Promotion of cooperation when benefits come in the future: a water transfer case

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Abstract

A country with a surplus river basin cooperates with a country with a deficit river basin in a joint investment project to build a water canal. The benefits from the existence of the canal are realized when cooperation halts and countries will have the opportunity to engage in a non-cooperative water market game. We define an imputation distribution procedure to share the "burden" from cooperation. Investment costs while cooperation must be allocated according with future benefits from the water market. With this principle we prove that the cooperative solution is time consistent. Further, we can obtain the instantaneous side-payment scheme which makes the imputation distribution procedure feasible.

JEL Classification: F18, C73.

Keywords: Cooperative differential game, non-cooperative differential game, imputation distribution procedure, instantaneous side-payment, time-consistent solution.

1 Introduction

This paper analyzes the dynamic cooperation between two countries or regions in order to build a canal which connects a donor river basin, with higher precipitation rates, and a recipient river basin, with greater water productivity. This joint investment program presents two main characteristics. Firstly, cooperation does not lead to an immediate reward, but only after a first period in which the two parts have to pay the costs of building the canal. Secondly, the delayed benefits from cooperation are known by cooperating agents and they are typically asymmetric. The efficiency gains linked to the flow of water from a surplus basin to a deficit basin with greater water productivity, can be realized thanks to the water market created by the water canal.

Some examples of already operative schemes or ongoing projects of water transfer can be found, usually within a specific country:[1] The Tagus-Segura Transfer Project in Spain, the Snowy River Scheme in Australia, the São Francisco Interlinking Project in Brazil, the Olmos Transfer Project in Peru or the South-North Water Transfer Project in China. Some of them are still ongoing projects, which full implementation may take some decades, or even end-up as fail projects. These projects have been promoted by a central government. Much less frequent are the examples of water transfers between river basins located in different countries, like the transfer from the Kosi river in Nepal to the Ganges in India and Bangladesh, or the Lesotho Highlands Water Project (drawn out by corruption) between Lesotho and South Africa (geographically condemned to get along with each other). This reflects the difficulties tied to the obliged cooperation between two governments who have to determine how to share the the costs of the joint project, and how to distribute these costs along the often lengthy construction period. Maybe better examples of these difficulties are the failed projects, like the Rhone-Barcelona aqueduct proposed to supply the city of Barcelona in Spain with water from the Rhone river in France (see, Lopes (2008)). We will focus on the economic aspects that help maintain the agreement to build the canal although, as pointed out by Lopes (2008), the obstacles to the transfer between countries are also political or institutional.

The bulk of the literature on river water management involving two regions/countries and non-cooperative game theory is on water-sharing under an upstream/downstream configuration (see, for example, Ambec and Ehlers (2008), Bhaduri and Barbier (2008), Ambec and Sprumont (2002), or Kilgour and Dinar (1995, 2001)). However, the problem of a water transfer between two river basins has some specificities not present in the upstream/downstream literature. The donor river basin must be characterized by a surplus of water inflows, and the recipient basin by a deficit. Besides this asymmetry, water productivity is higher in the recipient. Further two additional features are also present in most of the examples highlighted above. The recipient might have access to alternative sources of water, by investing in infrastructures which can help to increase available water for the economy (examples could be desalination plants, projects to save water, to reduce pipelines leaking, or investment in recycling). The transfer brings environmental consequences mainly, but not exclusively, for the donor. If the relative cost of the alternative water supplies is low, and/or the magnitude of the environmental damages linked to the water transfer is high, then the inter-basin transfer might be unfeasible.

For the particular example of the Tagus-Segura transfer, Ballestero (2004) presents a

[1]Some of these projects transfer water from one river basin to another river, some others transfer the water to dams in the mountains (for irrigation and to generate hydroelectricity), or towards a specific region for municipal water supply, industry and irrigation.

static demand-supply model, later extended to a dynamic setting in Cabo *et al.* (2014). This latter studies the interaction between donor and recipient regions as a non-cooperative differential game, which defines the water market as a bilateral monopoly. Further, it also includes the environmental damage caused by the transfer in the donor region, and the alternative water supplies available for the recipient. It is assumed nonetheless, that the infrastructure required to transfer the water between the two river basins is already operative.[2] Under this assumption the water market equilibrium is dynamically analyzed. By contrast, with a broader perspective, and particularly when the transfer involves governments from two different countries, we consider important to address the previous coordination problem associated with the joint investment required to built the canal.

The central question in static cooperative game theory of how to distribute the gains from cooperation between the cooperating players, is extended by the dynamic cooperative game theory to study the distribution of these gains not only among players but also across time. In particular, how to distribute the surplus from cooperation across time to guarantee that no player has an incentive to deviate from cooperation,at any point in time (the cooperative payoffs to go surpass the non-cooperative payoffs to go). This concept is usually referred to as time consistency.[3] A widespread mechanism to guarantee the time consistency of the cooperative solution is to select a solution concept specifying each player's share of the total cooperative payoff, and define a payoff distribution procedure, as stated in Petrosjan (1997), to decompose over time the individual total payoff, in such a way that time consistency is preserved (see Zaccour (2008) for a review). This is not the unique mechanism to guarantee time consistency,[4] but it constitutes the basis to implement a time consistent solution in this paper.

Contrary to the standard literature, in our setting cooperation does not lead to immediate payoffs gains. More to the contrary, cooperation to invest in the construction of the canal, represents a costs for both players, not only instantaneously, but maintained through the whole period of cooperation. In fact, the gains from cooperation only start to dwell once the water starts to flow through the canal, and with it the efficiency gains. But this is precisely the exact moment at which the joint investment cooperation halts. Therefore, the

[2]For this case study, 230 km network of canals, aqueducts and tunnels were built by the Spanish central government to transfer water from the Tagus basin in the center of Spain to the Segura basin in south-eastern Spain.

[3]As stated in Zaccour (2008), it has been also named as sustainability of cooperation, dynamic individual rationality, dynamic stability, durability of an agreement, or agreeable solution.

[4]Other mechanisms can be found in the literature, like incentive strategies proposed by Ehtamo and Hämäläinen (1986, 1989, 1993), or design the cooperative agreement satisfying the property of being at equilibrium (see, for example Rincón-Zapatero *et al.* (2000)).

first question that must be addressed is how to share the "losses" from cooperation when its benefits will only be materialized when cooperation ceases to exist, and the two parts engage in a non-cooperative trade relationship in the water market. Thus, assuming that the aggregated (discounted) gains from the existence of the water canal surpasses the global economic and environmental costs of the joint investment project, our main research question is: how shall the investment costs be shared between the two parts and distributed across time to guarantee the time consistency of the cooperate solution? That is, to guarantee that no player deviates from cooperation and the canal is actually finished.

To analyze this question we define a differential game with two different regimes. The two countries jointly invest to build the canal within a first period, which length will be determined by the intensity of the investment paths. The cooperative objective function includes current investment costs as well as future benefits. These latter in the form of a scrap function defined as the sum of the value functions of the two players in the subsequent game. Within this subsequent period of infinite length the canal is operating and the two countries play a non-cooperative differential game as in Cabo *et al.* (2014). By comparing this latter with a baseline scenario of no transfer, we observe asymmetric surpluses for the two players from the existence of the canal. Therefore, we can compute each player share in the total gains stemming from the existence of the canal. This share helps us to link current investment costs while cooperation and future benefits. The cooperative solution concept proposed in this paper is based on the central idea that at each time within cooperation, each player's payoffs to go must be equal to his payoffs to go in case of defection (and hence no transfer), plus a share of the total surplus to go. And this share is defined by the player's share in total gains from the existence of the canal (computed at the moment when the canal start to be operative). In consequence, the proposed method assigns each cooperating player a contribution (to the joint investment required to build the canal) dependent on his share in total gains (obtained in the second period when the canal is operative). With this principle we are capable to define an Imputation Distribution Procedure (IDP) specifying what each player should be contributing at each moment through the cooperative period in order to guarantee time consistency. Further, we also provide the instantaneous transfer scheme through the whole cooperative period that makes this IDP possible.

