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# 1 **Multi-scale statistical properties of disaggregated SMOS soil moisture** 2 **products in Australia**

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## 5 **Highlights**

- 6 ● Fractal and multifractal properties were observed on remotely sensed soil moisture products  
7 acquired from SMOS satellite (Soil Moisture and Ocean Salinity), over space scales ranging  
8 from the kilometric field scale to the continental scale
- 9 ● Two scaling regimes were noticed for the soil moisture data disaggregated with DisPATCH  
10 algorithm (Disaggregation based on Physical And Theoretical scale Change), with a scaling  
11 break observed at about ten kilometers
- 12 ● Fractality and multifractality were also found on remotely sensed vegetation indices and  
13 surface temperature

## 14 **Abstract**

15 Soil moisture has a strong impact on climate, hydrology and agronomy at different space scales,  
16 from the continent global scale to the local watershed. Passive microwave sensors, like SMOS  
17 satellite (Soil Moisture and Ocean Salinity), allow a global study of soil moisture on the entire globe.  
18 To have access to kilometric variability, disaggregation algorithms have been developed, such as the  
19 Disaggregation based on Physical And Theoretical scale Change (DisPATCH). This method improves  
20 the space resolution of SMOS soil moisture from 40 km to 1 km. To do this, it combines coarse-scale  
21 ( $\approx 40$  km) SMOS products with fine-scale ( $\approx 1$  km) optical/thermal data. Validation studies on specific  
22 scales showed the potential of DisPATCH to enhance the spatio-temporal correlation of  
23 disaggregated SM with in-situ measurements, under low-vegetated semi-arid regions. Although the  
24 efficiency of the method was revealed in these regions, no studies fully explored its statistical  
25 behavior over a continuum of space scales. In this paper, we studied and compared the spatial multi-  
26 scale statistics of the different input and output datasets involved in DisPATCH downscaling. To do  
27 this, we applied spectral and multifractal analysis on the respective products for the region of  
28 southeastern Australia, from June to December 2010. Fractal and multifractal properties (in the  
29 framework of the Universal Multifractal model) were observed on inputs of DisPATCH (SMOS soil  
30 moisture, MODIS vegetation indices and surface temperature), which confirmed and completed

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31 some results reported in existing literature. For the output disaggregated soil moisture, two scaling  
32 regimes were observed, with a transition scale observed at about ten kilometers. Considering  
33 spectral analysis, at large scales (> 10 km), disaggregated soil moisture was found to have the same  
34 scaling as the original SMOS soil moisture. On finer scales (< 10 km), a different behavior was noticed,  
35 with a higher value of the slope of the power spectrum. The same scale break was detected on  
36 statistical moments, showing that both spectral and multifractal properties of DisPATCh soil moisture  
37 are characterized by this twofold scaling signature.

## 38 **Keywords**

39 Soil moisture; Multi-scale analysis; Multifractals; Disaggregation; SMOS; DisPATCh

## 40 **1 Introduction**

41 Soil moisture (SM) is a key component of the climate system and is strongly heterogeneous, at  
42 many time and space scales. Interactions between land surface and atmosphere, such as water,  
43 energy and carbon fluxes, are strongly related to SM (Ochsner et al., 2013). It has a significant role in  
44 the water cycle as it impacts runoff, infiltration and evaporation processes. Thus, SM is an important  
45 variable in several scientific fields such as hydrology (Western et al., 2004), meteorology (Dai et al.,  
46 2004), climatology (Anderson et al., 2007) and water resource management (Engman, 1991).

47 SM is heterogeneously distributed at different space scales, from few centimeters to several  
48 kilometers. This variability is due to environmental factors impacting directly SM at specific scale  
49 ranges (Brocca et al., 2007; Crow et al., 2012; Jana, 2010; Vereecken et al., 2014). For instance, we  
50 could mention here soil properties (texture and structure) acting at the field scale, topography  
51 features at the watershed scale, land cover (vegetation) and meteorological forcing at the regional  
52 and continental scales.

53 Many ground measurement techniques have been developed to acquire highly resolved SM data  
54 sets, down to centimeters in space and minutes in time (for more details see Dobriyal et al., 2012;  
55 Robinson et al., 2008; Robock et al., 2000). Although these methods are recognized as reliable and  
56 easy to implement, they are not adapted to represent spatial heterogeneity of SM at regional and  
57 continental scales (Collow et al., 2012; Crow et al., 2012).

58 Regional and global scale variability of SM may be acquired and studied with the help of remote  
59 sensing. Different active and passive microwave satellites allow daily measurement of surface soil  
60 moisture in the first 5 cm of the soil column (Petropoulos et al., 2015; Wigneron et al., 2003). These  
61 satellites acquire SM information thanks to the relationship between the soil dielectric constant and

62 water content. Active microwave sensors measure the energy reflected by the soil after sending a  
63 microwave pulse to the surface (backscatter): we find C-band Synthetic Aperture Radars (SAR) like  
64 Sentinel 1 satellite (S1) from European Space Agency ([Wagner et al., 2009](#)) and C-band  
65 scatterometers like the Advanced Scatterometer (ASCAT; [Bartalis et al., 2007](#)). These active sensors  
66 can provide space resolution from few meters (S1) to tens of kilometers (ASCAT). Their main  
67 drawback is their sensitivity to vegetation and surface roughness, which can alter useful information  
68 (SM) in the signal measured. Passive sensors, however, are less sensitive to scattering conditions.  
69 They measure the self-emission from the land surface (radiances). Good results were obtained by C-  
70 and X- band radiometers like the Advanced Microwave Scanning Radiometer-Earth observing system  
71 (AMSR-E; [Njoku et al., 2003](#); [Owe et al., 2001](#)), or by L-band radiometers such as Soil Moisture and  
72 Ocean Salinity (SMOS; [Kerr et al., 2010](#)) and the recent Soil Moisture Active Passive (SMAP) mission  
73 ([Entekhabi et al., 2010a](#)).

74 L-band microwaves (1.4 GHz) have the benefit of little sensitivity to vegetation, providing optimal  
75 estimation of SM on a wider range of land cover conditions. L-band based satellite missions deliver  
76 SM products with a revisit time of 2 to 3 days. However, because of technological constraints, the  
77 spatial resolution is coarse (30-55 km), much coarser than the kilometer scale. This is a problem since  
78 hydro-agricultural applications need better resolved information, below kilometric space scales  
79 ([Walker and Houser, 2004](#)).

80 To address this issue, downscaling methods have been created to improve the low spatial  
81 resolution of satellite data. Downscaling algorithms are characterized by their input data (satellite  
82 products, auxiliary ground measurements, etc.) and by the type of method (physical or statistical).  
83 [Peng et al. \(2017\)](#) reviewed the several methods developed so far and proposed a three-group  
84 classification: satellite-based methods, methods using geo-information data and model-based  
85 methods.

86 The first group (I) gathers downscaling techniques which combines satellite passive microwave  
87 products with satellite highly resolved auxiliary data, such as radar or optical/thermal observations.  
88 This takes advantage of the assets of complementary remote sensing measurement techniques.  
89 Considering the fusion with high resolution radar, a change detection method was proposed by  
90 [Njoku et al. \(2002\)](#) to merge coarse-scale passive microwave soil moisture products and fine-scale  
91 active backscatter data. This technique consists in the linear relationship between soil moisture and  
92 backscatter data assuming the time-invariance of vegetation and surface roughness effects. The  
93 methodology was further tested in other experiments ([Narayan et al., 2006](#); [Piles et al., 2009](#)) and  
94 proved its efficiency for improving the spatial details of soil moisture. Statistical tools were also used

95 to combine active and passive products, such as Bayesian merging method (Zhan et al., 2006) or  
96 wavelet-based image enhancement method (Montzka et al., 2016). This kind of approach showed  
97 the great potential of radar for improving soil moisture resolution, in particular for higher vegetation  
98 water content and different land cover types (Akbar and Moghaddam, 2015). A possible limitation of  
99 this approach is the time lag between active and passive data, due to the low revisit rate of high  
100 resolution radar. Recently, SMAP satellite was launched to bypass this problem, embedding on board  
101 one radiometer and one radar (Das et al., 2011). Unfortunately, the radar failed and no  
102 active/passive combination could be performed. However, the previous studies made to prepare the  
103 mission showed good capacity to improve spatial resolution of satellite products by merging active  
104 and passive microwave data. Another type of satellite-based method is the combination of passive  
105 microwave data with optical and thermal remote sensing data. The interest is to have the additional  
106 information of high spatial resolution and short revisit time of the optical/thermal products. The  
107 concept is to use highly resolved vegetation cover and surface temperature products to downscale  
108 coarse-scale soil moisture product. Based on the surface temperature/vegetation index triangular  
109 feature space proposed by Carlson (1994, 2007), Zhan et al. (2002) and later Chauhan et al. (2003)  
110 developed and applied this method based on a polynomial function linking high resolution SM with  
111 surface temperature, vegetation cover and surface albedo. At coarse resolution, scaling factors  
112 (regression coefficients) are estimated from this polynomial function and then used at high  
113 resolution in the same function to calculate the high resolution SM using NDVI (Normalized  
114 Difference Vegetation Index) and LST (Land Surface Temperature) obtained from the LST/NDVI  
115 feature space. An improved version was proposed by Piles et al. (2011) using brightness  
116 temperatures instead of albedo, showing better results when comparing downscaled SM with in-situ  
117 measurements. For instance, this downscaling technique was used to improve the resolution of  
118 AMSR-E soil moisture merging it with optical/thermal data measured from MODIS (Moderate  
119 resolution Imaging Spectroradiometer) (Choi and Hur, 2012) or MSG-SEVIRI (Meteosat Second  
120 Generation Enhanced Visible and Infrared Imager) (Zhao and Li, 2013). The main problem in this  
121 methodology is the non-conservativity of SM between fine-scale and coarse-scale SM. Based on the  
122 same theory, other downscaling algorithms were proposed to relate the downscaled SM with coarse  
123 observations of SM. An operationally implemented method is the downscaling algorithm DisPATCh  
124 (Disaggregation based on Physical And Theoretical scale Change; Merlin et al., 2008a; Molero et al.,  
125 2016). This algorithm is more physical because it uses soil evaporation processes to connect  
126 optical/thermal and SM data. Different applications of DisPATCh were realized to increase the ~40  
127 km resolution of SMOS SM to 1 km and even 100 m respectively with MODIS (Merlin et al., 2012) and  
128 Landsat-7 (Merlin et al., 2013) products. The originality of the method is the estimation of a SM proxy  
129 called Soil Evaporative Efficiency (SEE; sections 4.2 and 6.2). The latter has the advantage, compared

130 to land surface temperature or evapotranspiration, to be more linked to SM and to be quite constant  
131 during the day. Some improvements still need to be made about the modelling of SEE, especially on  
132 elevation and illumination effects (Malbêteau et al., 2017) or soil properties and atmospheric  
133 conditions (Merlin et al., 2016). Comparable evaporation-based methods were developed using  
134 different proxies of SM such as the Soil Wetness Index (Kim and Hogue, 2012) or the Vegetation  
135 Temperature Condition Index (Peng et al., 2016), both applied in the simple downscaling method  
136 UCLA. We can also mention algorithms directly improving the resolution of brightness temperature  
137 products (instead of retrieved SM), based on the relation between daily temperature change and  
138 daily average SM (Song et al., 2014). Generally, these downscaling methods present a significant  
139 asset considering the time coherence between the merged products, but some limitations exist.  
140 Indeed, the cloud sensitivity of optical/thermal sensors makes the application of these methods  
141 possible only under clear-sky conditions (Djamai et al., 2016).

142 Since SM is directly linked to geoinformation data such as topography, soil properties and  
143 vegetation attributes (Werbylo and Niemann, 2014), a second group (II) of downscaling methods  
144 were also proposed. These methods take advantage of highly resolved geoinformation data (giving  
145 information on the local attributes of the zone studied), and could give access to very high spatial  
146 resolution of SM. Topography for example was often used in downscaling approaches as an auxiliary  
147 data (Busch et al., 2012; Pellenq et al., 2003). However, certain types of geoinformation data, like soil  
148 properties, are usually provided by ground observations, which are really specific to the studied area.  
149 Thus, the application is limited to local areas and it may not be suitable for global scale study of SM.

150 The third class (III) of methods concerns model-based downscaling techniques. There are two  
151 types of models used here. On the one hand, there are hydrological (land surface) models. These  
152 ones are more site-specific because they try to link coarse-scale remotely sensed SM and fine-scale  
153 parameters obtained from local land surface models. The downscaling can be done through  
154 optimization techniques (Ines et al., 2013), linear regressions (Loew and Mauser, 2008) or bivariate  
155 relationships (Verhoest et al., 2015). On the other hand, there are models that analyze and describe  
156 statistics across scales: they are more generic and try to preserve statistical properties across scales.  
157 For example, Kaheil et al. (2008) proposed a wavelet-based downscaling method in order to model  
158 spatial statistical properties of fine-scale SM thanks to coarse-scale airborne SM products. Other  
159 approaches are based on the scaling (or fractal) properties of SM across spatial scales. Bindlish and  
160 Barros (2002) proposed a fractal interpolation method applied on airborne SM products, measured  
161 from Electronically Scanned Thinned Array Radiometer (ESTAR). They used power spectra to  
162 represent the fractal behavior of SM, and could improve spatial resolution from 200 m to 40 m. A  
163 few years later, Mascaro et al. (2010) applied Log-Poisson multifractal cascades on remote sensing

164 SM to generate simulations of fine-scale SM. The challenge here is to preserve non-stationarity from  
165 coarse to fine scales. Nevertheless, particular efforts are made to overpass this problem. For  
166 example, [Kim and Barros \(2002a\)](#) adapted the fractal interpolation method applying a sliding window  
167 on specific parts of the original field. They could simulate fractal variability while taking into account  
168 the local statistics of the field.

