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Optimal timing of carbon sequestration policies

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Abstract

Carbon capture and storage (CCS) is one of the most promising abatement options to curb CO2 emissions of the energy sector. Usually, in models where the atmospheric carbon stock is constrained to not exceed a given ceiling and under constant average costs, it is never optimal to deploy CCS before the time at which this ceiling is reached. In this paper, we show that, when the CCS technology is submitted to decreasing returns to scale, abatement activities must begin earlier, i.e. before the climate constraint binds. It must also cease strictly before the end of the ceiling period.
1 Introduction

In order to reduce the anthropogenic CO$_2$ emissions generated by fossil fuel combustion, the economy has generally two options: improving the energy efficiency of the production system, or abating directly the carbon emissions that are released into the atmosphere. Currently, Carbon Capture and Storage (CCS) appears to be the most cost-effective available technology to achieve this second objective.$^1$

When the atmospheric carbon stock is constrained to not exceed a given stabilization cap, Chakravorty et al. (2006) show that abatement activities must be delayed up to the time at which this cap is reached. The same conclusion can be found in Lafforgue et al. (2008), who extend this model to take into account the limited capacity of the geological reservoirs within which captured emissions are stored. A common explanation of this result comes from the characteristics of the CCS cost function: both studies consider a constant average cost. Does this result remain valid when the CCS cost function exhibits more sophisticated properties? It may depend. For instance, Amigues et al. (2014) show that introducing a learning-by-doing process in CCS activities does not modify the general timing.

The present paper investigates this question of the optimal timing of CCS policy, relatively to the time at which carbon emissions are constrained by the ceiling, but by considering decreasing returns in CCS or, equivalently, by assuming increasing marginal costs.$^2$ We show that it is now optimal to capture emissions before being constrained by the atmospheric carbon cap. Under constant average cost, i.e. when the unit cost is the same at any point in time, the discounting argument implies to delay as much as possible any additional cost. But under increasing average cost, smoothing the carbon capture process allows to reduce its cost. This effect counterbalances the discounting effect. Last, since the use of fossil resources combined with CCS technology is under competition with carbon-free resources such as solar energy, we examine how a more or less costly clean substitute modifies the optimal paths when the cap constraint binds.

The paper is organized as follows. Section 2 presents the model. Section 3 introduces the optimal program of the social planner and describes the first-order conditions. Section 4 characterizes the qualitative properties of the typical optimal paths and Section 5 gives two examples of them. Section 6 concludes.

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$^1$According to Iseleg and Reischelstein (2011), CCS may have considerable potential of CO$_2$ reduction at a reasonable social cost, with a break-even carbon value for adopting this option amounting to $30 per ton in the case of coal-fired power plants, and to $60 in the case of natural gas plants. See also Herzog (2011) for a technical and economical presentation of this technology, and Kalkuhl et al. (2015) for a balanced account of its competitiveness.

$^2$To justify this assumption, note that the total CCS cost includes the cost of capture of carbon emissions at the source, the cost of transport through the network and the cost of storage in the reservoirs. Whereas the first component may benefit from technological improvements, the two other components generally exhibit increasing marginal cost. Then, for a given technological level of the capture process, each new unit of carbon which is abated is more costly than the previous one since it must be transported to a more distant reservoir and/or it must be stored in a reservoir where the pressure increases. An empirical justification of these scale effects can also be found in Bielicki (2008) or in Durmaz and Schroyen (2013).
2 Model and notations

We consider a model of final energy production from two primary energy sources, coal and solar. Coal is non-renewable and potentially carbon emitting. We call "dirty" coal the polluting final energy obtained from burning coal without abatement, and "clean" coal the final energy which is carbon-free thanks to the CCS technology. Solar energy is renewable and carbon-free. We denote respectively by $x_d(t)$, $x_c(t)$ and $y(t)$ their consumption at time $t$. Assuming perfect substitution, the total energy consumption is given by $q(t) = x_d(t) + x_c(t) + y(t)$. This consumption generates an instantaneous gross surplus $u(q)$, with $u(.)$ satisfying the standard properties of utility functions (strictly increasing, strictly concave and verifying the Inada conditions). We define $p(q) \equiv u'(q)$ as the marginal gross surplus, i.e. the energy consumer price.

Denoting by $X(t)$ the available stock of coal at time $t$ and by $X^0$ the initial reserves, the dynamics of coal extraction is given by:

$$
\dot{X}(t) = -[x_d(t) + x_c(t)], \quad X(0) \equiv X^0.
$$

The average extraction cost function $c(.)$, common to the two types of coal, is assumed to be strictly decreasing and convex in $X$, with $\lim_{X \to 0} c(X) = +\infty$.