The paper is organized as follows. Section **??** describes the problem and presents the model. Section **??** presents the main contribution of the paper: the definition of an imputation distribution procedure that guarantees the time consistency of the cooperative solution and which can be attained through an instantaneous side-payment scheme. Section **??** shows our results for a linear quadratic example. Finally Section **??** concludes the paper.

2 The model

This section describes a two-regimes differential game between a (donor) country/region which river basin is characterized by relatively high precipitation rates and relatively low productivity of water and a (recipient) country/region with lower precipitations and highly productive uses. If the two countries cooperate within a first period and build a canal, this would make possible a inter-basin water transfer henceforth. In this water market the donor will transfer surplus water for a price, although it will experience an environmental damage due to the deterioration of its water quality. The recipient will pay the price for the water transfer which can be utilized to enhance production, and will also allow reductions in the investments on subsidiary water production, water savings or water recycling. We assume henceforth that the savings from lower investments plus the increments in production in the recipient surpass the environmental losses in the donor plus the costs of building the canal, leading to a rise in the Kaldor-Hicks efficiency. This is a necessary condition for the regions to cooperate and build the canal.

We first present the main hypotheses to describe the economic and environmental aspects of the relationship connecting these two regions. Then we present the non-cooperative game played within a second period when the canal is already operative, and analyze the dynamic determination of quantity and price in the water market. Finally, we present the cooperative period when countries jointly invest to build the canal.

2.1 The donor and the recipient

This section describes the trade market made possible by the existence of the canal. We follow the main assumptions in Cabo *et al.* (2014). No uncertainty is included in the model, assuming that a constant surplus of water still remains in the donor region after covering demands. Without the canal this surplus flows through the donor's river basin. However, if an aqueduct is built, an amount $\tau(t)$ of the surplus could be transferred through to the recipient. Before the transfer, the inhabitants in the donor region would enjoy the environmental amenities or environmental services of what we denote a pristine river. By contrast, the decrease in the water level provoked by the water transfer would decrease these amenities at an increasing rate. Thus, the environmental amenities can be represented by $E(\tau(t))$, with $E(0) > 0$, $E'(\tau(t)) < 0$ and $E''(\tau(t)) < 0$. As compensation the donor receives a monetary payment, $p(t)$, from the recipient for each unit of water transferred. The instantaneous welfare function for the donor is then expressed as:[5]

$$F^d(p(t), \tau(t)) = E(\tau(t)) + p(t)\tau(t). \tag{1}$$

[5]Here and henceforth, superscript d/r refers to donor/recipient respectively.

Due to the relatively higher water productivity in the recipient, this region will be willing to pay for the water transferred from the donor. Nevertheless, apart from the transfer, the recipient has an alternative and often subsidiary way to increase its volume of available water. It can invest in the equipment required for either water saving (reducing water use or water leakage from the distribution network), water recycling (use of gray water), or water production (like desalination plants). The usable water capacity, $x(t)$, is defined as the capacity to produce, recycle or save water using current equipment (measured in cubic meters). Capacity increases with new investments and decreases with depreciation:

$$\dot{x}(t) = s(t) - \delta x(t), \qquad x(0) = x_0 \geq 0,$$

wit $\delta > 0$ the depreciation rate and $s(t)$ the investment to replenish and further increase current capacity.

In the recipient region, welfare comes from the amount of available water: either water transferred, $\tau(t)$, or the region's usable water capacity, $x(t)$. Welfare increases with the amount of available water at a decreasing rate. Investments in new capacity are increasingly costly, which can reflect increasing transaction costs and/or incremental cost of successive projects to produce, save or recycle water. Finally, instantaneous welfare decreases with transfer payments to the donor. Hence, the welfare function of the recipient is expressed by:

$$F^r(p(t), \tau(t), x(t), s(t)) = Q(\tau(t), x(t)) - p(t)\tau(t) - C(s(t)) \tag{2}$$

with $Q(\tau, x)$ concave in τ and x, $C'(s) > 0$ and $C''(s) > 0$. For shortness of presentation we assume that the water inflow in the recipient river basin is null. We believe that non of the results would be affected if instead, a constant inflow of water were assumed in the recipient river basin.

The problem must be solve backwards. First, we present the non-cooperative differential game of infinite duration, starting at moment T, when the canal starts to be operative. The value functions of donor and recipient in the water market game, played within period $[T, \infty)$, must be taken into account to compute the cooperative solution of a joint investment program to build the canal, within a first period $[0, T)$.

2.2 The non-cooperative water market game within $[T, \infty)$

This subsection presents the dynamic interaction between the donor and the recipient regions starting at time T when the canal starts to be operative. The time paths for the amount and the price of the water transfer are determined from the supply and demand decisions taken by donor and recipient as described in Cabo *et al.* (2014). The donor determines the supply of water, τ^d, in order to maximize the stream of welfare discounted at a constant rate, ρ, within an infinite time horizon:[6]

$$\max_{\tau} \int_{T}^{\infty} \left[E(\tau) + p\tau\right] e^{-\rho(t-T)} dt. \tag{3}$$

Correspondingly, the recipient must decide on the demand for water, τ^r, from the donor and on the investment, s, in usable water capacity, to maximize discounted welfare:

$$\max_{\tau, s} \int_{T}^{\infty} \left[Q(\tau, x) - p\tau - C(s)\right] e^{-\rho(t-T)} dt, \tag{4}$$

$$\text{s.t.: } \dot{x}(t) = s(t) - \delta x(t), \quad x(T) = x_T \geq 0. \tag{5}$$

The recipient is a farsighted player whose maximization problem is subject to the evolution of usable water capacity in (**??**). By contrast, the donor behaves as a static or myopic player.[7] Price is determined by the optimal supply and demand decisions and by the Market clearing condition:

$$\tau^d(p) = \tau^r(p).$$

Assuming that a feedback Nash equilibrium for this problem exits,[8] we denote $V^i(x_T)$ the value function of player $i \in \{d, r\}$, with an initial stock of usable water capacity given by $x(T) = x_T$. To have an insight of each player's incentive to invest in the construction of the canal, these values must be confronted with each player's accumulated gains in absence of an aqueduct to transfer the water, and under the assumption of an identical initial stock of usable water capacity, x_T.

With no water transfer, the donor would not face any optimization problem and his profit reads:[9]

$$V^d_{NT} = \frac{F^d(0,0)}{\rho}.$$

The recipient would choose the investment in usable water capacity to solve the optimal control problem:

$$V^r_{NT}(x_T) = \max_{s} \int_{T}^{\infty} F(0, 0, x, s) e^{-\rho(t-T)} dt = \max_{s} \int_{T}^{\infty} \left[Q(0, x) - C(s)\right] e^{-\rho(t-T)} dt, \tag{6}$$

subject to (**??**).

Condition 1 *We first assume*[10] *that there exists a set $A \subseteq R^+$ such that*

$$V^i(x_T) > V^i_{NT}(x_T), \quad \forall x_T \in A, \quad i \in \{d, r\}.$$

Condition **??** states that both players are better off if the canal exists. This gives them both an incentive to cooperate in a joint investment project to build the canal.

[6] Here and henceforth, the time argument is omitted when no confusion can arise.

[7] Problem (**??**) could be written as: $\max_{\tau} \left[E(\tau) + p\tau\right]$.

[8] The game is analytically solved in Cabo *et al.* (2014) under a linear quadratic specification for functions $E(\tau)$, $Q(\tau, x)$, and $C(s)$.

[9] subscript NT refers to "no transfer" scenario.

[10] For the linear quadratic game proposed in Cabo *et al.* (2014), Condition **??** is satisfied for $A = [1/\alpha, +\infty)$.

