A simple bio-economic model of soil natural capital

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Abstract

Relying on the conceptual framework of natural capital, this paper build a concept of soil natural capital and implement it using an optimal control model. Considering soil as an ecosystem, we build a simple bio-economic model with two interelated stocks (the soil organic matter and the stock of nutrients directly contributing to the plant's biomass elaboration). The production function is of Liebig type, a Linear one with plateau. The economic part of the model relies on the long term profit maximisation in the context of private management. We retained two controls: the mineral fertilization adding to the stock of nutrients, and the rate of biomass given back to soil to contribute to the soil organic matter.
By combining both controls, we identified management regimes and defined the set of stationaries states. Going beyong that standard step of analysis, we simulated optimal time path for different initial conditions and different set of parameter values. We specifically focused on the role of the price of fertilizers relative to the price of the agricultural products. Results show that private management of soil natural capital drives to the quasi depletion of soil organic matter. As a consequence, there is a need for public incentives to promote those ecosystems services non supported by market.

Keywords: Natural Capital, Optimal Control, Ecosystems Services, Environmental policy

Contents

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Introduction

Despite huge progress in plants genetic and plants protection, the production of food, fiber or biofuels by agriculture still relies on land as a complex resource made by the mix of an area and a volume of soil. Plants use photosynthesis to capture solar energy trough area and soil's volume holds the nutrients the plants could need to growth. This ecological mix has been regarded through the long history of soil sciences under various lenses Feller et al. (2012).

At the beginning of the XIXth century, the plant nutrition theory still focused on the role of humus (Humus being on organic compund) Thaer (1809). At the same time, soil fertility is regarded by economists as a natural resource inequally allocated to differents places. Observing the great range of soils productivity, even under the same climate, early economists postulated that differences in agricultural output comes from the differences in the soil's fertility. As a consequence, they elaborated the land rent theory. That view point remained unchanged, even after the demonstration by german scientists that plants use only simple elements like nitrogen, phosphorus and potash for their nutrition, and not directly soil's organic matter Sprengel (1838) Liebig (1840). That mineralist theory of plant nutrition had little impact on the way economists regarded the soil fertility, until the german chemists Haber and Bosch discovered the process of extracting nitrogen from atmosphere. From that innovation onwards, modern agriculture has been thought as a organic free agriculture Feller et al. (2012). As extractive industry is able to supply farmers with mineral fertilizers, agricultural production seems no longer to rely on natural processes. For economists, the neo-classical theory of production could be at that point substituted to the land rent theory. This substitution has been usefull and robust for decades, being reinforced by the success of what had been called "The Green Revolution" Borlaug (1970) Borlaug (2007). Because land, mineral fertilizers, pesticides and seeds could be found on markets, there is no need to look at the soils contents to explain differences in agricultural productivity. Farmers have only to allocate factors of production according to relative prices, productivity under the constraint of the production functions Heady (1952). As a consequence soils properties and soils services would be ousted from the research in agricultural economics for several decades.

Nevertheless, at the end of the XXth century, the awarness of several shortcommings, able to balance the benefits of modern agriculture, was increasing. First of all, land resources degradation by the very effects of agricultural intensification practices threats non only the ability of agriculture to afford food and fiber, but also impair many others ecosystem services. The first environmental crisis, known as a "dust bowl crisis", had already been experienced in the State before the second World War. The ecological and economical consequences have been so dramatic that the Federal Government created the first Public Agency to promote the soils conservation. The premiss of the soil natural capital concept have been elaborated at that time and legitimize the public intervention Bunce (1942) Hicks (1939a) Weitzell (1943). After the war, the memory of the dust bowl crisis has been soon discarded, and we observe that during the following decades up to 1980, no papers on the issue has been published. In the years 1980, Burt (1981) and McConnell (1983) published models of soil depletion by the effect of production intensification. They are the first papers using an intertemporal framework (dynamics programming or optimal control). After them, only a bunch of papers has been published on erosion issues, half of them being motivated by land degradation in developing countries Shortle and Miranowski (1987) Barbier (1997) Miranda (1992) Goetz (1997) Grepperud (1997) Brekke et al. (1999) Shiferaw and Holden (1999) Hediger (2003) Nakhumwa and Hassan (2011) Yirga and Hassan (2010). In all that papers, soil is modeled as a single state variable with a various dynamics, under the control of production intensity choices. They do not explicitly consider that farmers, by their choices and practices, are managing an natural capital.

At dawn of the XXIst century, the crisis induced by the competing uses of land for biofuel drawn attention on the fragility of organic free agriculture, and his dependance on non renewable (and sometime non substitutable) resources. As a consequence, concerns about the unsustainability of modern agriculture has been raising Griffon (2006) Conway (1997).

The convergence between specific issues in agriculture, and the international consensus on the role of carbon emissions on the climate change has driven to a new paradigm shift in the scientific conceptions of agricultural production, and more specifically, on the representation of soil's fertility. We will call the new paradigm Systemic. Systemic, because soil is now considered as an ecosytem of his own right. Desertification and land degradation has been promoted on the International Agenda, and become a main chapters in the MEA Millennium Ecosystem Assesment (2005).

Main changes in the representation of soil functions and services shared by soils scientists are related to the scope of soil services and to its structure and functions. Soils do not only provide support for food and fiber production, they are also home for a great biodiversity, they contribute to carbon sequestration, to regulate water cycle, and also give support to cultural services. The XXIst Century beginning, following Costanza et al. (1998) Costanza and Daly (1992), some soil scientists proposed to apply the concept of natural capital to the soil ecosystem Robinson et al. (2009), Dominati et al. (2010), Sanchez et al. (1997). "We define the soil natural capital as the stock of biotic and abiotic mass that contains energy and organization. Furthemore, the structure and functionality of soild accross the landscape facilitates needed process for the well-being of Humanity and the Earth system" Robinson et al. (2009). As soils scientists, Robinson, Dominati or Sanchez put emphasis on the components of the soil ecosystem. Naturally, even they have seen the very interest of the concept for public decision making and policy design, they are not in position to further develop corresponding models suitable for management.