Thus the stock of coal finally exploited is endogenously determined. Producing energy from clean coal is more expensive than from dirty coal since an additional CCS cost must be incurred. Assuming decreasing returns to scale in this technology, the average CCS cost function $a(.)$ is then strictly increasing and convex in $x_c$, and such that $a(0) \equiv a^0$. We define $ma(x_c) \equiv a(x_c) + x_c a'(x_c) > 0$ as the marginal CCS cost, with $ma(0) = a$ and $ma'(x_c) > 0$.

Let $Z(t)$ be the atmospheric carbon stock at time $t$, and $Z^0$ be the initial concentration inherited from the past. The dynamics of this stock is given by:

$$
\dot{Z}(t) = \zeta x_d(t) - \alpha(Z(t)), \quad Z(0) \equiv Z^0,
$$

where $\zeta$ is the unit carbon content of coal and $\alpha(.)$ is a natural regeneration function, assumed to be increasing and strictly concave in $Z$.

As in Chakravorty et al. (2006), the pollution stock is constrained to not overshoot a given critical threshold $\bar{Z}$:

$$
\bar{Z} - Z(t) \geq 0.
$$

An implication of this constraint is that, when the ceiling is reached, dirty coal consumption is constrained and, from (2), its maximal level amounts to $\bar{x}_d \equiv \alpha(\bar{Z})/\zeta$.

Last, the natural flow of solar energy is supposed to be large enough to supply the energy demand in the absence of coal. It is processed at a constant average cost $b$ which

\footnote{The underlying assumption is that coal deposits have different extraction costs depending upon various characteristics. Minimizing the discounted sum of these costs implies that they are exploited by increasing order of costs.}

\footnote{Examples of convex CCS cost functions can be found in Ayong le Kama et al. (2013), or Gerlagh and van der Zwaan (2006).}

\footnote{For a discussion on the different types of functions $\alpha(Z)$, see Toman and Withagen (2000).}
is larger than $c(X^0)$ to justify the use of coal, at least initially. When coal is no more competitive and its exploitation ceases, the remaining stock $X_b$ still underground must be such that $c(X_b) = b$. Then the optimal solar energy consumption amounts to $\tilde{y}$, solution of $u'(\tilde{y}) = b$.

### 3 Program of the social planner

The social planner problem consists in determining the path $\{(x_c(t), x_d(t), y(t)), t \geq 0\}$ that maximizes the discounted net surplus of energy users. Denoting by $\rho$ the discount rate, the optimal program is (we drop the time index when this causes no confusion):

$$
\max_{\{x_d,x_c,y\}} \int_0^\infty [u(x_d + x_c + y) - c(X)(x_d + x_c) - a(x_c)x_c - by] e^{-\rho t} dt
$$

subject to (1)-(3) and to the non-negativity constraints on $x_d$, $x_c$ and $y$. Let $\lambda_X$ and $-\lambda_Z$ be the costate variables of $X$ and $Z$ respectively. Let $\nu_Z$ be the Lagrange multiplier associated with the ceiling constraint on $Z$ and $\gamma$ be those corresponding to the non-negativity constraints on the control variables. The Lagrangian of the program is:

$$
\mathcal{L} = u(x_d + x_c + y) - c(X)(x_d + x_c) - a(x_c)x_c - by - \lambda_X[x_d + x_c] - \lambda_Z[\zeta x_d - \alpha(Z)]
+ \nu_Z[\bar{Z} - Z] + \gamma_d x_d + \gamma_c x_c + \gamma y y
$$

The first-order conditions (together with the usual transversality conditions and complementary slackness conditions, not mentioned here) are:

$$
\frac{\partial \mathcal{L}}{\partial x_d} = 0 \Rightarrow p = c(X) + \lambda_X + \zeta \lambda_Z - \gamma_d \tag{4}
$$

$$
\frac{\partial \mathcal{L}}{\partial x_c} = 0 \Rightarrow p = c(X) + \lambda_X + m a(x_c) - \gamma_c \tag{5}
$$

$$
\frac{\partial \mathcal{L}}{\partial y} = 0 \Rightarrow p = b - \gamma y \tag{6}
$$

$$
\dot{\lambda}_X = \rho \lambda_X - \frac{\partial \mathcal{L}}{\partial X} \Rightarrow \dot{\lambda}_X = \rho \lambda_X + c'(X)(x_c + x_d) \tag{7}
$$

$$
\dot{\lambda}_Z = \rho \lambda_Z + \frac{\partial \mathcal{L}}{\partial Z} \Rightarrow \dot{\lambda}_Z = [\rho + \alpha'(Z)]\lambda_Z - \nu_Z \tag{8}
$$

