Global Sensitivity Analysis of Poroelastic Soil Parameters

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ABSTRACT

In this proposal, we present a global sensitivity analysis method applied to the study of wave propagation occurring in a poroelastic material [Saltelli et al, 2008]. The global aim is to get relevant information in the investigation of the most influential poroelastic parameters. A sensitivity analysis allows to quantify the impact of the input parameter uncertainties on the mechanical model outputs. The analysis is based on the calculation of the Sobol indices or the partial variances that quantify the influence of each independent parameters, but also the possible interactions between these parameters, on the simulated pore pressure or solid displacement fields. The Extended FAST method is handled in combination with an efficient forward solution. The configuration under study concerns a fluid overlying a poroelastic two-dimensional half-space ground submitted to a transient excitation [Lefeuve-Mesgouez et al, 2012]. A line source emits cylindrical waves. The problem is first solved analytically with the stiffness matrix method, to get a forward solution [Mesgouez et al, 2009]. Since the global sensitivity analysis is based on the repetition of numerous forward problems, these ones have to be efficient from the numerical point of view. Excitation type and receiver locations concern laboratory experiment simulations, of typical distance about 1 meter. A 10% uniform distribution of uncertainties has been taken into account. The same kind of approach can be applied in civil engineering, aquifer or soil auscultation, or biomechanics, involving larger typical dimensions or other uncertainty distributions. Moreover, this approach is part of a broader methodology that aims at constructing an inversion procedure involving only the parameters of interest. Four influential parameters have been identified in the specific studied configuration, ρ_S, μ, ϕ and K_F. The methodology is generic but the results are linked to the characteristics of the problem under study.

INTRODUCTION AND PURPOSE

After describing the geometry and problem under consideration, the proposal briefly recalls the main characteristics of the forward problem. Then a specific section is dedicated to sensitivity analysis to underline the major points. A description of the practical implementation of a global sensitivity analysis is then summarized and discussed both in terms of a synthetic graphical description and ad hoc explanations. To finish, one example of results obtained is presented and specific comments are provided.

THE FORWARD PROBLEM CHARACTERISTICS

The 2D configuration under investigation is a fluid half-space over a homogeneous poroelastic half-space. The x and y geometrical axes point rightward and upward, respectively. A causal source point located in the fluid emits cylindrical waves. The fluid domain is governed by the acoustic equations and is supposed to be perfectly well-known. The poroelastic medium is modeled using the low-frequency Biot theory [Biot, 1956; Bourbié et al, 1987]. It corresponds to the medium to be investigated and the 10 independent physical parameters of the Biot theory involved are recalled in Figure 1 (STEP 1), with the targeted values corresponding to water-saturated sand [Denneman et al, 2002]. The fluid / porous interface I has been chosen in this study to be modeled as an "open-pore" interface. The approach uses a direct model based on integral transforms. Details are given in [Lefeuve et al, 2012; Mesgouez et al, 2009]. The model calculates for instance pressure, velocities, displacements or stresses in the porous medium or in the upper fluid.

THE SENSITIVITY ANALYSIS CONCEPTS

Due to the large number of unknown parameters and their associated uncertainties, the inversion process is tedious and difficult. In this context, sensitivity analysis (SA) may provide relevant information about the relationships between uncertain model input parameters and potentially observable outputs. This information may help in setting up a subtler strategy: *i)* to characterize the material by identifying the main parameters controlling the variability of the model output, or to modify the model itself by a reduction of the influential input parameters; *ii)* to help construct experimental designs for model inversion.

SA methods are traditionally divided into two families: local and global methods [Saltelli et al, 2008]. The local methods focus on the effect of a perturbation near a point of the factor space and the perturbation is applied factor by factor. They are called "One-At-a-Time" methods, are based on deterministic approaches and are applicable to costly models or to cases with many parameters. The global sensitivity analysis (GSA) methods incorporate the influence of the whole range of variation of model inputs and often evaluate the impact of the inputs while all the other parameters can vary. That is the main reason why we have chosen such approaches. Among global methods, variance-based methods are very popular. Their principle is to apportion the total variance of model outputs to the various input factors and to their interactions, given their uncertainty distributions. They can deal with nonlinear and non-additive models.