At that point of our investigation, we get a very surprising conclusion: today, soil scientists propose a definition of soil natural capital, and propose to take onto account in the soil's social management the bulk of goods and services provided by SES, but dont have the means to develop corresponding analytical models. Meanwhile economists have developped useful tools to take onto account intertemporal trade-off in the agricultural production, but they mainly rely on very simple, unidimensional, models of soil mainly defined by the depth or the volume of topsoil. Naturally, there is no contradiction between both positions, there is just a gap to be fulfilled. And some modelling hurdle ahead.

Our aims in that paper are to take advantage of the recent advances both in soil sciences and in optimal control theory, in order to elaborate an simple bio-economic model of soil natural capital. We will define soil natural capital (SNC) as an economical concept, an economical indicator usefull to evaluate the flows of goods and services provided by the soil considered as an ecosystem. SNC help in monitoring management actions (extracting, renewing, use conversion...) of private actors, and in designing public policies aimed at long term conservation of soils capacities. This definition relies on the economical appraisal of flows of goods and services over time, and she is different from the "naturalist definition" elaborated by soil scientists. Namely, she is not only related to the components of the SES, but to the capitalisation of services, evaluated either by markets or by others evaluation methods. Because the main services provided by SES beside food and fiber production are non markets services, and are by nature public goods, the value of SNC is different according to the private or social point of view. As long as intertemporal management is involved, we will rely on optimal control theory, and because main soil's ecosystem services (SESS) are related to soil's organic matter (SOM) Feller et al. (2012) Victoria (2012) Miles et al. (2009), we will focus on that stock. Moreover, we will rely on the simplest representation of SOM dynamics given by Hénin and Dupuis (1945). Humification process incorporates crops residues into SOM stock, while mineralisation process nurtur the second stock of directly assimilable nutrients. Because to the inherent complexity of SES functionning, we will not deal at this stage with others SESS beside the support to the agricultural production. Moreover, we will focus on private management, keeping the social value of SNC for further investigations. Assuming that the private management of soils aims at maximising the net present value of the soil's asset, we will make use of optimal control models, and will identify management regimes and stationnaries states. We then will proceeds to simulations of transitory regimes from differents initial situations.

Our paper is organized as follow:

In the first section, we will explain the model's structure, the resolution method and the results, and will give economical interpretation.

In the second section, we will look at the stationaries state corresponding to different management regimes.

We will devote the third section to the simulations of optimal time profiles for relevant initial conditions and sets of parameters values. We will give a specific attention to the impact of the prices of fertilizers.

Finally we will draw conclusions and trace some research perspectives on soil natural capital.

1 The model

1.1 The model's structure

The SNC model uses the optimal control theory to maximize over an infinite horizon the profits from the agricultural activity. The manager is entitled with two controls: one is the fertilizer application rate, and the other one is the rate of biomass restitution to soil. The soil ecosystem structure and dynamics is represented by two interelated stocks: the first one represents the SOM contents, the second is the soil's nutrients contents. Fertilizers just add to the second stock, while restitutions of crops residues to soil contributes, trough humification process, to the building of the first. Trough the mineralization process of SOM, the first stock contributes to the building of the stock of nutrients N.

According to the mineralist theory of plants nutrition, the crop's biomass depends only on the stock of nutrients. SOM plays yet no direct role in the production. As a consequence, the manager face an intertemporal trade-off between harvesting today all the marketable biomass, or leave a share to the soil, in order to spare in the future some fertilizer addition. In order to accomodate that trade-off, we directly model the biomass production, not only the grain or fiber yield. The biomass production function is piecewise linear, with a plateau, according to the limiting factor theory Paris (1992):

$$f(N) = \begin{cases} \beta N & \text{if} \quad N < \bar{N} \\ \beta \bar{N} & \text{if} \quad N \geq \bar{N}. \end{cases}$$

The first statement holds when the nutrients available are not sufficient to provide full growth to the crop; in that case, N is a limiting factor. The second one holds as soon as N is sufficient, while others factors like water or temperature remain limiting. In that conditions, the biomass production is constant, and the production's plateau occurs. The thresshold value for N will be noted $\overline{N}$. It's not necessary to add fertilizers as soon as the SOM stock is able to provides $\overline{N}$ or more to the plants.

β is the technical coefficient of transformation of nutrients N into biomass. ν coefficient express the corresponding consumption of nutrients. As $N \geq \check{N}$, the nutrients consumption remains constant and is value is $\overline{N}$:

$$\epsilon(N) = \begin{cases} \nu N & \text{if} \quad N < \bar{N} \\ \bar{N} & \text{if} \quad N \geq \bar{N}. \end{cases}$$

The soil's organic matter dynamics arises from two complex processes: the humification of the crop's residue restitutions, on one side, and his own degradation trough bacteria, known as the mineralization. The later mineralization contributes to the building of the nutrients stock, N . Fertilizers application, n, directly contribute to N, with some losses, so as the efficient application becomes $\chi(n)$. While the rate of biomass restitutions, k, contributes to N indirectly, trough M dynamics:

$$\dot{M} = kf(N) - \gamma M, \quad M(0) = M_0, \tag{1}$$

$$\dot{N} = \gamma M + \chi(n) - \epsilon(N), \quad N(0) = N_0, \tag{2}$$

The timepoint profit function is quadratic in k and n, due to operating variable costs for harvesting and incorporating crops remains to the soil. Fertilizers application is submitted to the same constraints, because application of one increasing quantity of fertilizer requires more time and energy.

$$a(1-k)f(N) - b\left[(1-k)f(N)\right]^2 - \Phi(n) \tag{3}$$

with:

a: price of biomass sold on markets
b : harvesting costs
Φ_1 : price of fertilizer
Φ_2 : applications costs

$$\Phi(n) = \Phi_1 n + \frac{\Phi_2}{2} n^2$$

The manager's problem is one of maximizing the present value of the flow of profits over one infinite horizon:

$$\max_{n \geq 0, k \in [0,1]} \int_0^\infty e^{-\rho t} \left[a(1-k)f(N) - b\left[(1-k)f(N)\right]^2 - \Phi(n)\right] dt \tag{4}$$

where

$$f(N) = \begin{cases} \beta N & \text{if} \quad N < \bar{N} \\ \beta \bar{N} & \text{if} \quad N \geq \bar{N}. \end{cases}$$

such that

$$\dot{M} = kf(N) - \gamma M, \quad M(0) = M_0, \tag{5}$$

$$\dot{N} = \gamma M + \chi n - \epsilon(N), \quad N(0) = N_0, \tag{6}$$

$$\epsilon(N) = \begin{cases} \nu N & \text{if} \quad N < \bar{N} \\ \bar{N} & \text{if} \quad N \geq \bar{N}. \end{cases}$$

In order to solves that intertemporal optimization problem, the manager could combine the controls in several ways. As a consequence, beside the interior solution ($n > 0; 0 < k < 1$), they are a bunch of others possible management regimes. Naturally, some of potential regimes do not have any practical chance to be implemented, because they induce charges and no profits.

