Pooling promises with moral hazard

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Abstract

In this paper, we extend the framework of Dubey and Geanakoplos (2002) to the case of moral hazard. Risk-averse consumers, who can influence the likelihood of states of nature by undertaking a hidden action, receive insurance by voluntarily participating in a pool of promises of deliveries of future uncertain endowments. In exchange, they gain the right to receive a share of the total return of the pool, in proportion to their promises. We first analyze the equilibrium properties of the model and then illustrate how an aggregate pool of promises of heterogenous consumers, differing in expected endowment, results in a welfare improvement over the two segregated pools.

Keywords: moral hazard, pool of promises, heterogeneous consumers.

JEL Classification: D3, D8, G2.

1 Introduction

In this paper, we study a model in which risk-averse consumers face uncertain endowments. Consumers can influence the likelihood of the states of nature by undertaking a costly action. Since the action is unverifiable, there is moral hazard. Contrary to the traditional literature on insurance with moral hazard (see e.g. Arnott and Stiglitz 1988), we do not consider that consumers buy insurance contracts from perfectly competitive insurance companies. Instead, we assume that consumers commit to contribute a fraction of their endowments to a common pool, and, therefore, gain the right to receive a fraction of the total return of the pool proportional to their promises.

In particular, and as in Dubey and Geanakoplos (2002), consumers take the return of the pool as given and they are free to choose how much to promise to the pool. This feature allows for the possibility that consumers, although equal ex-ante, choose to promise differently, and, as a consequence, choose different actions. We verify that this possibility actually occurs,

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as well as other possible equilibrium configurations in which all consumers make the same choice of action and promise. Additionally, we consider the case of ex-ante heterogeneous consumers, where one type of consumers has higher expected endowments than the other, conditional on choosing the same action. In this case, it could be conjectured that the wealthiest would prefer a pool only among themselves rather then a pool together with the poorest ones, as the latter would lower the pool's return. However, we provide an example showing that the wealthiest consumers have no loss in welfare by joining a pool to! gether with the poorest ones, while the latter are better off. The crucial feature of this result relies on moral hazard, that is, on the possibility of influencing the value of expected endowments by choosing different actions. In the aggregate pool, the proportion of the wealthiest consumers choosing an action which positively affects the return of the pool is increased. This example illustrates how such a pool of voluntary promises can be used for redistribution purposes, as opposed to compulsory systems. This is a crucial feature of our model in contrast with other contributions that consider mutual arrangements in which participants have to pay a uniform contribution to the pool, see e.g. Guinnane and Streb (2011).

The framework first proposed by Dubey and Geanakoplos (2002) had, as its main purpose, to overcome the problem of existence of equilibrium in the competitive model with adverse selection of Rothschild and Stiglitz (1976). Other authors, since that time, have been extending and applying their framework but most consider setups with adverse selection (see, among others, Martin (2007) and Fostel and Geanakoplos (2008)). To our knowledge our contribution is the first to consider a pool of promises as a means of insurance in the presence of moral hazard. In particular, we identify an equilibrium where ex-ante equal consumers end up choosing different actions and different consumption bundles, even though they are equivalent in terms of utility. This feature enables redistribution among consumers even when they are ex-ante heterogenous, a case that we also consider.

Our paper is set out as follows: in section 2, we introduce the model; in section 3, we present our results, illustrate them through examples, and discuss their main implications.

2 The model

We consider a pure exchange economy with a single consumption good. The economy is populated by a large number of ex-ante identical consumers, and it lasts for two periods $t = 0, 1$. At $t = 0$ there is no consumption, and at $t = 1$ each consumer has verifiable endowments that depend on a state of nature. There are two possible states $s = G, B$, and we let $w = (w_G, w_B) \in \mathbb{R}^2_+$ denote the vector of endowment, with $w_G > w_B \geqslant 0$.

Consumers may influence the likelihood of states of nature by undertaking an action $a \in \mathcal{A} = \{L, H\}$, which is not verifiable, and thus information is asymmetric. Let π_a denote the probability of the state G when action a is chosen, with $1 > \pi_H > \pi_L > 0$. The (dis)utility of the action is c_a, and we assume $c_H > c_L = 0$. The tradeoff is thus clear: on the one hand, undertaking action H increases the likelihood of the state G where endowment is higher but, on the other hand, it is costly since it requires higher effort.