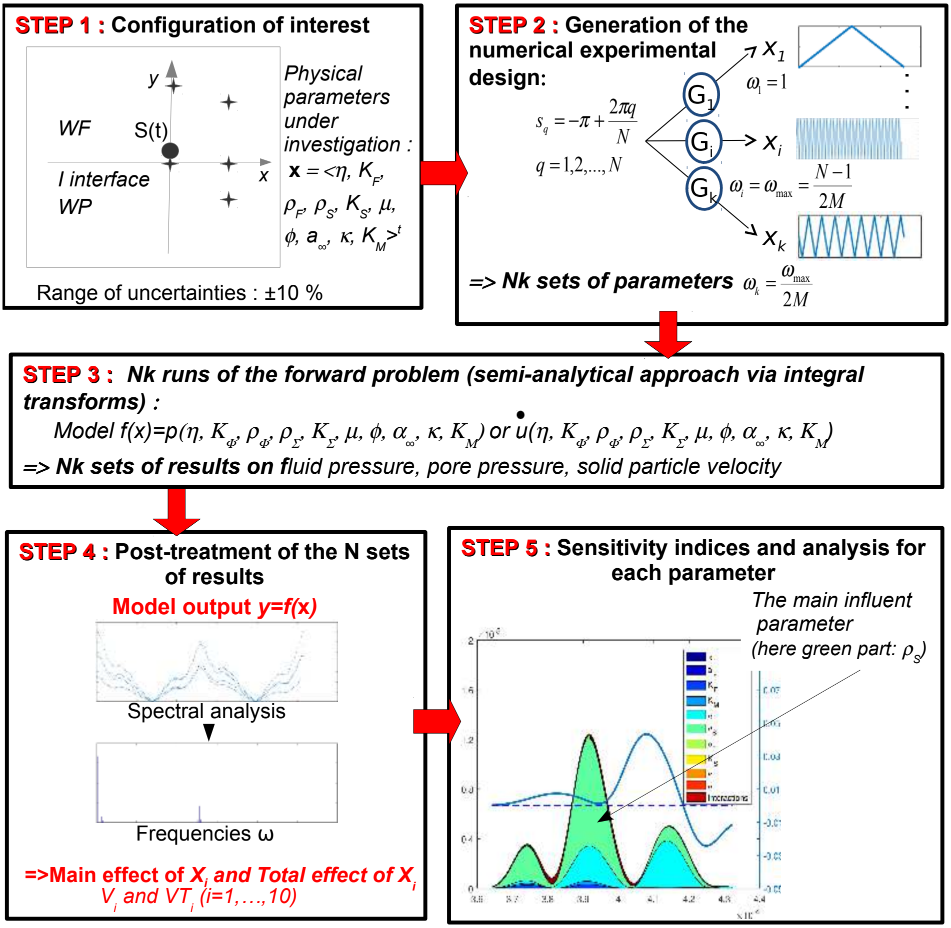

Figure 1. Geometry under study.

To the best of our knowledge, GSA techniques have, however, still rarely been applied to wave propagation in poroelastic environments.

Let us note y the output of the model (for instance, it can be the pressure or the vertical solid velocity in our configuration), $\mathbf{x} =< x_i >^t,\ i = 1, ...k$ the input parameters (here the 10 parameters of the Biot theory as described in Figure 1 - STEP 1) and f the model linking both y and $\mathbf{x}$: $y = f(\mathbf{y})$. Using ANOVA decomposition (also called High-Dimensional-Model-Representations (HDMR) [Rabitzi et al., 1999]) and under the hypothesis of independent random variables X_i, we can write:

$$
\begin{aligned}
\mathbb{V}(Y) &= \sum_i V_i + \sum_{1 \leqslant i < j \leqslant k} V_{ij} + ... + V_{1,2,...,k} \\
&\text{with} \\
V_i &= \mathbb{V}(\mathbb{E}(Y|X_i)) \\
V_{ij} &= \mathbb{V}(\mathbb{E}(Y|X_i, X_j)) - V_i - V_j \\
\cdots &
\end{aligned}
\tag{1}
$$