In order to solve the manager's problem, we now form the Lagrangean in his more general expression. From that Lagrangean, we will compute the first order conditions and will give economical interpretation. In a further step, we will seek if there exist stationaries states.

Controls	$n = 0$	$n > 0$
$k = 0$	Mining	Compensated Mining
$0 < k < 1$	Attenuated Mining	Complementarity
$k = 1$	Fallow	Improved Fallow

Table 1: Management regimes

1.2 Model's resolution

The Lagrangean:

$$L = a(1-k)f(N) - b\left[(1-k)f(N)\right]^2 - (\Phi_1 n + \tfrac{\Phi_2}{2}n^2) + \lambda\left[kf(N) - \gamma M\right] + \mu\left[\gamma M + \chi n - \epsilon(N)\right] +$$

$$+\lambda_{k=0}k + \lambda_{k=1}(1-k) + \lambda_{n=0}n,$$

where λ and μ are the adjoint variables corresponding to M and N respectively. $\lambda_{k=0}$, $\lambda_{k=1}$, $\lambda_{n=0}$ are the Lagrange multipliers corresponding to the constraints $k \geq 0$, $k \leq 1$ and $n \geq 0$ respectively:

$$\begin{aligned} \lambda_{k=0}k &= 0 & \lambda_{k=0} &\geqslant 0 \\ \lambda_{k=1}(1-k) &= 0 & \lambda_{k=1} &\geqslant 0 \\ \lambda_{n=0}n &= 0 & \lambda_{n=0} &\geqslant 0 \end{aligned}$$

First order conditions, when derivatives exists, are:

$$\frac{\partial L}{\partial k} = f(N)\left[-a + 2b(1-k)f(N) + \lambda\right] + \lambda_{k=0} - \lambda_{k=1} = 0 \tag{7}$$

$$\frac{\partial L}{\partial n} = -\Phi_1 - \Phi_2 n + \mu\chi + \lambda_{n=0} = 0 \tag{8}$$

$$\dot{\lambda} = \rho\lambda - \frac{\partial H}{\partial M} = (\rho + \gamma)\lambda - \gamma\mu \tag{9}$$

$$\dot{\mu} = \rho\mu - \frac{\partial H}{\partial N} = (\rho + \epsilon'(N))\mu - f'(N)\left[a(1-k) - 2b(1-k)^2 f(N) + k\lambda\right] \tag{10}$$

Remarque 1 *Note that when $0 < k < 1$ as $f(N) > 0$, equation (13) becomes*

$$-a + 2b(1-k)f(N) + \lambda = 0.$$

Replacing in (10) we have,

$$\dot{\mu} = (\rho+\epsilon'(N))\mu - f'(N)\left[(1-k)(a - 2b(1-k)f(N)) + k\lambda\right] = (\rho+\epsilon'(N))\mu + f'(N)\lambda.$$

1.3 Economic interpretation

As crops do not make a difference among nutrients coming from differents sources, the question arises to define the optimal combining of both controls. Its worth remarking that crop residues restitutions contributes with a certain timelag to provide nutrients. Naturally, the nutrients provided by the mineralization of SOM are not free, they have an opportunity cost represented by the lost value of biomass given back to soil.

The Lagrangean resolution in both controls n et k, gives the following first order conditions for optimality :

$$\frac{\partial L}{\partial n} = -\Phi_1 - \Phi_2 n + \mu\chi + \lambda_{(n=0)} = 0 \tag{11}$$

The multiplier (also called implicit price) associated to the corner solution ($n = 0$) will be positive when $n = 0$,and null nul otherwise.

If the manager chooses $n > 0$, the optimal value n^* should be :

$$n^* = \frac{\mu\chi - \Phi_1}{\Phi_2} \tag{12}$$

The use of fertilizers goes down to zero as their price goes close to the implicit price of the nutrients N. (We could remark that the implicit price is often called, in the resources economics, the user's cost). When the stock N, is important, the corresponding multiplier is low. As a consequence, it becomes more economical to prioritize the SOM as a source of nutrients, instead of fertilizers as soon as: $\mu\chi - \Phi_1 < 0$ and to use the control $n^* = 0$.

Now, solving the Lagrangean in k: gives the corresponding first order conditions:

$$\frac{\partial L}{\partial k} = f(N)\left[-a + 2b(1-k)f(N) + \lambda\right] + \lambda_{k=0} - \lambda_{k=1} = 0 \tag{13}$$

Assuming manager chooses the interior solution ($0 < k < 1$) we get:

$$\frac{\partial L}{\partial k} = f(N)\left[2bf(N)(1-k) + \lambda - a\right] = 0$$

This second condition states that the opportunity cost of the sold biomass depends on the gap between the price of the sold biomass and the implicit price of the SOM, M in our model. When M is high, his implicit price is low, and the opportunity cost of biomass restitutions becomes high.

Even restricted to interiors solutions, the above considerations could help in understanding the results we will get in looking for stationaries states in the following stage.

2 Parameters settings and sensitivity analysis of Steady States

We will perform the study of the SS taking onto account the trichotomy introduced by the mere existence of the plateau in the production function $N < \bar{N}$, $N > \bar{N}$,et $N = \bar{N}$. (see Annex B). In what follows we suppose $\frac{\nu}{\beta} < 1]$ because ν is lower than one and β is always greater than one.

