Risk preferences and optimal management of uneven-aged forests in the presence of climate change: a Markov decision process approach

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December 18, 2013

Preliminary version. Do not cite.

Abstract

Climatic changes will affect the occurrence probability of extreme windstorms. Consequently, management of uneven-aged forests can only be optimized correctly if changes in climatic conditions are considered. This article determines the optimal management regime of uneven-aged forests in the presence of climatic changes affecting windstorm occurrence probabilities, and takes into account the risk preferences of the forest owner. This study analyzes optimal harvesting of uneven-aged stands by applying a Markov Decision Processes (MDP) framework and an economic description of uneven-aged forestry. Two management types are considered: the exact uneven-aged forest management model in which the forest owner jointly manages all the different stand plots, and the independent uneven-aged forest management model that assumes that the forest owner separately and independently manages each plot of the forest. The MDP framework is applied to a non-industrial private forest owner located in North-East of France. We show that the forest owner tends to converge toward a forest structure which is very close to a normal forest. We also find that the independent model can be seen as a poor approximation of the exact model because of strong incidence in terms of optimal harvesting policy. We also show that the optimal decisions depend on risk preferences but not on the considered levels of windstorm probability possibly resulting from climate change.

Keywords: Forest management, uneven-aged forestry, climate change, optimal harvesting, single species stand, MDP.

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1 Introduction

Climate change is one of the most important challenges of the 21st century. The effects of climate change and climate variability on forest ecosystems are obvious and the impacts are unavoidable, requiring changes to forest management plans and practices (FAO, [11]). Adaptation is one of the two main responses to climate change (mitigation is the other one), and addresses the impacts of climate change. In the forest sector, adaptation encompasses changes in management practices designed to decrease the vulnerability of forests to climate change.

Climate change is among the factors that greatly complicate the economically rational management of forest resources. Among catastrophic events that affect European forests, windstorms are a major disturbance (Hanewinkel et al., [16]). Evidence suggests that climate change will increase the incidence and damages of windstorms. Therefore it is important to study to what extend forest management can be adapted to respond to the increase in windstorm risk as a result of changing climatic conditions. Forest management decisions may help to reduce the impact of windstorm risk. Consequently, over the last decades forest management under windstorms has received considerable attention in the literature.

The inclusion of risk and uncertainty in the economic analysis of forest management has a long tradition. There exist different theoretical methods for analysing the problem of optimal forest management under risk. Many works are based on the traditional Faustmann's model (Reed, [38]) but other frameworks such as dynamic programming, option approaches, etc. seem more adapted to the problem mentioned. More precisely, several approaches have been proposed to deal with the impact of catastrophic events on production and/or price: discrete time stochastic dynamic programing and Markov Decision Processes (MDP) (Lembersky and Johnson [24]), two-period models (Koskela and Ollikainen [23]), simulation models (Carmel *et al.* [7]), option theory models (Plantinga [35]), etc. Lots of papers deal with the impact of storm risk on forest management. Similarly, some surveys on this topic exist relating the different kinds of models and hypothesis (Pasalodos-Tato *et al.* [32]). Yousefpour *et al.* [50] provide a review of approaches of state-of-the-art methods for optimal decision making under risk and uncertainty in forestry. In the same vein, Hanewinkel *et al.* [16] present recent approaches to model the risk of storm to forests and their integration into simulation and decision support tools.

As forest management can be modeled as a controlled Markov process (Lohmander, [26]), we focus on the application of the MDP model for defining the optimal management of uneven-aged forests in the presence of climate change. MDP models offer a general means of reducing large and complex stochastic systems to elegantly simple probability matrices (Holling *et al.* [17]). The first application of MDP models to forestry[1] was proposed by Hool [18], and the first practical application was carried out by Lembersky and Johnson [24], who studied the optimum management of a stand with growth and price risks. Similar studies have followed, e.g., Kao [21], Teeter and Caulfield [47] and Pastor *et al.* [33]. Other studies have extended this approach to include ecological criteria (Lin and Buongiorno [25]), uneven-aged forest management (Kaya and Buongiorno [22]) or both (Rollin *et al.* [39]), as well as stochastic interest rates (Buongiorno and Zhou [6]; Zhou and Buongiorno [51]). However, the literature concerning the MDP model considers risk-neutral forest owners, and often assumes a single stand.

The impact of risk aversion on the forest owner's behavior has been demonstrated in different models (Ollikainen [31]; Gong [13]; Lönnstedt and Svensson [27]; Gong and Löfgren [14]; Alvarez and Koskela [2]; Andersson [3]). For example, in a Wicksellian single rotation framework, Alvarez and Koskela [2] indicate that higher risk aversion decreases the optimal harvesting threshold and thus shortens the expected rotation period. In the same way, in an expected utility maximization

[1] Williams [49] presents a general review of MDP applications in natural resources management.

model, Gong [13] shows that a risk-averse forest owner should harvest earlier than a risk-neutral one. In a multi age-class model, Tahvonen and Kallio [46] (in a more general model including also forest owners' consumption-saving decision making), Salo and Tahvonen [41] show that under strictly concave utility, it is optimal to adjust the age-class structure close to a normal forest distribution.

Several approaches have been proposed in the economic literature on forest management to address multiple tree age-class problems. The seminal paper on optimal uneven-aged forestry is by Adams and Ek [1]. They present a transition matrix model and dynamic optimization but apply several simplifications. Berck [5] or Johansson and Löfgren [20] have proposed a linear multiple age-class forest model while Lyon and Sedjo [28], Sedjo and Lyon [44] have developed a numerical method to solve an optimal control timber supply model based on a steady-state normal forest. Mitra and Wan [30] or Salo and Tahvonen [40] have proposed an analytical price support method and a dynamic programming model to address this multiple tree age-class issue. Salo and Tahvonen [41, 42] and Tahvonen [45] have considered Lagrangian function approaches. The main result of these studies is that, depending upon the characteristics of the model (discount factor level, utility form, initial age-class allocation, possibility of alternative land use, etc), the optimal long-run forest structure may either converge toward a normal forest age-class structure or toward an equilibrium with unevenly allocated tree age-class structures. All these articles consider a purely deterministic context. More recently, some authors (Tahvonen and Kallio [46]; Couture and Reynaud [8]; Goetz et al. [12]) have integrated stochastic factors into the mutliple tree age-class problem. As in the deterministic context, one of the main questions studied by these works is the convergence of the optimal forest age-class structure toward the normal forest distribution where the land area is evenly allocated over the existing age-classes. It is generally found that it is optimal to smooth the age-class composition. Moreover, including risk aversion changes the harvesting policy in the sense that, if the forest initially consists of just one age-class, it is always optimal to smooth the age-class structure and have more frequent cuttings from younger age-classes (Tahvonen and Kallio, [46]). To our knowledge no MDP model has analyzed the problem of optimal multiple tree age-class forest management including forest owners' risk aversion.

