Evaporation from heterogeneous and sparse canopies: on the formulations related to multi-source representations

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2   **Evaporation from heterogeneous and sparse canopies: on the formulations**

3   **issued from some multi-source representations**

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11   **Abstract**: Evaporation from heterogeneous and sparse canopies is often represented by multi-
12   source models which take the form of electrical analogues based upon resistance networks.
13   The chosen representation de facto imposes a specific writing on the composition of
14   elementary fluxes and resistances. The two- and three-source representations are discussed in
15   relation to several papers of the scientific literature where some ambiguities arise. Using the
16   two-layer model [Shuttleworth WJ, Wallace JS (1985). Q J Roy Meteorol Soc 111: 839–855]
17   and the clumped (three-source) model [Brenner AJ, Incoll LD (1997). Agric For Meteorol 84:
18   187–205] as a basis, it is shown that the stomatal characteristics of the foliage
19   (amphistomatous or hypostomatous) generate different formulations. New generic and more
20   concise equations, valid in both configurations, are derived. The differences between the
21   patch and layer approaches are outlined and the consequences they have on the composition
22   and formulation of component fluxes are specified. Then, the issue of calculating the effective
23   resistances of the single-layer model from multi-source representations is addressed. Finally,
24   a sensitivity analysis is carried out to illustrate the significance of the new formulations.

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1 **1. Introduction**

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3   The energy transfers within heterogeneous and sparse canopies are often represented

4 by multi-source models. The first models of this kind were designed for sparse canopies to

5 separately account for vegetation and substrate (soil) contributions (Shuttleworth and Wallace

6 1985; Choudhury and Monteith 1988; Shuttleworth and Gurney 1990). Subsequently, more

7 complex representations were proposed to account for heterogeneous canopies with various

8 components (main vegetation, herbaceous substrate, bare soil) (Dolman 1993; Huntingford et

9 al. 1995), but always keeping the Shuttleworth-Wallace model as a basis. In the "clumped"

10 model of Brenner and Incoll (1997), specially adapted to semi-arid shrub-lands, the substrate

11 layer is divided into two sub-layers: one corresponds to the soil under the foliage and the

12 other to the bare soil. The ERIN model of Wallace (1997) addresses the issue of evaporation

13 in multi-species canopies through an extension of the Shuttleworth-Wallace model. From

14 principles similar to those of Brenner and Incoll (1997), Verhoef and Allen (2000) developed

15 a multi-source model, where the substrate is divided into three sub-layers: grasses, herbs and

16 bare soil.

17   The multi-source models are generally represented by resistance networks, which

18 combine surface and air resistances within the canopy. They simulate fairly well evaporation

19 provided appropriate parameterizations for component resistances are available. However, it

20 appears that the writing of the total flux of evaporation from the component fluxes and the

21 combination of elementary resistances differs depending on the authors and can lead to

22 apparent inconsistencies and even inexact formulations. For instance, whereas in the first

23 models (e.g. Shuttleworth and Wallace 1985) the component fluxes of the multi-source

24 representation are simply added, in the subsequent ones (e.g. Dolman 1993) they are weighted

25 by the relative area of each source without clear justification. In line with the pioneering work

26 of Wallace and Verhoef (2000) on the modelling of interactions in mixed-plant communities,

27 the main goal of the current study is to carefully examine the formulation of these multi-

28 source models in order to clearly establish their common ground, the differences between

29 them and the correct way to formulate them. This examination is made through the most

30 emblematic ones (Shuttleworth and Wallace 1985; Brenner and Incoll 1997) and three

31 particular issues will be addressed in relation to the main objective.

32   The first issue is linked to the stomatal characteristics of the canopy (amphistomatous

33 versus hypostomatous leaves) and the impact they have on the basic equations of the multi-

34 layer representation. Although the Verhoef-Allen model accounts for differences in stomatal

1 characteristics, the introduced modifications are not systematically applied in many other

2 studies based on multi-source models. In the case of vineyard for example, several papers

3 apply the Shuttleworth-Wallace model (Ortega-Farias et al. 2007; Zhang et al. 2009; Ortega-

4 Farias et al. 2010) or the clumped model (Zhang et al. 2008; Poblete-Echeverria and Ortega-

5 Farias 2009) without correction for the hypostomatous characteristics. We propose here new

6 generic formulations which are valid in both configurations (amphi- and hypo-stomatous).

7 The second issue deals with the distinction between the patch (or uncoupled) approach

8 and the layer (or coupled) approach. As already explained by Lhomme and Chehbouni (1999)

9 and Daamen and McNaughton (2000), the choice of a coupled or uncoupled model has

10 significant consequences in the formulation of the basic equations. When a coupled (also

11 called interactive or layer) model is used, component fluxes are strictly additive, whereas in

12 an uncoupled (patch) model, component fluxes should be weighted by the relative area of

13 each patch. This point will be addressed and discussed in relation to the clumped model of

14 Brenner and Incoll (1997).

15 The third issue addressed here occurs when the relative complexity of a multi-source

16 model should be represented by a single-layer model, i.e., the "big leaf" model of Penman-

17 Monteith. How should one combine the component resistances of the multi-source approach

18 to calculate the effective resistances of the single-source? Since the aggregation procedures

19 proposed in the literature are diverse and raise questions (Were et al. 2007), the problem will

20 be discussed and solutions will be proposed.

21 The paper is divided into three main sections which examine successively the two-

22 layer representation (Shuttleworth and Wallace 1985), the "clumped" or three-source model

23 (Brenner and Incoll 1997) and the calculation of single-source effective resistances.

24 Numerical simulations and sensitivity analyses are presented in the last section to illustrate

25 the interest of the new formulations.

26

27 **2. A new and generic formulation of the two-layer model**

28

29 The two-layer model of Shuttleworth and Wallace (1985) represents the evaporation

30 from a stand of vegetation composed of two main sources: a substrate (which can be bare soil

31 or grass) and an upper canopy (main foliage). The corresponding resistance network is shown

32 in Figure 1. The resulting formulation for total evaporation slightly differs depending on the

33 distribution of stomata on the leaves. The amphistomatous case (stomata distributed on both

34 sides of the leaves) is the case implicitly considered in the original formulation (Shuttleworth

1   and Wallace 1985; Shuttleworth and Gurney 1990). More general equations are established

2   here: they take into account the two possible patterns of stomatal characteristics

3   (amphistomatous and hypostomatous).

4   In the two-layer model, total evaporation (λE) from the canopy is expressed as the

5   simple sum of two components: λE_f (foliage) and λE_s (substrate):

6

7   $$\lambda E = \lambda E_f + \lambda E_s. \tag{1}$$

8

9   Each evaporation component is calculated separately from equations of the Penman-Monteith

10 type involving the corresponding available energy (A_f or A_s) and the vapour pressure deficit

11 D_m at the mean canopy source height z_m, assumed to be located at the apparent sink for

12 momentum (zero plane displacement height d + roughness length z_0). The component

13 resistances are expressed per unit area of land surface. So, leaf stomatal resistance $r_{s,l}$ (one

14 side) and the corresponding leaf boundary-layer resistance for latent heat $r_{a,l}$ (one side) should

15 be divided by the transpiring surface expressed per unit area of land surface: *2LAI* for

16 amphistomatous leaves and *LAI* for hypostomatous leaves. For convenience we introduce the

17 parameter n ($n = 1$ for amphistomatous leaves and $n = 2$ for hypostomatous leaves), which

18 allows the bulk stomatal resistance to be written as

19

20 $$r_{s,f} = n\, r_{s,l} / (2LAI). \tag{2}$$

21

22 The foliage boundary-layer resistance for water vapour is written similarly

23

24 $$r_{a,f,v} = n\, r_{a,l} / (2LAI). \tag{3}$$

25

26 However, since each leaf side is a heat source, the foliage boundary-layer resistance for

27 sensible heat remains the same in both cases (amphistomatous or hypostomatous) and is

28 defined as

29

30 $$r_{a,f,h} = r_{a,l} / (2LAI) = r_{a,f,v} / n. \tag{4}$$

31

32 The hyperstomatous case (stomata only on the upper side of the leaves) is similar to the

33 hypostomatous case and the corresponding expressions of bulk resistances are identical.

