Modeling of submerged membrane bioreactors with a view to control

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Abstract: A simple dynamic model of an integrated membrane bioreactor (MBR) is developed for control purposes. This model consists of two parts: (a) a biological model based on one biomass growing on a limiting substrate in a chemostat (b) a model of the cake formation on the membrane. This model is compared in simulation to commonly accepted models. A preliminary analysis of the model is given, including equilibrium points and stability analysis.

Keywords: Mathematical Modeling; Membrane filtration, Bioreactor

INTRODUCTION

In the published literature, numerous articles deal with the detailed physical description of MBR, including aeration, cake formation, filtration and fouling [1], physical and bioprocess description[2] [3] [4]. However these models include many parameters which can be delicate to estimate and are in general too complex for process control. Concerning this latter issue, there are only a few proposals based on empirical approaches [5] or artificial neural network models [6]. The motivation of this study is to derive such a simplified integrated MBR model based on first-principle models and to analyze its dynamical behavior.

PROCESS DESCRIPTION

In this study, two reference models of the cake formation are considered [1] [7], as starting point for our simplified model. The first model has been proposed by Li and Wang [1], and predicts the main trends observed in experimental data. It covers reversible and irreversible cake layer formation, cake compression, pore blocking and the influence of the feed side hydrodynamics.

$$\begin{cases} \dfrac{dM_{sf}}{dt} = \dfrac{24CJ^2}{24J + K_1 G} - \dfrac{\beta(1-\alpha)GM_{sf}^2}{\gamma V_f t + M_{sf}} \\ G = \left(\dfrac{\rho_\omega g q_a}{1.05\mu_\omega e^{0.02C}} \right)^{1/2} \end{cases} \tag{1}$$

The second model is the cake formation model used in the simulation software GPS-X [7] [8] and is expressed by equation (2). It represents the attachment of the biomass on the membrane surface and detachment by backwash cycle and cross-flow aeration.

$$\frac{dm}{dt} = q_{perm} x_{liq} f_{cap} - q_{bw} x_{cake} f_{bw} - \frac{q_{cross}}{A_m} \frac{x_{cake}}{x_{cake} + K_{s,cake}} f_{cross} \tag{2}$$

TOWARDS SIMPLIFIED MODELS

For control purposes, a simplified model of the cake formation is proposed, which will be coupled with a simple dynamic model representing the main biological phenomena (represented by one or two biomasses). Most authors agree on the fact that the most important resistance is the cake resistance [5]. Hence, a possible simplification is to consider that the membrane resistance is mostly due to cake formation.

The proposed cake formation model is considered in agreement with Table 1.1 which shows the relation between the process variables and the cake formation dynamics, as reported in the published literature [1] [2] [7].

Table 1.1: Relation between process variables and cake dynamics $\left(\frac{dm}{dt}\right)$[1][2][7].

Cake Resistance		
	Effect	$\frac{dm}{dt}$
↑	Permeate Cycle	↑
↑	Backwash Cycle	↓
↑	Idle Cycle	↓
↑	MLSS	↑
↑	Crossflow Air	↓
↑	J(m(t))	↑
↑	Pressure	↕
↑	BckWsh Efficiency	↓
↑	Cake Compression	↑
↑	Membrane’s Area	↑

The first term in equation (3) represents the attachment on the membrane surface and contains $J(m(t))[m\,day^{-1}]$, the effluent flow rate, $X[g\,m^{-3}]$ the biomass concentration, and $A[m^2]$ the membrane surface area. The second term represents the cake detachment proportional to crossflow air ($\beta[m^{-1}]$ is the cross flow rate factor, $J_{air}[m\,day^{-1}]$ is the air flow, $m[g]$ is the cake mass and $K_{air}[g]$ is the half-saturation coefficient).

$$\frac{dm}{dt} = J(m)XA - \beta J_{air}\frac{m^2}{K_{air}+m} \qquad (3)$$

The flux $J(m(t))$ is ruled by Darcy’s law, equation (4), where $\Delta P(t)$ is the transmembrane pressure, η is the water viscosity, R_m is the membrane intrinsic resistance, ρ is the specific resistance and m_0 is the initial cake mass.

$$J(m(t)) = \frac{\Delta P(t)}{\eta\left(R_m + \rho\frac{(m(t)+m_0)}{A}\right)} \qquad (4)$$

Having proposed a membrane filtration model, the biological activity is described using a simple chemostat model involving one biomass growing on a limiting substrate.