2.3 The joint investment project to build the canal within $[0, T)$

Within a first period $[0, T)$, the two regions cooperate in a joint investment project to build the canal. At any time $t \in [0, T)$, each country i invests an amount $I^i(t)$ at a cost $C^i(I^i(t))$. The joint investment contributes to increase the length of the aqueduct already built, measured by the stock variable $K(t)$. The canal is finished once it reaches its full length, when the accumulated stock $K(t)$ reaches the value $\overline{K}$. Thus, the duration, T, of this cooperative period is determined by the intensity of investments, and by the magnitude of the canal represented by constant $\overline{K}$. Within this period, the cooperating agents must also decide the investment on alternative water supply, $s(t)$. The incentive to invest in the stock of usable water capacity is reduced by the expectations of a future water transfer of infinite duration. The cooperative maximization problem can be written as:

$$\max_{I^d, I^r, s, T} \int_0^T [F^d(0,0) + F^r(0,0,x,s) - C^d(I^d) - C^r(I^r)]e^{-\rho t} dt + SC(x(T)), \qquad (7)$$

$$\text{s.t.: } \dot{x} = s - \delta x, \qquad x(0) = x_0 \geq 0, \qquad (8)$$

$$\text{s.t.: } \dot{K} = I^d + I^r, \qquad K(0) = K_0, \;\; K(T) = \overline{K}, \qquad (9)$$

where the scrap value is given by the present value of the addition of the value functions of donor and recipient in the non-cooperative differential game played *a la* Nash from T to ∞, considering feedback strategies as presented in section **??**:

$$SC(x(T)) = [V^d(x(T)) + V^r(x(T))]e^{-\rho T}.$$

Assuming that this problem has a solution,[11] the value function of the cooperative game is denoted as:

$$V_{\mathrm{C}}(x_0) = \int_0^T [F^d(0,0) + F^r(0,0,x_{\mathrm{C}},s_{\mathrm{C}}) - C^d(I^d_{\mathrm{C}}) - C^r(I^r_{\mathrm{C}})]e^{-\rho t} dt + SC(x_{\mathrm{C}}(T)), \qquad (10)$$

where $s_{\mathrm{C}}(t)$, $I^d_{\mathrm{C}}(t)$ and $I^r_{\mathrm{C}}(t)$ are the optimal investment paths of the cooperative game (**??**)-(**??**), and $x_{\mathrm{C}}(t)$ the cooperative stock of usable water capacity which solves equation (**??**) for the optimal investment path $s_{\mathrm{C}}(t)$. The level $x_{\mathrm{C}}(T)$, reached by this stock when the players cooperate from 0 till moment T at which the canal is finished, also defines the starting stock of usable water capacity for the subsequent non-cooperative water market game. Note that this value function collects the accumulated payoff of the two regions within the first cooperative period, plus what they get within the subsequent non-cooperative game, included in the scrap value $SC(x(T))$.

A necessary conditions for the two players to agree to cooperate in the joint investment program is:

[11] For the lineal quadratic case, the solutions is computed in the Appendix.

Condition 2 *Overall global rationality or Kaldor-Hicks efficiency:*

$$V_C(x_0) > V^d_{NT}(x_0) + V^r_{NT}(x_0). \tag{11}$$

The accumulated payoffs globally for the two players if they coordinate their efforts and build a canal within a first period utilized henceforth, $V_{\mathrm{C}}(x_0)$, must be greater than the addition of the accumulated payoffs for the two regions if the canal is never initiated and no transfer ever take place, $V^d_{\mathrm{NT}}(x_0) + V^r_{\mathrm{NT}}(x_0)$.

Overall global rationality is a requirement for the agreement to arise. However, it cannot guarantee that an agreement for the joint investment project is reached and maintained until its finalization. Next section presents a mechanism which guarantees the time consistency of the cooperative solution. This property ensures the formation of a coalition to build the canal, and guarantees that no region has an incentive to deviate from the cooperative solution at any time before the completion of the canal.

3 Time consistency of the cooperative solution

After defining some notation and recalling the different definitions of rationality, this section first proposes a sharing rule and an associated side-payment to guarantee that globally, each region is better off if the canal is built. Secondly, this sharing rule is extended to an imputation distribution procedure to guaranty time consistency. This imputation is feasible based on an instantaneous side-payment scheme.

Next we define the instantaneous welfare for the two regions under the assumption that they behave optimally. First we consider the no transfer scenario (subscript NT), in which no water transfer ever takes place. Then, if players agree to jointly invest to build the canal, there is a fist period $[0, T)$, of cooperation (subscript C), in which both regions invest to build the canal, followed by a second period from T on (no subscript), when the water transfer is available and a water market is established.

$$w^d_{\mathrm{NT}} = F^d(0,0), \quad w^r_{\mathrm{NT}}(t) = F^r(0,0,x_{\mathrm{NT}}(t), s_{\mathrm{NT}}(t)), \quad \forall t \geq 0, \tag{12}$$

$$w^d_{\mathrm{C}}(t) = F^d(0,0) - C^d(I^d_{\mathrm{C}}(t)), \quad w^r_{\mathrm{C}}(t) = F^r(0,0,x_{\mathrm{C}}(t), s_{\mathrm{C}}(t)) - C^r(I^r_{\mathrm{C}}(t)), \quad \forall t \in [0,T], \tag{13}$$

$$w^d(t) = F^d(p(t), \tau(t)), \quad w^r(t) = F^r(p(t), \tau(t), x(t), s(t)), \quad \forall t > T, \tag{14}$$

All control and state variables are at their corresponding optimal values. Further, the price and quantity of the water market are also at their equilibrium values.

Remark 1 *In the no transfer scenario, the donor enjoys the constant environment amenities linked with a pristine river, while the recipient, with no possibility to get water from the donor, invests in its stock of water capacity.*

By contrast, if the regions agree to build the canal, within the first period of joint investment, the donor still enjoys the full environmental amenities and has to pay the investment costs of the infrastructure. The recipient also bears the investment cost associated with the construction of the canal, while its incentive to invest in the stock of water capacity is reduced by the expectation of future transfers. Finally, when the canal is built, the donor transfers water to the recipient at a price fixed by their dynamic interaction in the water market.

At any time t wihin the cooperative period $[0,T]$,we may define the payoffs to go when cooperating regions decide either to continue with cooperation or alternatively to suspend the agreement indefinitely. If regions maintain cooperation and finish the canal at T, allowing the transfer of water from this time on, under bilateral trade, the payoffs to go from this time $t \in [0,T]$ on, and for each region region $i \in \{d,r\}$ would read:

$$W^i(t) = \int_t^T w_{\mathrm{C}}^i(u)e^{-\rho(u-t)}du + \int_T^\infty w^i(u)e^{-\rho(u-T)}du,$$

or equivalently:

$$W^i(t) = \int_t^T w_{\mathrm{C}}^i(u)e^{-\rho(u-t)}du + V^i(x_{\mathrm{C}}(T))e^{-\rho(T-t)}, \quad \forall t \in [0,T],\ \forall i \in \{d,r\}. \tag{15}$$

Alternatively, regions might cease cooperation at this time t before the canal is finished, and stick to their no-transfer strategies henceforth (no water trading ever takes place). The payoffs to go starting at this time t would read:

$$W_{\mathrm{NT}}^i(t) = \int_t^\infty w_{\mathrm{NT}}^i(u;t)e^{-\rho(u-t)}du, \quad \forall t \in [0,T],\ \forall i \in \{d,r\}.$$

Or equivalently:

$$W_{\mathrm{NT}}^i(t) = \int_t^T w_{\mathrm{NT}}^i(u;t)e^{-\rho(u-t)}du + V_{\mathrm{NT}}^i(x_{\mathrm{NT}}(T;t))e^{-\rho(T-t)}, \quad \forall t \in [0,T],\ \forall i \in \{d,r\}. \tag{16}$$

with $w_{\mathrm{NT}}^d(u;t) = w_{\mathrm{NT}}^d$ constant, and $w_{\mathrm{NT}}^r(u;t) = F^r(0,0,x_{\mathrm{NT}}(u;t),s_{\mathrm{NT}}(u;t))$.