1   The combination equation (Penman-Monteith type) for the foliage takes the following

2   form derived in Appendix A:

3

4    $$\lambda E_f = \frac{\Delta A_f + \rho c_p D_m / r_{a,f,h}}{\Delta + \gamma (n + r_{s,f} / r_{a,f,h})}.$$

5   (5)

6

7   For the substrate, resistances are always expressed per unit area of land surface and the

8   boundary-layer resistance is assumed to be the same for sensible heat and water vapour.

9   Consequently, the corresponding combination equation is simply written as

10

11   $$\lambda E_s = \frac{\Delta A_s + \rho c_p D_m / r_{a,s}}{\Delta + \gamma (1 + r_{s,s} / r_{a,s})},$$ (6)

12

13   where $r_{s,s}$ is the substrate resistance to evaporation and $r_{a,s}$ the aerodynamic resistance

14   between the substrate and the source height. Expressing D_m as a function of D_a (vapour

15   pressure deficit at the reference height z_r) leads to (Shuttleworth and Wallace 1985, Eq. (8))

16

17   $$D_m = D_a + \left[\Delta A - (\Delta + \gamma) \lambda E \right] r_a / (\rho c_p).$$ (7)

18

19   The general expression of total evaporation λE is obtained by introducing Eq. (7) into Eqs. (5)

20   and (6). At this stage, two formulations of λE are possible: the one which follows the strict

21   formalism of Shuttleworth and Wallace's original equations and an alternative one,

22   considered as more synthetic, which is proposed hereafter.

23

24   *a. Original formulation*

25

26   If we respect the original formalism, the resultant equation is written as

27

28   $$\lambda E = C_f PM_f + C_s PM_s,$$ (8)

29

30   with

31

1 $$PM_s = \frac{\Delta A + \left\lfloor \rho c_p D_a - \Delta r_{a,s}(A - A_s) \right\rfloor/(r_a + r_{a,s})}{\Delta + \gamma \left[1 + r_{s,s}/(r_a + r_{a,s}) \right]},$$ (9)

2

3 $$PM_f = \frac{\Delta A + \left\lfloor \rho c_p D_a - \Delta r_{a,f,h}(A - A_f) \right\rfloor/(r_a + r_{a,f,h})}{\Delta + \gamma \left[(r_a + n r_{a,f,h} + r_{s,f})/(r_a + r_{a,f,h}) \right]}.$$ (10)

4

5 In these equations A represents the available energy for the whole stand (foliage and

6 substrate): $A = A_f + A_s$. The steps of the calculation are identical to those given in the original

7 article of Shuttleworth and Wallace (1985). The coefficients C_f and C_s are simple

8 combinations of the basic air and surface resistances. They are expressed as

9

10 $$C_f = \frac{R_s(R_f + R_a)}{R_f R_s + R_s R_a + R_f R_a},$$ (11)

11

12 $$C_s = \frac{R_f(R_s + R_a)}{R_f R_s + R_s R_a + R_f R_a},$$ (12)

13

14 with

15

16 $$R_f = (\Delta + n\gamma)r_{a,f,h} + \gamma r_{s,f},$$ (13)

17

18 $$R_s = (\Delta + \gamma)r_{a,s} + \gamma r_{s,s},$$ (14)

19

20 $$R_a = (\Delta + \gamma)r_a.$$ (15)

21

22 The two-layer representation of sparse canopies does not have the same mathematical form

23 for amphistomatous and hypostomatous canopies. Some adjustments should be made when

24 passing from one type of canopy to another: the PM_f and R_f terms undergo a change and it is

25 easy to verify that when $n = 1$ (amphistomatous case), the original equations of Shuttleworth

26 and Wallace (1985) are retrieved. Additionally, we have to note that the partition of the

27 original formulation into two "Penman-Monteith" type components (foliage and substrate) is

28 not really well-designed insofar as each part has a relatively complex form and does not

1 represent the respective component evaporation (foliage and substrate). This can be even

2 confusing and misleading. An alternative and simpler formulation is proposed hereafter.

3

4 ### b. Alternative formulation

5

6 By differently collecting the terms in the basic equations (the details of the calculation

7 are given in Appendix B) it can be shown that the total flux of evaporation can be written as

8

9 $$\lambda E = \frac{\Delta + \gamma}{\gamma}(P_f + P_s)\lambda E_p + \frac{\Delta}{\gamma}(P_f A_f r_{a,f,h} + P_s A_s r_{a,s})/r_a. \tag{16}$$

10

11 In Eq. (16) λE_p represents the potential evaporation from the sparse canopy expressed as

12

13 $$\lambda E_p = \frac{\Delta A + \rho c_p D_a / r_a}{\Delta + \gamma} \tag{17}$$

14

15 The other terms are defined as follows:

16

17 $$P_s = \frac{r_a R_f^{'}}{R_f^{'} R_s^{'} + R_a^{'} R_f^{'} + R_a^{'} R_s^{'}} \tag{18}$$

18

19 $$P_f = \frac{r_a R_s^{'}}{R_f^{'} R_s^{'} + R_a^{'} R_f^{'} + R_a^{'} R_s^{'}} \tag{19}$$

20

21 $$R_f^{'} = R_f / \gamma = r_{s,f} + (n + \frac{\Delta}{\gamma})r_{a,f,h} \tag{20}$$

22

23 $$R_s^{'} = R_s / \gamma = r_{s,s} + (1 + \frac{\Delta}{\gamma})r_{a,s} \tag{21}$$

24

25 $$R_a^{'} = R_a / \gamma = r_a(1 + \frac{\Delta}{\gamma}) \tag{22}$$

26

1   This new formulation of the two-layer model has three main advantages: (*i*) first it is more

2   concise than the traditional one; (*ii*) it avoids the confusion generated by separate evaporation

3   terms; (*iii*) it involves the "climatic demand" λE_p, which can be convenient and useful in

4   many applied studies (Lhomme 1997). We also note that Eq. (16) has correct asymptotic

5   limits. If there is no substrate evaporation (A_s is zero and $r_{s,s}$ is infinite), Eq. (16) reduces to

6   the conventional Penman-Monteith equation for a closed canopy with $n = 1$:

7

8    $$\lambda E = \frac{\Delta A + \rho c_p D_a /(r_a + r_{a,f,h})}{\Delta + \gamma \left[1 + r_{s,f} /(r_a + r_{a,f,h})\right]} \qquad (23)$$

9

10   **3. A reformulated clumped model**

11

12   ***3.1. Layer or patch approach?***

13

14   As already mentioned, the choice between the two representations is critical in the

15   sense that it leads to different manner of aggregating the elementary fluxes (simple sum or

16   area weighted addition). The choice should be dictated by the way the aerodynamic resistance

17   above the canopy (between z_m and z_r) is defined. If the patches of vegetation or bare soil are

18   large enough to allow different aerodynamic resistances to be defined for each patch, an

19   uncoupled representation should be chosen (see Fig. 2). On the other hand, if the different

20   sources are close to each other and do not allow the definition of separate aerodynamic

21   resistances, a layer approach should be preferred (see Fig. 1). Daamen and McNaughton

22   (2000) explained: "the patch model is fully justified at the scale where a boundary layer is

23   fully developed over each patch and edge effects between patches are insignificant, but as the

24   size of the patches decreases this model may be less valid". From a turbulent transfer

25   perspective, McNaughton and van den Hurk (1995) also showed that the coupled (interactive)

26   model is a simplification of more complex and realistic Lagrangian models and consequently

27   more widely applicable than the patch model.

28   The clumped model of Brenner and Incoll (1997), which is similar to the multi-species

29   canopy representation described by Wallace (1997), constitutes in fact a modified two-layer

30   model analogous to the Shuttleworth-Wallace model: the layer representing the soil surface is

31   divided into two sub-layers (soil under the foliage and bare soil) and the component fluxes

32   mix together at canopy source height before experiencing the same aerodynamic resistance

33   above the canopy (Fig. 3). It was used and reworked by Domingo et al. (1999) and Were et al.