In this paper, we analyze optimal forest harvesting including multiple age-classes, and forest owners' risk aversion. The problem of the forest owner is to manage a forest composed of different stand plots with different age-classes in the presence of windstorm risk. We adopt a MDP framework to model this problem. We consider two management types: the exact uneven-aged forest management model in which the forest owner jointly manages all the different stand plots, and the independent uneven-aged forest management model that assumes the forest owner separately and independently manages each plot of the forest. It is then possible to highlight the importance of managing the forest jointly. This framework generalizes existing MDP studies that assume a single stand and risk neutrality.

The MDP framework is applied to a non-industrial private forest owner located in North-East of France. We show that the forest owner tends to converge toward a forest structure very close to a normal forest where only the oldest tree age-class is harvested. At the long-run equilibrium, the areas allocated to age-classes are almost identical, except for the oldest age-class for which the allocated area is reduced. We obtain that there appears to be no difference between exact and independent models under risk neutrality while there are differences in terms of optimal policy and expected value when introducing risk aversion, whatever the number of plots. We also find that the independent model can be seen as a poor approximation of the exact model because of strong incidence in terms of policy and optimal harvesting, especially when risk aversion is considered. We also show that the optimal decisions depend on risk preferences but not on the considered levels of windstorm probability due to climate change.

The remaining of the paper is organized as follows. In Section 2, we describe the forest management model and we present the Markov Decision Processes model used. Section 3 deals with the numerical approach for solving the model calibrated for a non-industrial private forest owner located in North-East of France. We conclude by a brief summary of our findings.

2 The model

We consider a non-industrial private forest owner facing a dynamic problem of forest management under storm risk over an infinity of harvesting periods[2]. We assume that the private forest owner manages a forest composed of different stand plots with one unique species. Let n denote the number of stand plots. Each stand plot i ($i = 1, ..., n$) has trees of age s_i. Let assume that $s_i \in \{1, ..., m\}$ with 1 coding the earliest age-class and m coding the oldest one. m is the age after which trees do not improve their value anymore. Trees in age-class s_i are characterized by their timber content, V_{s_i}, measured in (m^3/ha) and which increases with the age-class, $V_{s_i} \geq V_{s_{i-1}}$ $, \forall s_i \leq m$.

The Markov Decision Processes (MDP) framework allows us to efficiently model and solve this sequential decision-making problem under uncertainty (Puterman [37]). In its classical formulation (Puterman [37]), an infinite-horizon stationary MDP is described by a four-tuple, $< S, A, p, r >$, where S represents the finite set of states that can be reached by the system, A represents the finite set of decisions that can be applied at each period, p is a state transition function, and r is a reward function.

2.1 State and action variables

The state of the n stand plots of the forest is represented by the vector $s = (s_1, ..., s_n)$ where $s_i \in \{1, ..., m\}$ is the age-class of stand plot i. Then, the space of states, S is defined by $S = \{1, ..., m\}^n$.

In each period, the forest manager has to decide on the number of stand plots of each tree age-class to be harvested. The decision for each stand is to do nothing, or to cut the stand and reforest immediately. Therefore an action a is represented as a vector of decisions $a = (a_1, ..., a_n)$ where $a_i \in \{1, 2\}$ with $a_i = 1$ if the stand plot i is not cut and $a_i = 2$ if the stand plot i is harvested. The space of actions, A, is defined by $A = \{1, 2\}^n$.

2.2 Risk

The stochastic environment of the forest-owner is described by a risk of stand plot destruction due to windstorms, depending only on the age-class of the stand, as suggested by Dhote [10].

Once forest-decisions are taken, the risk of destruction is taken into account. If the stochastic event does not affect age-class s_i then tree growth occurs ($s'_i = \min\{s_i + 1, m\}$).

Let p_s denote the probability of strong wind realization in a period of time. Given a strong wind occurrence, trees may or may not be destroyed. pd_{s_i} denotes the conditional probability of overturning a stand i of age-class $s_i \in \{1, \ldots, m\}$ given strong wind occurrence. Given strong wind occurrence, the probabilities of each stand being overturned are assumed independent, for simplification.

2.3 Returns

Empirical evidence (Schelhaas *et al.* [43]) suggests that in the case of strong windstorm, the entire production is not lost. Hence, we assume that in case of an age-class destruction, a proportion

[2]When the horizon is infinite, it can be shown that the value function of a stationary policy does not depend on time. For that reason, the time index does not appear in mathematical equations.

$\alpha \in [0,1]$ of the timber content can be recovered and sold by the forest-owner. α equal to 1 means that strong winds do not result in timber losses at all and the only impact of the stochastic event is to impose harvesting at a time which may not be optimal. On contrary, α equal to 0 means that a stand destroyed yields no revenue.

The gross net revenue generated at each time period is the sum of net revenue of all plots:$w = \sum_{s_i=1}^{n} w_i$. The net revenue of each plot depends on the decision of harvesting and on the occurrence of windstorm. The net revenue of the plot i harvested is: $area_i[V_{s_i}(T_{s_i} - C_h) - C_p]$ where $area_i$ is the area allocated at the plot i, T_{s_i} represents the timber price (in $euros/m^3$) for age-class s_i, C_h the harvesting cost, and C_p the planting cost, both costs being assumed independent of the age-class. The net revenue of the plot i not harvested and impacted by a storm is: $area_i[V_{s_i}(\alpha \cdot T_{s_i} - C_r) - C_p]$ where C_r is the recovering cost in case of the stochastic event occurrence. The planting cost depends upon the planted area and the harvesting and the recovering costs depend upon the timber content. In other cases, the net revenue is null.