Assuming that the two regions have agreed to cooperate from the beginning to time t prior to the completion of the canal, they have been investing I_{C}^d and I_{C}^r in the canal and s_{C} in the stock of usable water capacity. Hence, this stock has reached level $x_{\mathrm{C}}(t)$. If they maintain cooperation the optimal investment in usable water capacity is still given by s_{C} and this stock evolves to reach $x_{\mathrm{C}}(T)$ when the canal is finished. Conversely, if they halt cooperation, the optimal investment in this stock is now driven by s_{NT} and the stock evolves differently. Its initial value is given by $x_{\mathrm{C}}(t)$, and for that reason the optimal path followed by this stock in this second scenario will depend on the time t, when cooperation ended as well as the current time after defection $x_{\mathrm{NT}}(u;t)$, $u \geq t$. Obviously, at time T, the values $x_{\mathrm{NT}}(T;t)$ and $x_{\mathrm{C}}(T)$ are not necessarily the same.

Taking into account these definitions, at any time $t \in [0,T]$ we can define the surplus to go from cooperation from this time on for player i as:

$$S^i(t) = W^i(t) - W_{\mathrm{NT}}^i(t), \quad \forall t \in [0,T],\ \forall i \in \{d,r\}. \tag{17}$$

And the total surplus to go from cooperation from this time on as:

$$S(t) = W^d(t) + W^r(t) - W^d_{\mathrm{NT}}(t) - W^r_{\mathrm{NT}}(t) \quad \forall t \in [0,T]. \tag{18}$$

3.1 Overall rationality

With the notation above, Condition **??** can be written as $W^i(T) > W^i_{\mathrm{NT}}(T)$ or $S^i(T) > 0$ for $i \in \{d,r\}$ and $x_{\mathrm{C}}(T)$ in a nonempty set. Correspondingly Condition **??** of overall global rationality can be written as $W^d(0) + W^r(0) > W^d_{\mathrm{NT}}(0) + W^r_{\mathrm{NT}}(0)$ or $S(0) > 0$. The total investment costs of building the channel plus the environmental costs associated with the water transfer must be lower than the aggregate gains from the water transfer, which is equivalent as having a positive total surplus from cooperation. However, overall global rationality is not sufficient for cooperation, overall individual rationality is further required:

$$W^i(0) > W^i_{\mathrm{NT}}(0) \Leftrightarrow S^i(0) > 0, \quad \forall i \in \{r,d\}. \tag{19}$$

In general this condition is not fulfilled. Because we are assuming Condition **??** of overall global rationality, at least one of the regions is better off if the canal is jointly built (its opponent is worse off). We assume from now on that player i is worse off, while player $-i$ is better off:

$$S^i(0) \equiv W^i(0) - W^i_{\mathrm{NT}}(0) < 0, \qquad S^{-i}(0) \equiv W^{-i}(0) - W^{-i}_{\mathrm{NT}}(0) > 0. \tag{20}$$

Then, according to the Kaldor-Hicks compensation criterion, it is always possible to define a compensation payment or a side-payment from region $-i$ to region i, in the scenario in which the canal is built, that leaves everyone as well off and at least one better off as under the scenario in which the project is not implemented. Thus, the project leads to a Pareto improvement. The more straightforward and often used side-payment is to follow the egalitarian principle[12]. As equation (**??**) shows, the game analyzed here is particular in the sense that cooperation leads to immediate losses (investment costs to build the canal), and it is followed by a second non-cooperative period which will determine each player's gains from the existence of the canal. Instead of the egalitarian principle, our proposal is that each player contribution to the construction of the canal should be determined by his share in the total gains once the canal is operative. Thus, we define the i-th share in the total gains which stem from the canal as:

$$\phi^i = \frac{V^i(x_{\mathrm{C}}(T)) - V^i_{\mathrm{NT}}(x_{\mathrm{C}}(T))}{V^i(x_{\mathrm{C}}(T)) + V^{-i}(x_{\mathrm{C}}(T)) - V^i_{\mathrm{NT}}(x_{\mathrm{C}}(T)) - V^{-i}_{\mathrm{NT}}(x_{\mathrm{C}}(T))} = \frac{S^i(T)}{S(T)}, \quad i \in \{d,r\} \tag{21}$$

[12]In this case the side-payment is defined as a payment from $-i$ to i equal to half the surplus gained by $-i$ plus half the losses suffered by i when the canal is jointly built. This guarantees that each player attains an equal share of the global surplus from cooperation.

¿From Condicion **??** immediately follows that $\phi^i \in (0,1)$. This fraction ϕ^i is defined as the surplus for player i divided by the total surplus associated with the existence of the canal. From this definition, it is immediately obvious that $W^i(T) = W^i_{\mathrm{NT}}(T) + \phi^i S(T)$. This share is computed by comparing accumulated gains when the canal exists with accumulated gains if a canal is never built. Therefore, it does not take into consideration the costs incurred within the cooperating period. Our main idea is to define a global side-payment from player $-i$ to i in order to guarantee that this equation is also valid at the initial time. Each player's payoff to go under cooperation, including the side-payment, $v^i(0)$, must equate his payoffs to go in the case of no cooperation and no transfer plus a share of the total surplus to go at time 0. With this share given by the players gains attached to the existence of the canal, ϕ^i:

$$v^i(0) = W^i(0) + SD = W^i_{\mathrm{NT}}(0) + \phi^i S(0), \tag{22}$$

$$v^{-i}(0) = W^{-i}(0) - SD = W^{-i}_{\mathrm{NT}}(0) + \phi^{-i} S(0). \tag{23}$$

While $S(T)$ represents the surplus associated with the existence of the canal, $S(0)$ takes into account the gains from the existence of the canal, but also the investment costs within the first cooperative period. By construction, at time T each player receives what he would have gained without the canal, plus a different share of the global surplus from the existence of the canal. According to expressions (**??**)-(**??**) this statement must be equally valid at the beginning of the cooperative agreement. Thus taking into account all investment costs to build the canal, it must still be true that each agent gets what he would have gotten without the canal plus a share from the global surplus to go, identical to the share whe the canal is finished. In consequence, each player must contribute according to its future benefits stemming from the canal.

¿From (**??**)-(**??**) the global side-payment immediately follows:

$$SD = \phi^i[W^{-i}(0) - W^{-i}_{\mathrm{NT}}(0)] + \phi^{-i}[W^i_{\mathrm{NT}}(0) - W^i(0)], \quad i \in \{r, d\}, \tag{24}$$

defined as the addition of the i-th share of the total gains of region $-i$ plus the $-i$-th share of the losses of region i.

Corollary 2 *By labeling the donor as i and the recipient as $-i$, the side-payment from recipient to donor would read:*

$$SD = \phi^d[W^r(0) - W^r_{NT}(0)] + \phi^r[W^d_{NT}(0) - W^d(0)]. \tag{25}$$

Might the recipient had losses and the donor gains, then SD would be negative, and would represent a side-payment from donor to recipient.[13]

[13]We have assumed $S^i(0) < 0$ and $S^{-i}(0) > 0$, and hence the side-payment goes from player $-i$ (who is better

This subsection has presented one possible rule to share the global surplus from cooperation. The associated side-payment guarantees overall individual rationality. However, in a dynamic context, overall individual rationality is not sufficient for cooperation. Further, the cooperative solution must be time consistent. The basic idea behind the global side-payment will be utilized in the following subsection to distribute the side-payment as a continuous stream from 0 and until the canal is finished, in such a way that time consistency is attained.