169 The downscaling methods of the three groups presented above have their own advantages and  
170 disadvantages, with more or less efficiency according to specific surface or climate conditions. In this  
171 study, we focus on the evaluation of multi-scale variability of SM products generated by the method  
172 DisPATCH ([Merlin et al., 2008a](#); [Molero et al., 2016](#)). Despite its limitations related to cloud cover,  
173 this semi-physical downscaling algorithm combines low sensitivity to vegetation of L-band  
174 microwaves, high spatial resolution of optical/thermal data and it is dispensed from estimation errors  
175 commonly generated by land surface models. Several studies have been realized so far to evaluate  
176 and validate this method ([Malbêteau et al., 2016](#); [Merlin et al., 2013, 2015](#); [Molero et al., 2016](#)). In  
177 general, the assessment of downscaling algorithms is made comparing fine-scale output products  
178 with ground measurements. Different performance metrics are used, such as correlation, root mean  
179 square error or bias ([Albergel et al., 2013](#); [Al Bitar et al., 2012](#); [Entekhabi, 2010b](#)). More recently,  
180 [Merlin et al. \(2015\)](#) proposed a new metric that estimates the gain given by the downscaling method  
181 in terms of representativeness of downscaled data compared to non-downscaled data. To take into  
182 account scale mismatch between downscaled and ground measurements, upscaling techniques have  
183 been developed in order to bring downscaled and ground data together at common space scales  
184 ([Crow et al., 2012](#)). For example, [Merlin et al. \(2013\)](#) applied the DisPATCH algorithm on SMOS data  
185 while using both MODIS (Moderate resolution Imaging Spectroradiometer) and Landsat-7 auxiliary  
186 data. Coarse-scale satellite data, downscaled data and aggregated ground measurements were  
187 compared at three different scales: 40 km, 3 km and 100 m. Good results confirmed the potential of  
188 DisPATCH to improve the spatio-temporal correlation of remotely sensed SM with in-situ  
189 measurements. However, the drawback of these validation techniques is that they are restricted to  
190 specific scales. Thus, the validation of disaggregated SM products over a continuum of space scales  
191 has not been fully explored yet. Investigation of the multi-scale statistics and of possible scaling  
192 properties of these products could provide relevant information on this aspect.

193 During the last thirty years, several studies were carried out to describe the statistical properties  
194 of SM across spatial scales ([Famiglietti et al., 2008](#); [Rodriguez-Iturbe et al., 1995](#)). Different analytical  
195 methods were proposed. The most commonly used are spectral-wavelet analysis ([Si, 2008](#)) and  
196 multifractal analysis ([Kim and Barros, 2002b](#); [Mascaro et al., 2010](#); [Oldak et al., 2002](#)). In 1995,  
197 [Rodriguez-Iturbe et al.](#) highlighted for the first time the fractal behavior of SM from remote sensing:



198 the spatial variance of SM followed a power law decay as a function of aggregation scales ranging  
199 from 30 m to 1 km (Washita Experiment 1992, USA). Later studies showed that such a scaling  
200 behavior of SM variance could be extended to wider range of scales: up to regional scale (Hu et al.,  
201 1997) and even to continental scales (Rötzer et al., 2015). Similar research works demonstrated that  
202 increasing area extent (increasing size of the total area) induced the increase of SM variance  
203 according to a power law function (Famiglietti et al., 2008; Rötzer et al., 2015; Brocca et al., 2012).  
204 Moreover, in Oldak et al. (2002), the fractal scaling of SM was revealed to be multifractal: the power  
205 law was also applicable to the first six statistical moments of airborne SM products for scales ranging  
206 from hundreds of meters to tens of kilometers (Washita'92 Experiment and Southern Great Plains  
207 Experiment 1997, USA). Multifractal scaling was then detected in SM fields (Das and Mohanty, 2006;  
208 Kim and Barros, 2002b; Lovejoy et al., 2008; Mascaro et al., 2010). Since SM variability is directly  
209 related to the amount of soil wetness (Brocca et al., 2007; Famiglietti et al., 2008), it may be  
210 expected that scaling properties of SM may vary according to the state of SM. Indeed, when plotting  
211 SM variance power law in log-log coordinates, Rodriguez-Iturbe et al. (1995) and Manfreda et al.  
212 (2007) found that the corresponding slope of the curve was increased during drier periods, revealing  
213 seasonal variations of SM scaling (Rötzer et al., 2015). Moreover, it was observed that SM variability  
214 was not governed by a single scaling behavior, but by different scaling regimes depending on the  
215 range of scales. At the field scale, SM variability is mainly related to land surface characteristics such  
216 as soil properties or topography, whereas at larger scales it is impacted by meteorological quantities  
217 like rainfall or evapotranspiration (Cayan and Georgakakos, 1995; Entin et al. 2000). Studies based on  
218 semi-variograms (Ryu and Famiglietti, 2006; Korres et al., 2015) and spectral/moments analysis (Kim  
219 and Barros, 2002b) revealed the presence of scale breaks closed to this transition scale between land  
220 surface and meteorological regimes. Though, the aforementioned characteristics of SM highlight its  
221 complexity and its high degree of nonlinearity due to hydrometeorological processes acting at  
222 different space scales, attesting the necessity to better understand the scaling behavior of SM for  
223 applications such as data assimilation or downscaling (Rötzer et al., 2015).

224 In this paper, we propose an alternative and complementary method for verifying the  
225 multiscaling behavior of DisPATCH products. To do this, we studied and compared the statistical  
226 spatial properties across scales of the downscaled SM, the original SMOS SM and the MODIS auxiliary  
227 data, by applying spectral and multifractal analysis in the framework of the Universal Multifractal  
228 (UM) model (Schertzer and Lovejoy, 1987). The definition of multifractal formalism is given in section  
229 2, with a particular attention paid to UM parametrization. The methodology followed for multifractal  
230 (and spectral) analysis is detailed in section 3. Section 4 describes the case study and the data set.  
231 Then, the different results obtained from spectral and multifractal analysis are presented in section



232 5. Finally, section 6 proposes explanations to the multiscaling behaviors of DisPATCh SM, and a  
233 general conclusion of this study is given in section 7.

## 234 **2 Theory of multifractals**

235 During the last century, several studies showed that many geophysical processes could present  
236 scale invariance properties. This was first anticipated by [Richardson \(1922\)](#) in the case of turbulence:  
237 he described turbulent flows as cascade processes that transfer kinetic energy from large to small  
238 scales. Based on this approach, statistical models of turbulence were proposed such as the famous  
239 Kolmogorov law (1941) to describe velocity increments. Later research works generalized the study  
240 to take into account the heterogeneity of the energy flux ([Kolmogorov, 1962](#); [Obukhov, 1962](#);  
241 [Yaglom, 1966](#)). Multi-scale models such as multiplicative cascades were therefore proposed to  
242 reproduce scale invariance properties through the use of fractal geometry. Later, scale invariance  
243 was noticed in other geophysical fields: in his study of the coast of Britain, [Mandelbrot \(1967\)](#)  
244 revealed the presence of fractal properties in topography.

### 245 *2.1 From fractal sets to multifractal fields*

246 The concept of fractal dimension has been used in many works related to multi-scale analysis and  
247 geophysical modelling. Indeed, the term “fractal” refers to any entity (time series or 2D/3D random  
248 field) in which each part presents similar properties, geometrically or statistically, to the ensemble. In  
249 this manner, the structure of a fractal entity is characterized by scale invariance. Initially, the notion of  
250 fractal was introduced in the late 19<sup>th</sup> century in geometry with the creation of sets, i.e. mathematical  
251 objects, having unusual properties, especially a non-integer Hausdorff dimension, called later by  
252 Mandelbrot as “fractal dimension” ([Mandelbrot, 1967](#)). Scale invariance, in the statistical sense, was  
253 theoretically proposed by Kolmogorov in 1940 with the introduction of the fractional Brownian  
254 motion. This model could generate random time series whose trajectories present fractal properties  
255 in terms of statistical distribution. It illustrates the physical interest of fractal random processes, since  
256 Brownian motions are somewhat ubiquitous in physics. [Mandelbrot and Van Ness \(1968\)](#) made it  
257 famous by introducing it to more physical models. In particular, the first fractal stochastic models of  
258 topography were developed based on this theory ([Mandelbrot, 1975](#)).

259 These stochastic models aim to represent the simple scaling (monofractal) behavior of  
260 geophysical processes. In this context, the fractal dimension or scaling parameter is assumed to be  
261 unique, restricting multi-scale modelling to a specific class of variability. However, most geophysical  
262 processes are characterized by more complex statistics. In case of operational hydrology, rare and  
263 extreme events, present in precipitation or soil moisture for example, correspond to high order

264 statistics and need to be detected (Hubert et al., 1993). Therefore, multifractal models, characterized  
 265 by an infinite spectrum of fractal dimensions, have been proposed to account for a more exhaustive  
 266 set of statistics. Schertzer and Lovejoy (1987), based on the findings of Parisi and Frisch (1985),  
 267 initially established the multifractal formalism through the fundamental equation:

$$268 \quad Pr(\Phi_\lambda > \lambda^\gamma) \approx \lambda^{-c(\gamma)} \quad (1)$$

269 where  $\Phi_\lambda$  is a positive normalized random scalar process, time series or random field defined on  $R^2$   
 270 or  $R^3$ . The mean of the process is assumed to be statistically conserved across scales.  $\lambda$  is the  
 271 observation resolution, here defined as the inverse of the scale that can be seen as the sampling time  
 272 or pixel size for time and space domain processes respectively and  $\approx$  indicates an equality within the  
 273 limits of slowly varying functions. Eq.1 expresses the fact that for a multifractal process, the  
 274 probability of exceeding a threshold varies as a power law of the resolution with exponent  $c(\gamma)$ . This  
 275 exponent is called as fractal codimension of the process, depending on the amplitude of thresholds.  
 276 The thresholds are defined by the following power law:

$$277 \quad T_\lambda = \lambda^\gamma \quad (2)$$

278 with  $\gamma$  the notion of singularity, characterizing the amplitude of the process independently of the  
 279 scale. Each singularity is associated to a fractal codimension  $c(\gamma)$ , corresponding to a family of  
 280 thresholds of various amplitudes. From a more physical point of view, high singularities (detected by  
 281 high thresholds) are related to rare and extreme events, with high fractal codimensions and inversely  
 282 low (box-counting) fractal dimensions  $D_f$  (Mandelbrot, 1967). Indeed, the latter are related to the  
 283 dimension of space  $D$  through the relation  $c(\gamma) = D - D_f$ . Therefore,  $c(\gamma)$  can be described as a  
 284 codimension function, increasing with  $\gamma$ , which completely characterizes the multi-scale statistical  
 285 properties of the field  $\Phi_\lambda$ . In general, if the field is multifractal,  $c(\gamma)$  is found to be convex and  
 286 positive (with a fixed point  $C_1$  imposed by the condition of canonical conservation), whereas  
 287 monofractality is associated to the trivial case  $c(\gamma) = const$ .

288 Since probability distributions and statistical moments are related by a Mellin transform,  
 289 Schertzer and Lovejoy (1987) proposed an equivalent equation to (1):

$$290 \quad \langle \Phi_\lambda^q \rangle \approx \lambda^{K(q)} \quad (3)$$

291 where  $\langle \cdot \rangle$  is the statistical averaging operator,  $q$  is the order of the moment ( $q \geq 0$ ), and  $K(q)$  is the  
 292 moment scaling function. Eq.3 expresses that, for any fixed moment order, statistical moments and  
 293 resolution are linked through a power law. Singularities and moment orders are directly linked, since  
 294 the moment scaling function  $K(q)$  is the Legendre transform of the codimension function  $c(\gamma)$ .

295 Similarly to  $c(\gamma)$ ,  $K(q)$  is a convex function (with the special case  $K(1) = 0$  related to the conservation of  
296 the mean across scales), which entirely characterizes the multifractal field.

## 297 2.2 *Multiplicative cascades*

298 Multiplicative cascades are stochastic models that can be used to build multifractal fields.  
299 Cascades are multiplicative processes because they are defined by an iterative multiplicative  
300 construction: considering a two dimensional random signal (field), each pixel at resolution  $\lambda_{n+1}$  (with  
301  $n$  the construction level of the cascade) is the product of the embedding pixel at coarser resolution  
302 ( $\lambda_n$ ) multiplied by a random variable  $\mu\epsilon$ . This is described by the following equation:

$$303 \quad \Phi_{\lambda_{n+1}} = \mu\epsilon \times \Phi_{\lambda_n} \quad (4)$$

304 In this manner, the statistical properties of the field  $\Phi_{\lambda_{n+1}}$  are directly related to the statistical  
305 properties of the coarser field  $\Phi_{\lambda_n}$ . If all the multiplicative random variables used for each step of  
306 the iterative construction are independent and identically distributed, and distributed independently  
307 of the scale, the final field presents scale invariant properties.

308 Several models of cascades have been developed so far. First models were built within the  
309 framework of turbulence, such as the  $\alpha$ -model (Schertzer and Lovejoy, 1984) which corresponds to  
310 discrete construction of cascades: the multiplicative random variables are limited to two possible  
311 fixed values, respectively leading to increasing or decreasing pixel value when the resolution is  
312 refined. Later, more elaborated models were constructed generalizing the discrete case to  
313 continuous cascades (Dubrulle, 1994; Schertzer and Lovejoy, 1987, 1991, 1997; She and Levêque,  
314 1994). The latter are based on an infinite number of steps between any pair of resolutions, leading to  
315 continuity in scale. The benefit of continuous cascades is twofold. First, they can represent possibly  
316 more realistic structures by avoiding any arbitrary discretization of scales. Moreover, they often  
317 converge toward random processes which are characterized by a small number of degrees of  
318 freedom (special cases of log-infinitely divisible distributions). This is interesting considering that  
319 multifractal fields built by multiplicative cascade processes would otherwise need an infinite number  
320 of scaling parameters (one for each fractal dimension). For example, She and Levêque (1994)  
321 proposed a continuous cascade model based on Log-Poisson statistics, and Schertzer and Lovejoy  
322 (1987) used Log-stable random variables to build the Universal Multifractal model. In both models,  
323 only two fundamental parameters are needed to fully define multifractality.