1 (2007, 2008) for sparse bush vegetation in semi-arid Spain and by Zhang et al. (2008, 2009)

2 for vineyard in an arid region of China. Following the logic of the layer model, the total flux

3 of evaporation emanating from the canopy should be written in a simple additive form

4

5 $\quad \lambda E = \lambda E_f + \lambda E_{vs} + \lambda E_{bs}$ (24)

6

7 with subscript $i = f$ for the foliage, $i = vs$ for the soil below the vegetation and $i = bs$ for the

8 bare soil. This form of the conservation equation differs from the one used in the original

9 model of Brenner and Incoll (1997) where the total flux of evaporation is written as an area-

10 weighted form of the component fluxes

11

12 $\quad \lambda E = F(\lambda E_f + \lambda E_{vs}) + (1 - F)\lambda E_{bs}$,   (25)

13

14 F representing the fractional cover of the foliage. Although a formulation similar to Eq. (25)

15 has been used by Dolman (1993) and Verhoef and Allen (2000), we have to stress that this

16 form of the equation is not concordant with the resistance network representing the model

17 (Fig. 3). Given that all the component fluxes mix together at canopy source height and that a

18 sole aerodynamic resistance is defined above the canopy, a layer approach should be

19 preferred. We develop below the new equations of the clumped model when (*i*) the simple

20 additive form of the conservation equation is used instead of the area weighted form and (*ii*)

21 when the stomatal characteristics of the foliage are taken into account.

22

23 *3.2. New formulations*

24

25   The component resistances should be expressed in a way similar to the two-layer

26 model of Shuttleworth-Wallace (i.e. per unit area of land surface) since all the component

27 fluxes mix together at canopy source height. Foliage resistances (stomatal and boundary-

28 layer) have the same expressions as those of the two-layer model and are given by Eqs. (2),

29 (3) and (4). Substrate resistances, however, should have different expressions since the

30 exchange surfaces are not the same. If the resistances expressed per unit area of vegetated soil

31 or bare soil are written with the upper-script 1, the resistances should be divided by the

32 relative area of the corresponding surface (vegetated or bare soil) to obtain the component

33 resistances of the model (per unit area of land surface):

1

2 $$r_{s,vs} = r^1_{s,vs} / F \tag{26}$$

3

4 $$r_{a,vs} = r^1_{a,vs} / F \tag{27}$$

5

6 $$r_{s,bs} = r^1_{s,bs} /(1-F) \tag{28}$$

7

8 $$r_{a,bs} = r^1_{a,bs} /(1-F). \tag{29}$$

9

10 In fact, given the basic formalism (Ohm's law type) of component fluxes, dividing the

11 resistances by the fractional areas is equivalent to multiplying the corresponding fluxes by the

12 same quantity. In the original clumped model of Brenner and Incoll (1997), where a mix of

13 patch and layer approach is used, the surface and aerodynamic resistances of the substrates are

14 logically expressed per unit area of land surface and consequently they are not divided by

15 their relative area (*F* or *1-F*). This is consistent with the patch formulation of Eq. (25), but as

16 explained above, it is the approach itself which is questionable.

17   Each evaporation term can be expressed in the form of a Penman-Monteith equation

18 (Eqs. (5) and (6)), *n* having the same significance as in the Shuttleworth-Wallace model

19 (section 2). Replacing D_m by its expression as a function of D_a (Eq. (7)) leads to the following

20 formulation

21

22 $$\lambda E = C'_f PM_f + C'_{vs} PM_{vs} + C'_{bs} PM_{bs} \tag{30}$$

23

24 with

25

26 $$PM_i = \frac{\Delta A + \left[\rho c_p D_a - \Delta r_{a,i}(A - A_i)\right]/(r_a + r_{a,i})}{\Delta + \gamma\left[1 + r_{s,i}/(r_a + r_{a,i})\right]} \qquad i = \quad vs \quad or \quad bs \tag{31}$$

27

28 $$PM_f = \frac{\Delta A + \left[\rho c_p D_a - \Delta r_{a,f,h}(A - A_f)\right]/(r_a + r_{a,f,h})}{\Delta + \gamma\left[(r_a + n r_{a,f,h} + r_{s,f})/(r_a + r_{a,f,h})\right]} . \tag{32}$$

29

30

1   The three coefficients *C'* are simple combinations of the component air and surface

2   resistances and are detailed in Appendix C. We note that the modifications needed to take into

3   account the stomatal characteristics of the canopy (through the parameter *n*) are exactly

4   similar to the ones made to the Shuttleworth-Wallace model (Eq. (32) versus (10) and (57)

5   versus (13)).

6   As for the Shuttleworth-Wallace model, it is possible to obtain an alternative

7   formalism and to write more concisely the resultant expression of λE by collecting differently

8   the terms of the equation (derivation similar to that described in Appendix B). With the

9   coefficients *P'* defined in Appendix D, the equation equivalent to Eq. (16) in the two layer

10   model is written in the clumped model as

11

12    $$\lambda E = \frac{\Delta + \gamma}{\gamma}(P_f^{'} + P_{vs}^{'} + P_{bs}^{'})\lambda E_p + \frac{\Delta}{\gamma}(P_f^{'} A_f r_{a,f,h} + P_{vs}^{'} A_{vs} r_{a,vs} + P_{bs}^{'} A_{bs} r_{a,bs})/r_a, \qquad (33)$$

13

14   where λE_p represents the potential evaporation defined by Eq. (17). It can be verified that the

15   asymptotic limits of this formulation are correct. When the vegetated part of the substrate

16   tends to zero, $r_{s,vs}$ tends to infinite, $P_{vs}^{'}$ and A_{vs} tends to zero and Eq. (33) transforms into Eq.

17   (16).

18

19   **4. Big leaf model: aggregation of component resistances**

20

21   *4.1. The problem to solve*

22

23   In the modelling of surface fluxes at different scales, the canopy spatial

24   heterogeneities are often averaged by using simpler representations. One way to proceed is by

25   using a bulk-transfer approach based on a single-layer representation and the concept of kB^{-1}

26   introduced by Owen and Thomson (1963) and largely discussed since then (Garratt and

27   Hicks, 1973). However, this concept, which is defined as the logarithm of the ratio between

28   momentum and heat roughness length, is questionable and considered as not perfectly sound

29   from a physical standpoint (Verhoef et al. 1997; Lhomme et al. 2000). Another way to

30   proceed is by calculating effective parameters. Presently, the problem to solve is how to

31   correctly represent a heterogeneous canopy by a simple combination equation (Penman-

32   Monteith model) in which the effective surface and air resistances are expressed from the

1   component resistances of the multi-source model. The basic model (Monteith, 1965) is

2   commonly written as

3

4    $$\lambda E = \frac{\Delta A + \rho c_p D_a / r_a}{\Delta + \gamma(1 + r_s^e / r_a)} \ ,$$ (34)

5

6   where r_s^e is the effective surface resistance and r_a is the aerodynamic resistance calculated

7   between the mean source height of the canopy z_m and the reference height z_r (Fig. 4a). The

8   original Penman-Monteith equation is strictly valid for a full covering canopy (big leaf

9   model) and the surface resistance of the foliage is reduced to its stomatal component

10   (expressed from leaf stomatal resistance as $n \ r_{s,l}/2LAI$). The canopy boundary-layer resistance

11   for sensible heat and water vapour is neglected or assumed to be incorporated in the

12   aerodynamic resistance above the canopy (r_a).

13   When a two-layer or clumped approach is considered, it is clear that the effective

14   surface resistance (r_s^e) should be put in series with the aerodynamic resistance (r_a) above the

15   canopy. It is less clear, however, how the component surface and air resistances within the

16   canopy should be combined into the effective surface resistance. Were et al. (2007, 2008), for

17   instance, have tested with observed data of evaporation several aggregation procedures

18   previously discussed by Blyth et al. (1993): in parallel or in series, weighted by the relative

19   area F or not. The specific objective of this section is to examine this issue from a theoretical

20   standpoint and to identify the correct way of physically combining the component resistances

21   of multi-source models into the effective resistances of a single-layer model. The two-layer

22   case will be thoroughly examined. Then, we will show how the equations can be extended to

23   the clumped model.