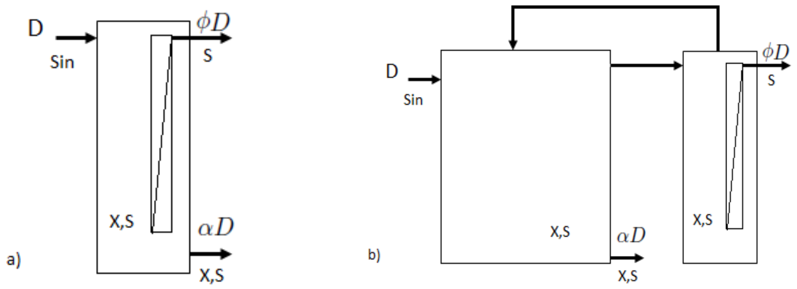

Figure 1.1: a) Simplified membrane bioreactor, $\boldsymbol{\phi}$ is the permeate flux factor and $\boldsymbol{\alpha}$ is the waste factor. b) Different representations for the same model.

The model is described by equations (5) which represent a membrane bioreactor independent of cake resistance, as in Figure 1.1. $D = \frac{Q_{in}}{V}$.

$$\begin{cases} \frac{dS}{dt} = -\frac{1}{Y}\mu(S)X + D(S_{in} - (\alpha+\phi)S) \\ \frac{dX}{dt} = (\mu(S) - \alpha D)X \end{cases} \tag{5}$$

The cake dynamics is then incorporated in this model with $\phi = \frac{J(m(t))A}{V}$.

$$\begin{cases} \frac{dS}{dt} = -\frac{1}{Y}\mu(S)X + \frac{Q_{in}}{V}S_{in} - \frac{Q_{in}}{V}\alpha S - \frac{J(m(t))A}{V}S \\ \frac{dX}{dt} = \left(\mu(S) - \alpha\frac{Q_{in}}{V}\right)X - \frac{J(m(t))A}{V}X + \beta\frac{J_{air}}{V}\frac{m^2}{K_{air}+m} \\ \frac{dm}{dt} = J(m(t))AX - \beta J_{air}\frac{m^2}{K_{air}+m} \end{cases} \tag{6}$$

PRELIMINARY MODEL ANALYSIS

Equation (5) is analyzed and the equilibrium points and Jacobian matrix are computed. The system has two equilibrium points. The washout $\left(\bar{X} = 0; \bar{S} = \frac{S_{in}}{\alpha+\phi}\right)$ and $\left(\bar{X} = \frac{Y(S_{in} - (\alpha+\phi)\bar{S})}{\alpha}; \bar{S} = \frac{\alpha\bar{D}K_s}{\mu_{max} - \alpha\bar{D}}\right)$, assuming that the process is modeled by Monod kinetics $\mu(\bar{S}) = \frac{\mu_{max}\bar{S}}{K_s+\bar{S}}$ and $\mu'(\bar{S}) = \frac{\mu_{max}}{K_s+\bar{S}} - \frac{\mu_{max}\bar{S}}{(K_s+\bar{S})^2}$. The Jacobian matrix is represented as:

$$\begin{pmatrix} \frac{1}{Y}\mu'(\bar{S})\bar{X} - \bar{D}(\alpha+\phi) & -\frac{1}{Y}\mu(\bar{S}) \\ \mu'(\bar{S})\bar{X} & 0 \end{pmatrix} \tag{7}$$

The characteristic polynomial is $\lambda^2 + \left(\frac{1}{Y}\mu'(\bar{S})\bar{X} + \bar{D}(\alpha+\phi)\right)\lambda + \frac{1}{Y}\mu'(\bar{S})\bar{X}\alpha\bar{D}$. A necessary and sufficient condition for a second order polynomial to have roots with all negative real parts is that all the coefficients or the polynomial have the same sign. This implies that $\frac{1}{Y}\mu'(\bar{S})\bar{X} + \bar{D}(\alpha+\phi)$ and $\frac{1}{Y}\mu'(\bar{S})\bar{X}\alpha\bar{D}$ must be positive. This condition will be fulfilled if and only if $0 < \alpha < \frac{S_{in}\mu_{max}}{\bar{D}(K_s+S_{in})}$ and $\phi = 1-\alpha$.