## 324 2.3 *Universal Multifractals*

325 Physically, multifractal fields built by Log-Poisson or Log-stable cascades have a high degree of  
326 generality in geophysics. Log-Poisson model has been successfully applied to different geophysical

327 variables such as rain (Deidda, 2000), or even soil moisture (Mascaro et al., 2010). However, this  
 328 model can have disadvantages of representing a restricted range of variabilities, which may make it  
 329 unsuitable for modeling processes with unbounded singularities. On the other hand, by assuming the  
 330 stability of the random variables and suitable renormalization, UM model is likely adapted for  
 331 characterizing a wide range of processes: topography (Lavallée et al., 1993), rain and clouds (Tessier  
 332 et al., 1993) and more recently soil moisture and vegetation optical indexes (Lovejoy et al., 2008).  
 333 Moreover, a possibly more immediate physical interpretation of the parameters is found in this  
 334 model. For mathematical and physical arguments supporting the universality of UM model, see  
 335 Schertzer and Lovejoy (1997); see also Gupta and Waymire (1997) for discussion about its generality.  
 336 UM model defines the moment scaling function using two “universal” parameters, through the  
 337 following equation (Schertzer and Lovejoy, 1987):

$$338 \quad K(q) = \frac{C_1}{(\alpha-1)} (q^\alpha - q) \quad (5)$$

339 where  $\alpha$  is the degree of multifractality of the field. It varies between 0 (monofractality) and 2 (log-  
 340 normality) and expresses how fast the codimension evolves as a function of the singularity. The  
 341 second parameter  $C_1$  is the codimension giving the dominant contribution to the mean value of the  
 342 field (related to moment of order 1):  $C_1 = K'(1)$ . Physically, it indicates inhomogeneity (dispersion)  
 343 of the field: it varies from 0 (homogeneous field) to the dimension  $D$  of the embedding space (very  
 344 intermittent field). Because of Legendre transform,  $c(C_1) = K'(1)$  is also defined as the fixed point  
 345 of the codimension function.

#### 346 2.4 FIF model

347 Generally, most of the geophysical fields are non-conservative, i.e. integrated processes defined  
 348 by a certain degree of fractional integration. This appellation comes from multifractal cascade  
 349 models: see Gagnon et al., 2006 for detailed explanations on this formalism. Thus, to account for a  
 350 wider range of processes, an extension of the UM model to non-conservative fields has been  
 351 proposed (Schertzer and Lovejoy, 1991): the Fractionally Integrated Flux (FIF) model. It expresses the  
 352 degree of fractional integration of the UM field, using a third parameter  $H$ . The latter is called the  
 353 order of integration and defines the non-conservativity of the field: in plain words, the larger is  $H$ , the  
 354 smoother is the field. The integrated flux is noted  $R_\lambda$  and is characterized by a power law variation of  
 355 its stationary increments:

$$356 \quad \Delta R_\lambda \approx \Phi_\lambda \Delta x^H \quad (6)$$

357 where  $\Delta R_\lambda$  are the increments (fluctuations of the flux) estimated over a varying window  $\Delta x$ , which  
 358 is equivalent to the space scale  $l$ . Note that when  $H = 0$ , the equation corresponds to the

359 conservative case  $\Phi_\lambda$ . Additionally, in the case of two dimensional fluxes, Eq.6 also applies for other  
360 directions (i.e.  $\Delta y$  increments), with the same exponent  $H$  if the process is isotropic.

361 Hereafter in this article, the appellation proposed in [Lovejoy and Schertzer \(2010\)](#) will be  
362 followed: non-integrated cascades will be called conservative fluxes, due to the conservation of the  
363 mean, and fractionally integrated “non-conservative” processes will be called “random fields” or  
364 simply “fields”.

### 365 **3 Multifractal analysis methodology**

366 The different techniques used to analyze the multi-scale properties of DisPATCH related products  
367 are detailed in this section. The methodology is based on the multifractal theory presented in the  
368 precedent section. Because our study treats only satellite images, we will focus on the two  
369 dimensional versions of these techniques.

#### 370 *3.1 Power spectrum: preliminary evidence of scaling*

371 Spectral analysis is a methodology often used in geophysics to characterize, in an easy and rapid  
372 way, some scaling properties of fields over different space scales ([Lovejoy et al., 2008](#)). Thanks to its  
373 high sensitivity to scale breaks, scaling regimes can be easily identified. In a first step, the two-  
374 dimensional power spectral density  $P(k_x, k_y)$  of the data under analysis,  $X$ , is estimated:

$$375 \quad P(k_x, k_y) = |\text{fft}(X)|^2 \quad (7)$$

376 with  $P$  the power spectral density defined on both vertical and horizontal image axis, corresponding  
377 respectively to  $k_x$  and  $k_y$  wavenumbers (spatial frequencies). Here, the estimation of the PSD is done  
378 through a two-dimensional *fft* or Fast Fourier Transform. Then, the one dimensional isotropic angle-  
379 integrated power spectrum  $E(k)$  is obtained ([Lovejoy et al., 2008](#); §8):

$$380 \quad E(k) = \int_{\|\vec{k}\|=k} P(\vec{k}) d\vec{k} \quad (8)$$

381 where  $k$  is the modulus of the wavenumber and  $\|\cdot\|$  is the Euclidean norm. Since it expresses space  
382 frequencies,  $k$  is directly related to the space resolution  $\lambda$ . If the process presents scaling properties,  
383 the spectrum should follow a power law, where  $\beta$  is the negative slope of  $E(k)$  on a log-log graph:

$$384 \quad E(k) \approx k^{-\beta} \quad (9)$$

385  $\beta$  is called the spectral exponent and is directly related to the FIF parameters through the equation:

$$386 \quad \beta = 1 + 2H - K(2) \quad (10)$$

387 In this manner,  $\beta$  also gives first indications about the possible conservative nature of the field, since  
 388 integrated flux ( $H > 0$ ) should correspond to spectral exponent greater than 1. Note that power  
 389 spectrum is a second-order statistic, hence the term  $K(2)$ .

### 390 3.2 Statistical moments: multifractal properties

391 To test the presence of multifractal properties in the data (Eq.3), statistical moments and moment  
 392 scaling function need to be estimated. To do this, different steps must be followed. First, the  
 393 underlying conservative field  $\Phi_{\lambda_{max}}$  has to be reconstructed from the data, at the maximum  
 394 observation resolution  $\lambda_{max}$ . Because the possible existence of a fractional integration of order  $H$   
 395 (Eq.6), a fractional derivative of the same order should be done. In this study, the modulus of the  
 396 gradient was applied to the data. Indeed, this operator provides a simple and good numerical  
 397 approximation of the fractional derivation without prior knowledge of  $H$  order (Lavallée et al., 1993):

$$398 \quad \Phi_{\lambda_{max}} = \sqrt{\left(\frac{\partial R_{\lambda_{max}}}{\partial x}\right)^2 + \left(\frac{\partial R_{\lambda_{max}}}{\partial y}\right)^2} \quad (11)$$

399 Once the conservative field is retrieved,  $\Phi_{\lambda_{max}}$  is normalized by its mean.

400 The second step involves the degradation of the field at lower resolutions  $\lambda < \lambda_{max}$ . It aims to  
 401 approximate the inversion of the stochastic multiplicative cascade by iteratively averaging the field at  
 402 coarser scales: each coarse pixel (level  $n$  of the cascade) is obtained by a simple average of  
 403 neighboring finer pixels (level  $n+1$ ). Note that each roughened pixel size is a power of two multiplied  
 404 greater than the observation scale  $l_{min} (= \lambda_{max}^{-1})$ .

405 Finally, empirical moments (i.e. computing  $q$ -th order moments in Eq.3 while replacing statistical  
 406 averages by empirical averages) are then computed for various orders and resolutions. On a log-log  
 407 graph, the different moments are plotted as a function of the resolution. If linearity is observed for  
 408 each moment curve, at least over a significant range of resolutions, Eq.3 is therefore verified, which  
 409 is the signature of multifractality. The empirical moment scaling function can be estimated, with  $K(q)$   
 410 corresponding to each linear fit of  $q$ th order moment. Afterwards, the universal parameters  $\alpha$  and  $C_1$   
 411 may be obtained by optimization according to the UM model form of  $K(q)$  (Eq.5).

### 412 3.3 Structure functions: some evidence of non-conservativity

413 A convenient way to reveal the non-conservative/fractionally integrated nature of the integrated  
 414 flux  $R_\lambda$  (Eq.6), is to compute its first order structure function:

$$415 \quad \Delta R_\lambda(\Delta x) = \langle |R_\lambda(x + \Delta x) - R_\lambda(x)| \rangle \quad (12)$$

416 If the flux is indeed non-conservative, the order of integration  $H$  should be the slope of the  
417 increments  $\Delta R_\lambda$ , plotted in a log-log graph as a function of space scale  $\Delta x$ . This technique will be  
418 used in this study to estimate the  $H$  parameter.

## 419 **4 Case study and data**

### 420 *4.1 C4DIS processor and satellites products*

421 The different products analyzed in this study are input and output data of C4DIS (CATDS level-4  
422 DISaggregation) processor (Molero et al., 2016). This processor includes the first operational version  
423 of the DisPATCh algorithm, taking into account the best configurations according to the latest studies  
424 (Merlin et al., 2010a, 2010b, 2013). Because the algorithm is still evolving, C4DIS products are called  
425 as “scientific” (Molero et al., 2016): users can have access on demand to the products over specific  
426 areas of the world.

427 As presented earlier, the downscaling method combines SMOS microwave data and MODIS  
428 optical/thermal data. The SM data is given by the SMOS Level-3 daily global SM product (reference:  
429 MIR CLF31A/D). This product is provided by the Centre Aval de Traitement des Données SMOS  
430 (CATDS), which is the French ground segment for SMOS Level-3 and Level-4 products. The SM data is  
431 acquired every day at a radiometric resolution that varies between 35 and 55 km, 40 km in average,  
432 from L-band brightness temperature measurements (Kerr et al., 2012; Wigneron et al., 2007). SMOS  
433 Level-3 products are delivered on the EASE (Equal Area Scalable Earth) grid, with a grid spacing of 25  
434 km  $\times$  25 km.

435 The optical/thermal products come from the MODIS sensor, embedded on both Aqua and Terra  
436 satellites. Two types of auxiliary data are used in DisPATCh. First, there is Land Surface Temperature  
437 (LST). It is extracted from the MODIS Level-3 daily products: MYD11A1 (Aqua) and MOD11A1 (Terra).  
438 These temperature products are estimated from thermal infrared radiances emitted from the surface  
439 (3-15  $\mu\text{m}$ ). Then, the second auxiliary data is Normalized Difference Vegetation Index (NDVI), given  
440 by the Level-3 16-day Terra product (MOD13A2). The vegetation index is computed from surface  
441 reflectances in red (0.7  $\mu\text{m}$ ) and near infrared (0.8  $\mu\text{m}$ ) wavelengths. Both LST and NDVI products are  
442 provided at 1 km resolution by the NASA Land Processes Distributed Active Archive Center (LP DAAC).  
443 They are presented on a sinusoidal grid, with a grid spacing slightly smaller than kilometer: 0.93 km  $\times$   
444 0.93 km (Solano, 2010; Wan, 2006). We may notice that LST products have daily time resolution,  
445 whereas NDVI products are representative of a period of 16 days.

446 Output DisPATCh products are generated every day by the C4DIS processor. Their resolution is  
447 that of MODIS products (1 km), and they are presented on an equal-spaced lat-lon WGS84 grid, with



448 a grid spacing of  $0.01^\circ$  ( $\approx 1.12$  km). For simplicity, in the following we'll make the approximation  $0.01^\circ$   
 449 = 1 km. One single downsampled image is the result of the combination of four downsampled SMOS  
 450 SM images, one MODIS NDVI image, and up to six MODIS LST images corresponding to 3 consecutive  
 451 days of Aqua and Terra acquisitions (for more details on the combination methodology see  
 452 [Malbêteau et al., 2016](#); [Merlin et al., 2012](#); [Molero et al., 2016](#)). In other words, in the final product,  
 453 each high resolution output pixel comes from the average of 24 possible disaggregated pixels (up to  
 454 24 SM-LST possible pairs). The advantage of this composition is that uncertainty in downsampled SM  
 455 can be potentially reduced and estimated, and time-coverage is improved ([Malbêteau et al., 2016](#)).

#### 456 4.2 DisPATCh algorithm

457 DisPATCh is based on a semi-empirical model that estimates the Soil Evaporative Efficiency (SEE)  
 458 from high resolution (HR = 1 km) LST and NDVI products. The method is based on the separation of  
 459 MODIS LST into its soil and vegetation components, respectively referred in this study as  $T_{s,HR}$  and  
 460  $T_{v,HR}$ . To do this, the approach relies on a variant of the trapezoid method from [Moran et al. \(1994\)](#)  
 461 which interprets the feature space defined by MODIS LST and NDVI-derived fractional vegetation  
 462 cover  $f_{v,HR}$ . The purpose here is to extract the soil temperature  $T_{s,HR}$  according to the following  
 463 equations:

$$464 \quad T_{s,HR} = (LST - f_{v,HR} * T_{v,HR}) / (1 - f_{v,HR}) \quad (13)$$

$$465 \quad \text{with } f_{v,HR} = (NDVI - NDVI_{soil}) / (NDVI_{veget} - NDVI_{soil}) \quad (14)$$

466 In equation 13, the vegetation temperature  $T_{v,HR}$  is calculated according to [Moran et al. \(1994\)](#).  
 467  $NDVI_{soil}$  and  $NDVI_{veget}$  in (14) are respectively the NDVI obtained from bare soil (set to 0.15) and  
 468 from full-cover vegetation (set to 0.90).

