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# Hypothesis assessment with qualitative reasoning: modelling a periodic spring

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## Abstract

In this paper we illustrate the use of qualitative reasoning models to assess the consistency of a hypothesis about the functioning of a black box system. To that purpose we have built a qualitative model of a mechanism for the periodic spring of Fontestorbes (Ariège, France) based on the siphon's hypothesis. The spring, called fountain locally, is a black box system because the mechanism is not accessible. Our model exhibits a cyclic behaviour that matches well with the periodic flow observed at Fontestorbes. It also generates the unexpected behaviour that was used to challenge the consistency of the hypothesis. The simulation of the expected behaviours appears to be a necessary condition to assert the consistency of the hypothesis while the generation of unexpected behaviours provide directions to challenge it. The mechanism at work characterizes a nonlinear relaxation oscillator, its modelling is our contribution to the modelling of natural phenomena.

## 1 Introduction

The ecological knowledge about a natural phenomenon can be sparse and the structure of the system hidden and not accessible. In such conditions the system appears more or less as a black box, for which only the inputs and outputs are known. Often, several hypotheses can be envisaged to explain the phenomenon, and there is a need to investigate only the ones that are causally sound and that can really explain the observed behaviour.

Qualitative reasoning [Weld and de Kleer, 1990] is an AI approach that provides means to capture the conceptual knowledge of experts, as qualitative models can in turn be used to run qualitative simulations and provide explanations of the results [Bredeweg and Salles, 2009]. Qualitative simulations allow to assess the consistency of a conceptual model from a causal point of view in providing a means to check if the model can effectively exhibit and explain the observed behaviour or not. Therefore qualitative models can support a critical assessment of the hypotheses.

The main goal of the work presented in this paper is to illustrate the use of a qualitative reasoning model to assess a

hypothesis about the functioning of a black box system. For that purpose we have been interested in the Fontestorbes fountain (Ariège, France). This fountain is a perfect example of a periodic spring (also called ebb and flow spring), namely a spring with a periodic rate of flow. It is a re-emergence that comes out of a mountain, and the mechanism behind this surprising phenomenon is hidden since the inner part is not accessible. From the middle of the 19th century scientists have proposed different models to explain the phenomenon. Until the work of Mangin in 1969 [Mangin, 1969], the mechanism was commonly associated to that of a "Tantalus cup" device, i.e. a reservoir fed continuously by an inflow and drained by a siphon. Under specific conditions the Tantalus cup behaves as a non-linear relaxation oscillator and the outflow is periodic.

We have developed a qualitative model to assess this siphon's hypothesis using Garp3, a qualitative reasoning environment [Bredeweg *et al.*, 2009].

The qualitative model of a relaxation oscillator is our additional contribution to the field, even if, given the main topic of the study, the model derived from a rather pragmatic view. Relaxation oscillations are actually associated to the modelling of important natural phenomena such as the dynamic of two populations competing for a shared resource (Gause's model) [Haefner, 2005] or the earth's climate [Whitethread *et al.*, 2001].

### 1.1 The periodic spring of Fontestorbes

The Fontestorbes fountain is characterized by a periodic flow during the low-water season, which starts in July and generally ends in November. The period of the oscillations is about one hour, on average the period of high outflow is about half as long as the period of low outflow, but they can also be very close. The perfect regularity of the oscillation and the importance of the flow at the outlet characterize the Fontestorbes fountain. When the shortage of water is particularly severe and the inflow decreases to a value of about 0.6 m<sup>3</sup>/s the oscillations stop and the outflow is constant again. To summarize, the water flow observed at the outlet of the fountain has the following characteristics:

- during the high-water season, no oscillations but a variation of the inflow and of the outflow accordingly;

- during the low-water season, periodic oscillations of the outflow;
- during the low-water season, the possibility for the oscillations to stop if the inflow becomes very low.

