Stabilizing the cake evolution for a class of submerged membrane bioreactors using a Lyapunov controller

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1 Introduction

The use of membrane bioreactor for wastewater treatment is a relativity new technology [3], that process usually runs in open-loop with empirical set-points. This work implements a Lyapunov controller to stabilize the evolution of the cake fouling and shows that the empirical methods implemented in the industry can be mathematically explained[1]. In control theory, a control-Lyapunov function $V(x;u)$, where x is the state and u the input, is a generalization of the notion of Lyapunov function $V(x)$ used in stability analysis[1].

2 Stabilizing cake fouling

Considering the nonlinear model of a submerged membrane bioreactor, which is based on mass balance equations and is described by [2]:

$$\begin{cases} \frac{d\beta}{dt} = -\gamma\beta \\ \frac{dS}{dt} = -\frac{1}{Y}\mu_S(S)X + \frac{Q_{in}}{V}(S_{in}-S) \\ \frac{dX}{dt} = \mu(S)X - \left(\frac{Q_w}{V} + \frac{Q_{cake}}{V}\right)X + \beta\frac{J_{air}}{V}\mu_m(m)m \\ \frac{dm}{dt} = Q_{cake}X - \beta J_{air}\mu_m(m)m \end{cases} \quad (1)$$

where β, S, X and m represent the cake evolution, substrate, biomass and cake attachment. Regarding process optimization the cake attachment should be controlled by permeate flow (Q_{cake}) and membrane crossflow (J_{air}), considered as process inputs. Given the state m and set point m^* with error $(m-m^*)$, a control-Lyapunov function is described as $V = \frac{1}{2}(m-m^*)^2$ which is positive definite for all $m \neq 0$. Computing the time derivative $\dot{V} = (m-m^*)(\dot{m}-\dot{m}^*)$ and adding a λ factor for convergence speediness and considering both inputs, Q_{cake} as input 1 (u_1) and J_{air} as input 2 (u_2) the following control law is computed.

$$u_1 = \frac{-\lambda(m-m^*)+\beta J_{air}\mu_m m}{X}, \quad u_2 = \frac{\lambda(m-m^*)+Q_{cake}X}{\beta\mu_m m} \quad (2)$$

Considering that m^* is larger than m, u_1 become negative value, which is physically impossible. On the other sense, supposing m^* is smaller than m, u_2 could become negative. A interesting way to compute these values is to considered a set of equation that works in each case and adding an α factor that cancels the some nonlinearities on the process, for simplification purposes. Dealing with this problem, the following control law (3) should compute Q_{cake} and J_{air} in relation with an α parameter. Thus, the inputs are linked to the biomass concentration and the cake dynamic.

$$\begin{cases} m \geq m^* & \bar{u}_1 = \frac{\alpha}{X} & u_2 = \frac{\lambda(m-m^*)+\bar{u}_1 X}{\beta\mu_m(m)m} \\ m \leq m^* & \bar{u}_2 = \frac{\alpha}{\beta\mu_m m} & u_1 = \frac{-\lambda(m-m^*)+\beta\bar{u}_2\mu_m m}{X} \end{cases} \quad (3)$$

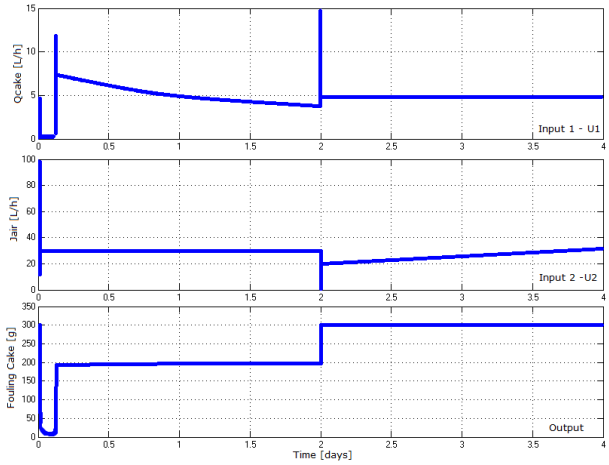

Figure 1: A step from 200g to 300g of m is implemented, considering $\alpha = 1000$.

The switch between Q_{cake} and J_{air} are empirically known in industrial practice. This study offers a way to understand and compute these values.

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