469 Then, MODIS-derived soil temperature  $T_{s,HR}$  allows to estimate  $SEE_{HR}$  at 1 km resolution  
 470 following the methodology proposed by [Merlin et al. \(2012\)](#):

$$471 \quad SEE_{HR} = (T_{s,max} - T_{s,HR}) / (T_{s,max} - T_{s,min}) \quad (15)$$

472 where  $T_{s,max}$  and  $T_{s,min}$  are endmembers estimated from the approximations of [Merlin et al. \(2013\)](#)  
 473 considering the relations between the minimum/maximum of LST and the associated  $f_{v,HR}$  (more  
 474 details can be found on these estimates in [Molero et al., 2016](#); p.4).

475 SEE is used to describe the spatial variability of SM within the low resolution (LR = 40 km) pixel  
 476 given by SMOS product. High resolution SM ( $SM_{HR}$ ) is linked to high resolution SEE ( $SEE_{HR}$ ) through  
 477 the linear model proposed by [Budyko \(1956\)](#) and [Manabe \(1969\)](#):

$$478 \quad SEE_{HR} = \frac{SM_{HR}}{SM_p} \quad (16)$$

479 where  $SM_p$  is a LR parameter depending on atmospheric conditions and soil properties. In the C4DIS  
 480 processor, this parameter is computed at low resolution at each execution from daily SMOS SM  
 481 ( $SM_{LR}$ ) and SEE averaged inside the LR pixel ( $SEE_{LR}$ ):

$$482 \quad SM_p = \frac{SM_{LR}}{SEE_{LR}} \quad (17)$$

483 The disaggregation is finally realized by applying a first order Taylor expansion to the SEE and SM  
 484 dataset. The downscaling relationship is written as:

$$485 \quad SM_{HR} = SM_{LR} + SM'(SEE_{LR}) \times (SEE_{HR} - SEE_{LR}) \quad (18)$$

486 with  $SM'(SEE_{LR})$  the partial derivative of SM relative to SEE computed at low resolution. Here, this  
 487 derivative simply equals the  $SM_p$  parameter estimated according to (17).

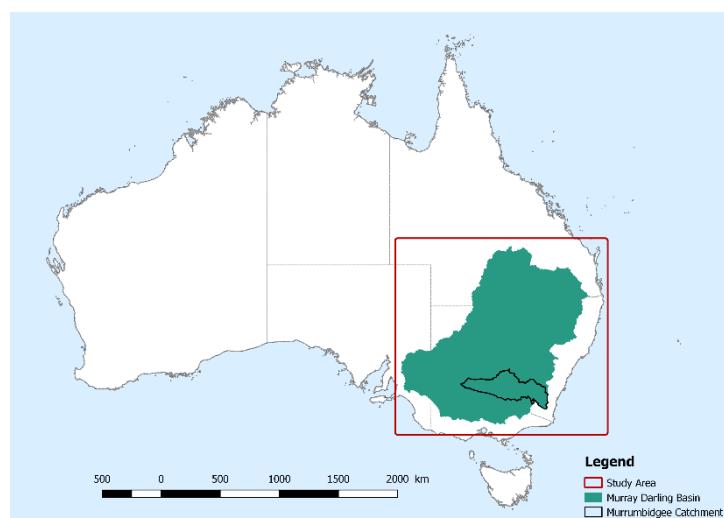
### 488 4.3 Study area

489 Australia is a wide country, with an area of almost 8 million km<sup>2</sup> and characterized by various  
 490 surface and climate conditions. Thus, it is a suitable area to study spatial variations of soil moisture  
 491 over a wide range of scales. Many studies on SM have been carried out in Australia in order to  
 492 monitor SM variability using ground, airborne and satellites data (Smith et al., 2012). Among others,  
 493 we can mention the National Airborne Field Experiment 2006 (NAFE'06; Merlin et al., 2008b) and the  
 494 Australian Airborne Calibration/validation Experiments for SMOS (AACES; Peischl et al., 2012). These  
 495 experiments were realized over the Murrumbidgee catchment (82 000 km<sup>2</sup>, Fig.1), located at the  
 496 southeastern part of Australia. Because of its variable climatic conditions (humid in the east, semi-  
 497 arid in the west), this region was used for validating satellites missions such as SMAP (Panciera et al.,  
 498 2014) or SMOS. SM products delivered by SMOS were assessed during the AACES experiments, which  
 499 took place in 2010 over two periods: January-February (AACES-1) and September (AACES-2). Wide  
 500 spatially distributed networks of in-situ measurements (OzNet hydrological monitoring network;  
 501 Smith et al., 2012) and transect flights (Polarimetric L-band Multibeam Radiometer; Peischl et al.,  
 502 2012) were used to validate SMOS data. In this context, benefiting from a dense SM dataset at  
 503 different space scales, some of the first applications of DisPATCH algorithm were realized during the  
 504 AACES experiments (Merlin et al., 2012). These works showed the efficiency of DisPATCH under low-  
 505 vegetated semi-arid areas, and its potential to evaluate coarse-scale SMOS products. Later studies  
 506 (Malbêteau et al., 2016; Molero et al., 2016) continued the evaluation and improvement of DisPATCH  
 507 algorithm over the Murrumbidgee catchment.

508 In this paper, DisPATCh analysis is made during the 7-month period from June to December 2010,  
509 taking advantage of previous DisPATCh studies over this period. We choose to extend the study area,  
510 from the Murrumbidgee catchment to the Murray Darling Basin (MDB, 1 million km<sup>2</sup>, Fig.1). The first  
511 reason of this choice is related to the main objective of the study, which is the analysis of DisPATCh  
512 related products over different space scales. Though, our study covers a large range of scales, from  
513 the pixel size (kilometer scale) to the full basin extent (1300 ×1400 km<sup>2</sup>), giving a new point of view  
514 considering DisPATCh validation. Moreover, spectral and multifractal tools presented in section 3  
515 cannot be properly applied if the data size is not sufficient enough. Because of its low resolution, it  
516 would be inappropriate to do multiscale analysis of SMOS SM over the Murrumbidgee catchment  
517 (“images” would be smaller than 5 × 5 pixels).

518 MDB is located in southeastern Australia and contains more than 20 catchments such as  
519 Murrumbidgee in its south part (Fig.1) The climate is sub-tropical in the North-East (average annual  
520 precipitation up to 1500 mm), semi-arid in the West (average annual precipitation less than 300 mm)  
521 and mostly temperate in the South (snowfall during winter on the peaks of the Great Dividing  
522 Range). Regarding to land use, West is made of wide plains essentially composed of saltbush  
523 shrublands and mulga lands. From South to North-East, there are the mountains of the Great  
524 Dividing Range reaching 2 300 m in altitude. Irrigation, dry land cropping and pastures are spread  
525 over the basin, but most of the irrigated areas are located in the South (like Murrumbidgee region).

526



**Figure 1.** The study area includes the Murray Darling Basin (1 million km<sup>2</sup>), southeastern Australia.

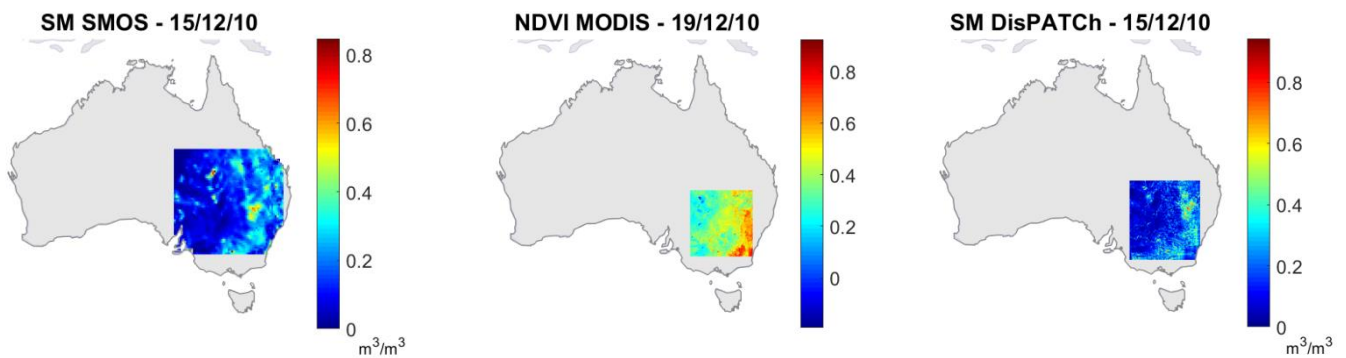
527

#### 528 4.4 Data preprocessing

529 Before applying the multi-scale analysis, preprocessing must be done on the different satellite  
530 products. The first preprocessing step is to handle the missing values. Because of technology or  
531 acquisition conditions, all satellite sensors provide products that present more or less missing values.  
532 These can be caused by failures in the data acquisition or delivering, or even voluntarily generated by  
533 the production center when discarding incorrect values. In our case, SMOS products can be affected  
534 by unauthorized emissions that cause radio frequency interference (RFI). SMOS SM used in this study  
535 are pre-filtered by CATDS in order to remove pixels with more than 10% RFI probability (Kerr et al.,  
536 2013; Olivia et al., 2012). Considering MODIS products, cloud pixels are also removed to avoid the  
537 impact of atmosphere on downscaled data. Though, missing values in output DisPATCh products are  
538 mainly caused by the accumulation of missing values coming from inputs. Thanks to the 24 averaged  
539 HR outputs combination implemented in C4DIS processor (section 4.1), the probability to get missing  
540 values in the final averaged downscaled product is reduced. In our study, we applied bilinear  
541 interpolation in each satellite image to fill in missing data (noted NaN). To do this properly, some  
542 conditions were established. To minimize the impact of data interpolation on spectral and  
543 multifractal analysis, each image with more than 40% of NaN were discarded. Moreover, in order to  
544 treat separately land-surface NaN values from sea areas located outside the continent, the latter  
545 were filled with zeros. Previous studies showed that biased multifractal parameters could be  
546 obtained from data containing significant proportion of zeros (De Montera et al., 2009; Verrier et al.,  
547 2010, 2011). Thus, we made sure to select images whose ground area contains a minimum of sea  
548 pixels (less than 10 %).

549 In a second stage, sub-images of  $2^n \times 2^n$  pixels need to be selected over the MDB area. To  
550 estimate statistical moments over different spatial resolutions, images must indeed be square, with a  
551 number of pixels equal to a power of two along each dimension (section 3). Because of different  
552 satellites projection grids and spatial resolutions, selected sub-images from different satellites do not  
553 cover exactly the same area and they do not completely match to the original MDB area. Figure 2  
554 presents examples of sub-images obtained for DisPATCh SM, SMOS SM and MODIS NDVI, whose size  
555 is respectively  $1024 \times 1024$ ,  $64 \times 64$  and  $1024 \times 1024$  pixels (for readability, during preprocessing all  
556 MODIS products were projected from sinusoidal to orthogonal lat/lon coordinates; Sohrabinia,  
557 2012). Considering the different grid spacing of the products and sub-images size condition, the sub-  
558 image selected for SMOS SM covers the entire MDB ( $1600 \times 1600 \text{ km}^2$ ), whereas the sub-images  
559 selected for DisPATCh and MODIS products are smaller (around  $1000 \times 1000 \text{ km}^2$ ). It is important to  
560 notice that, while they have similar spatial resolution and a same number of pixels, DisPATCh SM and  
561 MODIS images do not exactly correspond to the same ground area. This is caused by slightly different

562 grid spacing for the two products, 1 km for DisPATCh and 0.93 km for MODIS (Solano, 2010; Wan,  
 563 2006). For simplicity, we'll consider in the following that both DisPATCh and MODIS products present



**Figure 2.** Sub-images selected for each satellite product over the Murray Darling Basin.

564 a grid spacing of around 1 km.

565

566 Table.1 summarizes the main characteristics of our preprocessed satellite dataset. Two  
 567 important observations should be highlighted. First, considering their daily revisit time, few DisPATCh  
 568 SM and MODIS LST images are retained over the full June-December period: only 12 maps for  
 569 DisPATCh and around 70 maps for MODIS LST. This is directly related to the significant number of  
 570 missing values that is in average 30 % in these two types of products. Therefore, missing values in  
 571 downscaled SM seem to be mostly generated by those in LST products, probably due to the presence  
 572 of clouds in the data. Then, another point concerns the different surface areas of the preprocessed  
 573 products. Because they do not fully overlap, SMOS and DisPATCh sub-images may capture different  
 574 SM dynamics. Extreme events occurring in northern MDB are observed in SMOS data whereas it may  
 575 not be taken into account in DisPATCh data. However, we ensured that all products did have the  
 576 widest area in common, focusing on irrigated regions in the middle-south part of the basin (like  
 577 Murrumbidgee).

578

PRODUCTS	Revisit (day)	Effective resolution (km)	Grid spacing (km)	Surface area (km <sup>2</sup> )	Number of images (with NaN% < 40%)	Average NaN rate (%)
SM DISPATCH	1	1	1	1024 x 1024	12	32
SM SMOS	1	40	25	1600 x 1600	203	3
LST Aqua-MODIS	1	1	≈ 1	950 x 950	60	28
LST Terra-MODIS	1	1	≈ 1	950 x 950	82	26
NDVI Terra- MODIS	16	1	≈ 1	950 x 950	13	10

**Table 1.** Main characteristics of satellites products analyzed in this study. We also mentioned the surface area, the number of images conserved, and the average rate of missing values (without sea areas) of the dataset after preprocessing.