Several hypotheses have been formulated to explain the phenomenon. The first satisfactory explanation was the siphon's hypothesis proposed by Darcy in 1857. The mechanism supposes an underground reservoir continuously fed with water and connected to a channel with a siphon-shape, namely a reversed U-shape pipe. This type of device is usually known as a "Tantalus cup", which is a common device used in physics teaching to illustrate the principle of a non-linear oscillator (Fig. 1).

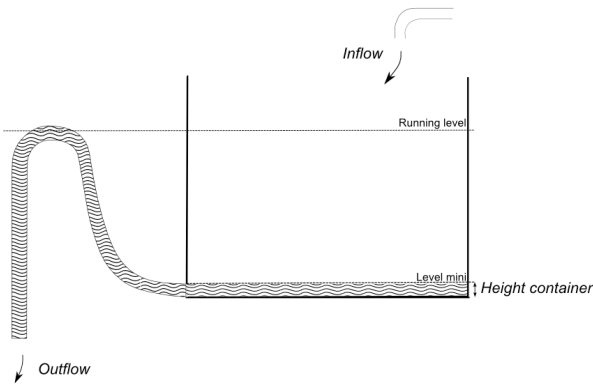


Figure 1. Schematic representation of a "Tantalus cup" device.

The hypothesis was quite convincing and appealing, so thereafter, it was considered as the explanation of the phenomenon. In 1969 Mangin [Mangin, 1969] used scale models to investigate the mechanism behind the periodic flow. His results led him to abandon the siphon's hypothesis for a model integrating an air-inlet on the output channel (not modelled in this paper). The non-matching behaviour exposed by Mangin (1969) is the following:

- under conditions of low-water season, a system analogue to a Tantalus cup would not systematically exhibit the expected oscillations.

His model matches better with the field observations and can be generalized to other periodic springs.

Even after Mangin's work the siphon's hypothesis is still used in certain studies to explain the mechanism of periodic springs (see for example [Bonacci and Bojanic, 1991]).

### Presentation of the siphon hypothesis

The hypothesis of the siphon ruled out by Mangin (1969) explains the oscillations of the outflow at the outlet of the fountain by the presence of a siphon. The mechanism is the following:

Starting from a state with no outflow coming out the siphon, the inflow fills the reservoir and the water level in the reservoir increases. At a given height of water, above *Running level* in Fig.1, the siphon activates. The outflow increases and eventually exceeds the inflow if the inflow

is low enough. If that occurs the water level in the reservoir decreases and so does the outflow. The level of water goes below the entry of the siphon's tube, namely below *Level mini* in Fig.1, air is sucked into the tube causing the complete unpriming of the siphon then the cycle repeats.

This type of device is commonly referred as a "Tantalus cup". The inflow is the control parameter that determines the way the device behaves.

The constant presence of residual outflow at the outlet of the fountain during the unpriming phase is explained by a secondary channel, not *per se* connected to the reservoir.

## 1.2 Qualitative reasoning

A strong feature of the qualitative reasoning is the ability to capture the conceptual knowledge about a phenomenon. Garp3 [Bredeweg *et al.*, 2009] provides a workbench to build qualitative models that also provides tools for simulating and inspecting the results. The ontology and the formalism implemented in Garp3 derive from typical approaches to qualitative reasoning [Bobrow, 1984].

A Garp3 model involves several ingredient types. *Entities* are the structural elements endogenous to the system. They are declared in Garp3 using a subtype hierarchy. *Agents* represent the exogenous structural elements. Entities and agents can be connected using structural relations, called *Configuration*. *Quantities* are the properties of *Entities* or *Agents*. *Quantities* are characterized by the couple  $\langle \text{magnitude}, \text{derivative} \rangle$ . The former represents the actual value of the quantity, the latter the direction of change. The domain of values of a quantity is called a Quantity Space (QS), it is a finite and discrete set of symbols. A QS is an abstraction or a mapping of a continuous numerical scale consisting of a succession of points (also called landmarks) and intervals. The derivatives have systematically the same QS:  $\{\text{min}, \text{zero}, \text{plus}\}$ , referring to decrease, steady and increase, respectively.