The problem of the forest owner is to maximize the expected utility of the consumption c. Assume in addition that the instantaneous utility of consumption is given by $U(c)$, where U is a continuous, twice differentiable, increasing and strictly concave utility function and c the level of consumption (net revenue of timber harvest, w) in period. In the numerical computation, we apply the specification $U(c) = c^{1-\beta}/(1-\beta)$ if $c > 0$, and $U(c) = -(-c)^{1-\beta}/(1-\beta)$ if $c < 0$, with $\beta \neq 1$. This utility function belongs to the Hyperbolic Absolute Risk Aversion family. Parameter β is the degree of relative risk aversion.

Therefore the reward function r of the forest is taken equal to $r = U(c)$.

2.4 Markov chain models of forest growth

Forest growth is described by the transition probabilities between forest states. In terms of the MDP framework, the application of an action may change the state of the forest. Furthermore, the state of the forest at the next period depends stochastically only on the state of the forest at the current period, and on the decision applied. The transition probabilities between forest states $p(s' \mid s, a)$ are defined as the probability that the system will change from state s to state s', given that action a has been applied.

The exact uneven-aged forest management model

In the exact uneven-aged forest management model, the forest owner manages all the different stand plots simultaneously. In this case, the state transition function is defined as:

$$p(s' \mid s, a) = \sum_{\epsilon \in \{0,1\}} \prod_{i=1}^{n} p_i(s'_i \mid s_i, a_i, \epsilon) p(\epsilon). \tag{1}$$

ϵ is a binary variable describing storm occurrence: $\epsilon = 1$ if there is a storm and $\epsilon = 0$ if there is no storm. The storm probability is defined by $p(\epsilon = 1) = p_s$ and $p(\epsilon = 0) = 1 - p_s$. $p_i(s'_i \mid s_i, a_i, \epsilon)$ is the probability the stand i's age-class changes from s_i to s'_i under action a_i, given storm event ϵ.

This transition probability is defined as follows:

- If the chosen action is to cut the stand ($a_i = 2$), then the next state is $s'_i = 1$, whatever the state s_i and whether or not a storm occurs :

$$p_i(s'_i \mid s_i, a_i = 2, \epsilon) = 1 \text{ if } s'_i = 1 \text{ and } 0 \text{ otherwise.}$$

- If the chosen action is to let the stand grow, and there is no storm ($\epsilon = 0$), then trees will grow, for sure :

$$p_i(s'_i \mid s_i, a_i = 1, \epsilon = 0) = 1 \text{ if } s'_i = \min\{s_i + 1, m\} \text{ and } 0 \text{ otherwise.}$$

- Finally, if the chosen action is to let the stand grow, and a storm occurs ($\epsilon = 1$), stand i will be overturned, for sure :

$$p_i(s'_i \mid s_i, a_i = 1, \epsilon = 1) = p_{d_{s_i}} \text{ if } s'_i = 1 \text{ and } 1 - p_{d_{s_i}} \text{ if } s'_i = \min\{s_i + 1, m\}, \text{ and } 0 \text{ otherwise.}$$

The above model is a Markov decision process, with state and action space sizes $|S| = m^n$ and $|A| = 2^n$. The explicit representation of transition probabilities by matrices therefore requires space $O(m^{2n}.2^n)$, which becomes prohibitive when the number of stands, n, is large.

An optimal policy of the PDM, $\pi^* : S \to A$, can be obtained from the following equation :

$$\pi^*(s) = \arg\max_a \left\{ \sum_{s'} p(s' \mid s, a) r(s, a, s') + \gamma V^*(s') \right\}, \tag{2}$$

where V^* is the unique fixed-point solution of the Bellman equation :

$$V(s) = \max_a \left\{ \sum_{s'} p(s' \mid s, a) r(s, a, s') + \gamma V(s') \right\}. \tag{3}$$

An optimal policy $\pi^* : S \to A$ is a state-dependent rule that gives the best action $\pi^*(s) \in A$ to take in state s. $V^*(s)$ is the value function of the optimal policy, giving the expected net present value of policy π^*, applied in forest state s. γ is the discount factor, and $r(s, a, s') = \sum_{i=1}^n r_i(s_i, a_i, s'_i)$ is the instant reward.

The independent uneven-aged forest management model

In the independent uneven-aged forest management model, the forest owner separately and independently manages each plot of the forest. In this case, the state transition function $p(s'|s, a)$ defined in equation 1 is approximated by the following factored transition function $p^I(s' \mid s, a)$:

$$p^I(s' \mid s, a) = \prod_{i=1}^{n} p_i^I(s'_i \mid s_i, a_i) \tag{4}$$

with $p_i^I(s'_i \mid s_i, a_i) = \sum_{\epsilon_i \in \{0,1\}} p_i(s'_i \mid s_i, a_i, \epsilon_i) p(\epsilon_i)$.

Now, it is assumed that "independent" storm events occur independently in each stand. This is, of course, unrealistic, but this assumption allows to efficiently compute approximate management policies, even for problems with many stands.

The factored transition function $p^I(s' \mid s, a)$ is an approximation of the transition function $p(s'|s, a)$. The infinite-horizon stationary MDP corresponding to this approximation is described by the four-tuple, $< S, A, p^I, r >$. It is easy to prove that the optimal policy of this PDM, π^{I*}, can be obtained by solving n independent and identical MDPs described by the four-tuple $< \{1, ..., m\}, \{1, 2\}, p_i^I, r_i >$.

The policy π^{I*} is then an approximation of the optimal policy π^* of the exact model. This policy has a double advantage: first, it can be concisely represented using the solution policies $\pi_i^*(s_i)$ of the independent MDPs, and second, these policies are identical, and can be obtained by solving a single MDP with sizes of the state and action state spaces equal to m and 2 respectively. However, in general, π^{I*} is suboptimal in the original exact model.

3 An application

3.1 Calibration of the model

The model has been calibrated to represent the behavior of a non-industrial private forest-owner located in Lorraine, a region located in North-East of France. The private forest estate is assumed to produce spruce, one of the most common stands observed in Lorraine. With 25 million cubic meters on the ground, Lorraine was the second devastated French region by the December 1999 exceptional storms.

Following Tahvonen and Kallio [46], we consider 5 tree age-classes, $m = 5$. Given the growth process of spruce, the time period represents a 20 year interval. Hence, the age-class s_i corresponds to trees of age $20 \times (s_i - 1)$ in stand i at the beginning of period, and $20 \times s_i$ at the end. The volume per ha and the price for each age-class for spruce in North-East of France are presented in Table 1.