## 579 5 Results

### 580 5.1 Spatial power spectra

581 Figure 3 shows the mean power spectra estimated over the full period (June-December 2010) of  
582 the different input and output products involved in DisPATCh (it represents an average spectrum  
583 based on individual spectra obtained within the period). Each spectrum is plotted in log-log  
584 coordinates, with horizontal axis converted into space scale  $l (= k^{-1})$ , expressed in kilometers.  
585 Considering SMOS SM and MODIS products, the mean spectra are found to be scaling over the entire  
586 range of scales. This is observed by a linear evolution of  $\log(E(k))$  (Eq.9), with coefficients of  
587 determination  $R^2$  greater than 0.9 for each spectrum (Table.2). Note that  $R^2$  is used as a measure of  
588 the goodness-of-scaling, estimated from the linear regression between  $\log(E(k))$  and  $\log(l)$ .  
589 However, a different behavior is noticed for the disaggregated SM spectrum. Two scale ranges seem  
590 to appear, with an increasing slope on scales lower than about ten kilometers. A segmentation  
591 algorithm was applied on this spectrum (D'Errico, 2017), which confirmed a scale break at  $l \approx 10$  km.  
592 According to the different values of spectral slopes obtained (Table.2), a three-group classification  
593 was proposed:

- 594 •  $\beta \approx 1$ : SMOS SM, MODIS vegetation index and disaggregated SM ( $l > 10$  km)

595 For these three products, the negative slope is found to be close to one. Though, according to  
596 Eq.10, this may reveal the conservative nature of the fields ( $H \approx 0$ ). Moreover, these values are  
597 quite similar to the estimates proposed in literature: Lovejoy et al. (2008) found  $\beta = 1.2$  for both  
598 vegetation and soil moisture indexes (from MODIS products, Guadalajara, central Spain, July  
599 2006). Previous studies on topography, especially on volcanic surfaces (Laferrière and Gaonac'h,  
600 1999), found comparable results with quite low degree of fractional integration. Since  
601 topography can affect the spatial distribution of SM and vegetation (Kim and Barros, 2002b), it is  
602 not surprising to observe similar scaling behavior between these fields.

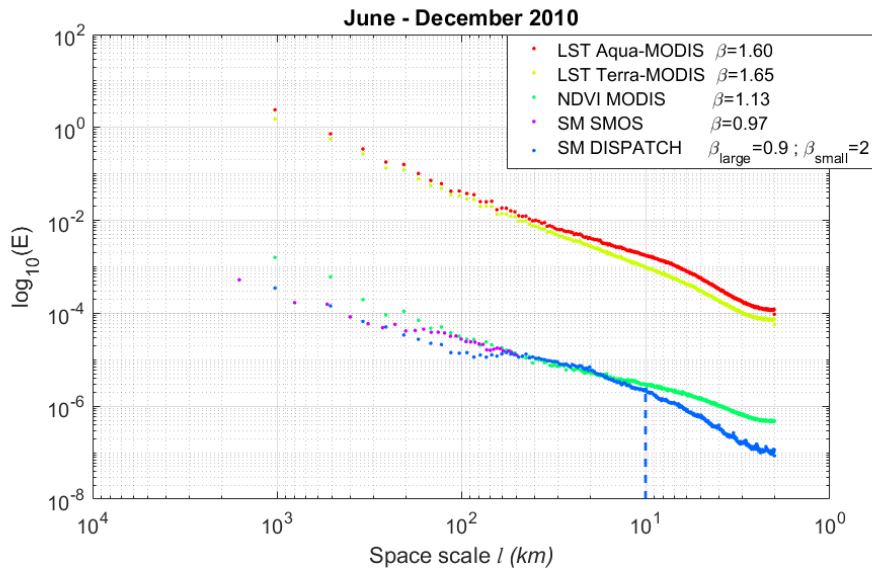
- 603 •  $1 < \beta < 2$ : MODIS surface temperature (from both Aqua and Terra satellites)

604 LST spectra have  $\beta$  values greater than 1. Here, surface temperature seems to correspond to a  
605 non-conservative field ( $H > 0$ ). These spectral slopes may be comparable to those obtained in  
606 literature on precipitation fields (Lovejoy and Schertzer, 2008), showing possible connections  
607 between the spatial distribution of surface temperature and that of rainfall, and therefore with  
608 the underlying (turbulent) atmospheric dynamic (Schmitt et al., 1993).

- 609 •  $\beta > 2$ : disaggregated SM ( $l < 10$  km)

610 On small scales, DisPATCh SM spectrum presents a relatively large slope, reflecting a high degree  
611 of fractional integration ( $H > 0.5$ ). To our knowledge, such high value of spectral exponent has

612 never been observed in previous studies on SM fields. However, comparable scaling was  
 613 obtained on SM time series, revealing spectral slopes greater than 2 (Katul et al., 2007).



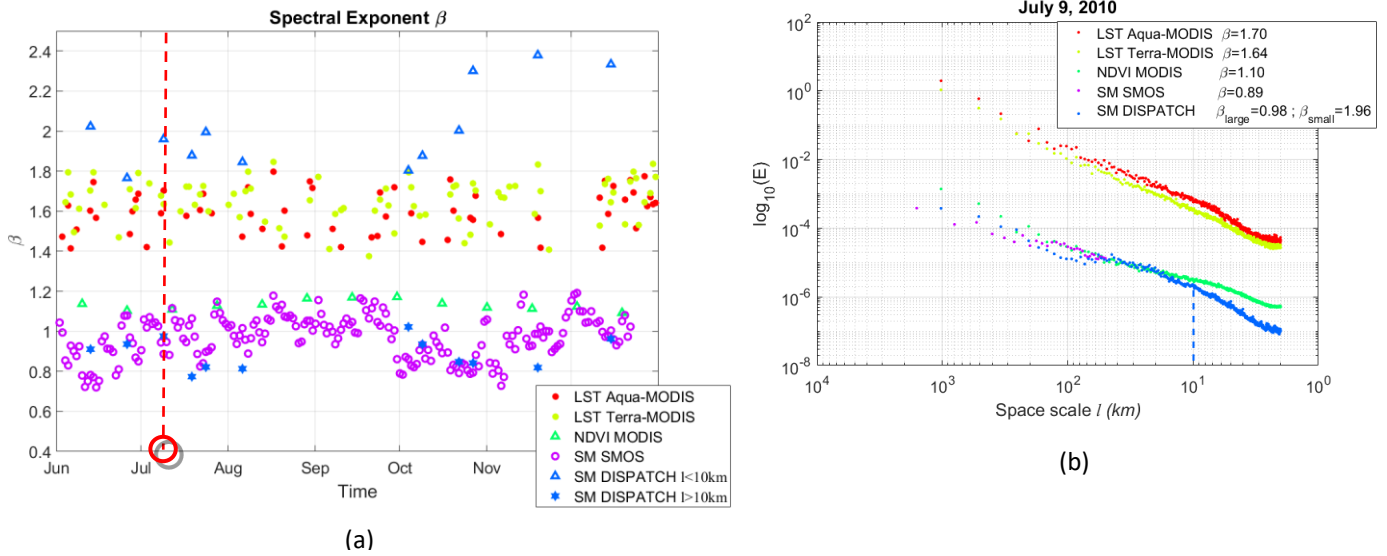
**Figure 3.** Mean angle-integrated power spectra of DISPATCH related products (over the full June-December period).  $\beta_{large}$  and  $\beta_{small}$  refer to disaggregated SM spectral exponents obtained respectively from scales  $l > 10$  km and scales  $l < 10$  km.

614 From these spectral observations, a similar scaling seems to appear between the original SMOS SM  
 615 and the disaggregated SM on scales greater than 10 km, but this behavior is found to change for  
 616 scales lower than about ten kilometers. A comment may also be made on LST power spectra and  
 617 their linear regressions : although  $R^2$  coefficients present good values on the entire range of scales ( $>$   
 618 0.9), a scale break may be observed at about the same spatial scale found for DISPATCH spectrum ( $l$   
 619  $\approx 10$  km). The scale break seems less pronounced but it could be related to that of DISPATCH. This  
 620 point will be discussed in section 6.2.

621 This twofold scaling regime of DISPATCH SM can be also observed on each specific date of the  
 622 study period (with  $R^2$  coefficients greater than 0.9 on almost all images and on both scale ranges).  
 623 Figure 4a shows the time series of the individual spectral exponents estimated for all products (i.e.  
 624 spectra computed for each image). From June to December, a significant difference of  $\beta$  values is  
 625 observed between the two scale ranges of disaggregated SM. For example, on July 9 (Fig.4b), power  
 626 spectra are found to be similar as mean ones presented above (Fig.3). In particular, the same scale  
 627 break is still observed for disaggregated SM at about ten kilometers. Another remark concerns the  
 628 amplitude of the scale break according to seasons. Figure 4a shows that, for disaggregated SM, the  
 629 difference between the spectral exponents of small scales and large scales (respectively blue triangle  
 630 and blue star symbols) is more important during the last three months of the period. At small scales,  
 631 the spectral slope suffers a drastic change from around 1.9 (Jun-Jul-Aug-Sept) to 2.3 (Oct-Nov-Dec,



632 i.e. spring and early summer in Australia). The amplitude of the scale break observed in DisPATCH SM  
 633 could be related to the seasonal conditions of the study area. This will be discussed in section 6.1.



**Figure 4.** (a) Time series of spectral exponents over the period June-December 2010, (b) angle-integrated power spectra obtained on July 9, 2010.

634

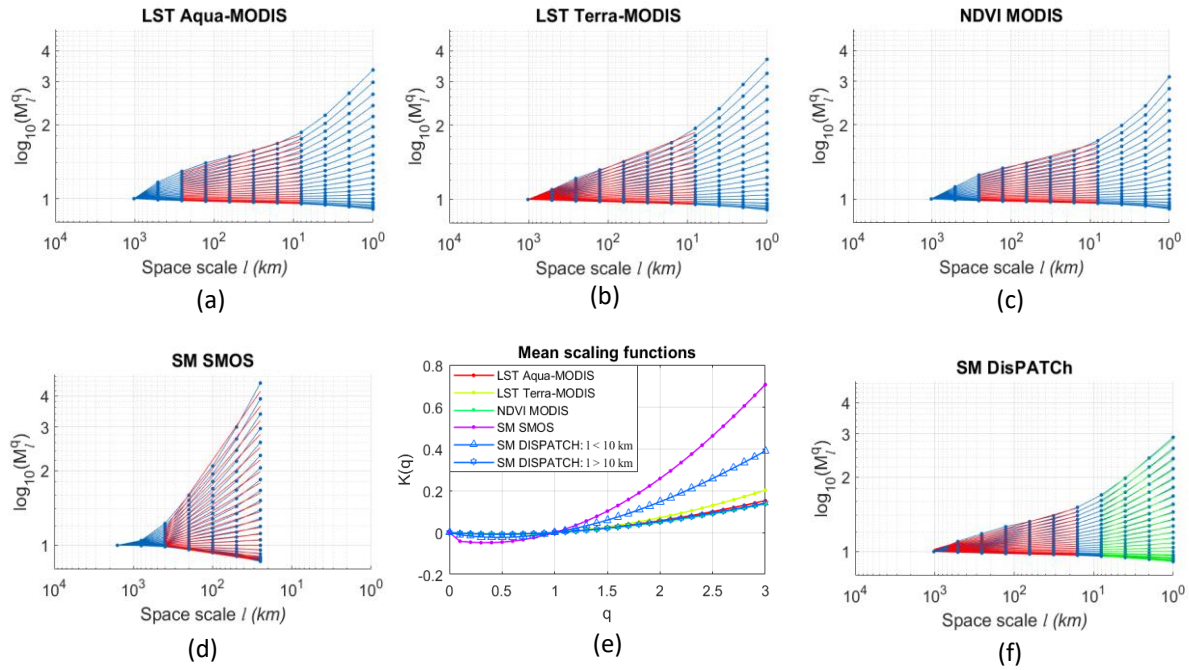
## 635 5.2 Multifractal analysis

636 The moments of the normalized absolute gradients were estimated at all accessible resolutions.  
 637 Since divergence for  $q$  greater than  $q_D \approx 3$  was reported in most of the literature (Hubert et al.,  
 638 2007), and because of sample size limitations, in this study moments were computed for orders set  
 639 from 0 to 3, in steps of 0.1. Figure 5 shows the mean moments over the 7-month period, plotted in  
 640 log-log coordinates as a function of the space scale  $l$  ( $= \lambda^{-1}$ ). For each product, multifractal regimes  
 641 are identified on specific scale ranges. The power-law described by Eq.3 is well verified over these  
 642 spatial scales, corresponding to a linear variation of  $\log(M_l^q)$  for all orders of moments  $\langle \Phi_\lambda^q \rangle = M_l^q$ .  
 643 This behavior means that a multifractal model is well adapted on the corresponding scale ranges.

644 Considering vegetation and temperature MODIS products, a scaling regime is found on scales  
 645 greater than 8 km (Fig.5a-c). On these scales, moments curves were fitted by linear regression (red fit  
 646 lines on Fig.5), and the corresponding scaling functions  $K(q)$  were computed (red, yellow and green  
 647 curves Fig.5e). UM parameters were then estimated applying (derivative-free) minimization method  
 648 between empirical scaling function  $K(q)$  and the model form of  $K(q)$  described in Eq.5. For the  
 649 vegetation, parameters values are found to be  $\alpha = 1.74$  and  $C_1 = 0.03$  (Table.2). They are quite close  
 650 to those estimated by Lovejoy et al. (2008) on similar NDVI MODIS products ( $\alpha = 2$  and  $C_1 = 0.06$ ). For  
 651 Aqua surface temperature, we found the same parameter values as the vegetation ones ( $\alpha = 1.7$  and

652  $C_1 = 0.03$ ), which is related to the very similar  $K(q)$  functions for all orders  $q$ . Slightly different  
653 parameters are found for Terra products ( $\alpha = 1.91$  and  $C_1 = 0.04$ ). This difference could be due to the  
654 different acquisition time of the two satellites (10:30 for Terra and 13:30 for Aqua). This may have  
655 some effect on the multiscaling behavior of surface temperature. Another reason to this difference  
656 could be the larger scaling regime considered for Terra: a multifractal behavior is observed on scales  
657 ranging from 8 km to 1024 km, against 8 km to 300 km for Aqua and NDVI products. Anyway, these  
658 results confirm (NDVI) and reveal (LST, not yet studied at this time) the multifractal properties of the  
659 considered MODIS products. In both cases, they are characterized by a high degree of multifractality  
660 ( $\alpha$  is close to 2, value corresponding to the log-normal case) and by a low dispersion of the field ( $C_1 <$   
661 0.1).