The key cause-effect relations are: direct influence ( $I+$ ;  $I-$ ) and qualitative proportionality ( $P+$ ;  $P-$ ). The former represents the cause of the changes, whereas the latter represents the propagation of these changes. The causal relations are used to compute the value of the derivatives during the simulation. The workbench allows the use of constraints such as value correspondences and inequalities  $\{<, \leq, =, \geq, >\}$  as well as the two arithmetic operations  $+$  and  $-$ .

Key ingredients of qualitative models in Garp3 are *Scenarios*, they specify the state of the system at the start of a simulation, and *Model fragments* (MFs), which capture chunks of knowledge. During a simulation, the reasoning engine calls the MFs to determine the current state of the system and the possible transitions. The end result is a state-graph representing the behaviour of the system along time. Different views are available to inspect the results such as the value history diagrams [Bouwer and Bredeweg, 2010].

## 1.3 Model goals

The qualitative model is built to assess the consistency of the siphon's hypothesis. In particular, the model and simula-

tion results should help to understand why the hypothesis was first accepted and finally ruled out.

## 2 Modelling of a Tantalus cup

### 2.1 Modelling choices

The Fontestorbes fountain has a fixed geometry, height and section of the pipe, volume and shape of the reservoir, but we have no information about its structure. Since this aspect of the system cannot be discussed, it is a natural assumption to consider an optimal geometry of the system for the sake of the hypothesis.

Our qualitative model is based on distinctive phases of the state of the system, characterized mainly by the state of the flow through the siphon. We distinguish three types of flow: (a) one-phase flow when the siphon is flooded, (b) two-phase flow (air+water) during the priming and the unpriming processes, and (c) no flow when the downstream leg of the siphon is empty.

### 2.2 Model overview

There are four main entities, *Liquid source*, *Reservoir*, *Siphon*, and *Liquid* (water in our context). In the case of a spring, *Liquid source* can be an underground river. It is external to the system because it affects the system without being affected in return.

Besides these four entities, the model involves 4 configuration types, 10 quantities, 29 model fragments and 3 scenarios corresponding to situations encountered at the Fontestorbes fountain but more if we investigate the possible behaviours of a Tantalus cup. The quantities of the model are listed in table 1.

Table 1. Quantities of the siphon model.

Quantity	Description
Inflow	The flow of water from the groundwater.
Height container	The reservoir water level.
Outflow	The flow outgoing the siphon and measured at the outlet of the fountain.
State of the siphon	Indication of the water flow regime in the siphon.
Balance	The net water flow in the reservoir.
Downward force	The force initiated by the difference of pressure between the reservoir and the siphon and the gravity.
Flow of air	The flow of air into the siphon.
Efficient height	The reservoir water level above the running level.
Running level	A fixed parameter of the model corresponding to the height of water required to prime the siphon.
Minimum level	A fixed parameter of the model corresponding to the height of the siphon entry.

The organization of the qualitative model is based on the distinctive phases that characterize the state system during

one oscillation, namely filling, priming, running, unpriming. During the filling and the running phases, the regime of the water flow is respectively stationary (no outflow) or quasi-stationary (siphon flooded, outflow respecting the Bernoulli's law). The priming and the unpriming are phases during which the regime is not stationary, in particular because both water and air flow through the siphon.

### 2.2 Modelling the filling and the running phases

*Filling* and *Running* are the most basic phases. They allow us to introduce the fundamental causal relations.

We define the variation of the water level in the reservoir as a balance (quantity *Balance*), namely the difference between *Inflow* and *Outflow* (Fig. 2). There is a positive proportionality (P+) between *Inflow* and *Balance* and a negative proportionality (P-) between *Outflow* and *Balance*. The MF "Liquid flow" (Fig. 2) represents closely the volume conservation relation that commands the system [Whitehead et al., 2001]:

$$S \frac{dz}{dt} = \text{inflow} - \text{outflow}$$

with  $z$  being the reservoir water level ( $z > 0$ ),  $S$  the section of the reservoir and  $t$  the time. The section is not represented by a quantity of the qualitative model so *Balance* influences directly (I+) *Height reservoir*. We assume that the conservation relation is always true and the corresponding MF always fires whatever the state of the siphon.