For situations where $N < \bar{N}$, among the three management regimes with fertilizers use, only one (n>0, K=0) give rise to a possible stationary state, depending on the conditions on the parameters values. The same result occurs for regimes without fertilizers use (n=0). In that case, the only SS arises from the regime (n=0, k=0), resulting in the complete depletion of both stocks M and N, and a null production for ever. For $N > \bar{N}$,, there are no SS, whatever n or K are. The form of the production function led to pay a special attention to the case et $N = \bar{N}$. In the management regime without biomass restitution to soil, and fertilizers use, there is a potential SS depending on the parameters values. In that case, the stock of SOM is fully depleted. Management regimes with biomass restitution are also possible depending on parameters values. When they do not use fertilizers, we get a SS with $N = \bar{N}$. and $M = \bar{N}/\gamma$. The study of the conditions for existence of SS emphasizes the importance of the model parameters. In the next paragraph, we will describe and legitimize our choices. We will then perform a sensitivity analysis of the existence of SS according to the parameters values, focusing specifically on the cost of the mineral fertilization.

2.1 Parameters settings

The set of parameters used in the following simulations encompasses both economical and agro-ecological parameters.

$a, b, \phi_1, \phi_2, \gamma, \chi, \nu, \beta, \overline{N}, M_0, N_0$

a and b are set as paramaters for the price of the biomass harvested and marketed, so as to provide gross production, ϕ_1 and ϕ_2 stand for the cost of mineral fertilizers acquisition and spreading. All the four parameters are set so as to keep a reasonable relationship between them $(a > b, \varphi_1 > \varphi_2)$ whatever the absolute value of a and φ_1 are. a has been set to 1 for being a reference for others prices and cost.

Among agro-ecological parameters, γ stands for the rate of degradation of soil organic matter M. We know that the soil organic matter dynamic is a very complex one, involving several stocks with short and long turn-over. In a simplifying assumption, we take an average value of $0,2$ for γ. That correspond to one average turn-over of 5 years. Smaller values of γ correspond to situations where the Soil organic matter turn-over is longer, so as the SOM stock provide little nutrients to plants. On the opposite, greater value describe situations where the SOM' contribution to the plant nutrition is higher and faster. χ is a coefficient of transformation of mineral fertilizer added into nutrients for plants. It correspond to losses arising from the fact that nitrogen is easily soluble in water, and leaking frequently occur. We set χ to 0.7. β is a coefficient of transformation of nutrients' stock N into biomass, we retained the value of 30 (1 unit of nitrogen give 30 units of biomass).

$\bar{N}$ is a value tresshold beyond which the yield of N is constant $(f(N) = \bar{N})$. According to agronomic experience, and considering the value of others economical parameters, $0,166$ looks a reasonable value for $\bar{N}$ (we have performed simulations with greater values).

While the initial values of M and N could vary in a wide a range so as to reflect the diversity of soils' fertility, we choose to keep them in a constant ratio M/N mimicking the C/N ratio in average soils (15). We then performed several simulations in order to browse situations ranging from that of fertile soils, rich in organic matter, up to poor soils, with depleted stocks of organic matter and nutrients (From 10 to 20).

To summ up:

$$a = 1, b = 0.1, \chi = 0.7, \gamma = 0.2, \nu = 1, \rho = 0.01, \bar{N} = 0.166, \beta = 30, \phi_1 = 0.02, \phi_2 = 0.1.$$

2.2 Sensitivity analysis according to ϕ_1 or ϕ_2

$$a = 1, b = 0.1, \chi = 0.7, \gamma = 0.2, \nu = 1, \rho = 0.01, \bar{N} = 0.166, \beta = 30, \phi_2 = 0.1.$$

Recall that : ϕ_1 is a relative price of fertilizers while ϕ_2 is a cost of delivering fertilizers to fields.

2.2.1 $N_0 < \bar{N}$, ϕ_1

$n = 0$ **et** $k = 0$ is a SS up to $\phi_1 > 20.79207921$

2.2.2 $N_0 = \bar{N}, \quad \forall t$, ϕ_1

$0 < k < 1 n > 0$

- $M > 0, \quad \forall \phi_1$
- $k > 0, \quad \forall \phi_1$
- $k < 0, \quad \phi_1 < 1.422714286$
- $n > 0, \quad \phi_1 < 0.02734$ then true while $\phi_1 < 0.02734$

$n = 0$

- $k = \frac{\nu}{\beta}, \quad \forall \phi_1$
- $M = \frac{\nu \bar{N}}{\beta}, \quad \forall \phi_1$
- $N = \bar{N}, \quad \forall \phi_1$

Remarque 2 *for* $\phi_1 = 0.02$ *there are two SS and we leaved aside the case where* $n = 0$.

$$a = 1, b = 0.1, \chi = 0.7, \gamma = 0.2, \nu = 1, \rho = 0.01, \bar{N} = 0.166, \beta = 30, \phi_1 = 0.02.$$

2.2.3 $N_0 < \bar{N}$, ϕ_2

The regime $(0 < k < 1; n = 0)$ is an SS $\forall \phi_2$

2.2.4 $N_0 = \bar{N}, \quad \forall t$, ϕ_2

The regime ($0 < k < 1$; $n = 0$) is a SS because conditions are not dependent of ϕ_2.

3 Optimal time profiles

3.1 Simulations organization

In order to build complete time profile resulting from the choice of a acceptable set of parameters, we should match the previous stationary states with transient regimes applied to any couple of initial values $(M(0) = M_0, N(0) = N_0)$. Indeed, it is natural to classify initial values according to the typology used in Table 1. In order to set the initial values of the M stock according to real situations, we will parameter them as follow :

Vary N_0 and M_0 keeping them in a ratio M/N of 10 $0.0649 < N_0 < 0.105$ before and $k > 1$ after $k < 0$

Vary N_0 and M_0 keeping them in a ratio M/N of 20 $0.044 < N_0 < 0.053$ before $k > 1$ after $k < 0$

Because we use optimality conditions for both SS and transient regime, we get an optimal trajectory corresponding to each initial conditions. In order to match transient regime and SS, we make use of continuity conditions (ref SS). We will perform simulations for situations where the mineral fertilizers are respectively cheap and expansive.

$N < \bar{N}$ à la fin Starting from $N_0 = \bar{N}$ and $M_0 = \frac{\bar{N}}{\gamma}$, we found a matching solution and the SS with

($0 < k < 1; n > 0$) is ever better than the SS with ($0 < k < 1; n = 0$)

In all cases, for the terminal stage, we get the control ($0 < k < 1; n = 0$).