662 SMOS SM products show good multifractal behavior too: moments are found to be well  
663 fitted ( $R^2 = 0.99$ , cf. Table.2), on most of the aggregation scales (apart from the 2 greatest scales,  
664 1600 km and 800 km). Scaling function was computed over spatial scales going from the 25 km  
665 observation scale to 400 km (purple curve in  $K(q)$  graph, Fig.5). Compared to MODIS products, a  
666 growing divergence is noticed between SMOS and NDVI/LST scaling functions, especially for orders  $q$   
667 greater than 1. This scaling behavior is confirmed by different UM parameters:  $\alpha = 1.46$  and  $C_1 = 0.16$ .  
668 To our knowledge, no application of the UM model has already been made on remotely sensed SM  
669 from passive microwaves. Therefore, it is difficult to compare these results with literature. However,  
670 although they didn't use the UM model, [Kim and Barros \(2002b\)](#) studied spatial scaling properties of  
671 passive microwave SM, estimated from airborne L-band radiometer (Southern Great Plains  
672 Experiment 1997, USA). They observed a multifractal scaling on a similar scale range (1.6 km to 250  
673 km), which is coherent with our results. [Lovejoy et al. \(2008\)](#) indeed applied the UM model, but on  
674 an optical SM index, estimated from MODIS reflectances ([Lampkin and Yool, 2004](#)). They found  $\alpha = 2$   
675 and  $C_1 = 0.05$  over lower spatial scales (0.5 km to 25 km). These parameter values are quite different  
676 from ours. The different scale range and the different study area (Guadalajara, central Spain, in  
677 [Lovejoy et al., 2008](#)) between their work and ours could be a possible explanation to this result.  
678 Another reason might be linked to the nature of the signal studied. Optical-estimated indexes, like  
679 MODIS SM index, are more sensitive to land cover such as vegetation ([Fabre et al., 2015](#); [Haubrock et](#)  
680 [al., 2008](#)), then "polluting" the scaling properties of SM.



**Figure 5.** Mean moments as a function of space scale  $l$  in km (a-d; f) and mean scaling functions (e) of DisPATCh related products (over the full June-December period). Scaling regimes are distinguished and fitted with the straight lines (linear regressions on moments graphs).

681

682 Focusing now on disaggregated SM products, a change of slope is noticed for each of the  
 683 statistical moments. The same segmentation algorithm was applied on all moment curves, revealing  
 684 a scale break at about ten kilometers. Two multifractal scaling regimes may be observed here,  
 685 confirming the twofold scaling behavior found in the power spectra. Considering larger scales ( $l > 10$   
 686 km, red fit lines on Fig.5), estimated UM parameters are:  $\alpha = 1.64$  and  $C_1 = 0.03$ . They are close to the  
 687 parameters found for our MODIS products (NDVI and LST), with a high degree of multifractality and a  
 688 low dispersion of the field. For smaller scales ( $l < 10$  km, green fit lines on Fig.5), the degree of  
 689 multifractality is almost unchanged ( $\alpha = 1.59$ ) compared to the large scales regime. However, the  
 690 dispersion parameter is increased ( $C_1 = 0.09$ ), which is three times the value obtained on greater  
 691 scales. Though, the difference between the two multifractal scaling regimes seems to be mainly  
 692 linked to the dispersion of SM through scales. If we refer to the multifractal analysis of MODIS SM  
 693 index made by [Lovejoy et al. \(2008\)](#), our estimates are coherent considering  $\alpha$  (for both ranges of  
 694 scales) and  $C_1$  (on large scales). [Lovejoy et al. \(2008\)](#) didn't notice any scale break, therefore it is  
 695 difficult to comment our estimate of  $C_1$  at small scales. Nevertheless, [Kim and Barros \(2002b\)](#)  
 696 observed a similar scale break (at about the same 10 km scale) on passive microwave SM. Indeed,  
 697 they noticed two scaling regimes from variance, spectra and moments graphs. The twofold scaling

698 behavior of DisPATCH SM products looks consistent with the scale break identified first by Kim and  
 699 Barros (2002b).

PRODUCTS	Grid spacing (km)	Scale range (km)	$\beta$	$R^2_{\beta}$	$\alpha$	$C_1$	$R^2_{K(q)}$	H	$R^2_H$
SM DISPATCH	1	[1 – 10]	2.01	0.97	1.59	0.09	0.97	0.45	0.99
		[10 – 1024]	0.89	0.92	1.64	0.03	0.96	0.15	0.98
SM SMOS	25	[25 – 1600]	0.97	0.94	1.46	0.16	0.99	0.29	0.98
LST Aqua-MODIS	$\approx 1$	$\approx [1 – 1024]$	1.60	0.98	1.7	0.03	0.96	0.26	0.98
LST Terra-MODIS	$\approx 1$	$\approx [1 – 1024]$	1.65	0.99	1.91	0.04	0.95	0.31	0.99
NDVI Terra- MODIS	$\approx 1$	$\approx [1 – 1024]$	1.13	0.98	1.74	0.03	0.96	0.15	0.99

**Table 2.** Scaling parameters obtained from multifractal analysis over the period June-December 2010.  $R^2$  coefficients were estimated from linear regressions on the specified scale range. Note that  $R^2_{K(q)}$  is the average of the coefficients obtained on every moment curves.

## 700 6 Discussion

### 701 6.1 A physically-explained twofold scaling behavior of soil moisture?

702 Since SM variability is impacted by several environmental factors (Brocca et al., 2007; Crown et  
 703 al., 2012), the scale break observed on disaggregated SM could be the result of processes acting at  
 704 different space scales. At finer scales ( $l < 10 \text{ km}$ ), spatial structure of SM is governed by infiltration  
 705 or runoff, which are mainly related to the soil properties (texture, structure) (Hawley et al., 1983;  
 706 Famiglietti et al., 1998). On the other hand, at larger scales ( $l > 10 \text{ km}$ ), SM variability is more  
 707 affected by evapotranspiration processes (Mohanty and Skaggs, 2001) or precipitation (Jackson et al.,  
 708 1999).

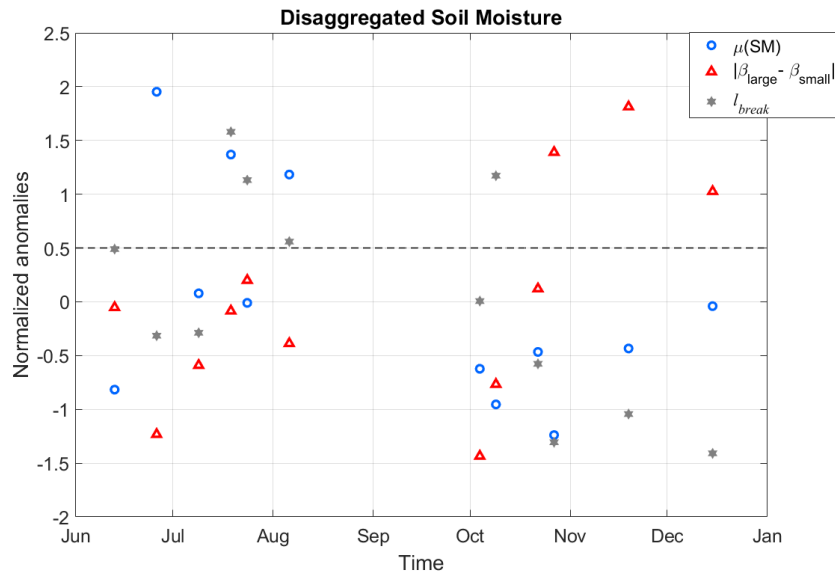
709 A similar scale break at  $\sim 10 \text{ km}$  was also noted by Kim and Barros (2002b) based on power  
 710 spectra and statistical moments of SM, estimated from airborne L-band radiometer (Southern Great  
 711 Plains Experiment 1997, USA). SM retrievals were obtained at 1 km nominal resolution from the  $T-\omega$   
 712 model (Jackson and Schmugge, 1991) which depends on Vegetation Water Content (VWC) estimates  
 713 based on NDVI. They observed that the relationship between the spatial structure of SM and  
 714 landscape characteristics was strongly modulated by the wetness of the soil. Indeed, they applied an  
 715 EOF analysis (Empirical Orthogonal Function) between SM and auxiliary data which are topography,  
 716 VWC and soil content. This revealed that SM was much correlated to topography during rain events,

717 whereas stronger correlation with vegetation (water content) was noticed during drier periods  
718 (mainly governed by evapotranspiration processes).

719 These results are interesting since other research studies also observed similar scale break in the  
720 case of precipitation products obtained from radar at 1 km resolution (southeastern France, [Gires et](#)  
721 [al., 2011](#)). Indeed, a transition in spectra and moments was noticed at about twenty kilometers (not  
722 far from our 10 km scale break). However, some limitations relative to radar data acquisition must be  
723 taken into account considering these results. Indeed, constraints due to algorithmic processing  
724 (change from polar to Cartesian coordinates, impact of missing data, temporal integration...) and to  
725 physics (attenuation by rainfall, etc.) may impact the scaling properties of precipitation radar images.  
726 Moreover, the Z-R relationship between radar reflectivity and rain rate ([Marshall and Palmer, 1948](#))  
727 remains somehow controversial, with a non-robust parameterization from a multi-scale point of view  
728 ([Verrier et al., 2013](#)). Thus, in this context, the scale break detected by [Gires et al. \(2011\)](#) may not be  
729 as relevant as it could be. However, they also analyzed the multifractal behavior of simulated  
730 precipitations generated on the same area, at  $\sim 2$  km resolution, from the Meso-NH atmospheric  
731 model ([Lafore et al., 1997](#)). The analysis revealed the presence of a comparable scale break at about  
732 30 km, which tends to show that this transition scale in precipitation data is not an artifact. Since  
733 rainfall is an important forcing of SM, it may be thought that a break in the rainfall spectra would  
734 affect the SM, in a more significant way when the rain event is important. Moreover, a theoretical  
735 model of SM in the time domain was proposed by [Katul et al. \(2007\)](#) to relate the scaling of  
736 precipitation to that of SM. The spectral exponents of these two variables were found to be  
737 connected over time scales finer than 7 days, through the simple equation:  $\beta_{SM} = \beta_P + 2$  (with  $\beta_{SM}$   
738 and  $\beta_P$  the negative spectral slopes of respectively soil moisture and precipitation time series).  
739 Despite these results were observed on time series, it may corroborate the possible dependence  
740 between the SM variability and that of heavy rainfall, even in the space domain.

741 Considering seasonal variations, the power spectra of disaggregated SM seem to reveal a  
742 pronounced twofold scaling behavior especially during spring and early summer (October to  
743 December period). Since DisPATCh images are mainly located over the middle-south part of the  
744 Murray Darling Basin, climate is then mostly temperate. Therefore, the last months of the study  
745 period correspond to a drier landscape. Thus, the two scaling regimes seem to be even more distinct  
746 when the soil is drier. To demonstrate this effect, the spatial mean of DisPATCh SM ( $\mu(SM)$ ) and the  
747 absolute difference  $|\beta_{large} - \beta_{small}|$  were computed for each disaggregated image. In Figure 6, the  
748 normalized anomalies of these two variables are in line with this hypothesis (blue and red circle  
749 symbols): a more pronounced twofold scaling behavior seems to be found on the driest days (Oct-  
750 Nov-Dec). [Kim and Barros \(2002b\)](#) noticed a similar behavior, with lower scaling differences during

751 rain events (observed on both spectra and moments of SM). In certain dates, they even noticed that  
 752 there was no scale break at all, corresponding to very high wetness conditions of the soil. Moreover,  
 753 we estimated the position of the scale breaks on each power spectrum during the period  
 754 (corresponding normalized anomalies plotted in gray star symbols, Fig.6). Although it is positioned on  
 755 average around 10 km (not shown here but the mean value over the full period was estimated at  $\sim$   
 756 13 km), the transition scale between the two scaling regimes seems to follow a decreasing trend as



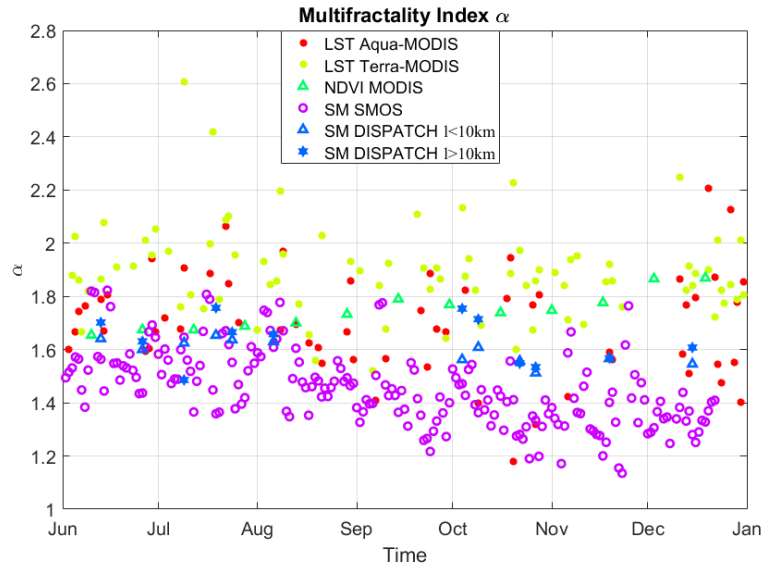
**Figure 6.** Time series over the period June-December 2010 of the normalized anomalies of the three following variables: the spatial mean of each DisPATCH SM product; the absolute difference between the two spectral exponents  $\beta_{\text{large}}$  and  $\beta_{\text{small}}$  (estimated respectively for  $l > 10 \text{ km}$  and  $l < 10 \text{ km}$ ); the position of the scale breaks estimated on each spectrum by the segmentation algorithm. The dotted line differentiates the two regimes: wetter trend from June to September and drier trend from October to December.

757 the soil is drying, with estimated scale breaks ranging from  $\sim 15 \text{ km}$  in wet period to  $\sim 12 \text{ km}$  in dry  
 758 period. A comparable behavior was observed by [Kim and Barros \(2002b\)](#), showing that the position  
 759 and the amplitude of the scale break in the scaling behavior of SM is dependent on the state of SM,  
 760 and thus on the hydrometeorological conditions like rain, evapotranspiration and infiltration  
 761 processes.