The integrated model equation (6), is analyzed and the equilibrium points are $\bar{S} = \frac{\alpha\bar{D}K_s}{\mu_{max}-\alpha\bar{D}}$, $\bar{X} = \frac{\left(\frac{\beta J_{air}\bar{m}^2}{K_{air}+\bar{m}}\right)}{J(\bar{m})A}$ and $\bar{m}$ is solution of $J(\bar{m})A\bar{X} - \beta J_{air}\frac{\bar{m}^2}{K_{air}+\bar{m}} = 0$. With $J(\bar{m}) = \frac{\Delta P}{\eta\left(R_m + \frac{\rho(\bar{m}+m_0)}{A}\right)}$ and $J(\bar{m})' = -\frac{\Delta P}{\left(\frac{\eta\rho}{A}\right)^2}$, the Jacobian matrix is given by

$$\begin{pmatrix} -\frac{1}{Y}\mu'(\bar{S})\bar{X} - \frac{Q_{in}}{V}\alpha - \frac{J(\bar{m})A}{V} & -\frac{1}{Y}\mu(\bar{S}) & -\frac{J(\bar{m})'A}{V}\bar{S} \\ \mu(\bar{S})'\bar{X} & -\frac{J(\bar{m})A}{V} & -\frac{J(\bar{m})'A}{V}\bar{X} + \beta\frac{J_{air}}{V} + \frac{2\bar{m}(K_{air}+\bar{m}) - \bar{m}^2}{(K_{air}+\bar{m})^2} \\ 0 & J(\bar{m})A & -\beta J_{air} + \frac{2\bar{m}(K_{air}+\bar{m}) - \bar{m}^2}{(K_{air}+\bar{m})^2} \end{pmatrix}$$

(8)

A necessary and sufficient condition for a third order Jacobian to be stable is to have a negative trace and determinant. Hence, to stabilize the cake dynamics J_{air} should be such that $J_{air} > \frac{2\bar{m}(K_{air}+\bar{m}) - \bar{m}^2}{\beta(K_{air}+\bar{m})^2}$ and to prevent the undesired equilibrium (washout), α should be such that $0 < \alpha < \frac{S_{in}\mu_{max}}{\bar{D}(K_s+S_{in})}$.

The model of Li and Wang (1) and the proposed model (3) have been implemented in Matlab/Simulink, and compared with GPS-X results. Figure 1.2 shows the dynamic behavior without crossflow air until day 7, followed by a period where the air flow is set and the resistance and cake formation decrease and the membrane flux increases. The prediction of the three models are very similar.

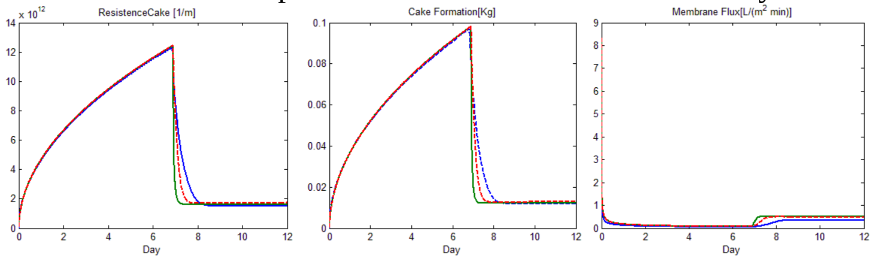

Figure 1.2: Comparison between the models. Red: Li Model, Blue: GPS-X model and Green: Proposed Model.

There is a relation between cake formation and the factor ϕ(m) that of permeate flux. Figure 1.3 shows cake formation and the factor ϕ(m) for different transmembrane pressure values. If the transmembrane pressure increases the cake formation increases, but the factor φ(m) decreases. The same behavior is observed in [2] [8].

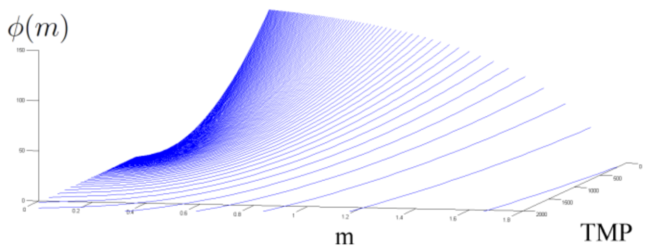

Figure1.3: Different values of TMP. Relation between cake, transmembrane pressure and ϕ.

CONCLUSIONS AND PERSPECTIVES

A simplified MBR model including a model of the cake formation, and a simple biological model based on the growth of a single biomass on a limiting substrate, has been proposed and compared to established, more detailed, models. A preliminary analysis of the equilibrium points and their stability is also presented. Further research entails pursuing the comparison with more detailed models, and demonstrating through model reduction, that the proposed model is indeed sufficient for control purposes.

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