where :
• V_i is the partial variance of Y attributed to the main effect of X_i, explained by the variations of X_i on its uncertainty domain independent of the variations of the other factors. $\mathbb{E}(Y|X_i)$ can be interpreted as the only function dependent on X_i that best approximates Y.
• V_{ij} is the partial variance of Y attributed to the second order effect of X_i and X_j, explained by the variations of X_i and X_j on their uncertainty domains but not by the sum of their main effects. It describes the interaction between X_i and X_j, i.e. the fact that the effect of X_i (resp. X_j) may depend on the values of X_j (resp. X_i).
The variance of Y can thus be expressed as a sum of individual contributions of the different factors and of their interactions.

As computing $(2^k - 1)$ parts of the variance of decomposition (1) is practically often intractable, [Homma et al, 1996] introduced the concept of total-effect. The part of the variance VT_i attributed to the total effect of X_i is:

$$
VT_i = V_i + \sum_{1 \leqslant i < j \leqslant k} V_{ij} + ... + V_{1,2,...,k} \tag{2}
$$

It includes the effect of X_i alone as well as interactions with any combination of the other parameters. The difference between VT_i and V_i is the part of the variance attributed to the interactions of all orders between X_i and the other factors. Estimation of the k pairs (V_i, VT_i) is often performed in practice since it yields a good and synthetic, although non exhaustive, characterization of the sensitivity pattern for a model, as mentioned by [Saltelli et al, 2008].

In the literature, variance-based sensitivity analyses are usually presented using Sobol indices. These indices, introduced in [Sobol, 1993], have been widely used these last twenty years for sensitivity analysis studies. They correspond to the normalization of the parts of the variance V_i, V_{ij}, ..., $V_{1,2,...,k}$ and VT_i through the total variance $\mathbb{V}(Y)$, as follows :
• the main sensitivity index of X_i:

$$
S_i = \frac{V_{X_i}(\mathbb{E}_{X_{\sim i}}(Y|X_i))}{V(Y)}
$$

• the total effect index:

$$ST_i = S_i + \sum_{1 \leqslant i < j \leqslant k} (S_{ij} + ... + S_{12...k})$$

which includes the effect of X_i alone and in interaction with any combination of the other parameters.
The main purposes when conducting a sensitivity analysis are :
• factor fixing: identify the factors which influence on output is negligible, ST_i very low ;
• factor prioritisation: identify the factor list to be estimated to restrict the uncertainty on output Y_j, S_i high.

PRACTICAL IMPLEMENTATION

The main steps of the methodology are summarized in Figure 1 and described as follows:
Step 1 : define the configuration of interest, the parameters which influence is to be analysed and the results the SA is constructed on. This step has been described in previous section concerning the forward problem. We briefly recall the main points.
$\Rightarrow$ Here we focus on the 10 parameters included in the Biot theory for a configuration of a fluid overlying a poroelastic half-space, with a transient solicitation in the fluid. The range of parameters variations covers a 10% uniform distribution of uncertainties, see [Dupuy, 2011]. The outputs to be treated are: the acoustic pressure p in the fluid and the porous medium, the solid velocity $\dot{\mathbf{u}} = (\dot{u}_x,\ \dot{u}_y)^t$ in the porous medium.
Step 2 : generate the numerical experimental design, with EFAST approach.
The main principle of the FAST method is to make the model input variables x i oscillating at different frequencies i and to estimate the impor- tance of the associated factors X i by scrutinizing the Fourier components of the model output at these frequencies [Saltelli et al., 1999], [Mara, 2009].
$\Rightarrow$ A scalar s controls the values of all the 10 parameters via functions G_i (here we have taken $G_i(z) = F_i^{-1}(\frac{1}{\pi}\arcsin(z) + \frac{1}{2})$, with F_i^{-1} the inverse cumulative distribution function of the probability distribution of X_i; s_q takes N values and thus $N \times k$ sets of parameter values are created. In each set, one parameter is given a high frequency (ω_{max}) to emphasize its behavior;
Step 3 : compute $N \times k$ runs of the forward problem with