Table 1: Characteristics of tree age-classes

s_i Age-class	V_{s_i} [a] Timber content (m^3/ha)	T_{s_i} [b] Timber price ($10^3 euros/m^3$)	pd_{s_i} Conditional probability of overturning [c] (%)
1	24.60	0.1303	1
2	112.20	1.3688	30
3	353.50	8.4133	65
4	601.40	22.0713	71
5	694.70	30.9836	72

[a]: Adapted from Vannière [48]. V_{s_i} corresponds to the timber content for 20*i year old trees.

[b]: From Guo [15]. Pt_{s_i} corresponds to the timber price for 20*i year old trees.

[c]: Using Picard et al. [34] and based on forest expert interviews.

The cost functions associated to forest management (planting, harvesting and recovering cost) are assumed to be linear and are derived from Guo [15]. The unit cost of planting, harvesting and recovering tree plots are respectively 2.1038 thousand euros per ha, 0.0037 and 0.0055 thousand euros per cubic meter. Last, in case of the stochastic event occurrence, 10% of the forest area can be recovered and sold (α is equal to 0.1).

According to Picard et al. [34], the annual probability of strong wind realization is $p_s = 3.1^0/_{00}$ for France. Assuming that strong wind occurrences are i.i.d, the probability of observing *at least* one strong wind during a twenty-year period is equal to 6.02%. As suggested by Dhote [10], the conditional probabilities of overturning for age-class s_i given strong wind occurrence vary from 1% for the first age-class to 72% for the fifth, see Table 1.

We have normalized the area of the forest land to 1 ha. This reflects the French situation since approximately 68% of non-industrial private forest-owners own less than one hectare. This assumption is not restrictive and higher forest areas may be considered. The forest-owner is also characterized by his risk preferences but there is currently no estimation available for this parameter in the case of small forest-owners. By reference to previous empirical studies, we have considered the following value $\beta = 0.5$. This value is consistent with the range of reported estimates in the economic literature. It corresponds to a low level of risk aversion. This case will be termed the *benchmark case* in the remaining of the article.

3.2 Solution method

Exact model Considering $n = 5$ plots and combining the stand states with the age-class states gives 3125 possible system states: $S = \{1, ..., 3125\}$. Each state s corresponds to a set of site age-classes $(s_1, ...s_5)$. The coding of the states is done in lexicographic ordering:

s	$(s_1$	s_2	s_3	s_4	$s_5)$
1	(1	1	1	1	1)
2	(1	1	1	1	2)
3	(1	1	1	1	3)
4	(1	1	1	1	4)
5	(1	1	1	1	5)
6	(1	1	1	2	1)
|		|			
3124	(5	5	5	5	4)
3125	(5	5	5	5	5)

The set of actions A is defined as $A = \{1, ..., 32\}$; each action a corresponds to a set of site action $(a_1, ..., a_5)$. The coding of actions is the following:

a	$(a_1$	a_2	a_3	a_4	$a_5)$
1	(1	1	1	1	1)
2	(1	1	1	1	2)
3	(1	1	1	2	1)
4	(1	1	1	2	2)
|		|			
32	(2	2	2	2	2)

The global transition probability table is defined from equation (1):

$$p(s' \mid s, a) = \sum_{\epsilon \in \{0,1\}} \prod_{i=1}^{n} p_i(s'_i \mid s_i, a_i, \epsilon) p(\epsilon).$$

It is assumed here that all individual transition tables p_i are equal for each site. They are defined by :

1. if the action is "cut" ($a_i = 2$), $p_i(1 \mid s_i, 2) = 1$ and $p_i(s'_i = 2, ..., 5 \mid s_i, 2) = 0$, $\forall i$,
2. if the action is "wait" ($a_i = 1$), $p_i(s'_i \mid s_i, a_i)$ is defined as $p_i(s'_i \mid s_i, 1) = M(s_i, s'_i)$ with matrix M defined as $M = (1 - p_s) \cdot P_0 + p_s \cdot P_1$, where

$$P_0 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \text{ and } P_1 = \begin{pmatrix} pd_1 & 1-pd_1 & 0 & 0 & 0 \\ pd_2 & 0 & 1-pd_2 & 0 & 0 \\ pd_3 & 0 & 0 & 1-pd_3 & 0 \\ pd_4 & 0 & 0 & 0 & 1-pd_4 \\ pd_5 & 0 & 0 & 0 & 1-pd_5 \end{pmatrix}$$

The size of the global transition probability table p is $3125 \times 3125 \times 32$. The problem can be solved using the classical policy iteration algorithm (Howard [19]). The optimization problem was solved numerically with MATLAB [29] using the MDP toolbox (http:inra.fr/mia/T/MDPtoolbox).

Aggregated model However, using the above defined MDP model, one cannot solve problems with more than 5 stands, due to the size of the generated transition table, p. Therefore, for larger problems, we have developed an equivalent MDP model of the forest management problem, which uses a more concise (aggregated) representation of the states and action spaces, and therefore transition matrix and reward function. This second model is used in the larger experiments.

Indeed, note that in the above state representation, where $s = (s_1, \ldots, s_n)$, with $s_i \in \{1, \ldots, m\}$, the state of each individual stand is not really relevant for the management problem: the only relations between stands are through the global storm event, and the reward aggregation function and in these relations the stand indices can be interchanged without any problem. Thus, the relevant information is the number of stands in each age-class, which can be represented by the vector $s^A = (s_1^A, \ldots, s_m^A)$, with $s_1^A + \ldots + s_m^A = n$. In the same way, the relevant action information is the number of stands cut in each age-class, which can be represented by action vector $a^A = (a_1^A, \ldots, a_m^A)$, with $0 \leq a_j^A, \leq s_j^A, \forall j = 1, \ldots, m$. With this equivalent "aggregated" representation, the state space size is reduced from $|S| = m^n$ to $|S^A| = C(n+m-1, n) = (n+m-1)!/(n!(m-1)!)$. The action space size is reduced accordingly, $|A^A|$ being upper bounded by $(n+1)^m$ when $|A| = 2^n$.

The complexity of the aggregated model is less sensitive to the increase in the number of stands, n, than the initial model. Indeed, the aggregated model complexity roughly increases linearly with n^m, when the initial model increases linearly with m^n. In practice, with the aggregated model, we are able to solve problems with 12 stands and 5 ages classes.

3.3 Results

In this section, we investigate how risk preferences and windstorm probabilities impact on the optimal forest owner decisions, and on the optimal long-run equilibrium.