Preferences are represented by an expected utility function $U(x, a) : \mathbb{R}^2_+ \times \mathcal{A} \to \mathbb{R}$, which depends on a state contingent consumption bundle $x = (x_G, x_B) \in \mathbb{R}^2_+$ and action as follows:

$$U(x, a) := \pi_a u(x_G) + (1 - \pi_a)u(x_B) - c_a \,, \tag{1}$$

with u twice differentiable, strictly increasing and strictly concave.

2.1 The pool of promises

Since consumers are risk-averse, they prefer to smooth their consumption across idiosyncratic states. This can be accomplished by pooling the risk associated with individual endowments. Indeed, we assume that each consumer faces uncertainty independently of other consumers. This assumption, in addition to the fact that there is a large number of consumers, rules out aggregated uncertainty.

Inspired by Dubey and Geanakoplos (2002), we propose the following insurance mechanism: at $t = 0$, each consumer voluntarily promises to make a delivery to a common pool, proportional to his endowment at $t = 1$. In exchange, at $t = 1$, the consumer receives a share of the total resources of the pool in proportion to his promise, and not to his actual delivery. More precisely, suppose that a fraction q of consumers choose $a = H$ and promise θ_H, while a fraction $1 - q$ choose $a = L$ and promise θ_L. In this case, total deliveries to the pool equal $q\theta_H\bar{w}_H + (1 - q)\theta_L\bar{w}_L$, where $\bar{w}_a = \pi_a w_G + (1 - \pi_a)w_B$ is the average (aggregate) endowment when action a is undertaken. Obviously, probabilities, and hence the fraction of consumers in each state, depend on the action chosen by consumers. Let κ denote the return per promise, given by:

$$\kappa = \frac{q\theta_H\bar{w}_H + (1 - q)\theta_L\bar{w}_L}{q\theta_H + (1 - q)\theta_L}. \tag{2}$$

Note that, since all consumers participate in the pool, the idiosyncratic uncertainty is wiped out, hence κ is not state contingent. Additionally, (2) implies that $w_B < \bar{w}_L \leqslant \kappa \leqslant \bar{w}_H < w_G$, and therefore that net deliveries to the pool $\theta_a(w_s - \kappa)$ are positive for consumers in the good state of nature, and negative for consumers in the bad state, irrespective of the action chosen. Indeed, state contingent consumption bundles are given by:

$$x_s = w_s - \theta(w_s - \kappa), \tag{3}$$

with $s = G, B$. Hence, consumers in state G consume less than their endowment, while those in state B consume more than their endowment. Therefore, the pool actually works as an insurance mechanism.

2.2 Consumers' problem

Consumers take the return per promise κ as given and choose their promises and actions so as to maximize expected utility. Formally, the consumers' problem can be written as follows:

$$\max_{\theta\in\Theta, a\in\mathcal{A}} \quad v(\theta, a) = \pi_a u(w_G - \theta(w_G - \kappa)) + (1 - \pi_a)u(w_B - \theta(w_B - \kappa)) - c_a, \tag{4}$$

where we have replaced (3) into (1), with $\Theta = [0, \bar{\theta}]$, and $\bar{\theta} = w_G/(w_G - \kappa)$ being the maximum value θ can take to ensure a non-negative x_G. In what follows, $\psi(\kappa) \subset \Theta \times \mathcal{A}$ denotes the set of solutions to problem (4). It is easy to verify that $\psi(\kappa)$ is not empty.

Note that $0 \leqslant \theta$ implies $x_G \leqslant w_G$, and therefore negative insurance is ruled out. Moreover, since $\bar{\theta} > 1$, overinsurance, that is $x_B > x_G$, is admitted. Also, state-contingent consumption levels are always non-negative. Indeed, $\theta \leqslant \bar{\theta}$ implies $x_G \geqslant 0$, and since $w_B < \kappa$, $\theta \geqslant 0$ also implies $x_B \geqslant 0$.