During the running phase the siphon is flooded and the outflow obeys the following law, in agreement with the Bernoulli's principle:

$$\text{outflow} = s \sqrt{2gz(t)}$$

with  $z > 0$ ,  $s$  the section of the siphon and  $g$  the acceleration due to gravity. This relation is captured by a positive proportionality (P+) from *Height reservoir* to *Outflow*.

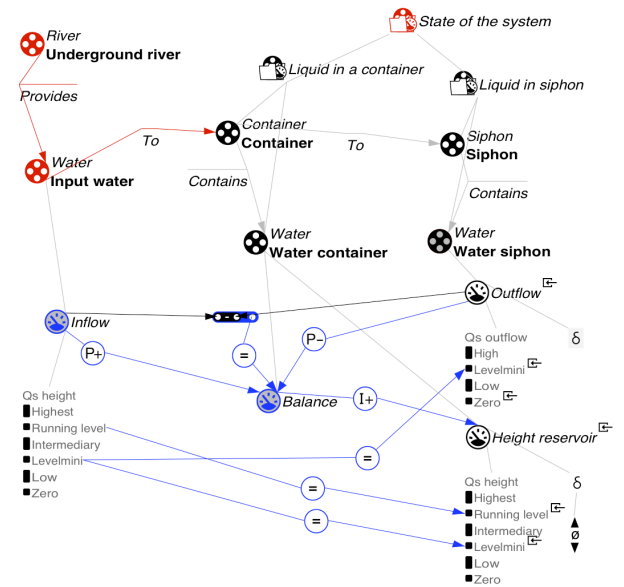


Figure 2. Definition of *Balance* and equality relations between the landmarks of *Inflow*, *Outflow* and *Height reservoir* quantity spaces.

### Adaptation to detailed quantity spaces

According to the structure of a Tantalus cup (Fig. 1), *Height reservoir* must reach two important landmarks, *Level mini* and *Running level*. The QS of *Height reservoir* is therefore  $\{zero, low, levelmini, intermediate, runninglevel, highest\}$ , where *zero*, *levelmini* and *runninglevel* are points and *low*, *intermediate* and *highest* are intervals. *Inflow* being the control parameter of the system, its QS should also be quite detailed. Finally, to be able to visualize the oscillation of the *Outflow* we need a QS with at least two intervals to distinguish low and high outflow.

These are detailed QS which is known to be a factor that makes the simulations more complex [Noble *et al.*, 2009]. With this system it is possible to establish links between the QS' of *Height reservoir*, *Inflow* and *Outflow*, because a part of the behaviour is driven by the two equations above.

To interpret the value of the three quantities we take as a reference the QS of *Height reservoir*. When the siphon is flooded the Tantalus cup, the outflow and the height of water in the reservoir are related by the two equations above, and the value of the outflow is given by:

$$\frac{outflow^2}{2gs^2} = z$$

with  $z$  being the height of water in the reservoir. This relation is bijective when  $outflow \geq 0$  which is the case here. Therefore we can assign to *Outflow* a QS which represents the projection of *Height reservoir*'s QS following this relation. In doing so we can draw equalities between the respective landmarks. Since *Inflow* and *Outflow* have the same dimension they can have the same QS, thus by transitivity the value of *Inflow* can be interpreted relatively to the QS of *Height reservoir*.

This means for example that *Outflow(levelmini)* is the value of the outflow when the water level in the reservoir reaches *Level mini* (see Fig.1), providing that the siphon is flooded. *Inflow(levelmini)* is the value of the inflow that stabilizes the system at *Height reservoir(levelmini)* and *Outflow(levelmini)* when the siphon is flooded.