$$f(\mathbf{x}) = p(\eta, K_F, \rho_F, \rho_S, K_S, \mu, \phi, a_\infty, \kappa, K_M)$$

if the pressure is under consideration. $\Rightarrow$ We obtain thus $N \times k$ series of results. Some repetitions with different beginning points can be considered to avoid aliasing.
Step 4 : post-treat the results with spectral analysis, EFAST analysis, to extract the main effect V_i and the total effect VT_i of each parameter under study.
Step 5 : present the results of the SA in a graphical form in order to get all the V_i, VT_i

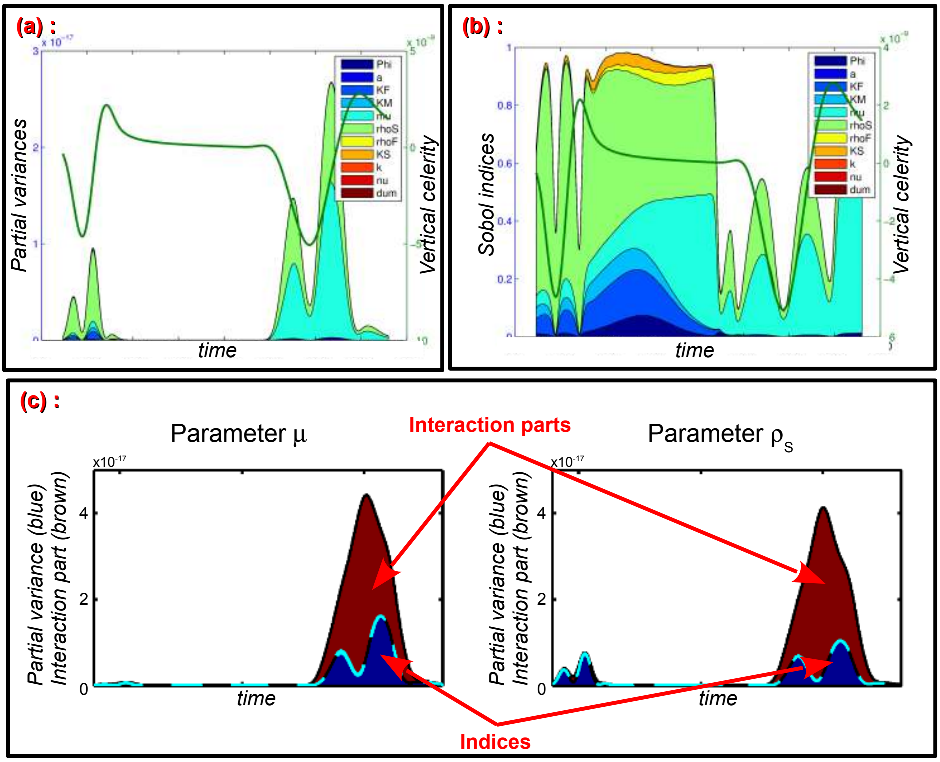

Figure 2. Example of results for a point located in the porous medium at $(x, y) = (0.3; -0.3)$**m.**

at a time and then analyse the main and total effect results.
Graphical representation of dynamic (resp. spatial) evolution of sensitivity indices allows having an exhaustive view of the parameter importance on the selected variable and of their temporal or spatial variability. The main influent parameter appears obviously in the graph (see Figure 1 - STEP 5). The analysis of different indices is discussed in next section.

EXAMPLE OF RESULTS

In this section, we illustrate the GSA with an example of results. Figure 2 presents partial variances obtained from the vertical solid velocity in the porous medium for point $(+0.3; -0.3)$ m.