3 Results and discussion

In equilibrium, consumers maximize their utility by taking as given the return of the pool, which is endogenously determined in a consistent way. Therefore, we propose the following definition of equilibrium:

Definition 3.1. *An equilibrium with a pool of promises is* $(\tilde{\theta}, \tilde{a}, \tilde{q}, \tilde{\kappa})$ *such that:*

1. $(\tilde{\theta}, \tilde{a}) \in \psi(\tilde{\kappa})$,
2. $\tilde{\kappa}$ *satisfies (2)*,
3. $\tilde{q}$ *satisfies:*

$$\begin{aligned}
&(a)\ \tilde{q} = 0 \quad \textit{if} \quad (\tilde{\theta}, \tilde{a}) \in \psi(\tilde{\kappa}) \quad \textit{implies} \quad \tilde{a} = L\,, && \text{(Action } L \text{ Equilibrium)}\\
&(b)\ \tilde{q} = 1 \quad \textit{if} \quad (\tilde{\theta}, \tilde{a}) \in \psi(\tilde{\kappa}) \quad \textit{implies} \quad \tilde{a} = H\,, && \text{(Action } H \text{ Equilibrium)}\\
&(c)\ \tilde{q} \in (0,1) \quad \textit{otherwise}\,. && \text{(Mixed Action Equilibrium)}
\end{aligned}$$

The above definition states that the equilibrium values of q must be properly related to the optimal choices of consumers. In particular, $q = 0$ ($q = 1$) can only arise in equilibrium if $a = L$ ($a = H$) is the optimal choice for every consumer. Similarly, for $q \in (0,1)$ to arise in equilibrium, both $a = H$ and $a = L$ must be optimal choices of consumers. In what follows, we first show that an action H equilibrium never arises (Proposition 1), and then we propose conditions for the existence of both the action L equilibrium and the mixed action equilibrium (Propositions 2 and 3, respectively).

Proposition 1. [Impossibility of a high cost action equilibrium]
There cannot be an equilibrium in which all consumers undertake the action H*, i.e., if* $(\tilde{\theta}, H, \tilde{q}, \tilde{\kappa})$ *is an equilibrium, then* $\tilde{q} \neq 1$.

Proof. Let $\phi(\kappa, a) \subset \Theta$ denote the solution set of $\max_{\theta \in \Theta}\ v(\theta, a)$, and $\chi(\kappa, \theta) \subset \mathcal{A}$ the solution set of $\max_{a \in \mathcal{A}}\ v(\theta, a)$. Both $\phi(\kappa, a)$ and $\chi(\kappa, \theta)$ are non empty and $\phi(\kappa, a)$ is a singleton, because of the strict concavity of u. Notice that $(\tilde{\theta}, \tilde{a}) \in \psi(\kappa)$ implies $\tilde{\theta} = \phi(\kappa, \tilde{a})$ and $\tilde{a} \in \chi(\kappa, \tilde{\theta})$. Now, suppose, by way of obtaining a contradiction, that $\tilde{q} = 1$. In this case, (2) implies $\kappa = \bar{w}_H$. If $(\tilde{\theta}, H)$ is an equilibrium choice, then $(\tilde{\theta}, H) \in \psi(\bar{w}_H)$. This implies, in particular, $H \in \chi(\bar{w}_H, \tilde{\theta})$ and, therefore, $v(\tilde{\theta}, H) \geqslant v(\tilde{\theta}, L)$. Moreover, $(\tilde{\theta}, H) \in \psi(\bar{w}_H)$ also implies $\tilde{\theta} = \phi(\bar{w}_H, H)$ and, therefore, $\tilde{\theta} = 1$.! In this case, however, $v(\tilde{\theta}, L) > v(\tilde{\theta}, H)$, which is the desired contradiction. $\square$

Proposition 1 states that if $q = 1$ and $a = H$, then κ does not satisfy (2). Indeed, if consumers anticipate the high return per promise $\kappa = \bar{w}_H$, which is implied by $q = 1$, their optimal choice is actually to over insure themselves and to choose $a = L$. In the next proposition, we state the condition under which action L equilibrium exists.