The following subsection extends the sharing rule, ϕ^i, to decompose the total cooperative payoff between regions and over the time interval $[0, T]$ in such a way to guarantee time consistency. We also present an instantaneous side-payment scheme which makes this distribution possible.

3.2 Time consistency

The cooperative solution is time consistent if individual rationality is satisfied in every subgame starting at any time along the cooperative solution.

Definition 1 *The cooperative solution with no prior side-payment would be time consistent if*

$$S^i(t) \equiv W^i(t) - W^i_{NT}(t) \geq 0, \quad \forall t \in [0, T], \ \forall i \in \{d, r\}.$$

This is clearly not the case since in (**??**) we are assuming that one of the players is initially worse off under cooperation. However, even if individual overall rationality were satisfied by both players, time consistency would not be guaranteed. More conditions are needed.

Condition **??** or Kaldor-Hicks efficiency assumption implies $S(0) > 0$. An additional necessary condition for time consistency is a non-negative surplus to go at any time $t \in [0, T)$, assumed henceforth:

Condition 3

$$S(t) = W^d(t) + W^r(t) - W^d_{NT}(t) - W^r_{NT}(t) \geq 0 \quad \forall t \in [0, T].$$

Given the specification of the problem, investment costs are distributed along the initial cooperative period. By contrast, the benefits from the existence of the canal arise when cooperation ends. As times runs within the interval $[0, T]$ costs are being paid while the benefits are yet to come. It is then very likely[14] that the surplus to go increases with time and hence $S(0) > 0$ should imply $S(t) > 0$ for any $t \in [0, T]$. If it is initially beneficial to build the canal, $S(0) > 0$, it seems plausible that it will be beneficial at any intermediate time to

off with the cooperative agreement) to player i (who is worse off). The same analysis is valid if instead $S^i(0) > 0$ and $S^{-i}(0) < 0$.

[14] Although it has not been proved.

continue with the construction of the canal. Therefore, Condition **??** can be regarded as either irrelevant or not much more demanding than Condition **??**.

In order to define a side-payment scheme within interval $[0,T]$, we first decompose the total players' payoffs to go under cooperation after the side-payment, $v^i(0)$, as the value function of each region at time T when the canal is finished, plus a stream of payoffs from 0 to T. This latter is denoted in the literature as an imputation distribution procedure (IDP), $\pi^i(t)$, $\forall t \in [0,T]$, which verifies:

$$v^i(0) = \int_0^T \pi^i(u)e^{-\rho u}du + V^i(x_{\mathrm{C}}(T))e^{-\rho T}, \quad \forall i \in \{d,r\}. \tag{26}$$

Definition 2 *Let us define the payoffs to go for an IDP, $\pi^i(u)$, as:*[15]

$$v^i(t) = \int_t^T \pi^i(u)e^{-\rho(u-t)}du + V^i(x_C(T))e^{-\rho(t-T)}, \quad \forall t \in [0,T], \ \forall i \in \{d,r\}, \tag{27}$$

assuming that cooperation has lasted till t, and $x_C(T)$ is the stock of water capacity at the time, T, when the canal becomes operative. This IDP is time consistent under condition:

$$v^i(t) = W^i_{NT}(t) + \phi^i S(t), \quad \forall t \in [0,T], \ \forall i \in \{d,r\}. \tag{28}$$

While the payoffs to go are defined in (**??**), time consistency requires that they satisfy (**??**). The proof that there exists an imputation distribution procedure which satisfies the two definitions for the payoffs to go in (**??**) and (**??**) is split in two parts. First, we prove that $v^i(T)$ satisfies both definitions. And second we prove that[16] $\dot{v}^i(t)$ coincide under the two expressions for any $t \in [0,T)$.

Lemma 3 *Expressions (**??**) and (**??**) evaluated at T, both lead to $v^i(T) = V^i(x_C(T))$.*

Proof. From (**??**) it is immediately obvious. From (**??**) it follows:

$$v^i(T) = W^i_{\mathrm{NT}}(T) + \phi^i S(T) = V^i_{\mathrm{NT}}(x_{\mathrm{C}}(T)) + \frac{V^i(x_{\mathrm{C}}(T)) - V^i_{\mathrm{NT}}(x_{\mathrm{C}}(T))}{S(T)} S(T) = V^i(x_{\mathrm{C}}(T)),$$

for all $i \in \{d,r\}$. ■

Proposition 4 *The values of $\dot{v}^i(t)$ for all $t \in [0,T)$ under the two definitions (**??**) and (**??**) are identical under the IDP:*

$$\pi^i(t) = w^i_{NT}(t) + \phi^i s(t) + \phi^i \Theta^{-i}(t) - \phi^{-i}\Theta^i(t), \quad \forall t \in [0,T), \ i \in \{r,d\}, \tag{29}$$

with $s(t) = w^i_C(t) + w^{-i}_C(t) - w^i_{NT}(t) - w^{-i}_{NT}(t)$, the instantaneous surplus from cooperation at time t (typically negative), and $\Theta^i(t) = \int_t^T \dot{w}^i_{NT}(u;t)e^{-\rho(u-t)}du + \left(V^i_{NT}\right)'_{x_{NT}} \dot{x}_{NT}(T;t)e^{-\rho(T-t)}$.

[15]See, for example, Zaccour (2007).

[16]A dot always represents the derivative w.r.t. t.

Proof. Computing the time derivatives in expressions (**??**) and (**??**) we get:

$$\dot{W}^i(t) = -w^i_{\mathrm{C}}(t) + \rho W^i(t),$$
$$\dot{W}^i_{\mathrm{NT}}(t) = -w^i_{\mathrm{NT}}(t) + \rho W^i_{\mathrm{NT}}(t) + \int_t^T \dot{w}^i_{\mathrm{NT}}(u;t) e^{-\rho(u-t)} du + \left(V^i_{\mathrm{NT}}\right)'_{x_{\mathrm{NT}}} \dot{x}_{\mathrm{NT}}(T;t) e^{-\rho(T-t)}. \quad (30)$$

And hence,

$$\dot{S}(t) = -s(t) + \rho S(t) - \left[\left(V^i_{\mathrm{NT}}\right)'_{x_{\mathrm{NT}}} + \left(V^{-i}_{\mathrm{NT}}\right)'_{x_{\mathrm{NT}}}\right] \dot{x}_{\mathrm{NT}}(T;t) e^{-\rho(T-t)} - \int_t^T \left(\dot{w}^i_{\mathrm{NT}}(u;t) + \dot{w}^{-i}_{\mathrm{NT}}(u;t)\right) e^{-\rho(u-t)} du$$

Taking this into account, the time derivatives of expressions (**??**) and (**??**) read:

$$\dot{v}^i(t) = -\pi^i(t) + \rho v^i(t), \quad (31)$$
$$\dot{v}^i(t) = -w^i_{\mathrm{NT}}(t) + \rho v^i(t) - \phi^i s(t) + \left[\phi^{-i}\left(V^i_{\mathrm{NT}}\right)'_{x_{\mathrm{NT}}} - \phi^i \left(V^{-i}_{\mathrm{NT}}\right)'_{x_{\mathrm{NT}}}\right] \dot{x}_{\mathrm{NT}}(T;t) e^{-\rho(T-t)}$$
$$-\phi^i \int_t^T \dot{w}^{-i}_{\mathrm{NT}}(u;t) e^{-\rho(u-t)} du + \phi^{-i} \int_t^T \dot{w}^i_{\mathrm{NT}}(u;t) e^{-\rho(u-t)} du. \quad (32)$$

And these two expressions equate for the IDP in (**??**) ■

Corollary 5 *By labeling the donor as i and the recipient as $-i$, the IDP can be written as:*

$$\pi^d(t) = w^d_{NT}(t) + \phi^d s(t) + \phi^d \Theta^r(t), \quad \forall t \in [0,T), \quad (33)$$
$$\pi^r(t) = w^r_{NT}(t) + \phi^r s(t) - \phi^d \Theta^r(t), \quad \forall t \in [0,T). \quad (34)$$

For the recipient $\Theta^r(t)$ represents the effect of a marginal delay in defection (or a marginal extension of cooperation), on the path of investment on usable water capacity and, in consequence, on the evolution of the stock of usable water capacity and hence, on the flow of instantaneous welfare of the recipient from this time t (of defection) on. One might expect that, as cooperation last longer, (t increases approaching T), the cooperative stock of water capacity constituted under cooperation would rise higher, which would reduce the incentive to invest in this stock in the after-defection-no-transfer scenario from t to T. This will reduce the costs of investment within this period. Conversely, a shorter non-cooperative period, $T-t$, would imply a smaller stock of water capacity at time T, which would imply a reduction in the payoffs to go from this moment on.[17]

As for the donor, since he has no decision to make after the moment t when cooperation stops, logically, there will be no marginal effects associated with delays in the time of defection, $\Theta^d(t) = 0, \forall t \in [0,T)$.