24

25   *4.2. General expressions for the two-layer model*

26

27   The Penman-Monteith model results from the combination of two basic Ohm's law

28   type formulations, one for sensible heat and the other for latent heat (Fig. 4b). Written with

29   effective resistances they read

30

31   $$H = \rho c_p (T_m - T_a)/(r_a + r_{a,h}^e) \ ,$$ (35)

32

1 $$\lambda E = (\rho c_p / \gamma)(e*(T_m) - e_a)/(r_a + r_v^e).$$ (36)

2

3 T_m is air temperature at canopy source height, $r_{a,h}^e$ is the effective resistance for sensible heat

4 (which includes only air resistances within the canopy) and r_v^e is the one for water vapour

5 (which includes air and surface resistances). Both resistances should be logically added to the

6 aerodynamic resistance above the canopy (r_a). Combining Eqs. (35) and (36) with the energy

7 balance equation results in the following combination equation

8

9 $$\lambda E = \frac{\Delta A + \rho c_p D_a /(r_a + r_{a,h}^e)}{\Delta + \gamma\left[(r_a + r_v^e)/(r_a + r_{a,h}^e)\right]}.$$ (37)

10

11 Assuming plant and soil to be isothermal (at effective temperature T_m), the effective

12 resistances should be calculated as the parallel sum of the component resistances expressed

13 per unit area of land surface. This means that for the two-layer model we have

14

15 $$\frac{1}{r_{a,h}^e} = \frac{1}{r_{a,f,h}} + \frac{1}{r_{a,s}} = \frac{2LAI}{r_{a,l}} + \frac{1}{r_{a,s}},$$ (38)

16

17 $$\frac{1}{r_v^e} = \frac{1}{r_{s,f} + r_{a,f,v}} + \frac{1}{r_{s,s} + r_{a,s}} = \frac{2LAI}{n(r_{s,l} + r_{a,l})} + \frac{1}{r_{s,s} + r_{a,s}}.$$ (39)

18

19 This set of equations constitutes the unique rationale to aggregate the elementary resistances

20 within the canopy in order to calculate the effective resistances of a big-leaf model from a

21 two-layer representation. Two remarks should be made, however. First, the effective

22 resistance for water vapour r_v^e is a complex arrangement of air and surface resistances and

23 does not allow air and surface components to be separated into two bulk resistances in series.

24 Second, the resultant combination equation does not have the common formalism of Eq. (34),

25 where the simple ratio of a surface resistance to an aerodynamic resistance appears in the

26 denominator of the equation, allowing the effects of the air to be separated from those of the

27 surface. Nevertheless, this simple ratio can be obtained if the air resistances within the canopy

28 are neglected, as is the case in the original Penman-Monteith equation. This approximation

29 can be justified since it is well known that evaporation depends much more on stomatal

1 resistance and *LAI* than on internal air resistances. With this approximation the effective

2 resistance for sensible heat $r_{a,h}^e$ is equal to 0. This means that Eq. (37) transforms into Eq. (34)

3 and the effective resistance for water vapour r_v^e becomes r_s^e defined as

4

5
$$\frac{1}{r_s^e} = \frac{1}{r_{s,f}} + \frac{1}{r_{s,s}} = \frac{2LAI}{n\, r_{s,l}} + \frac{1}{r_{s,s}}\,,$$
(40)

6

7 ### *4.3. Decoupling surface and air resistances (2-layer model)*

8

9 In order to obtain the familiar formalism of the big leaf model it is convenient to split

10 the effective resistance for water vapour transfer r_v^e into two resistances put in series: one

11 including only surface components and the other only air components:

12

13 $r_v^e = r_s^e + r_{a,v}^e$.
(41)

14

15 This assumption is physically valid to a first approximation if we consider that it roughly

16 respects the path followed by water vapour in its transfer from the plant to the open air. In this

17 case, the effective surface resistance r_s^e is logically defined as the sum of parallel resistances

18 and obtained from Eq. (40). On the other hand, the effective aerodynamic resistance $r_{a,v}^e$ can

19 take two forms depending on the stomatal characteristics of the foliage.

20 In amphistomatous canopies, the same expression for the effective aerodynamic

21 resistance (calculated as the sum of parallel resistances) can be applied simultaneously to

22 sensible heat and water vapour: $r_{a,v}^e = r_{a,h}^e$ given by Eq. (38). Putting Eq. (41) into Eq. (37)

23 leads to the common formalism of the Penman-Monteith model, where the ratio of a surface

24 resistance divided by an aerodynamic resistance appears in the denominator

25

26
$$\lambda E = \frac{\Delta A + \rho c_p D_a /(r_a + r_{a,h}^e)}{\Delta + \gamma\left[1 + r_s^e /(r_a + r_{a,h}^e)\right]}\,.$$
(42)

27

28 Although this formulation is not perfectly sound from a physical standpoint, it is certainly

29 more realistic than the one obtained by neglecting the air resistances within the canopy.

1   The hypostomatous case is more complicated since the effective aerodynamic

2   resistance for water vapour $r_{a,v}^e$ differs from the one for sensible heat $r_{a,h}^e$. For the two-layer

3   model these effective aerodynamic resistances are respectively calculated as

4

5   $$\frac{1}{r_{a,h}^e} = \frac{1}{r_{a,f,h}} + \frac{1}{r_{a,s}} = \frac{2LAI}{r_{a,l}} + \frac{1}{r_{a,s}} \quad ,$$ (43)

6

7   $$\frac{1}{r_{a,v}^e} = \frac{1}{r_{a,f,v}} + \frac{1}{r_{a,s}} = \frac{LAI}{r_{a,l}} + \frac{1}{r_{a,s}} \quad .$$ (44)

8

9   Following steps similar to those developed in Appendix A, the combination equation in the

10 hypostomatous case writes as

11

12 $$\lambda E = \frac{\Delta A + \rho c_p D_a / (r_a + r_{a,h}^e)}{\Delta + \gamma \left[(r_a + r_{a,v}^e + r_s^e) / (r_a + r_{a,h}^e) \right]}.$$ (45)

13

14 It is not possible to obtain the simple ratio of a surface resistance divided by an aerodynamic

15 resistance. The decoupling of surface and air resistances does not allow obtaining a "strict"

16 Penman-Monteith formalism.

17

18   ***4.4. Extension to the clumped model***

19

20   The equations developed for the two layer model can be easily extended to the

21 clumped model by splitting the substrate component into its two sub-components of area *F*

22 (vegetated soil) and *1-F* (bare soil) (see section 3.1). The resistances expressed per unit area

23 of substrate being denoted by the upper-script 1, Eqs. (38) and (39) should be rewritten with

24

25 $$\frac{1}{r_{a,s}} = \frac{F}{r_{a,vs}^1} + \frac{1-F}{r_{a,bs}^1} \quad ,$$ (46)

26

27 $$\frac{1}{r_{s,s} + r_{a,s}} = \frac{F}{r_{s,vs}^1 + r_{a,vs}^1} + \frac{1-F}{r_{s,bs}^1 + r_{a,bs}^1} \quad .$$ (47)

28

When the air resistances within the canopy are disregarded, $1/r_{s,s}$ in Eq. (40) should be replaced by Eq. (47) in which air resistances ($r_{a,vs}^1$ and $r_{a,bs}^1$) are put to zero. When surface and air resistances are decoupled, Eqs. (43) and (44) should be similarly modified by expressing $1/r_{a,s}$ as in Eq. (46).

The formulations developed in section 4 are based upon a strict physical background with different levels of approximation and should be preferred to the empirical combinations tested by Were et al. (2007, 2008).

5. Sensitivity analysis

Numerical simulations were undertaken to assess the sensitivity of evaporation to some incorrectness or approximations in the formulation of multi-source representation. For the sake of convenience the analysis was carried out on the basis of the two-layer model. The parameterizations used to formulate available energy and component air resistances are given in Appendix E. Calculations are made for a canopy with a height (z_h) equal to 1 m under the following meteorological conditions at a reference height of 2 m above the canopy: $R_n = 400$ W m^{-2}, $T_a = 25$ °C, $D_a = 10$ hPa, $u_a = 2$ m s^{-1}. These values were already used by Shuttleworth and Wallace (1985) in the sensitivity analysis of their model.

5.1. Amphistomatous versus hypostomatous leaves

In section 2 it was shown that the formulation of the two-layer model differs depending on whether the canopy is amphistomatous or hypostomatous. The practical problem arises when the common formulation (strictly valid for an amphistomatous foliage) is applied to a hypostomatous one. Our simulation consisted in comparing for a hypostomatous canopy the "true" evaporation rate λE_0, calculated using the right formulae (Eqs. (8) or (16) with $n = 2$), with the "erroneous" evaporation rate $\lambda E'$, calculated with the equations valid for an amphistomatous one (putting $n = 1$), except for the bulk stomatal resistance (Eq. (2)), which logically should be calculated as hypostomatous ($n = 2$). The error is generated in fact by an under-estimation of foliage boundary-layer resistance for water vapour $r_{a,f,v}$ (Eq. (3)): its hypostomatous value being twice its amphistomatous value (for instance, for $LAI = 3$, 166 s m^{-1} against 83 s m^{-1}). Fig. 5 illustrates the corresponding error made on the evaporation rate as a function of LAI for different values of leaf stomatal resistance ($r_{s,l}$) and substrate resistance

1   ($r_{s,s}$). Under the standard conditions specified above, the relative error, calculated as $\delta\lambda E =$

2    $(\lambda E' - \lambda E_0)/\lambda E_0$, is around 10 % for *LAI* greater than *1*, which is not negligible and justify the

3   use of different formulations.