Proposition 2. [Possibility of a low cost action equilibrium]
Let $\hat{\theta}_H = \phi(\bar{w}_L, H)$ *and* $\hat{\theta}_L = \phi(\bar{w}_L, L)$ *be the consumers' optimal promises when* $\kappa = \bar{w}_L$ *conditional on choosing, respectively,* $a = H$ *and* $a = L$*. If* $v(\hat{\theta}_L, L) \geq v(\hat{\theta}_H, H)$*, then a low action equilibrium exists.*

Proof. $\phi(\kappa, a)$ is introduced in Proposition 1. When $q = 0$, (2) implies $\kappa = \bar{w}_L$. If $\kappa = \bar{w}_L$, then consumers' optimal promise is $\hat{\theta}_L$ when $a = L$ and $\hat{\theta}_H$ when $a = H$. If $(\hat{\theta}_L, L)$ is preferred to $(\hat{\theta}_H, H)$, then indeed every consumer will choose $a = L$ and hence $q = 0$. □

Proposition 2 states that if all consumers choose $a = L$, then κ satisfies (2) and, thus, it identifies a possible equilibrium. However, we are also interested in the possibility of a mixed action equilibrium. Yet, since consumers are ex-ante equal, this can only happen if they are all indifferent to undertaking action H or action L. Proposition 3 states the condition under which this happens.

Proposition 3. [Possibility of a mixed action equilibrium]
If $v(\hat{\theta}_L, L) < v(\hat{\theta}_H, H)$, then a mixed action equilibrium exists.

Proof. When $v(\hat{\theta}_L, L) < v(\hat{\theta}_H, H)$, by adapting lemma 3.2 in Hellwig (1983) it is possible to show that there exist $\hat{\kappa} \in (\bar{w}_L, \bar{w}_H)$, $\theta_H < 1$ and $\theta_L > 1$ such that $\psi(\hat{k}) = \{(\theta_H, H), (\theta_L, L)\}$. In this case, by definition of equilibrium it must be that $\hat{\kappa}$ satisfies (2) and $q \in (0, 1)$. From (2) we get:

$$q = \left[1 + \frac{\theta_H\,(\hat{\kappa} - \bar{w}_H)}{\theta_L\,(\bar{w}_L - \hat{\kappa})}\right]^{-1} .$$

Since $\bar{w}_L < \hat{\kappa} < \bar{w}_H$, we immediately verify that $q \in (0, 1)$. □

Proposition 3 says that there exists $\hat{\kappa}$ such that consumers are indifferent between either action H or L when choosing two different promises. In this case, they split into the two actions in the proportion $q \in (0, 1)$ required to ensure that $\hat{k}$ satisfies (2).

Figure 1, inspired by Dubey and Geanakoplos (2002), illustrates a mixed action equilibrium. The initial contingent endowments are $(w_G, 0)$. Indifference curves are steeper when $a = H$ than when $a = L$. Therefore, they cross below the certainty line and make a kink.[1] Combining the two state contingent consumption levels, as given by (3), with a view to eliminating θ, we can relate x_B and x_G as follows:

$$x_B = \frac{(w_G - w_B)\kappa}{w_G - \kappa} - \left(\frac{\kappa - w_B}{w_G - \kappa}\right) x_G. \tag{5}$$

This equation shows that, by giving up $(w_G - \kappa)$ units of consumption in the state G, a consumer gets $(\kappa - w_B)$ units of consumption in the state B. In Figure 1, we plot three downward sloping lines corresponding to (5) when $\kappa = \bar{\kappa}, \hat{\kappa}, \underline{\kappa}$, where $\bar{\kappa} = \bar{w}_H$, $\underline{\kappa} = \bar{w}_L$, and $\hat{\kappa}$ is the value emerging in a mixed action equilibrium (Proposition 3).

Alternatively, we can relate the consumers' state contingent consumption levels, as given by (3), by eliminating κ:

$$x_B = x_G - (w_G - w_B)(1 - \theta). \tag{6}$$

This equation shows how much is left over for consumption in the bad state of nature for a promise θ . In particular, when $\theta = 1$, then $x_G = x_B$, and when $\theta < 1$ ($\theta > 1$), then $x_G > x_B$ ($x_G < x_B$). In Figure 1 we plot two of these curves: one associated with the action H promise; and the other associated with the action L promise. These are the upward sloping

[1] The locus of points where indifference curves corresponding to the same utility level cross is sometimes called the *switching locus*.