An crucial result from the definition of the IDP in (**??**) is that $\pi^i(t) + \pi^{-i}(t) = w^i_{\mathrm{C}}(t) + w^{-i}_{\mathrm{C}}(t)$. The instantaneous payoffs provided by the IDP for the two players matches the instantaneous joint cooperative payoff at any time $t \in [0,T)$. In consequence an instantaneous

[17]The complete effect of a marginal delay in defection on the payoffs to go for region i has two other components as shown in (**??**). The payoffs to go are reduced in the instantaneous payoff corresponding to time t, $-w^i_{\mathrm{NT}}(t)$, (if fact they are increased because the instantaneous payoff is typically negative). Secondly, the initial time t moves forward, payoffs to go are valued later and then the value increases by $\rho W^i_{\mathrm{NT}}(t)$.

or continuous side-payment from country $-i$ to country i, $sd(t)$, can be defined from 0 to T in such a way to guarantee that each country gets the instantaneous payoff defined by the IDP in (**??**).

Proposition 6 *The instantaneous side-payment from player $-i$ to player i is given by*

$$sd(t) = \phi^i[w_C^{-i}(t) - w_{NT}^{-i}(t)] + \phi^{-i}[w_{NT}^i(t) - w_C^i(t)] + \phi^i\Theta^{-i}(t) - \phi^{-i}\Theta^i(t). \quad (35)$$

which satisfies that[18] $\pi^i(t) = w_C^i(t) + sd(t)$ *and* $\pi^{-i}(t) = w_C^{-i}(t) - sd(t)$ *for all* $t \in [0, T)$.

Corollary 7 *Labeling the donor by i and the recipient by $-i$, then the instantaneous side-payment from recipient to donor can be written as:*

$$sd(t) = \phi^d[w_C^r(t) - w_{NT}^r(t)] + \phi^r[w_{NT}^d(t) - w_C^d(t)] + \phi^d\Theta^r(t), \quad \forall t \in [0, T). \quad (36)$$

A negative value of sd(t) would represent a side-payment from donor to recipient.

We can distinguish two parts in the instantaneous side-payment in (**??**). The first part is the addition of the donor's share, ϕ^d, of the recipient instantaneous gains from cooperation (typically negative due to the investment costs to build the canal) and the recipient's share, ϕ^r, of the donor instantaneous saving if they do not cooperate to build the canal (typically positive). This part corresponds to the instantaneous version of the global side-payment, SD, in (**??**). The only difference being that, in overall terms, the construction of the canal and its ulterior utilization are assumed to be beneficial for the recipient and detrimental for the donor; however in instantaneous terms, countries incur in costs throughout the cooperative period when the canal is being built. Therefore, the instantaneous payoffs under cooperation are typically lower than under no transfer.

The second part of this instantaneous side-payment is given by the donor's share of the marginal effect of a delay in defection on the stream of instantaneous payoffs for the recipient (assuming that cooperation ends before the construction of the canal), $\phi^d\Theta^r(t)$.

An alternative expression for the instantaneous side-payment in (**??**) would be:

$$sd(t) = \phi^d\left[\dot{W}_{\mathrm{NT}}^r(t) - \rho W_{\mathrm{NT}}^r(t) - \left(\dot{W}^r(t) - \rho W^r(t)\right)\right] + \phi^r\left[\dot{W}^d(t) - \rho W^d(t) - \left(\dot{W}_{\mathrm{NT}}^d(t) - \rho W_{\mathrm{NT}}^d(t)\right)\right] \quad (37)$$

Note that $\dot{W}_{\mathrm{NT}}^i(t) - \rho W_{\mathrm{NT}}^i(t)$ and $\dot{W}^i(t) - \rho W^i(t)$ denote the temporal evolution of the payoffs to go, not linked to discounting under either the no transfer solution or the cooperative scenario, at time $t \in [0, T)$. Thus, the first term in brackets in (**??**) represents the gap in the temporal evolution of the recipient's payoffs to go under no transfer and cooperation. That is how much faster the recipient's payoffs to go increase under the no transfer case than under cooperation.[19] Correspondingly, the second term in brackets shows how much faster

[18] Negative values of $sd(t)$ would represent side-payments from player i to player $-i$.

[19] In fact, one might expect that the payoffs to go would decrease under the no transfer solution and increase under cooperation. Hence, this term is likely to be negative.

the donor's payoffs to go are increased in the cooperative scenario than in the no transfer solution (a likely positive term).

4 Application: specific functional forms

The dynamic of the usable water capacity in (**??**) is linear. Therefore, to have an analytically tractable problem we consider here the following quadratic form for the instantaneous profit function, as in Cabo *et al.* (2014). For the donor,

$$F^d(p,\tau) = E^d(\tau) + p\tau = c\left(R - \frac{\tau^2}{2R}\right) + p\tau, \quad c > 0,$$

with $R > 0$ the constant water surplus in the donor's river basin. In the absence of water transfer, environmental amenities increase linearly with the water surplus: cR. Further, we consider that the marginal reduction in environmental amenities is inversely proportional to this surplus, while proportional to the water transferred: $c\tau(t)/R$.

For the recipient, output increases with at a decreasing rate the total quantity of water (either the usable water capacity or the transfer). This is represented by the quadratic term in $\tau + x$. Further, we also consider quadratic costs of investment in usable water capacity.

$$F^r(p,\tau,x,s) = d\left(\tau + x - \alpha\frac{(\tau+x)^2}{2}\right) - p\tau - \beta\frac{s^2}{2} \quad d,\alpha,\beta > 0.$$

For illustration purposes, we consider parameter's values[20] which guarantee conditions **??**, **??** and **??** under which both players are better off if a canal exists, the project to build the canal improves overall global welfare, and in fact, the surplus to go from cooperation is positive at any time within cooperation. The equilibrium for the non-cooperative game can be found in Cabo *et al.* (2014), and the cooperative solution is presented in the Appendix. For the chosen parameters we obtain:

$$T^* = 30.76,\ x_{\mathrm{C}}^T = 47.97,\ \phi^d = 0.344,\ \phi^r = 0.656, S(0) = V_{\mathrm{C}}(x_0) - \left(V_{\mathrm{NT}}^d(x_0) + V_{\mathrm{NT}}^r(x_0)\right) = 1972.79.$$

However, although we assume overall global rationality, we also consider that cooperation increases the recipient's accumulated welfare, $S^r(0) = V_{\mathrm{C}}^r(x_0) - V_{\mathrm{NT}}^r(x_0) = 2048.06$, but decreases the donor's accumulated welfare $S^d(0) = V_{\mathrm{C}}^d(x_0) - V_{\mathrm{NT}}^d(x_0) = -75.26$. In consequence, without side-payment, this latter would not agree to cooperate to build the canal.