3.3.1 Optimal policy in the benchmark case

We consider an initial land allocation where each tree age-class is attributed the same area and we characterize the optimal forest harvesting decisions. We first focus on the optimal forest harvesting decisions in the benchmark case.

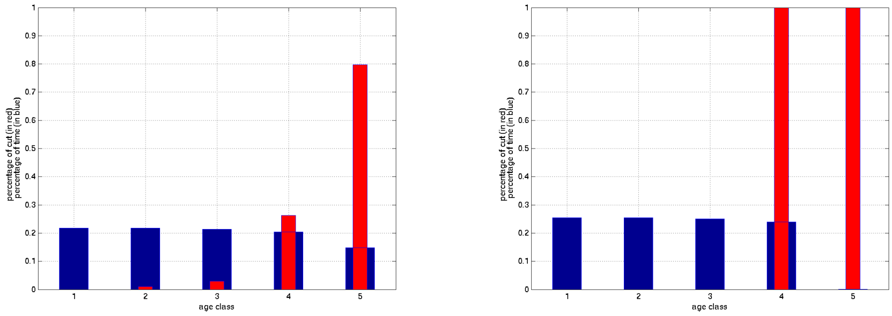
To illustrate the optimal policy in the benchmark case, we first present the case of a single plot. When the forest owner has only one plot, it is always optimal to harvest age-classes 4 and 5. In the long-run equilibrium the areas dedicated to age-classes 1 to 4 are almost the same and the area allocated to the fifth age-class is null. In this case, the optimal forest management does not tend toward a normal forest structure where only the oldest tree age-class is harvested, as generally obtained in the forest economics literature (see Salo and Tahvonen [40], and Couture and Reynaud [8] among others), but tends toward a restrictive normal forest where the oldest age-class is excluded from the forest structure but the principle of the normal forest is then respected among the remaining age-classes (see Table 2).

Table 2: Long-run equilibrium values of the areas dedicated to the different tree age-classes, and expected present values computed with the exact model for 1, 5 and 10 stands

Number of stand	Percentage of allocated area Age-classes					The expected present value of optimal policy per stand (EV^*/n)	The expected present value of maximum gross net revenue per stand
	1	2	3	4	5		
1 plot	0.2550	0.2549	0.2501	0.2400	0	$2.80 * 10^3$	$1.60 * 10^5$
5 plots	0.2175	0.2174	0.2132	0.2045	0.1474	$2.75 * 10^3$	$1.93 * 10^5$(+21%)
10 plots	0.2125	0.2114	0.2074	0.1981	0.1707	$1.95 * 10^3$	$1.96 * 10^5$(+22.4%)

When the forest owner has five plots, he can manage risk differently. He can expose the oldest age-class to the windstorm risk, in order to get higher returns (see Figure 1). The possibility of diversifying risks using the different plots allows the forest owner to harvest different age-classes

Figure 1: Optimal policies for the benchmark case with the exact model and with the independent model



a) Exact model b) Independent model

and to dedicate some area to the oldest age-class, unlike in the single stand case, or when stands are managed independently (since the latter case corresponds to 5 independently managed plots). In the long-run equilibrium, the proportion of stands allocated to each age-class 1 to 4 is still higher than that of the fifth age-class (see Table 2), however the latter proportion is strictly positive. More, it becomes optimal to smooth the age class structure if the number of stands increases. Indeed, the feature that the age class distribution may adjust close to a normal forest structure seems to be general.

This result highlights the impact of managing more than one plot on the optimal forest management policy. There is also an impact on the expected present value of maximum gross net revenue per stand. One can notice that the expected present value per stand is higher when the forest manager can spread risk between several stands. This value goes from $1.60 * 10^5$ per stand in the "independent" case, to $1.93 * 10^5$ in the "jointly managed" case. This shows that an independent management of stands is suboptimal: a loss of 21% is incurred for 5 plots, compared to the optimal (joint) management. Thus, there is a real *plus-value* to consider joint management policies in the case of storm risk, even if such policies are harder to compute.

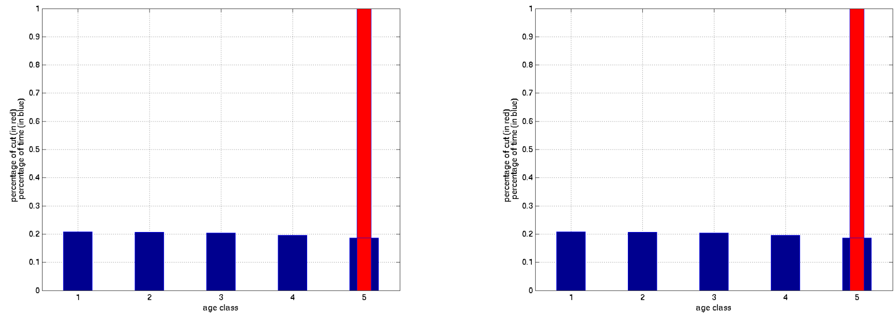
Finally, we assess the impact of risk preferences on the plus-value offered by a joint management of stands (see Figure 1). When risk-neutrality is assumed ($\beta = 0$), unlike in the benchmark case ($\beta = 0.5$) harvesting is restricted to the fifth age-class, in both independent and joint models (see Figure 2). Thus, it seems that different risk preferences may lead to different performances of the independent model optimal policies. We will study the risk preferences impact further in Section 3.3.3.

3.3.2 Assessment of the windstorm probability impact

Climate change is associated with increased climate variability and consequently with an increased frequency of extreme events such as heat waves, severe droughts and intense storms. For the studied area, no trend for an increase of the windstorm probability is demonstrated at the moment (Przyluski and Hallegatte [36]), however, it is still assumed that climate change and increased climate variability are expected to have repercussions on the windstorm probabilities. For forest owners, adapting to climate change is believed to require major adjustments in management practices. This is why we now investigate the impact of an increase in the windstorm probability on the long-run equilibrium in the MDP forest management model.

In addition to the initial value of the windstorm probability, we consider two other values ($p_s = 0.062$ and $p_s = 0.092$) that correspond to windstorm risk increased by 10% or 50%, respectively.

Figure 2: Optimal policies for the risk neutrality case with the exact model and with the independent model



Exact model Independent model

Such values are consistent with the expert forecasts of increase in the frequency of extreme climatic events due to climate change. Figure 3 shows that such variations in the windstorm probability (from the benchmark case) do not have much impact in terms of optimal policy and optimal forest structure. This can maybe be explained by the still relatively small values for the windstorm probability. However, it is important to note that these values are much larger than those by climatologists forecasts. This conclusion is also confirmed for the independent model (see Figure 4).