762

763 To go further on the dependences between seasons and SM scaling, [Kim and Barros \(2002b\)](#)  
 764 observed that multifractality was almost always involved on scales smaller than 10 km, whatever the  
 765 dryness of the soil. However, on scales greater than 10 km, multifractality was found to become  
 766 monofractality, especially during drier conditions. At large scales (between 25 km and 400 km), a  
 767 comparable effect was noticed on our SMOS SM products (purple circles, Fig.7): the multifractality

768 index  $\alpha$  is decreased from around 1.6 (June) to 1.3 (December), which may reflect a moderate  
 769 decrease of multifractality during the study period. Therefore, multifractal properties of SM at large  
 770 scales seem to be related to the soil dryness. This may give complementary explanations to the



**Figure 7.** Time series of the multifractality index  $\alpha$ , over the period June-December 2010.

771 twofold scaling behavior of SM. On the other hand, considering DisPATCh SM, a rather constant  
 772 evolution of  $\alpha$  is noticed on both small scales (blue triangle symbols) and large scales (blue star  
 773 symbols). The first case confirms the idea that multifractality is not dryness-dependent on smaller  
 774 scales, whereas the second is in contradiction with this assumption. Thus, the latter should be  
 775 considered cautiously to explain the scaling properties of SM.

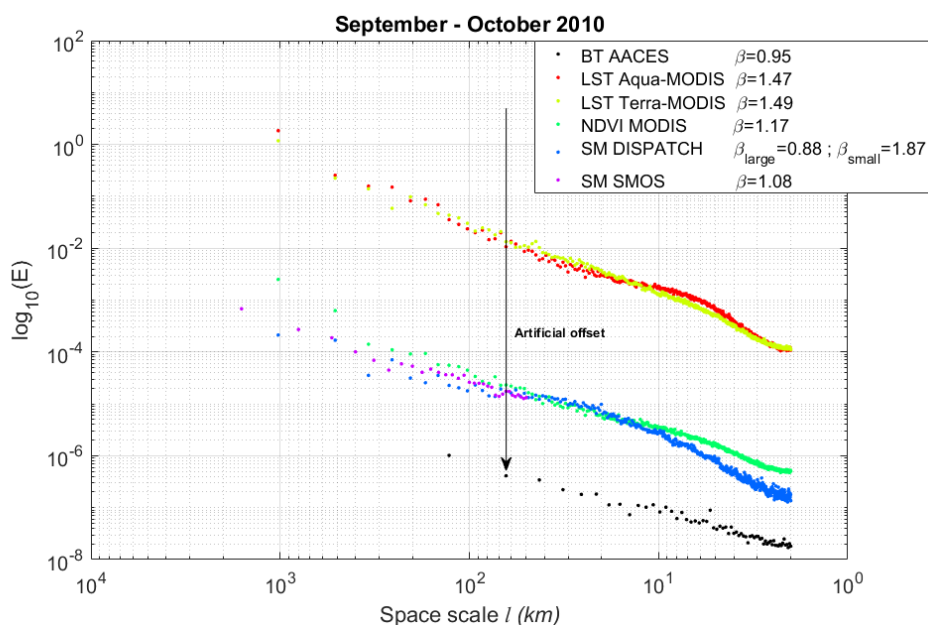
776

## 777 6.2 A model-induced twofold scaling behavior of soil moisture?

778 In relatively recent works (Mascaro et al., 2010; Mascaro and Vivoni, 2012), scale invariance and  
 779 multifractality were noticed from SM products measured from airborne L-band radiometers (Southern  
 780 Great Plains 1997 and 1999 Experiments, USA). In these studies, a Log-Poisson multifractal model was  
 781 applied (She and Levêque, 1994), and a single scaling regime was observed on statistical moments,  
 782 from 0.8 km to 25.6 km scales. Although this result confirms the multifractal properties of SM on  
 783 space scales similar to ours, it refutes the existence of two scaling regimes. No scale break was  
 784 observed at about ten kilometers. Since the Log-Poisson model is based on a similar universal theory  
 785 as the UM model (continuous cascades), it is somewhat unexpected not to detect the same transition  
 786 on comparable SM products (same technology and same scale range).



787 To investigate if this difference could be related to the case study (different areas or periods), we  
 788 compared our DisPATCH products to fine scale airborne data acquired during the AACES-2 mission  
 789 (Peischl et al., 2012). This mission was performed in September 2010, during which transect flights  
 790 were carried out over the Murrumbidgee catchment. Brightness Temperatures (BT) were acquired  
 791 from L-band radiometer (on both H- and V-polarizations), at a nominal 1 km spatial resolution. The  
 792 study area was divided in 5 patches of  $50 \times 100 \text{ km}^2$ , each corresponding to a single flight day (13, 16,  
 793 19, 21, 22 September). We gathered these patches into one single BT image, and we selected a sub-  
 794 image of  $128 \times 128 \text{ km}^2$ . To verify the presence of two scaling regimes in the data, we applied spectral  
 795 analysis on both H-polarized and V-polarized BT sub-images. In Figure 8, the power spectrum of H-  
 796 polarized BT was compared to the power spectra of DisPATCH related products on equivalent period.  
 797 The spectra of each satellite product available between the 13 and 22 of September were averaged  
 798 together. Since no DisPATCH SM products were pre-selected on this period (because of too many  
 799 NaN), we chose the nearest available product, which corresponds to 4 October. In Figure 8, one single  
 800 linear fit is observed on BT power spectrum, over the entire scale range (from 1 km to 128 km) and  
 801 with a spectral slope equivalent to that of SMOS SM spectrum ( $\beta \approx 1$ ). Note that V-polarized spectrum  
 802 was not plotted here, but it was found to be very similar to the H-polarized one.

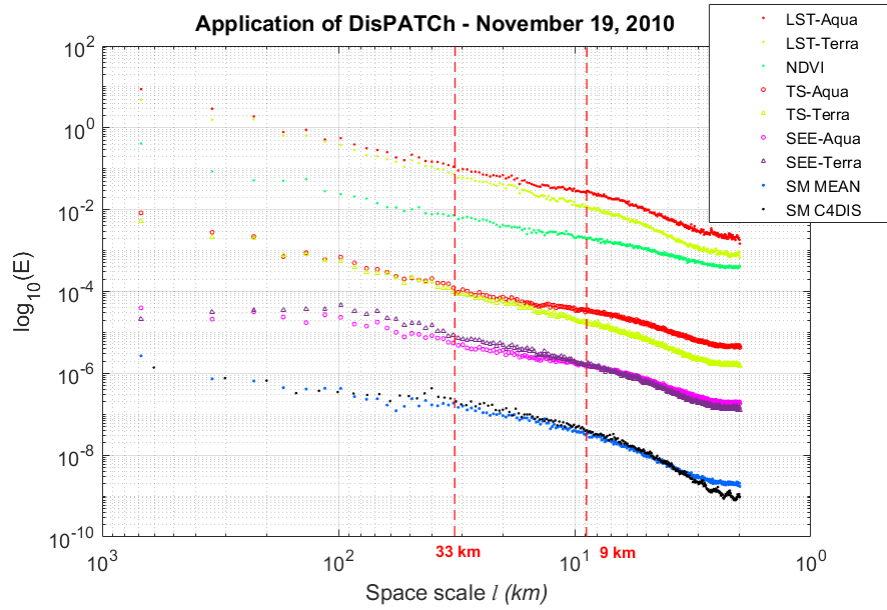


**Figure 8.** Angle-integrated power spectra of Brightness Temperature (BT) and DisPATCH related products, obtained on the period September-October 2010. For better visualization and comparison, BT power spectrum was shifted down (black arrow on the graph).

803 Different scaling behaviors were noticed for AACES BT and DisPATCH SM, on similar area and  
 804 similar period. This may sustain the idea that the scale break observed at 10 km could be caused by  
 805 the DisPATCH model and, specifically, by the way in which the multi-scale properties of each product

806 are mixed in the algorithm. To verify this hypothesis, a *simplified version of the C4DIS processor* was  
807 implemented in order to study the multi-scale behavior of the different variables combined and  
808 generated through the algorithm. To do this, the method proposed by [Molero et al. \(2016\)](#) was  
809 followed, which includes the two main steps described in section 4.2: (1) the estimation of  $SEE_{HR}$   
810 variable (Soil Evaporative Efficiency) from MODIS products (Eq.13-15) and (2) the proper  
811 disaggregation process of SM from SMOS products,  $SEE_{HR}$  and  $SM_p$  parameter (Eq.17-18).

812 According to this method, our algorithm was applied on SMOS and MODIS products acquired on  
813 November 19, 2010. A sub-area was selected ( $\approx 700 \times 700 \text{ km}^2$ ) in order to have a smaller number of  
814 missing data, and thus to get the minimum impact of gap-filling on the studied products. Figure 9  
815 shows the power spectra obtained from the input products of DisPATCh (LST, NDVI), intermediate  
816 products ( $T_s$ , SEE) and output product (SM MEAN). The latter product is the average of the 6  
817 disaggregated SM images obtained from the 6 SM-LST combinations (see section 4.1). Here, just one  
818 SMOS image of  $\approx 25 \text{ km}$  grid spacing was combined with the MODIS products. Indeed, both cases with  
819 one SMOS image and four downsampled ones were implemented (section 4.1 and [Molero et al.,](#)  
820 [2016](#)), and no significant differences were observed between the final products and between their  
821 power spectra. Therefore, for simplicity of implementation, only the case of one SMOS image was  
822 considered here. For comparison, the power spectrum of C4DIS SM product acquired on the same  
823 date and on the same sub-area was also plotted here. The segmentation algorithm used in section 5  
824 was applied on each power spectrum. A geometric mean was estimated from the different scale  
825 breaks obtained, revealing two averaged scale breaks which are nearly common to all spectra: the  
826 first at almost ten kilometers ( $l = 9 \text{ km}$ ) and the second at about thirty kilometers ( $l = 33 \text{ km}$ ). To  
827 evaluate the link between the multi-scale behavior of each product, spectral exponents were  
828 estimated on the two following scale ranges: from 33 km to 9 km (*large scales*) and from 9 km to 1  
829 km (*small scales*). Comparing our SM MEAN product with SM C4DIS product (Table.3), a very similar  
830 scaling is observed on *large scales* ( $\beta_{large} \approx 1.3$ ). On *small scales*, high spectral exponents are found,  
831 with  $\beta_{small} \approx 2$  for SM MEAN and  $\beta_{small} = 2.86$  for SM C4DIS. These different spectral slopes on finer  
832 scales could be related to the non-implementation of some filtering steps in our algorithm which are  
833 indeed coded in C4DIS processor: corrections of topography effects, filtering LST data with low  
834 quality, etc. ([Molero et al., 2016](#)).



**Figure 9.** Angle-integrated power spectra of some of the input, intermediate and output products obtained from our implementation of DisPATCH on November 19, 2010. Power spectrum of the final C4DIS product is also plotted here. For better visualization and comparison, the positions of power spectra according to Y axis were modified.

835

836 Despite these small differences between SM MEAN and SM C4DIS spectra, the scale break  
837 remains noticeable, as it seems to be on the other products of the algorithm. To demonstrate this,  
838 the absolute difference  $\Delta\beta = |\beta_{large} - \beta_{small}|$  was computed as an indicator of the amplitude of  
839 the scale break. Values greater than 0.6 were found for LST (Aqua and Terra),  $T_s$  (Aqua) and SEE  
840 (Aqua) products. These results seem to reveal that MODIS LST products would be the cause of the  
841 scale break located at about ten kilometers in the disaggregated SM product. This scale break would  
842 propagate in the algorithm through the estimation of  $T_s$  and SEE. A possible explanation to this scale  
843 break in LST products may be related to the physical nature of the signal used. Indeed,  
844 optical/thermal sensors can be characterized by modified spectral slopes near the satellite  
845 resolution. Moreover, this effect seems more important on Aqua LST ( $\Delta\beta = 0.83$ ) than on Terra LST  
846 ( $\Delta\beta = 0.62$ ). Similar differences can be observed between the mean power spectra of Aqua and Terra  
847 LST over the full period (Fig.3 in section 5.1). Thus, it may be thought that the amplitude of the scale  
848 break could be related to the diurnal cycle of surface temperature. Since surface temperatures  
849 measured from Aqua are acquired at the hottest hours of the day (13:30), there might be a  
850 correlation between the amplitude of scale break and the level of surface temperature.

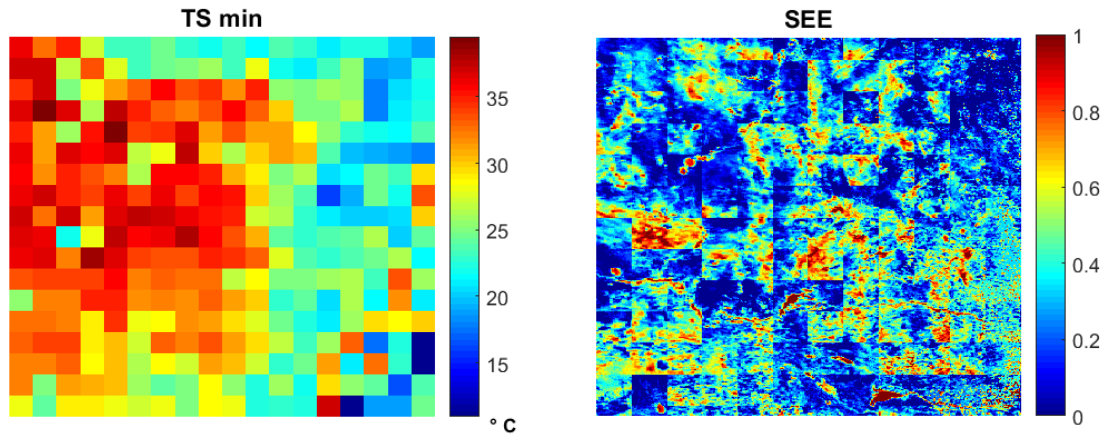
Scale range (km)	[33 - 9]		[9 - 1]		$\Delta\beta$
Spectral exponent	$\beta$	$R^2_{\beta}$	$\beta$	$R^2_{\beta}$	
LST - <i>Aqua</i>	1.08	0.97	1.91	0.98	0.83
LST - <i>Terra</i>	1.39	0.97	2.01	0.98	0.62
NDVI	0.93	0.94	1.22	0.98	0.29
TS - <i>Aqua</i>	0.90	0.95	1.55	0.98	0.65
TS - <i>Terra</i>	1.33	0.98	1.79	0.98	0.46
SEE - <i>Aqua</i>	0.88	0.96	1.61	0.98	0.73
SEE - <i>Terra</i>	1.25	0.97	1.83	0.98	0.58
SM MEAN	1.22	0.96	2.08	0.97	0.86
SM C4DIS	1.29	0.96	2.86	0.98	1.57

**Table 3.** Spectral exponents obtained from spectral analysis of DisPATCH related products on November 19, 2010 (Fig.9).  $R^2$  coefficients were estimated from linear regressions on the specified scale range.  $\Delta\beta = |\beta_{large} - \beta_{small}|$  is used as an indicator of the amplitude of the scale break, with *large* and *small* referring respectively to [33 – 9] km and [9 – 1] km scale ranges.