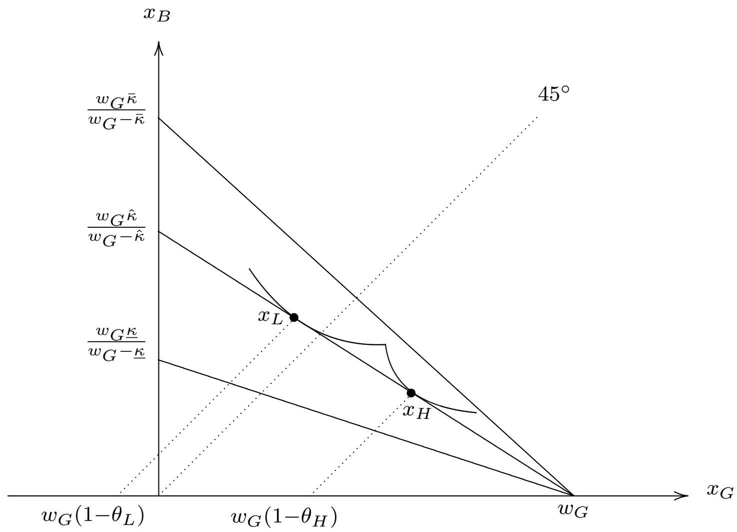

Figure 1: Mixed action equilibrium

curves, respectively below and above the 45° line. The mixed action equilibrium admissible consumption bundles are those at the intersection of the two lines identified by equations (5) and (6).

3.1 Examples of mixed action equilibria

We present three specific examples of mixed pools of promises. In the first two, we illustrate how mixed action equilibria appear. The third example aims to show the Pareto dominance of a mixed pool of promises among rich and poor individuals when compared to the two segregated pools of rich on the one side and poor individuals on the other.

Example 1: Let $u(x) = \log(x)$, $w = (1.5, 0)$, $c_H = 0.21$, and $(\pi_H, \pi_L) = (2/3, 1/3)$. In the mixed action equilibrium, $\hat{\kappa} = 0.52$ and $\hat{q} = 0.1$. In this case, $\psi(\hat{k}) = \{(0.51, H), (1.02, L)\}$. The level of utility achieved is $v(\theta_H, H) = v(\theta_L, L) = -0.65$, where $\theta_H = 0.51$ and $\theta_L = 1.02$.

Example 2: Let $u(x) = x^\gamma/\gamma$ with $\gamma = 0.5$, $w = (1, 0)$, $c_H = 0.163$, and $(\pi_H, \pi_L) = (2/3, 1/3)$. In the mixed action equilibrium, $\hat{\kappa} = 0.4$ and $\hat{q} = 0.56$. In this case, $\psi(\hat{k}) = \{(0.23, H), (1.21, L)\}$. The level of utility achieved is $v(\theta_H, H) = v(\theta_L, L) = 1.27$, where $\theta_H = 0.23$ and $\theta_L = 1.21$.

Example 3: Suppose there exist two equally sized groups of poor ($\mathcal{P}$) and rich ($\mathcal{R}$) consumers, with contingent endowments equal to, respectively, $w^{\mathcal{P}} = (1.5, 0)$ and $w^{\mathcal{R}} = (2, 0)$. Furthermore, we assume that, for individuals in both groups, $u(x) = \log(x)$, and $(\pi_H, \pi_L) = (2/3, 1/3)$. Finally, while we maintain $c_L^{\mathcal{R}} = c_L^{\mathcal{P}} = 0$, we assume that it is less costly for the rich to undertake $a = H$: $c_H^{\mathcal{R}} = 0.2$ and $c_H^{\mathcal{P}} = 0.21$. This assumption is natural when interpreted in terms of a better education that wealthier people receive in preventing health accidents (see Smith 1999 for a survey on the relation between wealth and health outcomes, and Case et al. 2002 and Currie 2009, which explore empirically the direction of the causality).

Consider first isolated pools of rich and poor individuals. Poor consumers alone face the same problem as in example 1 and, therefore, the same mixed action equilibrium emerges. On the other hand, the pool of rich consumers generates the following mixed action equilibrium: $q = 0.1$, $\hat{\kappa} = 0.7$, and $\psi(\hat{k}) = \{(0.51, H), (1.02, L)\}$. Rich consumers achieve higher utility: $v(\theta_H, H) = v(\theta_L, L) = -0.35$, where $\theta_H = 0.51$ and $\theta_L = 1.02$.