*Inflow* has the same QS as *Height reservoir* and *Outflow*'s QS is  $\{zero, low, levelmini, high\}$ . The 3 quantities share the same QS, therefore there are equalities between the landmarks of the three QS (Fig. 2). This reduces the complexity of the simulation and facilitates easier interpretation of the results.

### 2.3 Modelling the priming of siphon

The priming of a siphon is a complex phenomenon [Cognet *et al.*, 1993] and we cannot pretend to represent exactly the underlying mechanism. In this work we try to capture the principle of the priming process.

First, once *Height reservoir* is equal or above *runninglevel* the water flows through the siphon according to the water level in the reservoir. This is represented by a positive proportionality (P+) between *Height reservoir* and *Outflow*. In the meantime the *State of the siphon*, namely the

level of filling of the siphon's tube, is positive proportional (P+) to *Outflow*.

However, a single proportionality is not sufficient to explain the fact that the outflow can exceed the inflow, going beyond an immediate equilibrium point (when *Inflow* = *Outflow*). An additional process is involved. We have implemented the idea that the priming of the siphon results from the sudden action of a *Downward force* that results from the storage of energy during the filling phase. In this situation, the reservoir is analogous to a charged capacitor, which suddenly delivers a current. The *Downward force* is the difference between *Height reservoir* and *Outflow*. The difference expresses a pressure difference between the bottom of the reservoir and the crest of the tube that causes the water to go upward in the upstream leg of the siphon. The difference expresses also the storage of a potential energy that causes the water to go downward in the downstream leg of the siphon, due to the gravity.

*Downward force* accelerates the outflow (I+ with *Outflow*) when the siphon is completely or almost flooded, that is when *State of the siphon* = *full*. *Outflow* catches up the actual value of *Height reservoir* almost instantaneously. This is partially captured by the following inequality:

$$\partial(\text{Height reservoir}) < \partial(\text{Outflow})$$

*Downward force* becomes zero when *Height reservoir* = *Outflow*, i.e. when the regime is quasi-stationary and the system in the running phase.

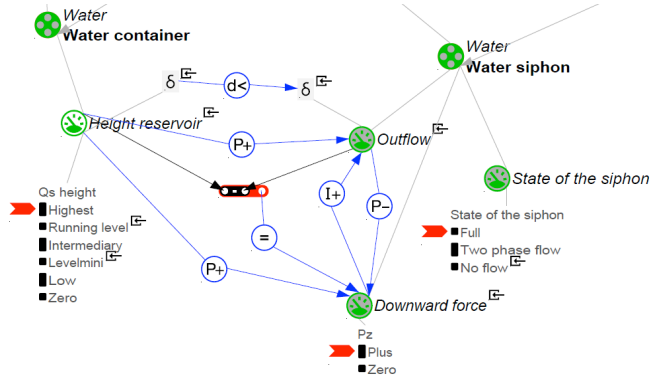


Figure 3. Definition and conditional influence of the *Downward force* during the siphon's priming.

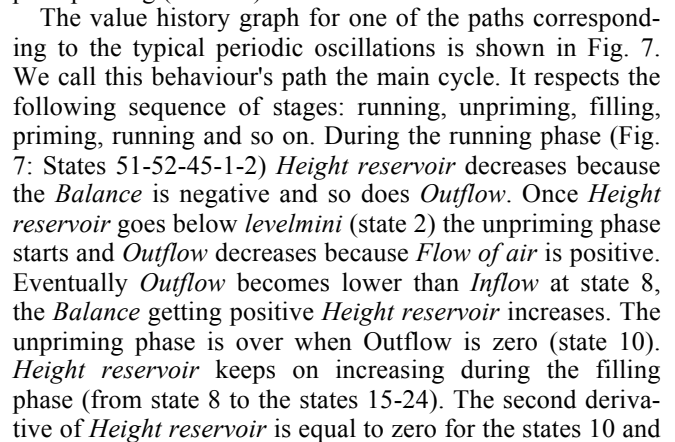
### 2.4 Modelling the unpriming of siphon