As usual in this type of ceiling models, we have three main periods: a pre-ceiling phase (gradual ascent of the atmospheric carbon stock to the stabilization cap), a ceiling period (binding ceiling constraint) and a post-ceiling phase (no more environmental externality). Let $t_Z$ and $\bar{t}_Z$ be the dates at which the ceiling constraint begins and ceases to bind, respectively. The use of the three energy options is driven by their relative competitiveness, determined by comparing their respective full marginal costs as given by the right hand

---

6 Using $-\lambda_Z$ as the costate variable of $Z$, we can directly interpret $\lambda_Z \geq 0$ as the social marginal cost of pollution. Note that, in the associated decentralized economy, the optimal carbon tax per unit of coal would be $\zeta \lambda_Z$. 

---

3
side of (4)-(6), according to the Herfindahl rule: \(^7\) "The cheapest, the first". Then, dirty coal must be used initially. We denote by \(t_x, t_c, t_X\) and \(t_y\), the times at which, respectively, clean coal exploitation begins, clean coal exploitation ceases, coal exploitation is over and solar energy exploitation begins.

Let \(p^F \equiv c(X) + \lambda_X\) be the part of the full marginal cost of coal including the extraction cost and the scarcity rent, common to the two types of coal. Time differentiating \(p^F\) and using (7), we get that \(p^F\) is always increasing as long as coal is used: \(p^F(t) = \rho \lambda_X(t) > 0\), \(\forall t < t_X\). For \(t < t_Z\), since \(\nu_Z = 0\), (8) implies that \(\lambda_Z(t) = \lambda_{Z0} e^{A(t)}\), with \(\lambda_{Z0} = \lambda_Z(0)\) and \(A(t) \equiv \int_0^t [\rho + \alpha'(Z(\tau))]d\tau\). Once the ceiling constraint is no longer active, after time \(\hat{t}_Z\), \(\lambda_Z\) must be nil.

4 Qualitative properties of the optimal paths

Under constant average CCS cost, Lafforgue et al. (2008) conclude that clean coal exploitation must occur at the earliest at the beginning of the ceiling period and that its consumption rate must be decreasing. In addition, clean coal and solar energy are never used at the same time. As we shall show now, these findings are no longer valid under decreasing returns to scale in the CCS technology. To establish this result, we need the three following lemmas.

**Lemma 1** The exploitation of solar energy begins neither before the ceiling period, nor before the beginning of the clean coal exploitation: \(t_y \geq \max \{t_Z, \hat{t}_Z\}\).

**Proof**: i) Assume that, at some time \(t'\), with \(t' < t_Z < t_X\), solar energy is competitive: \(b \leq \min \left\{p^F(t') + \zeta \lambda_{Z0} e^{A(t')}, p^F(t') + ma(x_c(t'))\right\}\). Since \(p^F\) increases for any \(t < t_X\) and since \(A\) also increases for any \(t < t_Z\), then we must have \(b < p^F(t) + \zeta \lambda_{Z0} e^{A(t)}\), \(\forall t \in (t', t_Z)\). Dirty coal is thus not competitive relatively to solar energy although it could be competitive, or not, relatively to clean coal. Whatever the case, only carbon-free energy (solar or clean coal) is used between \(t'\) and \(t_Z\), implying that \(Z\) decreases. Hence \(Z(t_Z) < \bar{Z}\), which is a contradiction.

ii) Assume now that there exists a time interval \((t', \hat{t}_c)\) during which solar energy is more competitive than clean coal: \(b < p^F(t) + a\), \(\forall t \in (t', \hat{t}_c)\). Since \(p^F\) increases and \(ma(x_c) > a\), this inequality holds even after \(\hat{t}_c\), meaning that once solar energy is competitive relatively to clean coal, it remains competitive forever. Then, clean coal exploitation cannot occur after the beginning of the solar energy exploitation.

**Lemma 2** If clean coal is used during the ceiling period, it is never optimal to delay its exploitation after \(t_Z\). Moreover, its exploitation must cease before \(t_Z\).