We now consider the possibility that the two groups form a common pool. Let $q^{\mathcal{R}}$ and $q^{\mathcal{P}}$ denote the proportion of rich and poor consumers choosing $a = H$. Moreover, we distinguish promises made by poor ($\theta^{\mathcal{P}}$) and rich ($\theta^{\mathcal{R}}$) consumers. On the other hand, both types of consumers benefit from the same return per promise from the common pool. As the two groups of consumers are of equal size, it is clear that the return per promise in this case is:

$$\kappa = \frac{\sum_i q^i \theta_H^i \bar{w}_H^i + \sum_i (1 - q^i) \theta_L^i \bar{w}_L^i}{\sum_i q^i \theta_H^i + \sum_i (1 - q^i) \theta_L^i}\,, \tag{7}$$

where $i = \mathcal{R}, \mathcal{P}$.[2] In this case, a mixed action equilibrium is characterized by $q^{\mathcal{P}} = 0$, $q^{\mathcal{R}} = 0.8$, $\hat{\kappa} = 0.7$, i.e., the return per promise is as high as the one of the pool of the rich alone, but higher than the return per promise of the pool of the poor alone. Moreover, $\psi^{\mathcal{P}}(\hat{k}) = \{(1.25, L)\}$ and $\psi^{\mathcal{R}}(\hat{k}) = \{(0.51, H), (1.02, L)\}$. Facing the same return per promise, the rich have no reason to choose differently, and therefore end up with the same level of utility. The poor consumers, on the other hand, face a higher return per promise, and, therefore, they promise more than when forming a pool of promises alone as in example 1: $v(\theta^{\mathcal{P}}, L) = -0.32$, where $\theta^{\mathcal{P}} = 1.25$.

The economy therefore gains from two different effects. Firstly, rich consumers are more active in preventing the bad state of nature, and this process increases the aggregate expected endowments. Secondly, rich consumers bear a lower cost in preventing the bad state of nature and this reduces the economy's overall cost of preventing accidents. In other words, the rich can, at no cost, redistribute towards the poor because they are wealthier and are more able to prevent bad outcomes.

3.2 Discussion

We analyze the pool of promises in a setting with ex-ante moral hazard, in which agents affect the probability distribution of events. This additional freedom allows that, besides the low effort equilibrium, it is also possible that economies end up in a mixed action equilibrium with some consumers undertaking action H. When a heterogeneous population is considered, we show how the rich, who are also more able, can redistribute towards the poor at no cost, i.e., the heterogenous pool Pareto dominates the two segregated pools.

The implementation of a mixed equilibrium is a natural question to raise. One can think of a pool organizer as allocating consumers to promise levels according to the q that guarantees a consistent return per promise. Again, consumers are completely indifferent to this process since, whatever their action, they end up with the same level of utility.

In our view this framework is of particular interest in developing countries. As Pauly et al 2006 suggest, it seems reasonable to think of insurance cooperatives as an adequate form of insurance organization for these countries. In fact, on the one hand, tax systems are often more deficient, which compromises a compulsory public insurance scheme. On the other hand, the population of these countries is poorer and more often excluded from the market. In developing countries, mutual insurance solutions have indeed emerged for smaller

[2] See the Appendix for the analytical derivation of the equilibrium of the pool of promises among rich and poor individuals.

communities. For example, Cabrales et al (2003) analyze a specific mutual fire insurance scheme used in Andorra, De Weerdt and Dercon (2006) find evidence of risk-sharing across networks within a village of Tanzania, and Murgai et al (2002) study water transfers along two water courses in Pakistan. Additionally, we argue that a voluntary mutual insurance scheme, such as the pool of promises, could be implemented at the national level.

However, for an application to developing economies, it seems reasonable to extend this model so that it encompasses aggregate uncertainty. Another interesting extension is to consider the possibility of limiting promises. Limiting promises has the same effect that partial insurance has in standard models of moral hazard: it makes incentive compatible a high cost action enhancing consumers' welfare. However, in a heterogenous pool, the consequences of limiting promises are not as straightforward.