The unpriming of the siphon is caused by the air sucked in the siphon once the entry of the tube is no longer covered by the water. In the model this happens when *Height reservoir* is below *levelmini* (see Fig. 1). The water level in the reservoir determines the extent of the opening for the air. Therefore the magnitude of *Flow of air* is captured by the following difference:

$$\text{Flow of air} = \text{Minimum level} - \text{Height reservoir}$$

*Minimum level* is a quantity whose unique value is *levelmini*, it is a constant parameter of the system.





14 indicating a linear increase at a rate equal to *Inflow*. The priming phase stretches over the part from the states 15-24 to the state 50. During this stage *Height reservoir* is equal to *highest*. Up to state 24 *Outflow* increases because of *Height reservoir*.

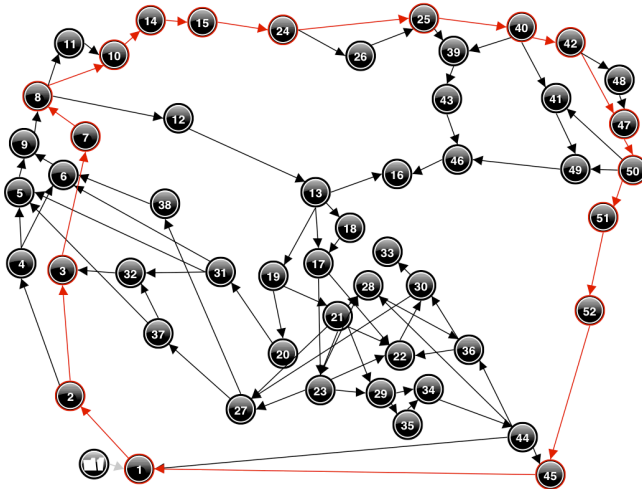


Figure 6. State-graph from simulation of the fountain during the low-water season. The expected periodic behaviour is the pathway in red.

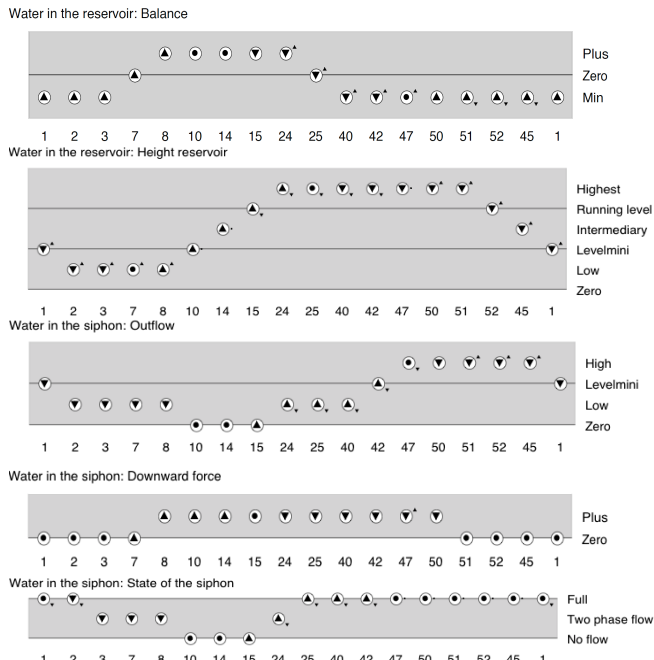


Figure 7. Value history diagrams of the main cycle from the low water season simulation.

Then, at state 25 *Inflow* = *Outflow*, *Height reservoir* is stable and the siphon is flooded (*State of the siphon* = *full*). In that case *Outflow* increases under the influence of *Downward force*, which prevails over *Height reservoir* during the following states. When *Downward force* is zero and *State of*

*the siphon* is *full*, the system is running (from state 51 to state 2); then the cycle repeats.