Matching with :

- $N_0 > \bar{N}$ with $k = 0$ et $n = 0$
- $N_0 < \bar{N}$ with $k = 0$ et $n = 0$
- $N_0 < \bar{N}$ with $0 < k < 1$ et $n > 0$

$N = \bar{N}$ at the end. At the end, the control used is : ($0 < k < 1$; $n > 0$).

Matching with :

- $N_0 > \bar{N}$ avec $k = 0$ et $n > 0$

3.2 Low price of fertilizers ($\phi_1 = 0.02$)

3.2.1 $N_0 < \bar{N}, 0 < k < 1, n = 0$ then $N_0 = \bar{N}, k > 0, n > 0$

Cf. figures 1, 2, 3, 4.

At the beginning, the restitutions to soil is high and close to 0.8. Then the restitutions decrease sharply, and the fertilizers are used as substitute. The fertilizers used stabilize around 0.036. A quantity equivalent to 0.216 of the value of $\bar{N}$. At that point, almost all the biomass produced is harvested and exported. The stock of SOM, M, increases at the beginning and reaches the value of 0.9, before being dropped to a lower value. In a long run, the SOM is kept at a very low level, and the ratio M/N stabilizes around 4.

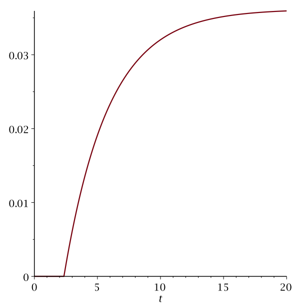

Figure 1: Cheap fertilizers $N_0 < \bar{N}$, $n^*(t)$.

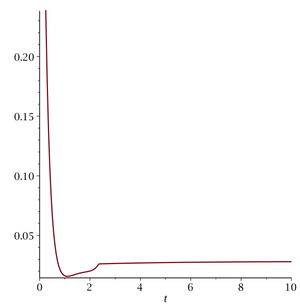

Figure 2: Cheap fertilizers $N_0 < \bar{N}$, $k^*(t)$.

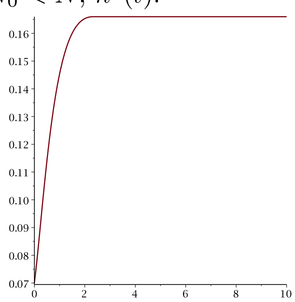

Figure 3: Cheap fertilizers $N_0 < \bar{N}$, $N^*(t)$.

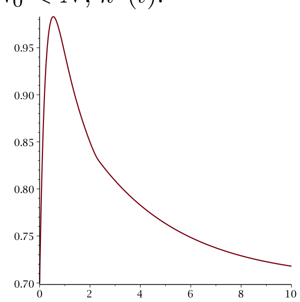

Figure 4: Cheap fertilizers $N_0 < \bar{N}$, $M^*(t)$.

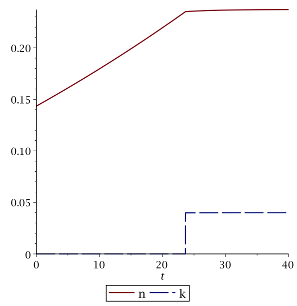

Figure 5: Cheap fertilizers $N_0 > \bar{N}$, $n^*(t)$ and $k^*(t)$.

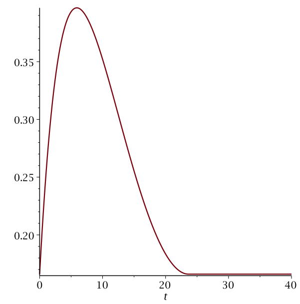

Figure 6: Cheap fertilizers $N_0 > \bar{N}$, $N^*(t)$.

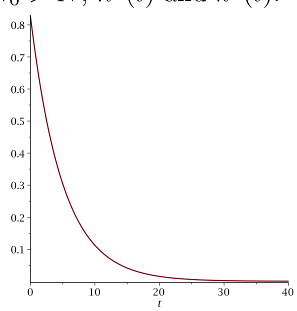

Figure 7: Cheap fertilizers $N_0 > \bar{N}$, $M^*(t)$.

3.2.2 $N_0 > \bar{N}, k = 0, n > 0$ then $N_0 = \bar{N}, k > 0, n > 0$

Cf. figures 5, 6, 7.

As N_0 increases, more time profiles of n and k go further on the right, and then, more the matching becomes later.

Varying N_0 and M_0 while keeping the M/N ratio at 10 $0.166 = \bar{N} < N_0 \leqslant 0.681$ before $\bar{N}$ after $n < 0$

Varying N_0 et M_0 while keeping the M/N ratio at 20 $0.166 = \bar{N} < N_0 \leqslant 0.356$ before $\bar{N}$ after $n < 0$

In the first stage, one uses the natural fertility of soil, without giving back anything. This is a mining strategy. As soon as SOM, the initial fertility, is depleted, and when the nutrients contents N reaches the plateau's value, $\bar{N}$, the manager stabilizes the fertility with low biomass restitutions and an quite important addition of fertilizers (0.25) greater than the plateau's value (0.166). In the long term, the SOM is depleted, and the ratio of M/N is close to 0. Varying the initial conditions of fertility, we get different optimal time profile, and use different controls along them.

3.3 High price of fertilizers ($\phi_1 = 0.2$)

$N_0 > \bar{N}, (k = 0, n = 0)$ then $N = \bar{N}, (0 < k < 1; n > 0$- then $N = \bar{N}, (0 < k < 1, n = 0)$ $N(0) = 0.2, M(0) = 2.5$ Matching is possible in three steps.

Cf. figures 8, 9, 10, 11.

In that case, we already demonstrated that the only SS remaining is the so-called "double turnpike". $N = \bar{N}$ et $M = \frac{\bar{N}}{\gamma}$. The mere existence of that turnpike makes the search of transitory solution more complex. It will be impossible to match initial and terminal conditions in two steps only. The price of fertilizers ϕ_1 don't affect that SS because $n = 0$, but it could impact the transitory regimes, using fertilizers before reaching the turnpike. At the turnpike, controls are ($n = 0$;$k = \frac{1}{\beta}$). The intuition for the matching procedure is as follow: because the stock M dynamics is under the control of k (degradation rate being constant as an ecological parameter), if we keep k at zero, we will drive M to zero. As soon as we will reach the value $M = \frac{\bar{N}}{\gamma}$, we will maintain that level by combining $k > 0$ et $n > 0$.

