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# Stabilizing the cake evolution for a class of submerged membrane bioreactors using a Lyapunov controller

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## 1 Introduction

The use of membrane bioreactor for wastewater treatment is a relatively new technology [3], that process usually runs in open-loop with empirical set-points. This work implements a Lyapunov controller to stabilize the evolution of the cake fouling and shows that the empirical methods implemented in the industry can be mathematically explained<sup>1</sup>. In control theory, a control-Lyapunov function  $V(x; u)$ , where  $x$  is the state and  $u$  the input, is a generalization of the notion of Lyapunov function  $V(x)$  used in stability analysis[1].

## 2 Stabilizing cake fouling

Considering the nonlinear model of a submerged membrane bioreactor, which is based on mass balance equations and is described by [2]:

$$\begin{cases} \frac{d\beta}{dt} = -\gamma\beta \\ \frac{dS}{dt} = -\frac{1}{Y}\mu_S(S)X + \frac{Q_{in}}{V}(S_{in} - S) \\ \frac{dX}{dt} = \mu(S)X - \left(\frac{Q_w}{V} + \frac{Q_{cake}}{V}\right)X + \beta J_{air}\mu_m(m)m \\ \frac{dm}{dt} = Q_{cake}X - \beta J_{air}\mu_m(m)m \end{cases} \quad (1)$$

where  $\beta$ ,  $S$ ,  $X$  and  $m$  represent the cake evolution, substrate, biomass and cake attachment. Regarding process optimization the cake attachment should be controlled by permeate flow ( $Q_{cake}$ ) and membrane crossflow ( $J_{air}$ ), considered as process inputs. Given the state  $m$  and set point  $m^*$  with error  $(m - m^*)$ , a control-Lyapunov function is described as  $V = \frac{1}{2}(m - m^*)^2$  which is positive definite for all  $m \neq 0$ . Computing the time derivative  $\dot{V} = (m - m^*)(\dot{m} - \dot{m}^*)$  and adding a  $\lambda$  factor for convergence speediness and considering both inputs,  $Q_{cake}$  as input 1 ( $u_1$ ) and  $J_{air}$  as input 2 ( $u_2$ ) the following control law is computed.

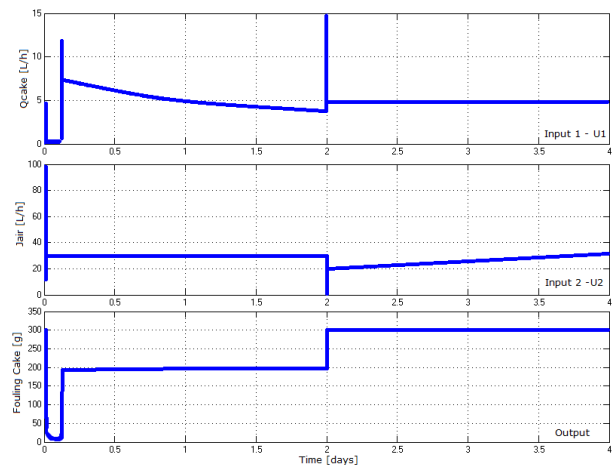
$$u_1 = \frac{-\lambda(m - m^*) + \beta J_{air}\mu_m m}{X}, \quad u_2 = \frac{\lambda(m - m^*) + Q_{cake}X}{\beta\mu_m m} \quad (2)$$

Considering that  $m^*$  is larger than  $m$ ,  $u_1$  become negative value, which is physically impossible. On the other sense,

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supposing  $m^*$  is smaller than  $m$ ,  $u_2$  could become negative. A interesting way to compute these values is to considered a set of equation that works in each case and adding an  $\alpha$  factor that cancels the some nonlinearities on the process, for simplification purposes. Dealing with this problem, the following control law (3) should compute  $Q_{cake}$  and  $J_{air}$  in relation with an  $\alpha$  parameter. Thus, the inputs are linked to the biomass concentration and the cake dynamic.

$$\begin{cases} m \geq m^* & \bar{u}_1 = \frac{\alpha}{X} & u_2 = \frac{\lambda(m - m^*) + \bar{u}_1 X}{\beta\mu_m(m)m} \\ m \leq m^* & \bar{u}_2 = \frac{\alpha}{\beta\mu_m m} & u_1 = \frac{-\lambda(m - m^*) + \beta\bar{u}_2\mu_m m}{X} \end{cases} \quad (3)$$



**Figure 1:** A step from 200g to 300g of  $m$  is implemented, considering  $\alpha = 1000$ .

The switch between  $Q_{cake}$  and  $J_{air}$  are empirically known in industrial practice. This study offers a way to understand and compute these values.

## References

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