Estimation in functional convolution model

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Abstract. *Currently large amounts of longitudinal data are acquired in biological experimentation. Exploiting these large datasets is challenging to produce new knowledge. The objective of this work is to study the relationship between a functional covariate* $X(t)$ *and a functional output* $Y(t)$ *through the convolution model* $Y(t) = \int_0^t \theta(s)X(t-s)ds + \varepsilon(t)$*. This model is derived from the historical functional linear model introduced in Malfait and Ramsay (2003). It allows to study the influence of the history of* X *on* $Y(t)$*, where* $\varepsilon(t)$ *is the noise. The objective is to estimate the unknown function* θ*, with a procedure which is rapid to calculate and adapted to the regularity of data. This model is promising to deal with daily curves of leaf elongation rates of plants, based on environmental variables.*
We use the Fourier Transform to estimate the unknown operator of this functional regression model. The Fourier Transform of the convolution model results in the well-known functional concurrent model $\mathcal{Y}(t) = \beta(t)\mathcal{X}(t) + \epsilon(t)$*. As noticed in Ramsay and Silvermann (2005), many functional linear models can be reduced to this form. In order to estimate the unknown function* β*, we extended the ridge regression method to this functional data framework. The estimator is a follows :* $\beta_n = \frac{\sum_{i=1}^n \mathcal{Y}_i\mathcal{X}_i}{\sum_{i=1}^n |\mathcal{X}_i|^2 + \lambda_n}$*, where* $(\mathcal{X}_i, \mathcal{Y}_i)$ *is the Fourier Transform of an i.i.d sample* (X_i, Y_i) *and* $\lambda_n > 0$*. We established good asymptotic statistical properties for this estimator, that allows good properties for the estimation of the unknown function* θ *in the initial convolution functional model.*
Both estimators showed also their high accuracy in simulation in fitting the unknown functions, despite a low signal-to-noise ratio. Data are observed on a grid of discrete points. The associated Fast Fourier Algorithm allows high speed computing.

Keywords. *Functional data; Convolution model; Concurrent model; Fourier transform.*

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