851

852        Considering the scale break observed at about 30 km, this one may not be related to the multi-  
853 scale properties of MODIS products but possibly to the combination of different products defined on  
854 different grid spacings. Indeed, DisPATCH algorithm combines and creates products which have  
855 either the grid spacing of MODIS data ( $\approx 1$  km) or the grid spacing of SMOS data ( $\approx 25$  km). For  
856 example, the estimation of SEE (Eq.15) combines end-members ( $T_{s,max}$  and  $T_{s,min}$ ) defined on the  
857 SMOS grid, with another product ( $T_s$ ) defined on the MODIS grid. As seen on Figure 10, a footprint of  
858 SMOS pixels is then observable on the resulting image of SEE. This property is due to the resampling  
859 strategy of SMOS data and to the end-members that are defined on the SMOS grid. This systematic  
860 footprint is visible in the real domain but can also have an impact in the Fourier domain. Indeed,  
861 sharp transitions at the SMOS pixels limits may create spurious convolutions by cardinal sine-like  
862 functions which may affect the spectrum. On the disaggregated SM, this effect can generate an  
863 imperfect transition between the part of the spectrum related to SMOS SM ( $l > 25$  km) and the part  
864 related to MODIS products ( $l < 25$  km). Regarding the behavior of SEE power spectra on scales  
865 greater than thirty kilometers (Fig.9), a lower spectral slope is observed ( $\beta \approx 0.5$ ) comparing to that  
866 obtained on finer scales ( $\beta \approx 1$  for [33 – 9] km). This could be related to the oversampling of SMOS

867 data, generating harmonics on fine scales and therefore not including variability on large scales. In  
868 this manner, not only large scales but even fine scales could be affected by this effect. The latter may  
869 also contribute, in a way, to the accentuation of the spectral drop observed at finer scales on the  
870 final disaggregated SM product.



**Figure 10.** Images corresponding to  $T_{s,min}$  and  $SEE_{HR}$  products obtained from our implementation of DisPATCh on November 19, 2010.

871

872

## 873 7 Conclusion

874 During the last century, several studies were carried out to investigate the scaling properties of  
875 SM. Very diversified technologies were used to access and study the spatial structure of SM: airborne  
876 microwaves products, satellite optical indices, etc. Moreover, different approaches have been  
877 considered, such as power spectra, statistical moments, fractal dimensions, and even different types  
878 of cascade models (Log-Poisson, Universal Multifractal, and even no explicit parameterization in  
879 some cases...). In this study, we analyzed the multifractal behavior of remotely sensed SM products  
880 over space scales ranging from the kilometric field scale to the continental scale. Universal  
881 Multifractal model was applied for the first time on SMOS SM data, giving access to large scale  
882 variabilities of SM, over the Australian landscape. Fractal and multifractal properties were observed,  
883 which confirmed and completed some results reported in existing literature.

884 The relevant aspect of the present work may be the multi-scale analysis of the outputs of the  
885 disaggregation algorithm DisPATCh (Merlin et al., 2008a; Molero et al., 2016). This deterministic  
886 algorithm improves the space resolution of SMOS SM products from 40 km to 1 km. To do this, it

887 combines coarse-scale SMOS SM with fine-scale ( $\approx 1$  km) MODIS optical/thermal data. Although  
888 several validation studies have been realized on this downscaling method ([Malbêteau et al., 2016](#);  
889 [Merlin et al., 2012, 2013, 2015](#); [Molero et al., 2016](#)), none fully explored its statistical behavior over a  
890 continuum of space scales. In this context, we applied fractal and multifractal analysis on the  
891 different products involved in DisPATCH algorithm, including disaggregated (and original) SM  
892 products, and MODIS auxiliary data which are vegetation indices (NDVI) and surface temperatures  
893 (LST).

894 Input products of DisPATCH revealed relatively good scaling properties over the considered scale  
895 ranges. Indeed, NDVI, LST and original SM were characterized by a power law evolution of their  
896 power spectra and statistical moments, meaning respectively fractality and multifractality. However,  
897 a specific scaling behavior was noticed for the output disaggregated SM. Two scaling regimes were  
898 obtained, with a transition scale observed at about ten kilometers, on both spectra and moments.  
899 Considering spectral analysis, on large scales ( $l > 10$  km), disaggregated SM was found to have the  
900 same scaling as the original SM measured from satellite. On finer scales ( $l < 10$  km), a different  
901 behavior was noticed, with an increasing slope of the power spectrum. Similar scale break was  
902 detected on statistical moments, showing that both spectral and multifractal properties of DisPATCH  
903 SM are characterized by this twofold scaling signature.

904 Two possible arguments were given to explain the specific scaling of the disaggregated SM. First,  
905 a more physical interpretation may indicate that this twofold scaling behavior would be related to  
906 the real properties of SM. As it was previously observed by [Kim and Barros \(2002b\)](#), such scale break  
907 would be reflective of nonlinear hydrometeorological processes (rainfall, infiltration,  
908 evapotranspiration) acting at different space scales and modulated by terrain, soils and vegetation  
909 distributions. The spatial structure of SM may be more impacted by infiltration or runoff at the field  
910 scale, whereas it would be mainly controlled by evapotranspiration or precipitation at the  
911 regional/continental scale. A more significant scaling transition was observed on the driest days  
912 (early summer), which may support the link between SM and external forcing agents such as  
913 precipitation.

914 A second explanation would be more algorithmic and directly related to the processing of the  
915 different products used and created within the algorithm. The model used in DisPATCH would  
916 generate SM whose statistics are not properly distributed across scales. This may occur at two levels  
917 in the algorithm. First, some MODIS products properties (such as breaks in the scaling) may be  
918 retrieved in the final DisPATCH products. Indeed, a spectral drop at about the same ten kilometers  
919 scale was detected on LST power spectra. Although it is less pronounced than on disaggregated SM,

920 this scale break may be introduced by MODIS LST and amplified by the disaggregation model. Since  
921 one single scaling regime was noticed on Brightness Temperature (BT) products acquired in the L-  
922 band over the same area and the same period, these observations suggest that the unexpected  
923 scaling in MODIS products would be caused by the technology specific to optical/thermal sensors.  
924 Then, another impact of the algorithm on the multi-scale properties of SM may be related to signal  
925 processing artifacts occurring with the combination of several products defined with different grid  
926 spacings. This combination is required to permit conservativity between input and output SM  
927 products. However, from a signal processing point of view, this could create systematic footprints on  
928 the final image (i.e., visible SMOS pixels in the downscaled products) and therefore affect the power  
929 spectrum (convolutions by cardinal sine-like functions).

930 At this point, it is difficult to determine which of the physical or algorithmic factors would be at  
931 the origin of this twofold scaling behavior. Though, a plausible hypothesis may be that both factors  
932 could affect the scaling of disaggregated SM. Indeed, a scale break at about the SMOS SM resolution  
933 could be initially produced by combination artifacts, which would be more or less amplified in the  
934 algorithm according to seasonal conditions, resulting in moving the scale break during the period to  
935 finer scales.

936 Further work need to be addressed to fully explain these results, in particular to determine to  
937 what extent each of the two factors impacts the scaling of DisPATCh SM. Complementary auxiliary  
938 data should be compared to our products. Indeed, an EOF or comparable analysis made on DisPATCh  
939 SM and topography, vegetation water content or soil content would provide relevant information  
940 about the connection between the spatial variability of these products and help with interpretation.  
941 Moreover, it would be interesting to verify if precipitation products can be characterized by a similar  
942 scale break on equivalent space scales and over the same area (Murray Darling Basin). However, it  
943 must be considered that such a comparison might be complex to interpret since, to our knowledge,  
944 no theoretical model has been proposed yet to relate the spatial scaling properties of SM and that of  
945 rainfall (as it was already done in the time domain by [Katul et al., 2007](#)). In the same way, the  
946 comparison between DisPATCh SM and airborne BT is not that trivial, because relatively complex  
947 operations are involved to get inverted SM from BT. To illustrate this, [Mascaro and Vivoni \(2012\)](#)  
948 noticed monofractality from BT data, whereas multifractality was observed from the corresponding  
949 inverted SM data. The scaling properties of BT could be affected during the inversion process,  
950 explaining why the single scaling we observed on BT does not imply single scaling of DisPATCh SM.  
951 Therefore, multifractal analysis of proper fine scale SM products may clarify this idea and help  
952 validating DisPATCh SM variability.



953 In the hypothesis of a model-induced scale break, current work is engaged to quantify the effect  
954 of MODIS products and the trace of pixel SMOS on different dates and on proper operational  
955 conditions (analysis of products used and generated within the C4DIS processor). An application of  
956 DisPATCh using Landsat-7 auxiliary data instead of MODIS products was realized by [Merlin et al.](#)  
957 [\(2013\)](#), allowing a disaggregation process at sub-kilometric scales (100 m). Since Landsat-7 provides  
958 optical/thermal data with higher resolution than MODIS, it could be interesting to verify if both  
959 Landsat-7 and the resulting disaggregated SM product would be characterized by a similar scale  
960 break, but shifted on finer scales than the 10 km scale observed on MODIS. Therefore, the results  
961 obtained could help to quantify the real impact of optical/thermal auxiliary data on the multi-scale  
962 properties of DisPATCh SM. On a more operational point a view, if this impact is confirmed, the  
963 results obtained may help to define a specific scale below which the variability generated by the  
964 disaggregation model may not be as reliable as it should be. Concerning the impact of SMOS pixels  
965 footprint effects on the disaggregated product, a solution could be to filter out the sharp transitions  
966 at SMOS pixels limits. However, this should be done with caution since such filters may excessively  
967 attenuate the variance at smaller scales. Another way to investigate our observations of DisPATCh  
968 SM is to focus on its dynamical behavior over different aggregation scales. Indeed, one of the main  
969 problem in downscaling a dynamical behavior arises from the fact that the dynamical behavior of an  
970 aggregated signal can be approximated by the same deterministic equation structure only when the  
971 aggregated area is phase-synchronized ([Mangiarotti et al., 2016](#)). Considering this issue, the  
972 applicability of deterministic downscaling methods like DisPATCh may not be that obvious over  
973 certain spatial scales, leading to several difficulties and, perhaps, contributing to explain the scaling  
974 irregularities observed in this study.

975 Finally, a possibility could be to compare the SM variability produced by DisPATCh with that  
976 created by fractal stochastic downscaling methods. Based on scaling properties, these methods  
977 preserve the probability distribution from large to fine scales. In precipitation, several studies applied  
978 these algorithms on rainfall data ([Rebora et al., 2006](#); [Sharma et al., 2007](#)). Research works proposed  
979 methods developed on multiplicative cascade such as Log-Poisson ([Deidda, 2000](#)) or UM model  
980 ([Gires et al., 2012](#)), revealing some potential to quantify uncertainty and representativeness errors  
981 between coarse-scale and in-situ measurements. Concerning SM downscaling, some studies used  
982 such fractal-based methods ([Bindlish and Barros, 2002](#); [Kim and Barros, 2002a](#); [Mascaro et al., 2010](#)).  
983 In our study, it may be interesting to apply this kind of method on SMOS products. This would  
984 consists in injecting in the UM model the values of  $\alpha$  and  $C_1$  parameters obtained from SMOS  
985 products on large scales, and then continuing the cascade at higher resolutions. Following this  
986 procedure, the fine-scale field will have the same scaling properties as the coarse-scale one.

987 However, since the disaggregation is based on random generator, an ensemble of possible fields can  
988 be proposed from just one pair of  $\alpha/C_1$  parameter. Therefore, this kind of methodology may not be  
989 fully suitable in the case of operational hydro-agricultural applications, in particular when  
990 determining the position of the extremes. To overcome this inconvenient, a combination of the two  
991 approaches may be an interesting compromise between statistical scaling and evaporation-based  
992 determinism. For example, in DisPATCh algorithm, an idea might be to find a modified estimator of  
993 SEE that would be used in the disaggregation equation (18). This modified SEE would be computed by  
994 applying a 2D filter on the original SEE, which would be actually equivalent to perform a fractional  
995 integration of order  $\Delta H = H_{requested} - H_{non-filtered}$  with  $H_{requested}$  and  $H_{non-filtered}$  measured respectively  
996 from SMOS soil moisture (at large scales) and from non-filtered SEE (for scales under 10 km). Doing  
997 this, the spectral slope of SEE may be adjusted, like that of the final disaggregated soil moisture.  
998 Thus, coarse-scale and fine-scale fields could be related through a common degree of fractional  
999 integration, which may contribute to limit the twofold scaling behavior observed on the  
1000 disaggregated product. In practice, this modification would not be easy to implement since the  
1001 filtering should be properly dimensioned in order to affect only the small scales, between 1 and 10  
1002 km. Moreover, this texture-based image correction may impact the physical properties of SEE, so  
1003 there would be a compromise to be made on this aspect.

1004

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