During the priming phase of the main cycle, 5 branching behaviours lead to the end state 16. They represent the result of an incomplete priming. In state 16 the system is stable, the siphon is not full, *Height reservoir* = *running level* and *Inflow*=*Outflow*=*low*. This happens when *Height reservoir* prevails over *Downward force*.

There is a branching at state 8 that is the start of many alternative behaviours. They can be viewed as deviations from the main cycle resulting from the incompleteness of the unpriming process. Most of the pathways are small cycles depicting short oscillations of the outflow. The value history graph for one of these pathways is shown in Fig. 8. Starting from the unpriming phase (states 5-9-8), *Height reservoir* reaches then goes beyond *levelmini* (states-12-13). The *Flow of air* stops and *Outflow* increases under the influence of *Downward force*. From state 19 onwards the *Balance* becomes negative and *Height reservoir* decreases. When it goes below *levelmini* (state 5) the cycle repeats.

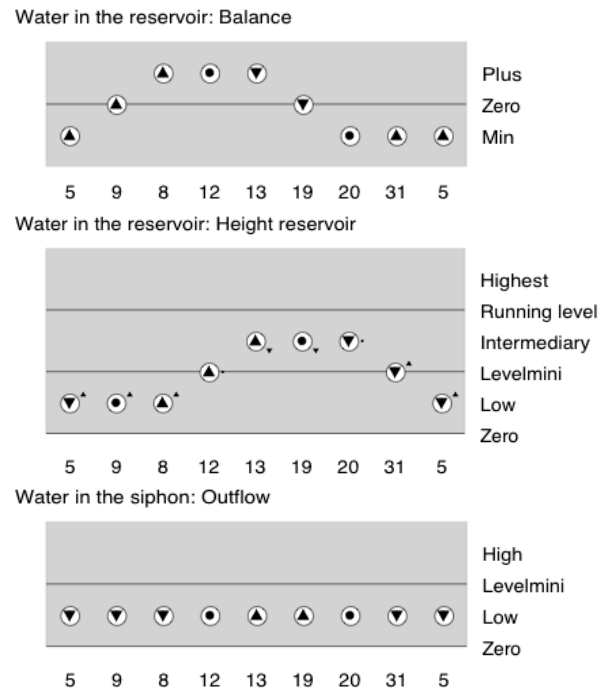


Figure 8. Value history diagrams of one of the cycles originating from an incomplete unpriming.

## 4 Assessment of the hypothesis

Our qualitative model clearly generates the behaviour expected by the siphon's hypothesis. The main cycle represents the typical behaviour of a Tantalus cup. It could explain the oscillations of the flow observed at Fontestorbes providing that a secondary channel exists to account for the fact that the outflow never ceases at the outlet. The fact that the priming phase of the main cycle might end up in a stable state (state 16, Fig. 6) matches well with the behaviour of the fountain when the shortage of water is severe.

On the other hand the short oscillations resulting from the incomplete unpriming match well with the unstable behaviour observed by Mangin (1969) and Whitehead *et al.*, (2001) on small-scale models. This reflects a transition behaviour that appears with a Tantalus cup when one reduces the inflow to obtain the periodic outflow [Whitehead *et al.*, 2001]. The transition phase should occur at Fontestorbes given the slow evolution of the inflow. However oscillations of the outflow are observed at Fontestorbes although the siphon's hypothesis would have predicted a transition behaviour. This finally discards the siphon's hypothesis.

The system needs to be modified so to not exhibit the transition behaviour but instead stable periodic oscillations of the outflow. As a matter of fact it is noticeable that the short oscillations resulting from an incomplete unpriming (Fig. 8) would fit well with the observed oscillations of the Fontestorbes' flow if the amplitude of the oscillations of *Outflow* was higher. To allow *Outflow* to reach a higher value *Height container* should increase while *Outflow* would remain less than *Inflow* until that the siphon primes when *Height container* is *High*. This is impossible with the Tantalus cup presented Fig. 1. As proposed by Mangin (1969) an air-inlet connected to the siphon's crest maintains a stable low outflow. The flow of air in the channel balances out the positive effect of *Balance* and/or *Downward force* so that *Outflow* remains stable and less than *Inflow* allowing *Height container* to increase. The air-inlet would be connected to the main reservoir and its inlet would be situated at the *Running level*. Once *Height container* reaches *Running level* the flow of air stops because the water covers the air-inlet, the siphon primes normally and the cycle repeats.