The expected present values of optimal policies for the "realistic" storm probabilities are also shown in Table 3.

Table 3: Expected present values per stand for the different levels of windstorm probability with the exact model for 5 stands and the aggregated model for 10 stands

Probability of windstorm	Expected present value of optimal policy per stand (Variation compared to the benchmark case)		Expected present value of maximum gross net revenue per stand (Variation compared to the benchmark case)	
	5 stands	10 stands	5 stands	10 stands
$p_s = 0.062$ (benchmark case)	$2.75 * 10^3$	$1.95 * 10^3$	$1.93 * 10^3$	$2.00 * 10^3$
$p_s = 0.068$	$2.73 * 10^3$	$1.94 * 10^3$	$1.91 * 10^3(-1.0\%)$	$1.94 * 10^3(-0.9\%)$
$p_s = 0.092$	$2.67 * 10^3$	$1.89 * 10^3$	$1.84 * 10^3(-4.6\%)$	$1.87 * 10^3(-4.5\%)$

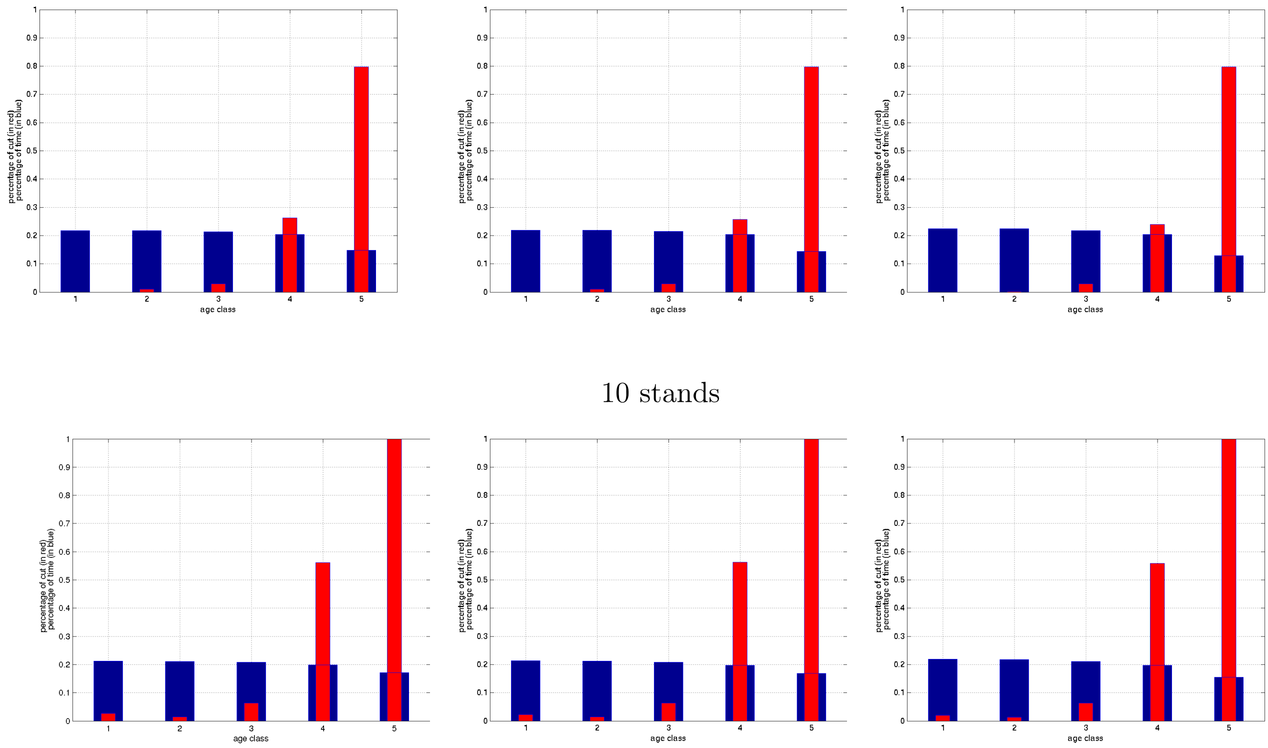
We observe that, even if the optimal policy stays roughly the same, an increase in the probability of windstorm will result in a decrease in the maximum expected present value. Given that the optimal policy is unchanged, this result is very intuitive. This reduction in the expected present value of the maximum gross net revenue (shown in Table 3) can be used to model the forest owner's willingness-to-pay to be covered against the risk increase. For example, in the case of windstorm probability rising to 0.092, the forest owner's willingness-to-pay could reach up to 4.6% of the expected present value of the forest, under the optimal management policy.

Figure 3: Impact of the windstorm probability on the percentage of harvesting and on the percentage of time spend in each age-class with the optimal policy with the exact model for 5 and 10 stands

Probability of windstorm equal to 0.062	Probability of windstorm equal to 0.068	Probability of windstorm equal to 0.092

5 stands

10 stands



3.3.3 Assessment of the risk preference impact

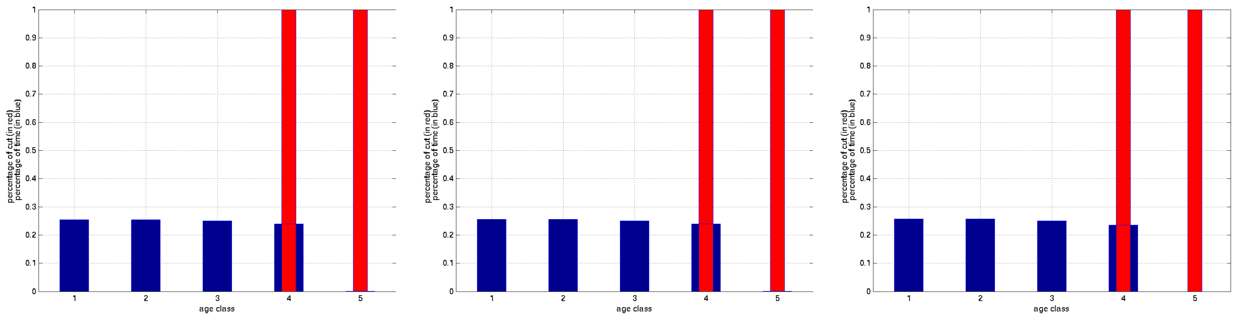
In this subsection, we simulate the model for various levels of the Arrow-Pratt constant relative risk-aversion coefficient (β), and we analyze the resulting optimal forest management policy. We measure the impact of varying the degree of risk aversion on the management strategy of a forest owner, and the long-term age profile of the forest. For this purpose, we consider three values for this coefficient, the first two corresponding to a decrease in risk aversion ($\beta = -0.5$: corresponding to a risk seeking owner, and $\beta = 0$: as a risk neutral owner) and one consisting in an increase in risk aversion ($\beta = 0.9$: corresponding to a strong risk averse owner).