The surplus to go under cooperation is depicted in Figure **??**, which shows that Condition **??**, $S(0) > 0$, and **??**, $S(t) \geq 0$, for all $t \in [0, T]$, are satisfied.

Figure **??** displays the payoffs to go if the countries accept cooperation, W^i, and under the no transfer scenario (when no canal is ever built) W_{NT}^i. This figure shows that at the

[20] $c = 4.7084$, $d = 2.1275$, $\rho = 0.001$, $\delta = 0.1$, $R = 593.67$, $\alpha = 0.0135$, as in Cabo *et al.* (2014). And $x(0) = 0$, $z^d = z^r = 10\ K_0 = 0, \bar{K} = 100$.

beginning of the cooperative period, the donor's payoffs to go would be higher if he deviates from cooperation to the no transfer scenario. Thus, the cooperative solutions is not time-consistent, and would never be implemented unless an adequate mechanism gives the donor an incentive to cooperate.

Figure **??** shows how the payoffs to go are modified if the instantaneous side-payment in (**??**) is implemented. The payoffs to go after the side-payment, $v^i(t)$, correspond to the IDP in (**??**). These are greater than their respective payoffs to go under the alternative no transfer scenario in case of defection. Therefore, this IDP is time consistent. Countries will stick to their cooperative investment strategies and the canal will be finished.

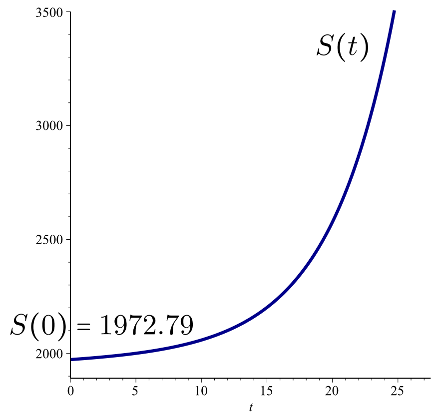

Figure 1: Surplus to go at t.

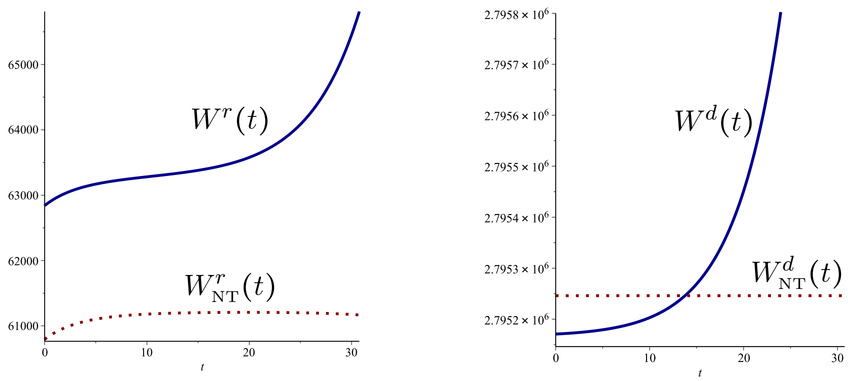

Figure 2: Payoff to go at t, under cooperation vs. no transfer.

5 Conclusions

This paper studies the dynamic interaction between two countries with differences in precipitation rates and water productivity. Water inflows are relatively higher in the river basin

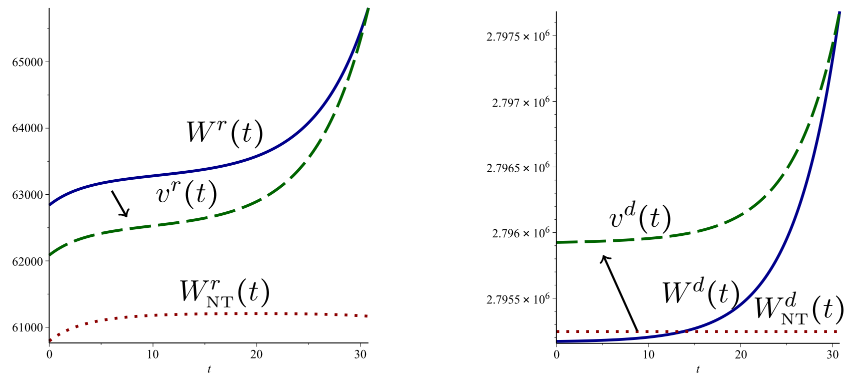

Figure 3: Payoff to go after side-payment at t, under cooperation vs. no transfer.

located in the country labeled as the donor than in the recipient country. After satisfying local demand the donor still enjoys a flow of water surplus (assumed constant for simplicity). Although this surplus is not utilized in the productive process, it is valuable because it helps to preserve the environmental amenities provided by the river. Correspondingly, water is highly productive in the recipient economy (one might think of different uses like irrigation, industry, municipal water supply, etc.) but the river basin in this country suffers from a chronic shortage of water. This country has the possibility to invest in alternative water supplies like desalination plants, water savings or water recycling. Might the donor's river basin transfer part of its water surplus to the recipient's, overall productivity could be increased. Thus, the existence of a canal allowing this water transfer would create a water market increasing overall productivity. The recipient would be willing to pay a price for the water its economy demands. Correspondingly, the donor would be willing to accept a price for the excess water transferred, as long as this price offsets the environmental losses associated with the transfer. Therefore, if the productivity of water in the recipient's economy is sufficiently high with respect to the losses in the environmental amenities provided by the river in the donor's country, then the existence of the canal, and the water market it makes possible, may signify a gain for both regions. Nonetheless, there is no reason for these gains to be necessarily symmetric.

This paper has addressed a question previous to the existence of the canal: Assuming that the existence of the canal involves an asymmetric win-win situation for the two regions, under which conditions would they agree to a first period of cooperation in a joint investment project to build the canal? To analyze this question, we define a two-regimes differential game between the two countries. Within a second period, when the canal exists, a non-cooperative differential game of infinite duration describes the water market they are engaged in. However, for the canal to exist, they must play a cooperative differential game within a first period to jointly build the canal.

This is an example of a cooperative game in which the benefits from cooperation only come at the end of the game, when cooperation stops. Cooperation does not imply an immediate reward but a burden: the investment costs of building the canal. Further, the asymmetric benefits from the cooperation are not defined by the cooperative agreement, but they are given by a subsequent non-cooperative differential game describing the water market made possible by the canal. Therefore, the standard question in cooperative game theory on how to share the gains from cooperation, is changed to a new question: how to share the burden from cooperation taking into account future uneven gains? This sharing rule must guarantee that the canal is actually finished. That is, it must be defined in such a way that the cooperative solution satisfies the property of time consistency.

We assume first that the surplus from cooperation is positive (initially and at any time within the cooperative period), and second that both players gain with the existence of the canal. In overall terms (for the whole time period) we propose a sharing rule at the initial time (when they start building the canal) which assigns each country its overall gains in the case of no transfer, plus a share of the global surplus from cooperation. This share is defined by the country's share in total gains from the existence of the canal (attained at time T when the canal is finished). A global side-payment is defined to guarantee that this share is attained.

This sharing rule is extended from overall to instantaneous terms. Thus we define an imputation distribution procedure, which satisfies that at any time within cooperation each country's payoffs to go equate what it would get with no transfer plus the previously-proposed share of the global surplus to go. For this sharing rule we are able to define the instantaneous payoffs each country should get and, more importantly, an instantaneous side-payment under which the sharing mechanism is fulfilled. Thus, by definition, at any time within the cooperative period, every country is better off by continuing with cooperation than by defecting, and hence the agreement is time consistent.