\(^7\)Herfindahl (1967). See also Chakravorty et al. (2008).
Proof: i) Assume that clean coal exploitation begins strictly after $t_Z$. We can thus consider two time intervals within the ceiling period, $(t_Z, t')$ and $(t', t'')$, with $t_Z < t' < t'' \leq t_Z$, during which $x_c = 0$ and $x_c > 0$, respectively. During the first time interval $(t_Z, t')$, since solar energy cannot be exploited before clean coal (cf. Lemma 1), only dirty coal is used and we have: $p^F + \zeta \lambda_Z = u'(\bar{x}_d) \leq p^F + a$. Since $p^F(t_Z) + \zeta \lambda_Z(t_Z) = u'(\bar{x}_d) \leq p^F(t_Z) + a$ and since $p^F$ is increasing, then:

$$
\lim_{t \to t'} \{p^F(t) + \zeta \lambda_Z(t)\} = u'(\bar{x}_d) < \lim_{t \to t'} \{p^F(t) + a\} 
$$

(9)

Consider now the second interval $(t', t'')$. Since both dirty and clean coal are used, we get:

$$
\lim_{t \to t''} \{p^F(t) + \zeta \lambda_Z(t)\} = \lim_{t \to t'} \{p^F(t) + am(x_c(t))\} \geq \lim_{t \to t'} \{p^F(t) + a\} 
$$

(10)

(9) and (10) imply that $\lambda_Z$ is not continuous at time $t'$, which is not possible. Then, clean coal exploitation cannot begin strictly after $t_Z$.

ii) Since $\lambda_Z(t_Z) = 0$, the full marginal cost of dirty coal at time $t_Z$ is simply $p^F(t_Z)$. Hence there exists a time interval $(t_Z - \Delta, t_Z)$, with $0 < \Delta < t_Z - t_Z$, during which $p^F + a > p^F + \zeta \lambda_Z$. During this time interval, clean coal is necessarily less competitive than dirty coal and it is then no longer used.

Lemma 3 When clean coal is exploited during the ceiling period, its consumption rate decreases. When solar energy is exploited simultaneously, then its consumption rate increases.

Proof: First, assume that only coal is exploited during the ceiling period. Then, (5) implies $u'(x_c + \bar{x}_d) = p^F + ma(x_c)$. Time differentiating this equation, substituting for $\dot{p}^F$ and rearranging, we get: $\dot{x}_c = \frac{\rho \lambda x}{w'(x_c + \bar{x}_d) - ma'(x_c)} < 0$. Assume now that solar energy is also exploited during the same period. Then, from (5) and (6), we have $b = p^F + ma(x_c)$, which implies: $\dot{x}_c = -\frac{\rho \lambda x}{ma'(x_c)} < 0$. In this case, since $x_c + \bar{x}_d + y = \bar{y}$ is constant and since $\dot{x}_c$ is negative, then we have $\dot{y} > 0$.

During the ceiling period, clean coal production always decreases. Contrary to the constant CCS cost case, when the solar cost is low, both clean and dirty coals can now be exploited simultaneously with solar energy. Furthermore, since the average CCS cost increases, assuming that clean coal is not exploited before the ceiling period would imply that the shadow cost of the pollution stock is discontinuous at time $t_Z$, which is clearly not optimal. Consequently, Proposition 1 shows that it is optimal to deploy the CCS option before being constrained by the ceiling.\(^8\)

\(^8\)It can be easily shown that, in a deterministic model, an abatement program is optimal if and only if the ceiling is attained, even at a single point in time. Then we are sure that the ceiling constraint binds, even if this abatement program starts before.
**Proposition 1** Under decreasing returns to scale in the CCS technology, clean coal exploitation begins during the pre-ceiling period: \( t_c \leq t_Z \). During this period, its consumption rate increases.

**Proof:** i) If clean coal is used during the ceiling period, its exploitation must begin at time \( t_Z \) at the latest (cf. Lemma 2). Hence there exists some time interval \((t_Z, t_Z + \Delta)\), \( \Delta > 0 \), during which \( p^F + \zeta \lambda = p^F + ma(x_c) \). Since \( x_c \) decreases within this interval (cf. Lemma 3) and since \( ma \) is increasing in \( x_c \), then:

\[
\lim_{t \downarrow t_Z} \left\{ p^F(t) + \zeta \lambda_Z(t) \right\} = \lim_{t \downarrow t_Z} \left\{ p^F(t) + ma(x_c(t)) \right\} > p^F(t_Z) + a \tag{11}
\]