4

5   *5.2. Effective resistances*

6

7   The aim of the exercise is to compare the evaporation rate calculated using the big leaf

8   model and different approximations for the effective resistances with the "true" evaporation

9   rate (λE_0) calculated using the two-layer model. The simulations are made here for an

10   amphistomatous canopy. Three levels of approximation are considered: (1) λE_1 is calculated

11   by the general expressions (Eqs. (37), (38), (39)), which are supposed to be the most accurate

12   ones; (2) λE_2 is obtained by the expressions derived by neglecting the air resistances within

13   the canopy (Eqs. (34), (40)); (3) λE_3 when surface and air resistances are decoupled (Eqs.

14   (42), (38), (40)). In each case, a relative error ($\delta\lambda E_i$) is calculated as in section 5.1 under the

15   weather conditions specified above. Fig. 6 illustrates the results obtained as a function of *LAI*

16   for different values of leaf stomatal resistance ($r_{s,l}$) and substrate resistance ($r_{s,s}$). It is clear

17   that λE_1 yields the most accurate results with a relative error ($\delta\lambda E_1$) generally less than 5 %.

18   λE_3 provides a relatively good approximation mainly for high *LAI* (> 2) and high substrate

19   resistance with a relative error generally lower than 5 %. λE_2 yields the less accurate

20   estimates, mainly when the substrate resistance is low or nil; nevertheless, the approximation

21   can be acceptable for large LAI, provided substrate resistance be large enough.

22

23   **6. Conclusion**

24

25   In several papers of the scientific literature, the multi-source representation of

26   heterogeneous and sparse canopies raises some questions in relation to the correct formulation

27   of the physical processes. When multi-source models are used to represent evaporation, some

28   basic principles should be respected in writing the equations. The resistance network chosen

29   to represent the physical processes necessarily dictates the correct formalism for combining

30   the component fluxes and resistances. In relation to that, three particular points have been

31   addressed. The first point stresses and details the fact that the basic formulations and the

32   resulting equations slightly differ when the foliage is amphistomatous and hypostomatous.

33   Generic equations, valid in both situations, have been established and it has been found that

34   not taking into account the differences can lead to errors on the evaporation rate of about 10

1   %. The second point concerns the conservation equation. If all the component fluxes mix

2   together at canopy source height and experience the same aerodynamic resistance above the

3   canopy, they should be simply added and not weighted by the relative area of each

4   component. The weighing by the relative area should be applied not to the fluxes but to the

5   component resistances. The third point deals with the effective resistances which allow a

6   multi-source model to be reduced to a single-layer model. It is shown that the way of

7   combining the elementary resistances within the canopy to formulate the effective air and

8   surface resistances is physically determined as the parallel sum of component resistances:

9   here also the hypostomatous case slightly differs from the amphistomatous one. More simple

10   and operational formulations of the effective resistances can be obtained by means of some

11   legitimate assumptions with different degrees of confidence illustrated by numerical

12   simulations.

13

14   **Nomenclature**

15

16   A: available energy of the whole crop (W m^{-2})

17   A_f: available energy of the foliage (W m^{-2})

18   A_s: available energy of the substrate (W m^{-2})

19   A_{vs}: available energy of the vegetated soil (W m^{-2})

20   A_{bs}: available energy of the bare soil (W m^{-2})

21   R_n: net radiation of the whole crop (W m^{-2})

22   G: soil heat flux (W m^{-2})

23   H: sensible heat flux from the complete canopy (W m^{-2})

24   λE: latent heat flux from the complete canopy (W m^{-2})

25   H_i: component heat flux ($i = f, s, vs, bs$) (W m^{-2})

26   λE_i: component latent heat flux ($i = f, s, vs, bs$) (W m^{-2})

27   D_a: vapour pressure deficit at reference height (Pa)

28   D_m: vapour pressure deficit at canopy source height (Pa)

29   T_a: air temperature at reference height (°C)

30   T_m: air temperature at canopy source height (°C)

31   T_i: surface temperature of component i ($i = f, s, vs, bs$) (°C)

32   u_a: wind speed at reference height (m s^{-1})

33   e_a: vapour pressure at reference height (Pa)

34   e_m: vapour pressure at canopy source height (Pa)

1     $e^*(T)$: saturated vapour pressure at temperature T (Pa)

2   c_P: specific heat of air at constant pressure (J kg^{-1} °C^{-1})

3   ρ: air density (kg m^{-3})

4   γ: psychrometric constant (Pa °C^{-1})

5   Δ: slope of the saturated vapour pressure curve (Pa °C^{-1})

6

7   *Canopy structural characteristics*:

8   d: canopy displacement height (m)

9   F: fractional cover of foliage (dimensionless)

10   LAI: leaf area index (m^2 m^{-2})

11   n: parameter with value of 1 for amphistomatous and 2 for hypostomatous foliage

12   z_r: reference height (m)

13   z_h: mean canopy height (m)

14   z_m: mean canopy source height ($= d+z_0$) (m)

15   z_0: canopy roughness length (m)

16

17   *Component resistances*:

18   r_a: aerodynamic resistance between the source height and the reference height (s m^{-1})

19    $r_{a,f,h}$: bulk boundary-layer resistance of the foliage for sensible heat (s m^{-1})

20    $r_{a,f,v}$: bulk boundary-layer resistance of the foliage for water vapour (s m^{-1})

21    $r_{a,s}$: aerodynamic resistance between the substrate and the source height (s m^{-1})

22    $r_{a,vs}$: aerodynamic resistance between the vegetated soil and the source height (s m^{-1})

23    $r_{a,bs}$: aerodynamic resistance between the bare soil and the source height (s m^{-1})

24    $r_{s,f}$: bulk stomatal resistance of the foliage (s m^{-1})

25    $r_{s,s}$: substrate resistance to evaporation (s m^{-1})

26    $r_{s,bs}$: bare soil resistance to evaporation (s m^{-1})

27    $r_{s,vs}$: vegetated soil resistance to evaporation (s m^{-1})

28    $r_{s,l}$: leaf stomatal resistance (one side) (s m^{-1})

29    $r_{a,l}$: leaf boundary-layer resistance for sensible heat and water vapour (one side) (s m^{-1})

30   r_s^e : effective surface resistance to water vapour within the canopy (s m^{-1})

31   r_v^e : effective resistance to water vapour transfer within the canopy (s m^{-1})

32    $r_{a,h}^e$: effective air resistance to sensible heat transfer within the canopy (s m^{-1})

33    $r_{a,v}^e$: effective air resistance to water vapour transfer within the canopy (s m^{-1})

1

Appendix A: Derivation of Eq. (5)

3

4   The steps of the derivation are the same as those used for deriving the Penman-

5 Monteith equation. Sensible and latent heat fluxes emanating from the foliage are written

6 respectively (Fig. 1)

7

8 $H_f = \rho c_p (T_f - T_m)/r_{a,f,h}$,   (48)

9

10 $\lambda E_f = (\rho c_p / \gamma)(e*(T_f) - e_m)/(r_{s,f} + r_{a,f,v})$.   (49)

11

12 T_f is foliage temperature and $e*(T_f)$ is the saturated vapour pressure at temperature T_f.

13 Linearizing the difference of saturated vapour pressure between the foliage and the canopy

14 source height and combining Eqs. (48) and (49) with the energy balance ($A_f = H_f + \lambda E_f$)

15 leads to

16

17 $\lambda E_f = \dfrac{\Delta A_f + \rho c_p D_m / r_{a,f,h}}{\Delta + \gamma \left[(r_{s,f} + r_{a,f,v})/r_{a,f,h} \right]}$.   (50)

18

19 Given that $r_{a,f,v} = n\ r_{a,f,h}$ (Eq. (4)), we obtain (Monteith and Unsworth 1990, p.188)

20

21 $\lambda E_f = \dfrac{\Delta A_f + \rho c_p D_m / r_{a,f,h}}{\Delta + \gamma \left[n + r_{s,f} / r_{a,f,h} \right]}$ (51)

22

23 with $n = 1$ and 2 respectively for amphistomatous and hypostomatous leaves.