- $N_0 > \bar{N}$
  - 1^{st} step : drop up to $\bar{N}$ using ($k = 0$; $n = 0$)
  - 2^{nd} step : From that point onwards, use ($(0 < k < 1$; $n > 0)$ to reach $\bar{M}$ while holding $\bar{N}$
  - 3^{rd} Hold the double turnpike with the corresponding control ($n = 0$; $k = \frac{1}{\beta}$)

  As a matter of fact, we proceeded backward, from $\bar{N}$ et $\bar{M}$, then only keeping $\bar{N}$ before the last step.

- $N_0 < \bar{N}$ In that case, we have not found optimal time profile matching terminal SS and initial conditions in three steps, and we stopped here.

In the context of our model, it appears to be of good economic rationality to deplete the natural soil's fertility, in the same way it is rational to exhaust a non renewable natural resource. SOM appears in our model exactly as a cheap fertilizer. The price of fertilizer does impact the time profile of their use. When they are high, they are used only temporary in the median stage.

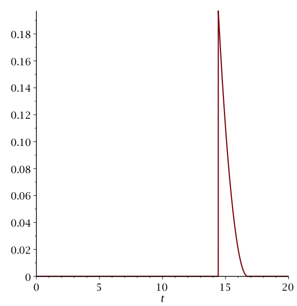

Figure 8: Expensive fertilizers, $N_0 > \bar{N}$, $n^*(t)$.

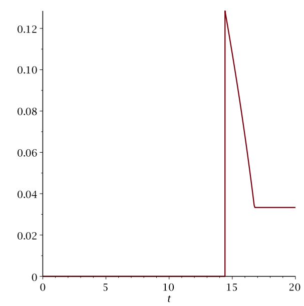

Figure 9: Expensive fertilizers, $N_0 > \bar{N}$, $k^*(t)$.

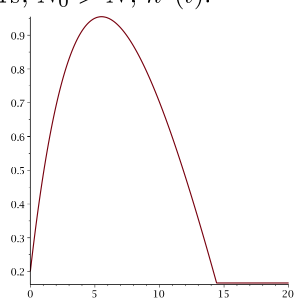

Figure 10: Expensive fertilizers, $N_0 > \bar{N}$, $N^*(t)$.

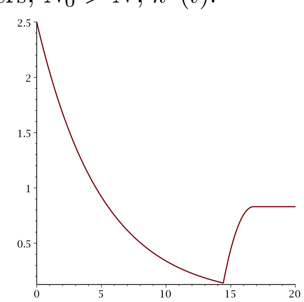

Figure 11: Expensive fertilizers, $N_0 > \bar{N}$, $M^*(t)$.

Conclusions and perspectives

In this paper, we have explored the potential of optimal control models to analyse intertemporal trade-off involved in the management of soil's ecosystem by farmers. To do that, we choose to rely on new approaches proposed by soil's scientists, placing the SOM at the core of our model. At this first stage of a research program on soil natural capital, we adopted the viewpoint of the mineralist theory of the plants nutrition: SOM does not plays another role than provisionning the nutrients stock. In the model, manager has two controls to drive the flow of profits over an infinite horizon. She could directly contributes to the nutrients stock by adding fertilizers, or indirectly by contributing to the stock of SOM, by giving back to the soil a part of the produced biomass. While fertilization has a direct cost, the mainenance of the SOM trough crop residues restitutions has an opportunity cost only. The production function is coherent with the Liebig's theory of limiting factors. The production reach a plateau as long as the level of the nutrients stock N is greater than a thresshold $\overline{N}$. We first solved the manager's problem using the Lagrangean method for interior solution $(n > 0; 0 < k < 1)$ and give economic interpretation useful for it. Nevertheless, because there are two controls with a range of values, we have examined the full range of theoretically possible management régimes. We then search for existence of stationaries states among these regimes. Moreover, we performed our search for all the possible values of N compared to the thresshold value $\bar{N}$.(corresponding to the plateau of the biomass production function). Among the 18 theoretically cases, 12 could never drive to SS. For values of the stock of nutrients above the thresshold $\bar{N}$, SS are never possible (that sounds like easy to understand). In all the cases where the biomass is entireley given back to soil there is no profit, and the regime has no economic viability: corresponding solutions could not be stationnaries and terminals. Among the six remaining régimes, stationaries states could be found, but depend on the set of parameters. For that reason, we carefully choose the model's parameters, relying to do that on agronomical and soils sciences. Eventually, our actual choices of parameters exclude all the SS with $N < \bar{N}$. At the end, the only remaining case able to give rise to SS is $N = \bar{N}$. The lessons are that, for a given parameters set, the existence of SS depends on the functional form of the production function we have adopted. While our choice is coherent with agronomical knowledge and easily tractable in the context of intertemporal optimisation, one could ask wether anoter choice would change the essence of our conclusion. Further research would be needed to answer the question.

Naturally, it's not satisfactory to stop investigations just after completing the study of SS. In the real world, and for policy purposes, it could be necessary to take onto account the variety of time path profile starting from different situation of soil's fertility and facing different economic situations. To look at optimal time profiles, we implemented simulations for situations with a high and low price of fertilizers (as compared to the price of agricultural output). We observed a path dependancy, because the terminal SS are different according to the initial situations. The fertilizers price impact the form of the optimal time profile of the control: when the prices of fertilizers are high, they are used temporalely and in the median stage of the horizon.

Beacuse the paper is just a first stage of a research program on soil's natural capital, we focused our attention on the case of the private management, and, in our representation of the soil functionning, we rely on the mineralist theory stricto sensu. Moreover, we adopted an utilitarian perspective in computing the value of soil natural capital. The conclusion follows: in long term, the SOM vanish, and, naturally, all the bulk of others ecosystem services. For that reason, in a further stage of research, we will investigate a quite different model, where the SOM would play a direct role in the production of biomass. Both stocks being complementaries, one could expect differents results about SS.

