Inference for epidemic data using diffusion processes with small diffusion coefficient

Romain Guy, Catherine Laredo, Elisabeta Vergu

HAL Id: hal-02746403

https://hal.inrae.fr/hal-02746403

Submitted on 3 Jun 2020

Inference for epidemic data using diffusion processes with small diffusion coefficient

Romain Guy[1,2], Catherine Laredo[1,2], Elisabeta Vergu[1]

[1] UR341 Mathématiques et Informatique Appliquées
INRA, F-78350 Jouy-en-Josas, France
[2] Laboratoire de Probabilités et Modèles Aléatoires
Université Paris Diderot, France

January 17, 2011

Abstract

One of the simplest and most appropriate models for the study of epidemic spread is the SIR (Susceptible-Infectious-Removed) model, in which the successive transitions of individuals between states are described in various mathematical frameworks. Epidemic data being often partially observed and also temporally aggregated, parametric inference through likelihood-based approaches is rarely straightforward, whatever the mathematical representation used. Although these last years methods based on data augmentation, able to deal with different patterns of missingness were developed, they do not provide systematic solutions since they are limited by the amount of augmented data and hence by computation times. In this context, diffusion processes provide a good approximation of epidemic dynamics and, due to their analytical power, allow shedding new light on inference problems of epidemic data. Indeed, the normalization by the population size N of the continuous time Markov jump process leads to an ODE system. Before passing to the limit, the SIR dynamics is described by a bidimensional diffusion with small diffusion coefficient proportional to $\epsilon = 1/\sqrt{N}$.

We consider here a multidimensional diffusion $(X_t^\epsilon)_{t\geq 0}$ with drift coefficient $b(\alpha, X_t^\epsilon)$ and diffusion coefficient $\epsilon\sigma(\beta, X_t^\epsilon)$. The diffusion is discretely observed at times $t_k = k\Delta$ on a fixed time interval $[0, T]$ with $T = n\Delta$. We study Minimum Contrast Estimators (MCE) derived from a Gaussian process approximating $(X_t^\epsilon)_{t\geq 0}$ for small ϵ. We obtain consistent and asymptotically Gaussian estimators of (α, β) as $\epsilon \to 0$ and $\Delta = \Delta_n \to 0$ under the condition "$\epsilon^2 n$" bounded , and for fixed Δ consistent and asymptotically Gaussian estimator of α. Our results extend [GC90] to multidimensional diffusions with arbitrary diffusion coefficient and [GS09] to low frequency data. Finally, we simulate various epidemics dynamics with Markov jump process, we compare our MCE to other commonly used estimators: least squares for noisy observed ODE, maximum likelihood for Markov jump processes. Our findings are very promising: on low and high frequency data, our estimator is very close to the estimator based on the complete observation of the Markov jump process . We are currently extended these results to the more realistic case of partially observed data.

References

[GC90] V. Genon-Catalot. Maximum contrast estimation for diffusion processes from discrete observations. *Statictics*, 21:99–116, 1990.

[GS09] A. Gloter and M. Sorensen. Estimation for stochastic differential equations with a small diffusion coefficient. *Stoch. Proc. Appl.*, 119:679–699, 2009.