24

Appendix B: Derivation of Eq. (16)

26

27   Introducing Eq. (7) into Eqs. (5) and (6) and adding the two equations gives

28

$$\lambda E = \frac{\Delta A_f + \left\{\rho c_p D_a + \left[\Delta A - (\Delta + \gamma)\lambda E\right]r_a\right\}/r_{a,f,h}}{\Delta + \gamma(n + r_{s,f}/r_{a,f,h})}$$
$$+ \frac{\Delta A_s + \left\{\rho c_p D_a + \left[\Delta A - (\Delta + \gamma)\lambda E\right]r_a\right\}/r_{a,s}}{\Delta + \gamma(1 + r_{s,s}/r_{a,s})}. \tag{52}$$

Introducing the terms $R_f{}'$ and $R_s{}'$ defined by Eqs. (20) and (21) yields

$$\gamma\lambda E = \frac{\Delta A_f r_{a,f,h} + \left\{\rho c_p D_a + \left[\Delta A - (\Delta + \gamma)\lambda E\right]r_a\right\}}{R_f{}'} + \frac{\Delta A_s r_{a,s} + \left\{\rho c_p D_a + \left[\Delta A - (\Delta + \gamma)\lambda E\right]r_a\right\}}{R_s{}'}. \tag{53}$$

By collecting the terms in λE we obtain

$$\lambda E\left[\gamma R_f{}' R_s{}' + (\Delta + \gamma)(R_f{}' + R_s{}')r_a\right] = \Delta A(R_f{}' + R_s{}')r_a$$
$$+ \rho c_p D_a(R_f{}' + R_s{}') + \Delta(A_f r_{a,f,h} R_s{}' + A_s r_{a,s} R_f{}'). \tag{54}$$

Introducing the term $R_a{}'$ defined by Eq. (22) gives

$$\lambda E\gamma\left[R_f{}' R_s{}' + R_a{}'(R_f{}' + R_s{}')\right] = (R_f{}' + R_s{}')r_a\left[\Delta A + \rho c_p D_a/r_a\right] + \Delta(A_f r_{a,f,h} R_s{}' + A_s r_{a,s} R_f{}'). \tag{55}$$

Eq. (B4) can be transformed into

$$\lambda E = \left(\frac{\Delta + \gamma}{\gamma}\right)\frac{(R_f{}' + R_s{}')r_a}{R_f{}' R_s{}' + R_a{}'(R_f{}' + R_s{}')}\left(\frac{\Delta A + \rho c_p D_a/r_a}{\Delta + \gamma}\right) + \frac{\Delta}{\gamma}\left(\frac{R_s{}' A_f r_{a,f,h} + R_f{}' A_s r_{a,s}}{R_f{}' R_s{}' + R_a{}'(R_f{}' + R_s{}')}\right). \tag{56}$$

Taking into account the definitions of P_s and P_f given by Eq. (18) and (19) leads to Eq. (16).

Appendix C: Coefficients of the modified clumped model (conventional formulation)

We define:

$$R_f = (\Delta + n\gamma)r_{a,f,h} + \gamma r_{s,f}, \tag{57}$$

$$R_{vs} = (\Delta + \gamma) r_{a,vs} + \gamma\, r_{s,vs}, \tag{58}$$

$$R_{bs} = (\Delta + \gamma) r_{a,bs} + \gamma\, r_{s,bs}, \tag{59}$$

$$R_a = (\Delta + \gamma) r_a. \tag{60}$$

The coefficients of Eq. (30) are written as

$$C'_f = R_{vs} R_{bs} (R_f + R_a) / DE, \tag{61}$$

$$C'_{vs} = R_f R_{bs} (R_{vs} + R_a) / DE, \tag{62}$$

$$C'_{bs} = R_{vs} R_f (R_{bs} + R_a) / DE, \tag{63}$$

DE being defined as

$$DE = R_{vs} R_f R_{bs} + R_f R_{bs} R_a + R_f R_{vs} R_a + R_{vs} R_{bs} R_a. \tag{64}$$

Appendix D: Coefficients of the modified clumped model (alternative formulation)

We put

$$R'_f = r_{s,f} + (n + \Delta/\gamma) r_{a,f,h} \tag{65}$$

$$R'_{vs} = r_{s,vs} + (1 + \Delta/\gamma) r_{a,vs} \tag{66}$$

$$R'_{bs} = r_{s,bs} + (1 + \Delta/\gamma) r_{a,bs} \tag{67}$$

$$R'_a = (1 + \Delta/\gamma) r_a \tag{68}$$

1    $$DE' = R_f' R_{vs}' R_{bs}' + R_f' R_{bs}' R_a' + R_f' R_{vs}' R_a' + R_{vs}' R_{bs}' R_a' .$$ (69)

2

3   The coefficients of the resultant equation are written as

4

5    $$P_f' = r_a R_{vs}' R_{bs}' / DE'$$ (70)

6

7    $$P_{vs}' = r_a R_f' R_{bs}' / DE'$$ (71)

8

9    $$P_{bs}' = r_a R_f' R_{vs}' / DE'$$ (72)

10

11   **Appendix E: Parameterizations used in the simulation process**

12

13   The parameterization used to simulate the component air resistances and the

14   distribution of available energy within the canopy are taken and adapted from Shuttleworth

15   and Wallace (1985), Choudhury and Monteith (1988) and Shuttleworth and Gurney (1990).

16   The net radiation reaching the substrate R_{ns} is calculated from the net radiation above the

17   canopy R_n following Beer's law

18

19    $$R_{ns} = R_n \exp(-0.7 LAI) .$$ (73)

20

21   Soil heat flux is calculated as a given fraction of R_{ns} ($G = 0.2\ R_{ns}$). Consequently, available

22   energies are obtained as: $A = R_n - G$, $A_f = R_n - R_{ns}$ and $A_s = R_{ns} - G$. For the sake of

23   convenience, the aerodynamic resistance above the canopy (r_a) is calculated in neutral

24   conditions. It is expressed as a function of wind speed u_a at reference height z_r

25

26    $$r_a = (1/k^2 u_a) \ln^2 \left[(z_r - d)/z_0 \right],$$ (74)

27

28   where $d = 0.63\ z_h$, $z_0 = 0.13\ z_h$ and k is von Karman's constant. The aerodynamic resistance

29   between the substrate (with a roughness length $z_{0s} = 0.01$ m) and the canopy source height (d

30    $+ z_0$) is calculated as

31

$$r_{a,s} = \frac{z_h \exp(\alpha_w)}{\alpha_w K(z_h)} \{\exp[-\alpha_w z_{0s}/z_h] - \exp[-\alpha_w(d+z_0)/z_h]\}, \tag{75}$$

where $\alpha_w = 2.5$ (dimensionless) and $K(z_h)$ is the value of eddy diffusivity at canopy height (Lhomme et al. 2000). Leaf boundary-layer resistance (one side) is expressed as a function of the wind speed at canopy height $u(z_h)$ as

$$r_{a,l} = \frac{\alpha_w[w/u(z_h)]^{1/2}}{2\alpha_0[1-\exp(-\alpha_w/2)]}. \tag{76}$$

w is leaf width (0.01 m) and α_0 is a constant equal to 0.005 (in m s$^{-1/2}$) (Lhomme et al. 2000).

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17
18

1   **Figures captions**

2

3   Fig. 1. Resistance networks and potentials for a two-layer representation (Shuttleworth-

4   Wallace model) of convective fluxes (sensible and latent heat) from a sparse canopy (see the

5   list of symbols).

6

7   Fig. 2. Patch representation of evaporation from a sparse crop: schematic diagram showing

8   the resistance network and potentials: subscript 1 is for foliage component and subscript 2 for

9   substrate component.

10

11   Fig. 3. Resistance networks and potentials for a clumped representation (Brenner-Incoll

12   model) of convective fluxes (sensible and latent heat) from a heterogeneous canopy (see the

13   list of symbols).