Assume now that, before \( t_Z \), clean coal is not competitive yet: \( p^F + \zeta \lambda \leq p^F + a \). Hence:

\[
\lim_{t \downarrow t_Z} \left\{ p^F(t) + \zeta \lambda_Z(t) \right\} \leq p^F(t_Z) + a \tag{12}
\]

(11) and (12) imply that \( \lambda_Z \) is discontinuous at time \( t_Z \), which is not possible.

ii) When clean and dirty coals are exploited simultaneously before \( t_Z \), then (5) and (4) imply \( p^F + \zeta \lambda_Z \geq p^F \). Hence:

\[
\dot{x}_c = \frac{\zeta \lambda_Z e^A}{ma(x_c)} > 0.
\]

---

5 Examples of optimal paths

We have shown that it can be optimal to use clean coal and solar energy simultaneously during the ceiling period. Then, two main scenarios have to be considered, depending on whether the cost of solar energy is high or low.

In the high solar cost scenario, characterized by \( b > u'(\bar{x}_d) \), solar energy is never competitive during the ceiling period. A typical path is illustrated in Figure 1.9

Initially only dirty coal is exploited. Its consumption decreases and the energy price increases, due both to the resource scarcity rent and to the rising carbon shadow cost, up to the time \( t_c \) at which clean coal becomes competitive, i.e. at which \( p^F + \zeta \lambda = p^F + a \). The next phase is still below the ceiling, but with both types of coal used simultaneously, which implies \( \zeta \lambda = ma(x_c) \). The shadow cost \( \lambda \) being increasing and the returns to scale in CCS, decreasing, dirty coal exploitation decreases and, simultaneously, clean coal exploitation increases during this second phase. Clean coal consumption attains its maximum at time \( t_Z \) when the pollution stock reaches the ceiling. Next, during the first phase at the ceiling, both types of coal are still used. However, clean coal consumption decreases down to 0 and, simultaneously, the energy price steadily increases up to \( u'(\bar{x}_d) \), which is attained at time \( \bar{t}_c \). During the second ceiling phase, only dirty coal is exploited:

---

9Figures 1 and 2 read as follows. The top panel draws the optimal price path (bold line) as the lower envelop of the respective full marginal cost curves of each energy option. The bottom panel depicts the composition of the energy portfolio, the bold line being the total energy consumption \( q \).
Figure 1: Optimal paths - The high solar cost case
$x_d(t) = \bar{x}_d$. At the end of this phase, the pollution stock decreases below the stabilization cap and $\lambda_Z = 0$ forever. The next phase is characterized by an energy production supplied by dirty oil and an increasing energy price, up to the time $t_y$ at which $X = X_b$ and $p^F = b$. Then coal exploitation ceases and the demand is exclusively supplied by solar energy: $y(t) = \bar{y}$, $\forall t > t_y$.

When the cost of solar energy is low enough, for $b < u'(\bar{x}_d)$, it can be optimal to deploy this energy during the ceiling period, simultaneously to clean coal. This scenario is illustrated in Figure 2.

As previously, clean coal exploitation starts before the stabilization cap has been reached. But now, the period at the ceiling includes three phases. During the first one, the two types of coal are used, the production of clean coal being decreasing up to the time at which the solar energy becomes competitive. The next phase is a phase during which the three types of energy are used simultaneously, a substitution from clean coal production toward solar energy occurring until the end of the abatement efforts. The third phase combines solar energy and dirty coal until the coal grade $X_b$ is attained. Then, coal exploitation ceases for good and the energy consumption is supplied by solar energy only. To understand this complex pattern, remark that clean coal energy is submitted to the increasing scarcity of the non-renewable resource, in addition to decreasing returns to scale. This explains why solar energy, while more costly than fossil fuel based energy, replaces progressively clean coal.

6 Conclusion

Operation scale is a main challenge for emissions mitigation technologies. This paper has explored this issue in the time-to-build context of a transition between fossil fuels and carbon-free energy and by assessing the consequences of long run decreasing to scale in abatement. We have concluded that the abatement process must start strictly before the climate constraint begins to bind. Since the first unit of abatement is always the cheapest one, discounting argument calls for a CCS deployment as early as possible. This stands in sharp contrast with the usual conclusions derived from constant average abatement cost models. Moreover, until the climate constraint binds, the abatement rate must increase over time. As usual in this type of model, the optimal policy can be decentralized by taxing the pollution emissions. The unitary tax rate $\lambda_Z$ would be increasing up the time at which the cap constraint is effective and next decreasing down to zero during the ceiling period.
Figure 2: Optimal paths – The low solar cost case
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