As long as the whole society is concerned by the sustainable management of SNC, long term management models such one we have developped in this paper are useful to measure the distance between private and social optimal time profiles. Because the prices on the land market are directed by a lots of factors, they do not reflect the social value of the soils uses. The inclusion of values related to the bunch of soil's ecosystem services in suitable models of SNC could help in designing policies aimed at a sustainable management of that vital natural capital.

Appendices

A Steady states study

We will perform the study of the SS taking onto account the trichotomy introduced by the mere existence of the plateau in the production function $N < \bar{N}$, $N > \bar{N}$,et $N = \bar{N}$. In what follows we suppose $\frac{\nu}{\beta} < 1$ because ν is lower than one and β is always greater than one.

Remarque 3 *Note that we can prove easily that when $0 < k < 1$ (see remark 1) or when $N > \bar{N}$ (because $f'(N) = 0$), $\dot{\lambda} = \dot{\mu} = 0$ implies $\lambda = \mu = 0$. Moreover, $\mu = 0$ implies that $n > 0$ is not possible at the SS because, in this case $n > 0$ is in contradiction with equation (8).*

A.1 $N > \bar{N}$

There do not exist SS when $N > \bar{N}$.
In fact by remark 3 the solution of first order conditions gives: $\lambda = \mu = n = 0, \lambda_{n=0} = \phi_1$. From (5) and (6) we obtain: $M = \frac{\bar{N}}{\gamma}, k = \frac{1}{\beta}$. But $k = \frac{1}{\beta}$ is not a solution of (13).

A.2 $N < \bar{N}$

$0 < k < 1$

- There do not exist SS with $0 < k < 1$ and $n > 0$. See remark 3.
- $0 < k < 1$ and $n = 0$ If $\frac{1}{2}\frac{a}{b(\beta-\nu)} < \bar{N}$, the SS verify:

$$k = \frac{\nu}{\beta}, \quad M = \frac{1}{2}\frac{\nu a}{b(\beta-\nu)\gamma}, \quad N = \frac{1}{2}\frac{a}{b(\beta-\nu)}$$

$k = 0$

- $n > 0$. If

$$\chi\beta a - \phi_1\rho - \phi_1\nu > 0, \quad -\beta\gamma + \nu\rho + \nu\gamma + \rho\gamma + \rho^2 > 0, \quad \frac{\chi(\chi\beta a - \phi_1\rho - \phi_1\nu)}{\phi_2\nu\rho + \phi_2\nu^2 + 2b\beta^2\chi^2} < \bar{N}$$

then the SS is:

$$n = \frac{\nu(\chi\beta a - \phi_1\rho - \phi_1\nu)}{\phi_2\nu\rho + \phi_2\nu^2 + 2b\beta^2\chi^2}, \quad N = \frac{\chi(\chi\beta a - \phi_1\rho - \phi_1\nu)}{\phi_2\nu\rho + \phi_2\nu^2 + 2b\beta^2\chi^2}, \quad M = 0, \quad \mu > 0, \lambda > 0.$$

$$\lambda_{k=0} = \frac{\chi\beta(\chi\beta a - \phi_1\rho - \phi_1\nu)(-\beta\gamma + \nu\rho + \nu\gamma + \rho\gamma + \rho^2)(\phi_2\nu a + 2b\beta\chi\phi_1)}{(\rho+\gamma)(\phi_2\nu\rho + \phi_2\nu^2 + 2b\beta^2\chi^2)^2}$$

- $n = 0$. If $\varphi_1 - \frac{a\beta\chi}{\rho+\nu} \geq 0$ then

$$M = N = 0, \quad \mu = \frac{a\beta}{\rho+\nu} > 0, \quad \lambda = \frac{a\beta\gamma}{\nu\gamma + \rho^2 + \rho\gamma + \nu\rho} > 0, \quad \lambda_{n=0} = \varphi_1 - \frac{a\beta\chi}{\rho+\gamma}, \quad \lambda_{k=0} = 0.$$

$k = 1$

- $n > 0$. It is not possible because:

$$\lambda = 0,\ M = \frac{-\chi\beta\phi_1}{\gamma\phi_2(-\beta+\nu)},\ N = \frac{-\chi\phi_1}{\phi_2(-\beta+\nu)},\ \mu = 0,\ n = \frac{-\phi_1}{\phi_2}.$$

- $n = 0$ gives

$$\lambda = \mu = M = N = \lambda_{k=1} = 0, \quad \lambda_{n=0} = \phi_1.$$

A.3 $N = \bar{N}$

The question in that case is now: it is possible to stay for ever in $N = \bar{N}$?
In this case we suppose $N = \bar{N}$ and $\dot{N} = 0$ for all t. $\dot{N} = 0$ implies that

$$n = \frac{\bar{N} - \gamma M}{\chi}.$$

The problem is:

$$\max_{k\in[0,1]} \int_0^\infty e^{-\rho t}\left[a(1-k)\beta\bar{N} - b\left[(1-k)\beta\bar{N}\right]^2 - \phi_1\frac{\bar{N}-\gamma M}{\chi} - \frac{\phi_2}{2}\left(\frac{\bar{N}-\gamma M}{\chi}\right)^2\right]dt \tag{14}$$

such that

$$\dot{M} = kf(N) - \gamma M, \quad M(0) = M_0.$$

The Lagrangian is:

$$H_N = a(1-k)\beta\bar{N} - b\left[(1-k)\beta\bar{N}\right]^2 - \phi_1\frac{\bar{N}-\gamma M}{\chi} - \frac{\phi_2}{2}\left(\frac{\bar{N}-\gamma M}{\chi}\right)^2 + \lambda(k\beta\bar{N} - \gamma M) + \lambda_{k=0}k + \lambda_{k=1}(1-k).$$

and first order conditions are:

$$-a\beta\bar{N} + 2b(\beta\bar{N})^2(1-k) + \lambda\beta\bar{N} + \lambda_{k=0} - \lambda_{k=1},$$

$$\dot{\lambda} = (\rho+\gamma)\lambda - \frac{\gamma}{\chi}\left(\phi_1 + \phi_2\frac{\bar{N}-\gamma M}{\chi}\right).$$

$0 < k < 1$

$0 < k < 1$ gives:

$$k = \frac{\chi^2(\rho+\gamma)(2\beta\bar{N}b - a) - \gamma(\phi_1\chi + \phi_2\bar{N})}{\beta\bar{N}(2\chi^2 b(\rho+\gamma) + \phi_2\gamma)}, \quad M = \frac{k\beta\bar{N}}{\gamma}, \quad n = \frac{\bar{N}(\gamma - k\beta)}{\gamma\chi}.$$

Remember conditions (I not write them) in order to have $0 < k < 1$ and $n > 0$.