To extend the effect of *Flow of air* and let it counterbalance the forces that would prime the siphon, the qualitative model can be modified by calculating *Flow of air* as follows:

$$\text{Flow of air} = \text{Running level} - \text{Height reservoir}$$

This simple adaptation of the model of the siphon's hypothesis generates the expected behaviour even if more work is needed to reduce the ambiguity resulting from the change.

## 5 Assessing a hypothesis with a qualitative model

This paper illustrates the application to a real case of a qualitative reasoning approach to assess a hypothesis about the mechanism of a black box system. To build the present model we followed the main steps of the approach presented in Bredeweg *et al.*, (2008).

The qualitative model reproduces the reasoning a domain researcher could make to investigate this type of system. Assessing a hypothesis with a qualitative model is interesting providing that the underlying reasoning of the scientist can be captured by the formalism. In the case of the Fontestorbes fountain the hypothesis of the siphon results from careful causal reasoning. Our results show that a qualitative model build with the Garp3 formalism can provide an useful support with that respect. Hypotheses resulting from a rela-

tive or absolute order of magnitude reasoning would call for more complex qualitative reasoning formalisms.

The agreement between the results of the qualitative simulation and the observed behaviour is analysed according to the inherent limits of qualitative reasoning models. Qualitative simulation is sound but incomplete [Struss, 1988, Travé-Massuyès *et al.*, 2003]. Consequently the qualitative simulation of the behaviour of a system generates all the qualitative equivalents of the real behaviours, providing that the causal model is correct. However spurious behaviours can also be generated as well as a potentially large number of nonspurious behaviours resulting from the incomplete specification of the processes [Travé-Massuyès *et al.*, 2003]. The fact that a qualitative simulation is sound implies that the generation of the expected behaviour is a necessary condition to assert the relevance of a hypothesis. More precisely the possible situations and interpretations of the results of the simulation fall into three groups:

1. the simulation does not exhibit the expected behaviours;
2. the simulation exhibits the expected behaviours but other unexpected behaviours are also generated;
3. the simulation exhibits only the expected behaviours.

The point 1 means that either the qualitative model of the mechanism is incorrect or the hypothesis is inconsistent with the observation. The point 3 is the reverse situation, considering that a qualitative simulation is sound it means that the expected behaviours are the only one consistent with the qualitative model. The point 2 is the situation we encountered with the siphon's hypothesis and most likely the most frequent of the three. It means that the hypothesis is plausible but an examination of the unexpected behaviours needs to be conducted before ruling them out.

## 6 Conclusion

A qualitative model of a "Tantalus cup" was build to assess the siphon's hypothesis that was initially formulated to explain the periodic flow of the Fontestorbes fountain.

Our qualitative model successfully generates a simulation of the periodic oscillations of the outflow that matches well with the observations *in situ*. It generates also a behaviour must likely unexpected or neglected by the advocates of the siphon's hypothesis that turned out to be its flaw.

The assessment of a hypothesis using a qualitative model supposes the comparison between a qualitative simulation and observations of the real behaviour. The simulation of the behaviour that is consistent with the observations is a necessary condition to assert the causal consistency of the hypothesis. The simulation of unexpected behaviours provides directions that the one should examine carefully to challenge the hypothesis.

The other main result of the work is the qualitative model of the "Tantalus cup" itself, i.e. a classical non-linear relaxation oscillator. Although build with a pragmatic view, the qualitative model implements generic mechanisms which can be reused for systems with similar properties.



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