14

15   Fig. 4. Resistance network and potentials for a one-layer representation (Penman-Monteith

16   model) of convective fluxes (sensible and latent heat) from a full covering canopy: a) basic

17   Penman-Monteith model; b) Penman-Monteith model with effective parameters accounting

18   for air resistances within the canopy.

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20   Fig. 5. Variation as a function of canopy *LAI* of the relative error $\delta\lambda E$ (expressed in

21   percentage) made on the evaporation rate of a hypostomatous canopy when calculated with

22   the two-layer equations valid for an amphistomatous one: a) for different values of leaf

23   stomatal resistance $r_{s,l}$ (with $r_{s,s} = 500$ s m^{-1}); b) for different values of substrate resistance $r_{s,s}$

24   (with $r_{s,l} = 400$ s m^{-1}).

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26   Fig. 6. Variation as a function of canopy *LAI* of the relative error $\delta\lambda E$ (expressed in

27   percentage) made on the evaporation rate when calculated with a big leaf model and effective

28   resistances estimated with different levels of approximation: a) with the physically soundest

29   formulation ($\delta\lambda E_1$); b) when air resistances within the canopy are neglected ($\delta\lambda E_2$); c) when

30   surface and air resistances are decoupled ($\delta\lambda E_3$).
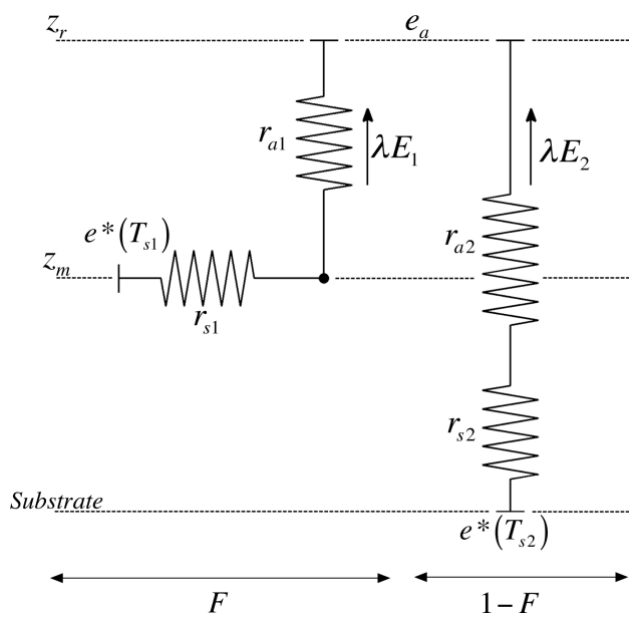
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3 Figure 1. Resistance networks and potentials for a two-layer representation (Shuttleworth-

4 Wallace model) of convective fluxes (sensible and latent heat) from a sparse canopy (see the

5 list of symbols).

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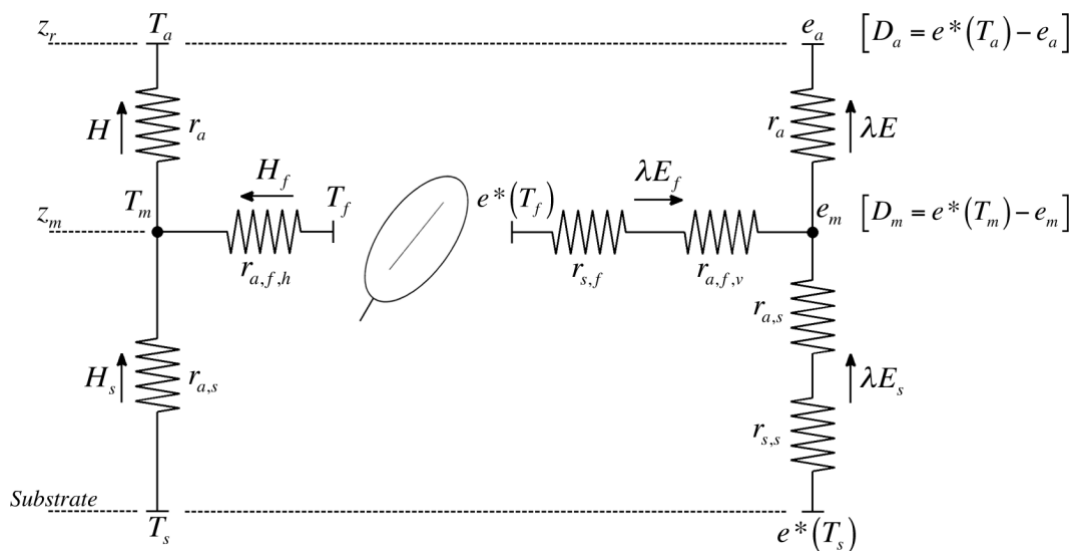
3   Figure 2. Patch representation of evaporation from a sparse crop: schematic diagram showing

4   the resistance network and potentials: subscript 1 is for foliage component and subscript 2 for

5   substrate component.

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3 Figure 3. Resistance networks and potentials for a clumped representation (Brenner-Incoll

4 model) of convective fluxes (sensible and latent heat) from a heterogeneous canopy (see the

5 list of symbols).

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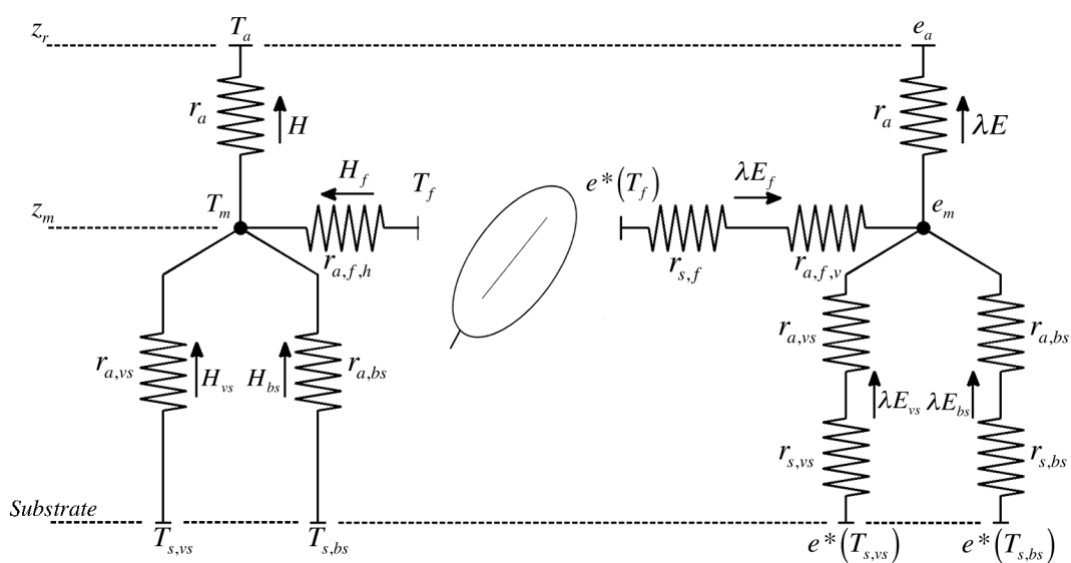
3   Figure 4. Resistance network and potentials for a one-layer representation (Penman-Monteith

4   model) of convective fluxes (sensible and latent heat) from a full covering canopy: a) basic

5   Penman-Monteith model; b) Penman-Monteith model with effective parameters accounting

6   for air resistances within the canopy.