$k = 0$

$k = 0$ gives in particular

$$M = 0, \quad \lambda_{k=0} = \frac{\beta\bar{N}\left[\chi^2(\rho+\gamma)(-2\beta\bar{N}b + a) - \gamma(\phi_1\chi + \phi_2\bar{N})\right]}{\chi^2(\rho+\gamma)}.$$

In this case $\lambda_{k=0}$ must be greater than zero according to the parameter's value.

$k = 1$

$k = 1$ gives

$$M = \frac{\beta\bar{N}}{\gamma}, \quad \lambda_{k=1} = -\frac{\beta\bar{N}\left[a\chi^2(\rho+\gamma) + \phi_2\beta\gamma\bar{N} - \gamma(\phi_1\chi + \phi_2\bar{N})\right]}{\chi^2(\rho+\gamma)}.$$

In this case $\lambda_{k=1}$ must be greater than zero.

$n = 0$

The case $n = 0$ is not in the analysis given above.

$$k = \frac{1}{\beta}, \quad N = \bar{N}, \quad M = \frac{\bar{N}}{\gamma}$$

$n = 0$ with $k = 0$ or $k = 1$ are not possible.

B Steady states summary

n>0, k=0 $\lambda_{k=0} > 0$ $\lambda_{k=0} = \frac{A.B.C.\beta\chi}{(\rho+\gamma)(\phi_2\nu\rho+\phi_2\nu^2+2b\beta^2\chi^2)^2}$ $n = \frac{\nu C}{\phi_2\nu\rho+\phi_2\nu^2+2b\beta^2\chi^2}$ possible if $\chi\beta a - \phi_1(\rho+\nu) > 0$ et $-\beta\gamma+\nu\rho+\nu\gamma+\rho^2+\rho\gamma > 0$	**n>0, 0<k<1** impossible $n = -\frac{\phi_1}{\phi_2}$	**n>0, k=1** impossible $n = -\frac{\phi_1}{\phi_2}$	$f(N) = \beta N$ $N < \bar{N}$
n=0, k=0 $M = 0$ $N = 0$ $\lambda_{k=0} = 0$ $\mu = \frac{a\beta}{\rho+\nu}$ $\lambda = \frac{a\beta\gamma}{\nu\gamma+\rho^2+\nu\rho+\rho\gamma}$ possible if $\lambda_{n=0} = \phi_1 - \mu\ \chi$ also $\phi_1 - \frac{a\beta\chi}{\rho+\nu} > 0$	**n=0, 0<k<1** $\lambda = 0$ $\mu = 0$ $\lambda_{n=0} = \phi_1$ $M = \frac{1}{2}\frac{\nu a}{b(\beta-\nu)\gamma}$ $N = \frac{1}{2}\frac{a}{b(\beta-\nu)}$ possible if $N = \frac{1}{2}\frac{a}{b(\beta-\nu)} < \bar{N}$	**n=0, k=1** $M = 0$ $N = 0$ $\lambda = 0$ $\mu = 0$ $\lambda_{k=1} = 0$ $\lambda_{n=0} = \phi_1$ incredible because benefit=0	
n>0, k=0 impossible $n = -\frac{\phi_1}{\phi_2}$	**n>0, 0<k<1** impossible $n = -\frac{\phi_1}{\phi_2}$	**n>0, k=1** impossible $n = -\frac{\phi_1}{\phi_2}$	$f(N) = \beta\ \bar{N}$ $N > \bar{N}$
n=0, k=0 impossible $N = 0$ or $N \geqslant \bar{N}$	**n=0, 0<k<1** $k = \frac{1}{\beta}$ impossible k is not a solution of $\frac{\delta L}{\delta k}$	**n=0, k=1** impossible $\lambda_{k=1} = -a\beta\ \bar{N}$	
n>0, k=0 $M = 0$ $\lambda_{k=0} = \frac{\beta\bar{N}\left[\chi^2(\rho+\gamma)(-2\beta\bar{N}b+a)-\gamma(\phi_1\chi+\phi_2\bar{N})\right]}{\chi^2(\rho+\gamma)}$ possible if $\chi^2(\rho+\gamma)(-2\beta\bar{N}b+a) - \gamma(\phi_1\chi+\phi_2\bar{N}) > 0$	**n>0, 0<k<1** $M = \frac{k\beta\bar{N}}{\gamma}$ $n = \frac{\bar{N}(\gamma-k\beta)}{\gamma\chi}$ $k = \frac{\chi^2(\rho+\gamma)(2\beta\bar{N}b-a)-\gamma(\phi_1\chi+\phi_2\bar{N})}{\beta\bar{N}(2\chi^2b(\rho+\gamma)+\phi_2\gamma)}$ possible if $\frac{\bar{N}(\gamma-k\beta)}{\gamma\chi} > 0$ et $k > 0$ et $k < 1$	**n>0, k=1** $M = \frac{\beta\bar{N}}{\gamma}$ $\lambda_{k=1} = -\frac{\beta\bar{N}\left[a\chi^2(\rho+\gamma)+\phi_2\beta\gamma\bar{N}-\gamma(\phi_1\chi+\phi_2\bar{N})\right]}{\chi^2(\rho+\gamma)}$ possible if $-\beta\bar{N}\left[a\chi^2(\rho+\gamma)+\phi_2\beta\gamma\bar{N}-\gamma(\phi_1\chi+\phi_2\bar{N})\right] > 0$	$f(N) = \beta\ \bar{N}$ $N = \bar{N}$
n=0, k=0 incompatible $k = \frac{1}{\beta}$	**n=0, 0<k<1** $k = \frac{1}{\beta}$ $N = \bar{N}$ $M = \frac{\bar{N}}{\gamma}$	**n=0, k=1** incompatible $k = \frac{1}{\beta}$	

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