The forest age-class profile remains rather stable under this range of risk-aversion parameter's values, but the management strategies change drastically (see Figure 5 for the exact model). This complex behavior is difficult to explain at first glance. As Couture and Reynaud [8] indicate, the behavior of a forest owner faced with a dynamic problem of forest management under risk depends on three effects. The first one, the wealth effect, corresponds to an incentive to harvest in order to increase the revenue resulting from timber harvesting. The second effect, the risk effect, makes the forest owner to reduce the risk exposure of the forest and then to harvest in order to diminish future potential damage. The last effect, the continuation effect, is driven by the forest owner's tendency to reduce harvesting in order to smooth the utility over time.

Figure 4: Impact of the windstorm probability on the percentage of harvesting and on the percentage of time spend in each age-class with the optimal policy with the independent model for 5 stands

Probability of windstorm equal to 0.062	Probability of windstorm equal to 0.068	Probability of windstorm equal to 0.092

5 stands



In the light of these remarks, one can notice that for risk-neutral or risk-averse owners, the more risk-averse the owner is, the more trees in intermediate age-classes are cut. Consequently, the proportion of stands in the latest age-class is decreased. However, this reduction is mitigated with a larger number of stand plots, and it becomes optimal to cut much older age classes. Harvests are principally from the two oldest age classes that are strongly harvested. It seems that in this case, the risk and wealth effects dominate. On the contrary, risk-seekers ($\beta = -0.5$) tend to have a more balanced harvesting strategy, cutting trees in all age-classes. As a result, the age profile of the forest is more balanced. This trend is confirmed with a larger number of plots except that risk seekers increase the harvest of the riskiest age class. For risk-seekers, the continuation effect seems to dominate.

With the independent model, the impact of risk preferences on optimal policies differs (see Figure 6). Optimal policies correspond to very simple threshold policies, and only the threshold changes. Risk-seeking or risk neutral managers only harvest oldest trees. In this case, the structure of the forest converges toward a normal forest where only the oldest tree age-class is harvested. When the level of risk-aversion increases, the threshold decreases: In the limit, a strongly risk-averse manager ($\beta = 0.9$) is not ready anymore to take any risk, and thus trees are cut as soon as possible. Note that the difference between the independent and the exact model is particularly sensitive to variations in the risk-aversion coefficient.

The results of this section show first that the optimal management depends on risk preferences. It is thus fundamental to incorporate the forest owner's risk aversion into forest management modeling. Furthermore, incorporating risk aversion also makes it important to use an exact model. The independent model results differ greatly from the exact model results.

Our results are in line with previous works integrating risk aversion into forest management. They conclude that risk preferences have a strong impact on optimal forest policies (Couture and Reynaud [8], [9]; Andersson [3]; Alvarez and Koskela [2]). The additional result that we prove is that taking explicitly into account multiple stands into the model is necessary in order to derive correct conclusions about risk aversion. Inclusion of risk attitudes into forest owners' decision management under uncertainty can help understand why non-industrial private forest owners harvesting deviates from net present value maximization (Andersson [3]). This calls for a deeper understanding of forest

Figure 5: Impact of the risk preference on the percentage of harvesting and on the percentage of time spent in each age-class under the optimal policy for the exact model