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6 Appendix

6.1 Solving a LQ game when cooperation is maintained

To find a solution to the problem described in (**??**), (**??**) and (**??**) we define the Hamiltonian:

$$H(x,K,s,I^r,I^d,\lambda,\mu) = F^d(0,0) + F^r(0,0,x,s) - C^d(I^d) - C^r(I^r) + \lambda(s - \delta x) + \mu(I^d + I^r).$$

Assuming quadratic costs, first order conditions give:[21]

$$\lambda = \beta s, \quad \mu = z^d I^d = z^r I^r,$$
$$\dot{K} = \tfrac{z^d+z^r}{z^d z^r}\mu,$$
$$\dot{\mu} = \rho\mu,$$
$$\dot{x} = \tfrac{\lambda}{\beta} - \delta x,$$
$$\dot{\lambda} = (\rho+\delta)\lambda - d(1-\alpha x),$$

$$H(x(T), K(T), s(T), (I^r)(T), (I^d)(T), \lambda(T), \mu(T)) - \rho[V^d(x(T)) + V^r(x(T))] = 0. \quad (38)$$

The system of differential equations in x, λ

$$\begin{pmatrix} \dot{x} \\ \dot{\lambda} \end{pmatrix} = \begin{pmatrix} -\delta & 1/\beta \\ \alpha d & \rho+\delta \end{pmatrix} \begin{pmatrix} x \\ \lambda \end{pmatrix} + \begin{pmatrix} 0 \\ -d \end{pmatrix}, \quad (39)$$

can be solved separately, obtaining:

$$\begin{pmatrix} x \\ \lambda \end{pmatrix} = C_1 \begin{pmatrix} \xi_x^1 \\ \xi_\lambda^1 \end{pmatrix} e^{r_1 t} + C_2 \begin{pmatrix} \xi_x^2 \\ \xi_\lambda^2 \end{pmatrix} e^{r_2 t} + \begin{pmatrix} x_p \\ \lambda_p \end{pmatrix},$$

where

$$x_p = \frac{d}{\alpha d + \beta\delta(\rho+\delta)}, \quad \lambda_p = \frac{d\beta\delta}{\alpha d + \beta\delta(\rho+\delta)}$$

and $\bar{\xi}^1 = (\xi_x^1, \xi_\lambda^1)$ and $\bar{\xi}^2 = (\xi_x^2, \xi_\lambda^2)$ are the eigenvectors associated to eigenvalues:

$$r_1 = \frac{\beta\rho - \sqrt{\beta(\rho+\delta)^2 + 4\alpha d\beta}}{2} < 0, \quad r_2 = \frac{\beta\rho + \sqrt{\beta(\rho+\delta)^2 + 4\alpha d\beta}}{2} > 0$$

Together with equation (**??**) conditions for this problem are: $x(0) = x_0$, $\lambda(T) = \frac{\partial SC}{\partial x}(x(T))$. Taking into account a lineal-quadratic differential game as described in Cabo *et al.* (2014), defining the water market after the finalization of the canal, the value functions for donor and recipient will be second order polynomials: $V^i(x) = a^i x^2/2 + b^i x + c^i$, $\forall i \in \{d, r\}$. Therefore, $(\partial SC/\partial x)(x(T)) = \partial[V^d(x(T)) + V^r(x(T))]/\partial x$ can be written as $ax(T)+b$, where $a = a^d + a^r$ and $b = b^d + b^d$, then these two conditions can be written as:

$$C_1\xi_x^1 + C_2\xi_x^2 = x_0 - x_p$$
$$C_1(\xi_\lambda^1 - a\xi_x^1)e^{r_1 T} + C_2(\xi_\lambda^2 - a\xi_x^2)e^{r_2 T} = ax_p - \lambda_p + b,$$

and the constants can be computed as functions of T: $C_{1\mathrm{C}}(T)$ and $C_{2\mathrm{C}}(T)$. For any $t \in [0, T]$, the optimal paths for x and λ would be:

$$x_{\mathrm{C}}(t) = C_{1\mathrm{C}}(T)\xi_x^1 e^{r_1 t} + C_{2\mathrm{C}}(T)\xi_x^2 e^{r_2 t} + x_p \quad (40)$$

$$\lambda_{\mathrm{C}}(t) = C_{1\mathrm{C}}(T)\xi_\lambda^1 e^{r_1 t} + C_{2\mathrm{C}}(T)\xi_\lambda^2 e^{r_2 t} + \lambda_p \quad (41)$$

[21] Similarly, one might consider lineal quadratic investment costs.

Similarly, we can compute the optimal capital stock and its costate:

$$K_{\rm C}(t) = \frac{\bar{K}-k_0}{e^{\rho T}-1}e^{\rho t} + \frac{k_0 e^{\rho T}-\bar{K}}{e^{\rho T}-1},$$
$$\mu_{\rm C}(t) = \frac{z^d z^r}{z^d+z^r}\frac{(\bar{K}-k_0)\rho}{e^{\rho T}-1}e^{\rho t}.$$

And the optimal time T at which the canal must be finished given by equation (**??**)

6.2 Solving a LQ game with defection at time t

We analyze the no water transfer scenario when players have cooperated up to $t \in [0,T)$. This is the standard problem but starting at $(t, x_{\rm C}(t))$. The donor faces no optimization problem, while the recipient solves the problem:

$$\max_s \int_t^\infty \left(d(x - \frac{\alpha}{2}x^2) - \frac{\beta}{2}s^2\right)e^{-\rho u}du, \quad \dot{x} = s - \delta x.$$

The dynamics of the state and co-state variables is identical as in the cooperative problem, stated in (**??**), which solution reads:

$$\begin{pmatrix} x(u) \\ \lambda(u) \end{pmatrix} = C_1 \begin{pmatrix} \xi_x^1 \\ \xi_\lambda^1 \end{pmatrix} e^{r_1(u-t)} + C_2 \begin{pmatrix} \xi_x^2 \\ \xi_\lambda^2 \end{pmatrix} e^{r_2(u-t)} + \begin{pmatrix} x_p \\ \lambda_p \end{pmatrix}, \quad u > t.$$

where

$$x_p = \frac{d}{\alpha d + \beta\delta(\rho+\delta)}, \quad \lambda_p = \frac{d\beta\delta}{\alpha d + \beta\delta(\rho+\delta)}$$

and r_1, r_2, $\bar{\xi}^1$ and $\bar{\xi}^2$, are the eigenvalues and eigenvectors already obtained in the cooperative case. The transversality condition $\lim_{u\to\infty}\lambda(u)x(u)e^{-\rho(u-t)} = 0$, requires $C_2 = 0$. This together with condition $x(t) = x_{\rm C}(t)$ gives $C_1 = (x_{\rm C}(t) - x_p)/\xi_x^1$ and the solutions of x and λ for any time u after t when cooperation halts:

$$x_{\rm NT}(u;t) = (x_{\rm C}(t) - x_p)e^{r_1(u-t)} + x_p, \tag{42}$$
$$\lambda_{\rm NT}(u;t) = (x_{\rm C}(t) - x_p)\frac{\xi_\lambda^1}{\xi_x^1}e^{r_1(u-t)} + \lambda_p. \tag{43}$$

We can now compute the derivative of $x_{\rm NT}(T;t)$ with respect to the time t when cooperation ceases, used in the definition of $\Theta^i(t)$ in Proposition **??**.

$$\frac{d}{dt}(x_{\rm NT}(T;t)) = \dot{x}_{\rm C}(t)e^{r_1(T-t)} - r_1[(x_{\rm C}(t) - x_p]e^{r_1(T-t)}$$
$$= [s_{\rm C}(t) - (r_1+\delta)x_{\rm C}(t) + r_1 x_p]e^{r_1(T-t)} = [\lambda_{\rm C}(t)/\beta - (r_1+\delta)x_{\rm C}(t) + r_1 x_p]e^{r_1(T-t)}$$

where $x_{\rm C}$ and $\lambda_{\rm C}$ are given in (**??**) and (**??**), while T is obtained from (**??**).