be interesting to verify if both

959   Landsat-7 and the resulting disaggregated SM product would be characterized by a similar scale

960   break, but shifted on finer scales than the 10 km scale observed on MODIS. Therefore, the results

961   obtained could help to quantify the real impact of optical/thermal auxiliary data on the multi-scale

962   properties of DisPATCh SM. On a more operational point a view, if this impact is confirmed, the

963   results obtained may help to define a specific scale below which the variability generated by the

964   disaggregation model may not be as reliable as it should be. Concerning the impact of SMOS pixels

965   footprint effects on the disaggregated product, a solution could be to filter out the sharp transitions

966   at SMOS pixels limits. However, this should be done with caution since such filters may excessively

967   attenuate the variance at smaller scales. Another way to investigate our observations of DisPATCh

968   SM is to focus on its dynamical behavior over different aggregation scales. Indeed, one of the main

969   problem in downscaling a dynamical behavior arises from the fact that the dynamical behavior of an

970   aggregated signal can be approximated by the same deterministic equation structure only when the

971   aggregated area is phase-synchronized (Mangiarotti et al., 2016). Considering this issue, the

972   applicability of deterministic downscaling methods like DisPATCh may not be that obvious over

973   certain spatial scales, leading to several difficulties and, perhaps, contributing to explain the scaling

974   irregularities observed in this study.

975   Finally, a possibility could be to compare the SM variability produced by DisPATCh with that

976   created by fractal stochastic downscaling methods. Based on scaling properties, these methods

977   preserve the probability distribution from large to fine scales. In precipitation, several studies applied

978   these algorithms on rainfall data (Rebora et al., 2006; Sharma et al., 2007). Research works proposed

979   methods developed on multiplicative cascade such as Log-Poisson (Deidda, 2000) or UM model

980   (Gires et al., 2012), revealing some potential to quantify uncertainty and representativeness errors

981   between coarse-scale and in-situ measurements. Concerning SM downscaling, some studies used

982   such fractal-based methods (Bindlish and Barros, 2002; Kim and Barros, 2002a; Mascaro et al., 2010).

983   In our study, it may be interesting to apply this kind of method on SMOS products. This would

984   consists in injecting in the UM model the values of α and C_1 parameters obtained from SMOS

985   products on large scales, and then continuing the cascade at higher resolutions. Following this

986   procedure, the fine-scale field will have the same scaling properties as the coarse-scale one.

987   However, since the disaggregation is based on random generator, an ensemble of possible fields can

988   be proposed from just one pair of α/C_1 parameter. Therefore, this kind of methodology may not be

989   fully suitable in the case of operational hydro-agricultural applications, in particular when

990   determining the position of the extremes. To overcome this inconvenient, a combination of the two

991   approaches may be an interesting compromise between statistical scaling and evaporation-based

992   determinism. For example, in DisPATCh algorithm, an idea might be to find a modified estimator of

993   SEE that would be used in the disaggregation equation (18). This modified SEE would be computed by

994   applying a 2D filter on the original SEE, which would be actually equivalent to perform a fractional

995   integration of order $\Delta H = H_{requested} - H_{non\text{-}filtered}$ with $H_{requested}$ and $H_{non\text{-}filtered}$ measured respectively

996   from SMOS soil moisture (at large scales) and from non-filtered SEE (for scales under 10 km). Doing

997   this, the spectral slope of SEE may be adjusted, like that of the final disaggregated soil moisture.

998   Thus, coarse-scale and fine-scale fields could be related through a common degree of fractional

999   integration, which may contribute to limit the twofold scaling behavior observed on the

1000   disaggregated product. In practice, this modification would not be easy to implement since the

1001   filtering should be properly dimensioned in order to affect only the small scales, between 1 and 10

1002   km. Moreover, this texture-based image correction may impact the physical properties of SEE, so

1003   there would be a compromise to be made on this aspect.

1004

1016   **References**

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