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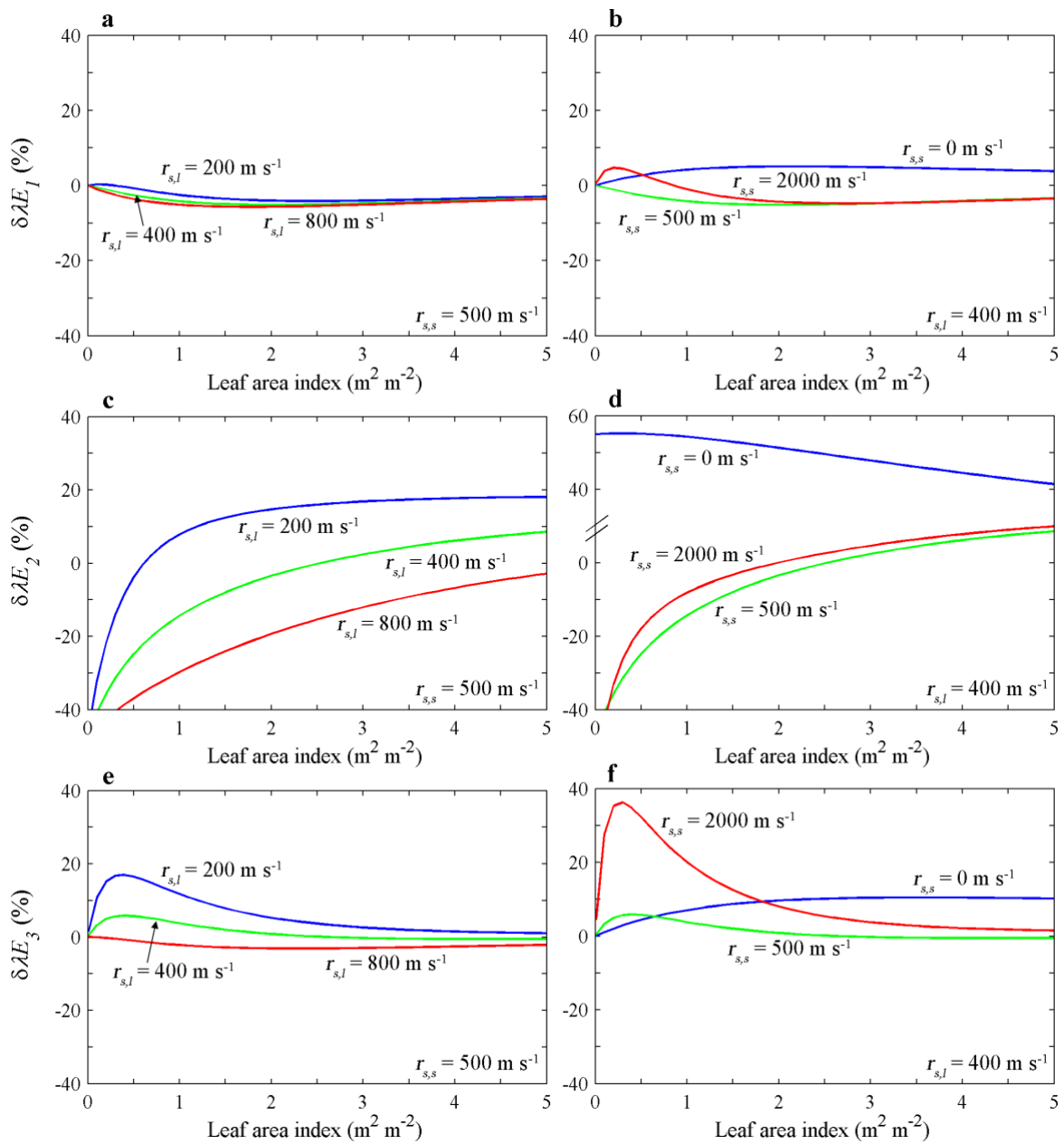

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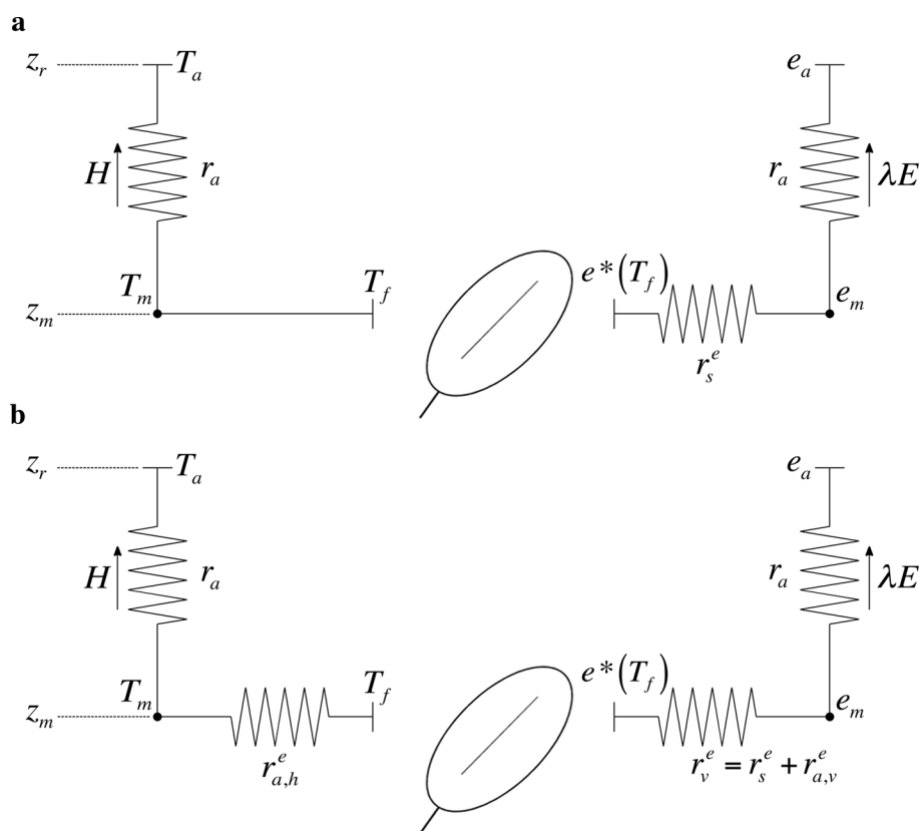
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Figure 5. Variation of the relative error $\delta\lambda E$ (expressed in percentage) made on the evaporation rate of a hypostomatous canopy when calculated with the two-layer equations valid for an amphistomatous one. The variation is plotted as a function of canopy *LAI*: (a) for different values of leaf stomatal resistance $r_{s,l}$ (with $r_{s,s} = 500$ s m^{-1}); (b) for different values of substrate resistance $r_{s,s}$ (with $r_{s,l} = 400$ s m^{-1}).

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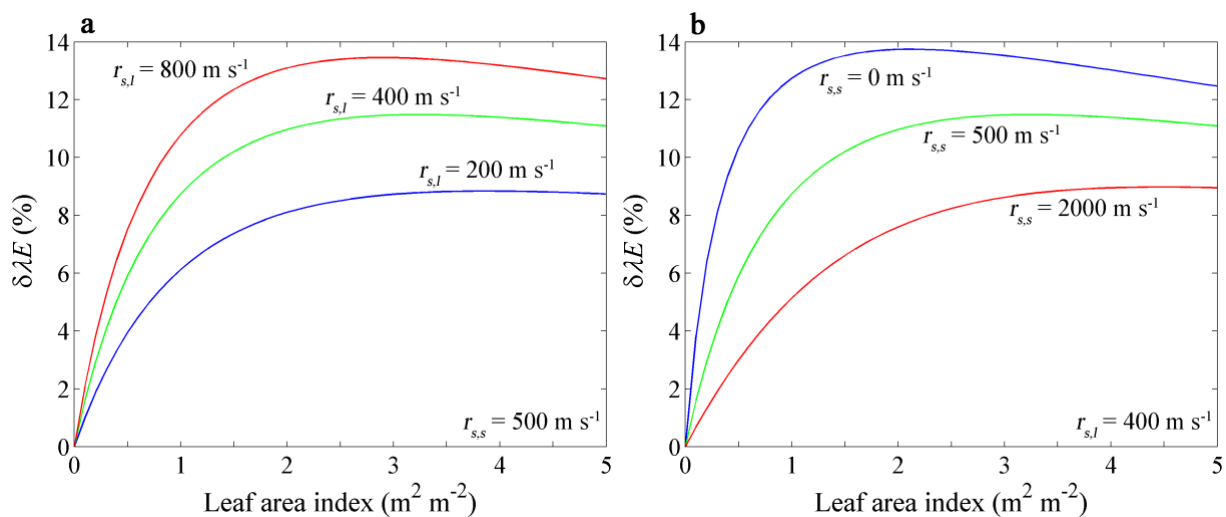
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Fig. 6. Variation of the relative error $\delta\lambda E$ (expressed in percentage) made on the evaporation

rate when calculated with a big leaf model and effective resistances estimated with different

levels of approximation: (a) and (b), with the physically soundest formulation ($\delta\lambda E_1$); (c) and

(d), when air resistances within the canopy are neglected ($\delta\lambda E_2$); (e) and (f), when surface and

air resistances are decoupled ($\delta\lambda E_3$). The variation is plotted as a function of canopy LAI.

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