Appendix

Let $\lambda^{\mathcal{R}}$ and $\lambda^{\mathcal{P}}$ denote, respectively, the proportion of rich and poor consumers, with $\lambda^{\mathcal{R}} + \lambda^{\mathcal{P}} = 1$. Moreover, let $\bar{w}^{\mathcal{R}}$ and $\bar{w}^{\mathcal{P}}$ be the expected endowments of, respectively, the former and the latter, with $\bar{w}^{\mathcal{R}} > \bar{w}^{\mathcal{P}}$. Note that the total expected endowments that an aggregate pool can guarantee to its members ($\lambda^{\mathcal{R}}\bar{w}^{\mathcal{R}} + \lambda^{\mathcal{P}}\bar{w}^{\mathcal{P}}$) is lower than $\bar{w}^{\mathcal{R}}$, the expected endowment a segregated pool of rich alone can guarantee to its members.

When an aggregated pool is formed, its return per promise depends on the deliveries of both types of consumers as follows:

$$\kappa = \frac{\sum_i \lambda^i q^i \theta^i_H \bar{w}^i_H + \sum_i \lambda^i (1-q^i)\theta^i_L \bar{w}^i_L}{\sum_i \lambda^i q^i \theta^i_H + \sum_i \lambda^i (1-q^i)\theta^i_L} \, . \tag{8}$$

We propose the following definition of equilibrium of the aggregated pool:

Definition 3.2. *An equilibrium with aggregate pool of promises is* $(\tilde{\theta}^i, \tilde{a}^i, \tilde{q}^i, \tilde{\kappa})$*, such that, for* $i \in \{\mathcal{P}, \mathcal{R}\}$*:*

(1) $(\tilde{\theta}^i, \tilde{a}^i) \in \psi^i(\tilde{\kappa})$,

(2) $\tilde{\kappa}$ *satisfies (8)*,

(3) $\tilde{q}^i$ *satisfies:*

(a) $\tilde{q}^i = 0$ *if* $(\tilde{\theta}^i, \tilde{a}^i) \in \psi^i(\tilde{\kappa}) \Rightarrow \tilde{a}^i = L, \forall i$,
(b) $\tilde{q}^i = 1$ *if* $(\tilde{\theta}^i, \tilde{a}^i) \in \psi^i(\tilde{\kappa}) \Rightarrow \tilde{a}^i = H \; \forall i$,
(c) $\tilde{q}^i \in (0,1)$ *otherwise*.

In the case of the heterogeneous pool of promises, an equilibrium with a mixed (aggregate) pool of promises is such that $q = \sum_i \lambda^i q^i \in (0,1)$. Also let $\hat{\kappa}^i$ represent the critical return per promise of group $i \in \{\mathcal{P}, \mathcal{R}\}$ above (below) which type i consumers choose to do action L (H). It is straightforward to check that $\hat{\kappa}$ is increasing in endowment, for $u(x) = \log(x)$, as used in examples 1 and 3. Thus, $\hat{\kappa}^{\mathcal{R}} > \hat{\kappa}^{\mathcal{P}}$. Consequently, considering a candidate equilibrium κ, one of the following configurations may occur:

1. $\hat{\kappa}^{\mathcal{P}} < \hat{\kappa}^{\mathcal{R}} < \kappa$, and both poor and rich choose action L. Hence, $q = 0$.
2. $\hat{\kappa}^{\mathcal{P}} < \hat{\kappa}^{\mathcal{R}} = \kappa$, and poor choose action L while rich are indifferent. Hence, $q^{\mathcal{P}} = 0$, $q^{\mathcal{R}} \in (0,1)$, and $q \in (0,1)$.
3. $\hat{\kappa}^{\mathcal{P}} < \kappa < \hat{\kappa}^{\mathcal{R}}$, and poor choose action L while rich choose action H. Hence, $q^{\mathcal{P}} = 0$, $q^{\mathcal{R}} = 1$, and $q \in (0,1)$.

4. $\hat{\kappa}^{\mathcal{P}} = \kappa < \hat{\kappa}^{\mathcal{R}}$, and poor are indifferent while rich choose action H. Hence, $q^{\mathcal{P}} \in (0,1)$, $q^{\mathcal{R}} = 1$, and $q \in (0,1)$.
5. $\kappa < \hat{\kappa}^{\mathcal{P}} < \hat{\kappa}^{\mathcal{R}}$, and both poor and rich choose action H. Hence, $q = 1$.

Note that case 5 can never arise in equilibrium, as follows from Proposition 1. In the text we illustrate case 2 type of equilibrium.

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