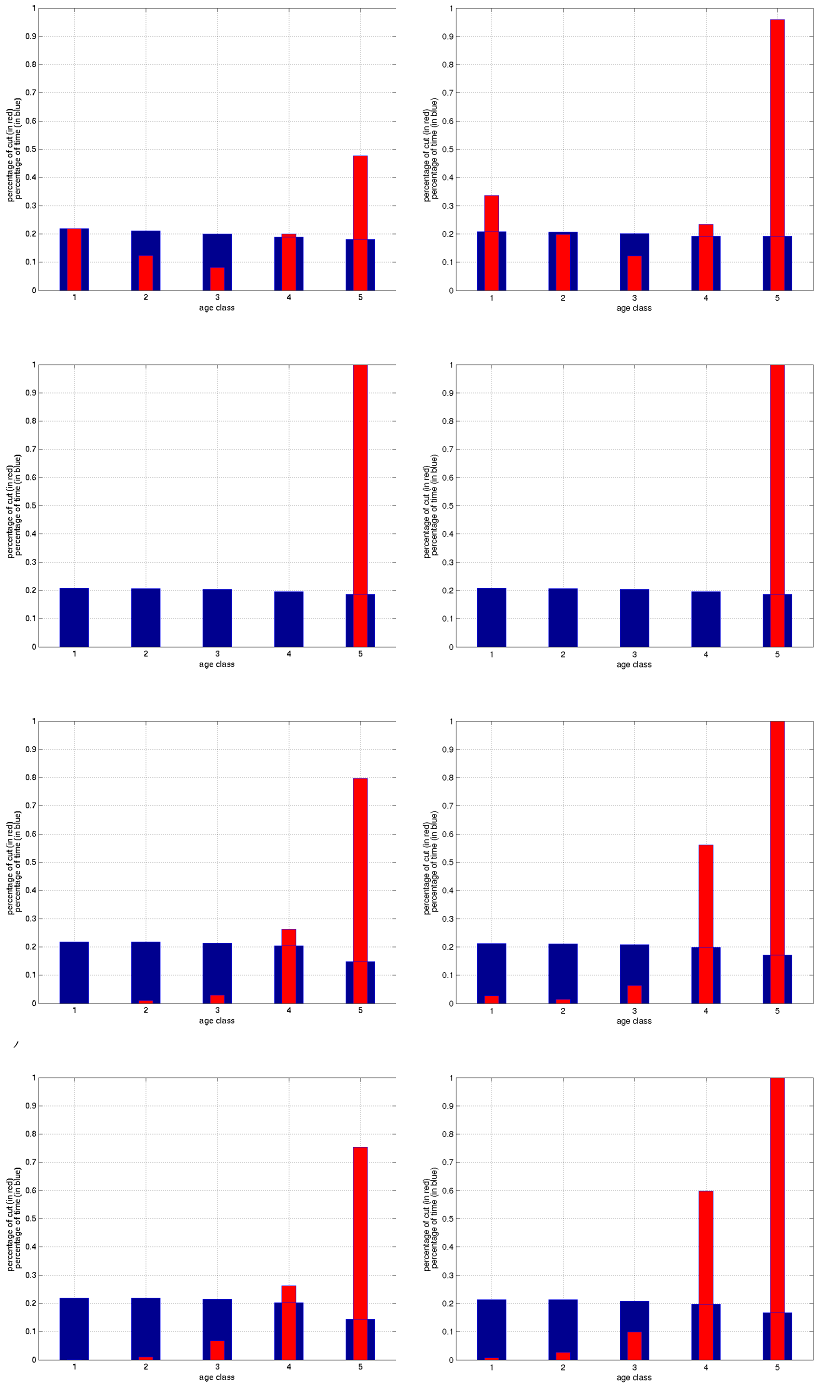
5 stands 10 stands

Risk seeking $\beta = -0.5$

Risk neutral $\beta = 0$

Risk averse $\beta = 0.5$ (benchmark c

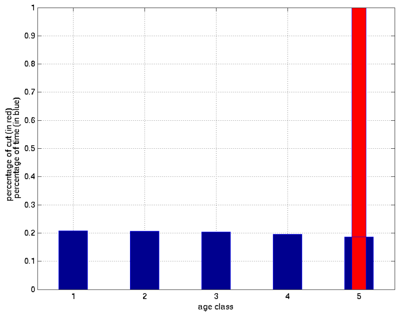
Strong risk averse $\beta = 0.9$



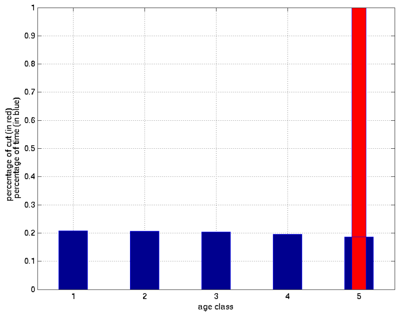
owner risk preferences as recently done by Andersson and Gong [4].

Figure 6: Impact of the risk preference on the percentage of harvesting and on the percentage of time spent in each age-class with the optimal policy for the independent model

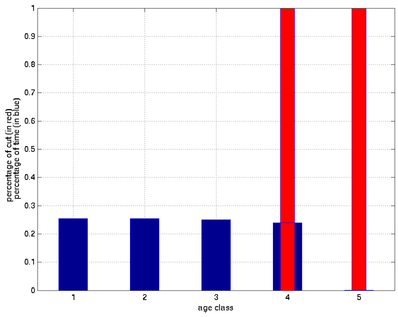
5 stands



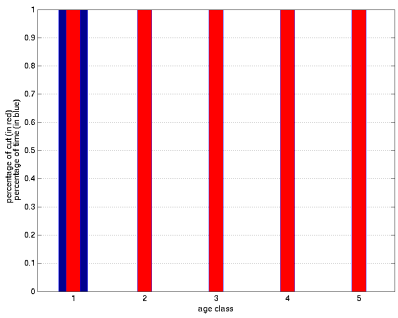
Risk seeking $\beta = -0.5$



Risk neutral $\beta = 0$



Risk averse $\beta = 0.5$ (benchmark case)



Strong risk averse $\beta = 0.9$

4 Conclusion

We have used a MDP model with an expected utility criterion for analyzing the forestry decisions of a non-industrial private forest owner facing a windstorm risk due to climate change. The problem of the forest owner is to manage a forest composed of different stand plots with different age-classes in the presence of windstorm risk. We determinate optimal forest harvesting including multiple age-classes, and forest owners' risk aversion using a MDP framework. Our model generalizes existing MDP studies that assume a single stand and risk neutrality. We consider two management types: the exact uneven-aged forest management model in which the forest owner jointly manages all the different stand plots, and the independent uneven-aged forest management model that assumes the forest owner separately and independently manages each plot of the forest. We have applied this framework to the management of a French forest-owner facing a windstorm risk. In this context, we have conducted sensitivity analyzes on the effects of risk preferences and windstorm probabilities on the optimal policies.

Summary of findings

The first and main finding of this study is to have shown the importance of an explicit multi-stand modeling of forests dynamics for their management. Indeed, we have found that:

- Optimal multi-stand management strategies differ from single-stand management strategies. While the latter are simple "threshold" strategies, the structure of the former is more complex.
- The age-class profiles differ between multi-stand and single-stand managed forests: The former one is more balanced at the forest scale, than the latter.
- The expected present value per stand of forests per stand is significantly higher under a multi-stand optimal management strategy than under a single stand strategy.
- The analysis of the risk-preference impact on optimal management strategies is significantly altered when single-stand strategies are considered. Such a study should be led under the multi-stand management model only.

The second finding concerns risk-preference. We have found that the sensitivity to the risk management is rather complex. The age-class profile is not significantly altered under optimal policies in different risk-preference conditions. However, it seems that the further we get from risk neutrality, the less balanced the profile becomes. In terms of optimal management strategies, this strongly depends on the risk preference parameter value. Our second finding is still in the line of already existing works on risk preference.

Our last finding concerns the impact of increased storm risk on forest management. We have not found a significant impact of an increased storm risk on the optimal forest management strategy, which could maybe seem a bit counter-intuitive. However, even if the optimal strategies are rather insensitive to increased storm risk, the corresponding expected present value was found to be clearly sensitive to an increase in storm risk, as expected.

Future work

There are several possible directions for future researches. The first extension of this study would be to consider the possibility for the forest owners to adopt prevention and coverage measures against the windstorm risk such as private insurance, self-insurance activity, self-protection actions or public funds. Another extension could be to introduce into the model the consumption-saving decisions in order to analyze the linkage between these decisions and forest management. Finally, another way to extend this research would be to introduce the non-timber benefits of the forest into the objective function, making it possible for a new trade-off to appear between timber and non-timber production. Moreover, in the case of storm occurrence, other losses could be considered in terms of carbon storage, biodiversity, etc.

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