Figure 2a presents the partial variances V_i of the different parameters involved in the Biot theory. Also, the green solid curve illustrates the response in terms of vertical velocity for the specific point under study. In this case, we can visualize both the $P-$

wave and the $S-$ wave arriving at 2 distinct times. Concerning the $S-$ wave, only Lamé constant μ and solid density ρ_S have influence on the response. This is not a surprise since the $S-$ wave is not supported by the fluid part of the medium. On the contrary, for the $P-$ wave, even if parameters μ and ρ_S have a strong influence, other parameter influences can be visualized such as ϕ and K_F.
Figure 2b presents the same results with Sobol indices (non-dimensional results): it allows a better view of the parameter impact concerning the $P-$ wave and helps in identifying the influence of all the parameters even those of smaller influence. Also, it gives an information on the interaction existing between the parameters since the sum of all contribution is equal to one when no interaction occurs. That is clearly not the case here. Figure 2c quantifies the interaction part compared to the main effect, here for parameters μ and ρ_S.

Note that although Sobol indices are widely used in sensitivity analysis, the interpretation has to be done with great care because these indices emphasize the information when the signal is small and can be difficult to be enough reliable. In such a study, the partial variance is more adequate. Moreover, to handle multivariate model outputs (pressure, velocity), using parts of the variance is useful to directly appreciate the variations in time and space of the total and partial variances when Sobol indices naturally hide this information owing to the normalization.

The authors also insist on the following point : the methodology of SA is generic but results are not : they are strictly linked to the configuration under study and a SA has to be conducted for each specific case.

CONCLUSION

Sensitivity analyses have been widely used and are efficient tools for exploring model behaviors under input uncertainties. However such analyses are rarely used in the context of wave propagation in poroelastic environments. We have thus proposed to illustrate the use of GSA in this context. Two points are of importance : to be able to identify the parameters of high influence, but also to get information on the interaction between parameters. Nevertheless, if the methodology is generic, the results are not and a SA has to be conducted for each configuration. In the configuration under study, a fluid on a poroelastic half-space, four influential parameters have been identified: ρ_S, μ, ϕ and K_F.

REFERENCES

Biot M.A. (1956) “Theory of propagation of elastic waves in a fluid-saturated porous solid. I: Low-frequency range.” J. Acoust. Soc. Am. 28-2, 168-178.

Bourbié T., Coussy O. and Zinszner B. (1987). *Acoustics of Porous Media*, Gulf Publishing Company.

Denneman A.I.M., Drijkoningen G.G., Smeulders D.M.J. and Wapenaar K. (2002). “Reflection and transmission of waves at a fluid / porous medium.” Geophy., 282-291.

Dupuy B. (2011). “Propagation des Ondes Sismiques dans les Milieux Multiphasiques Hétérogènes : Modélisation Numérique, Sensibilité et Inversion des Paramètres Poroélastiques.” PhD Thesis, University Grenoble.

Homma T., Saltelli A. (1996). “Importance measures in global sensitivity analysis of nonlinear models.”. Reliab. Eng. Syst. Safe., 52(1), 1-17.

Lefeuve-Mesgouez G., Mesgouez A., Chiavassa G. and Lombard B. (2012). “Semi-analytical and numerical methods for computing transient waves in 2D acoustic/poroelastic stratified media.” Wave Motion, 49-7, 667-680.

Mara T.A. (2009). “ Extension of the RBD-FAST method to the computation of global sensitivity indices”. Reliab. Eng. Syst. Safe. 94(8), 1274-1281.

Mesgouez A. and Lefeuve-Mesgouez G. (2009). “Transient solution for multilayered poroviscoelastic media obtained by an exact stiffness matrix formulation.” Int. J. Numer. Anal. Meth. Geomechanics, 33, 1911-1931.

Rabitz H., Alis O.F. (1999). “General foundations of high-dimensional model representations”. J. Math. Chem., 25(2-3), 197-233.

Saltelli A., Tarantola S., Chan K.P.S. (1999). “Quantitative Model-Independent Method for Global Sensitivity Analysis of Model Output”. Technometrics 41, 39-56.

Saltelli A., Ratto M., Andres T., Campolongo F., Cariboni J., Gatelli D., Saisana M. and Tarantola S. (2008).“Global Sensitivity Analysis: The Primer.” John Wilez & Sons, Chichester, UK,.

Sobol I.M., (1993) “Sensitivity estimates for nonlinear mathematical models.” Math. Model. Comput. Exper., 1, 07-414.