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Dynamic management of water transfer between two interconnected river basins

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Abstract

The paper analyzes the dynamic interaction between two regions with interconnected river basins. Precipitations are higher in one river-basin, the productivity of the water is higher in the other river-basin. While a water transfer increases overall productivity, the donor must take into account environmental considerations. The recipient faces a tradeoff between accepting the transfer and investing in alternative water supplies to build up a higher capacity of usable water. We analyze the design of this transfer in a dynamic modeling approach, comparing solutions with different informational structure to the cooperative solution. Contrary to standard games, where decision variables differ among players, we assume that the two players take decisions on the water transfer. The equilibrium between supply and demand determines the optimal transfer amount and price. If the problem was set as a static game, the non-cooperative solution would match the cooperative solution. However, in a more realistic dynamic setting, where the recipient uses a feedback information structure, the cooperative solution will not emerge as the equilibrium solution. The transfer amount is lower than in the cooperative case, while investment in usable water capacity is higher. We finally compare numerically our results to the Tagus-Segura water transfer in Ballestero (2004).

1 Introduction

This paper studies the interaction between two neighboring regions with differences in water inflows and in the productivity of water. If the river-basin in one region receives higher levels of rainfall while the productivity of water is higher in its counterpart (examples: higher fertility of the irrigated soil, or demand from highly productive activities like tourism), then the existence of a transfer infrastructure which enables a transfer of water from the former to the later would increase overall productivity. Independently of whether the two river-basins are located in the same or in two adjacent countries, using such an aqueduct to conduct the water from the donor to the recipient would help increase efficiency, and therefore can be regarded as a good decision (from a central government) or agreement (between the parts), from an economic point of view.

Because water inflows in the donor are large, there exists a water surplus after covering the demands in this region. The water transfer does not harm the donor's economy as long as it does not exceed the water surplus. Nevertheless, the water transfer reduces the water level in the donor's river-basin, causing an environmental degradation, reducing the donor's welfare. Thus, the water transfer improves productivity in the recipient worsening the environmental constraint in the donor (for environmental effects of water transfers, see, for example Kumar (2006)). A transfer payment must be set up to compensate the donor for forgone benefits from holding to the water resource.

The transfer payment of the water transfer can be settled by a central planner, who must determine how to share the gainings from cooperation, or through a bargaining process between the two regions in a water market. Considering that the water transfer is freely agreed between the two regions, the transfer price will be determined through a demand-supply setting, in a market (see for example Ballestero (2004)). More specifically, we consider a bilateral monopoly with a sole water seller and a sole water buyer, like in Lekakis (1998).

Addressing water scarcity through inter-basin water transfer is just one possibility for the recipient. Water-savings, water recycling or water production from desalination plants are alternatives to increase the supply of usable water which should be considered when long-term decisions on water supply are taken. The combination of inter-basin water transfer with desalination comes as the best supply oriented policy in a case study for the south-East of Spain in Bravo et al. (2010). Each of these three alternatives requires investment in infrastructure. This gives the problem a dynamic dimension: an intertemporal trade-off between current water transfer payments and investments to increase the size of this infrastructure, to build up a higher capacity of usable water.

The water transfer from the Tagus basin in the center of Spain to the Segura basin in the south-eastern Spain is a good illustration. While the Segura basin is an arid area with low precipitations and high evapotranspiration, the Tagus basin is a more humid area. The productivity of water also differs between the two regions. If irrigated, the fertility of the soil in the south-east of Spain is very high. Moreover this region receives many tourists which put a pressure on the demand for water. The disequilibrium in productivity of water and rainfalls led the central government in Spain to built an aqueduct (230 km of canals, aqueducts and tunnels) to transfer water from the Tagus to the Segura basin. The water quality in the Tagus basin has deteriorated with the transfer, which has led the regional government in the Tagus basin to complain for the transfer, opening a national political debate on alternatives like desalination plants.

Other examples of inter-basin water transfer already in operation or in project can be found (see, for example, Bhaduri and Barbier (2008)). Transfers can be set up within the same country (as in the Snowy River Scheme in Australia, the São Francisco Interlinking Project in Brazil, the Olmos Transfer Project in Peru, the South-North Water Transfer Project in China, or the Archeelos Diversion in Greece), or between countries (as the Kosi in Nepal to Ganges in India and Bangladesh, or the Lesotho Highlands Water Project between Lesotho and South Africa, to give just some examples).

Most of the existing literature refers to water-sharing by two neighboring countries that share the same river: the upstream and the downstream country. For an overview of this literature see Ambec and Ehlers (2008), Bhaduri and Barbier (2008), Ambec and Sprumont (2002), or Kilgour and Dinar (1995, 2001). These models seek for efficient water sharing agreements which allow the downstream country to compensates the upstream country for using more water. An inter-basin water transfers shares with these models the price charged by the donor for the water transferred to the recipient. However, in our model, the environmental problems caused by the water transfer between two river-basins differ from those linked to the management of a single river. With an inter-basin water transfer the two regions face very different environmental constraints. In particular, the transfer only decreases the water level in the donor river, which faces the highest environmental restriction.

Moreover, in our model, the recipient does not only rely on the water released by the donor, but it is entitled to invest in alternative water supplies, which gives the model a dynamic dimension. The dynamic interaction between donor and recipient is analyzed as a deterministic¹ differential game with an infinite time horizon. Contrary to the standard formulation, with player-specific decision variables, here the two regions decide on the amount of water transfer. Donor and recipient decide on the supply and demand for water transfer as a function of its price.

¹To address this problem, we do not to focus on inter-annual and intra-annual variations of hydrological resources, which can be modeled with stochastic models (see Ballestero (2004)).

The equilibrium of this bilateral monopoly will determine price and quantity of the transfer. A second characteristic of our model is the coexistence of a static donor with a dynamic recipient with an intertemporal dilemma between paying the price of the water transfer or investing to improve the capacity of usable water. For such a game, we prove that the supply and demand for water transfer do not differ when we move from open-loop players, who commit at the initial time to a certain path of actions, to more realistic feedback players who, at each time adjust their behavior to the information available on the level reached by the capacity of usable water. However, a feedback recipient would reduce the demand for water transfer as his own water supply gets bigger, thus he becomes aware of the negative relation between the capacity of usable water and the price of the transfer. This gives the recipient a market power, he invests more in water infrastructure, hence demands less water from the donor and pays a lower price for it. In either case, with or without commitment, the existence of an aqueduct to conduct water from donor to recipient at a market price makes both players better off.

A static analysis would ignore the recipient's dynamic dilemma between current water transfer payments or investments in water infrastructure. In this setting, the decentralized and the cooperative solutions coincide. Likewise, the open-loop solution of the non-cooperative dynamic game is also Pareto efficient. However, without an institution enforcing commitment, a feedback recipient invests more in capacity of usable water, demands less water and pays a lower price than what is socially optimal. Thus, the conclusion of Pareto efficiency would be misleading in a dynamic setting.

For the case study of Tagus-Segura transfer, Ballestero (2004) determines the quantity and price of the water transfer by simulating the stochastic supply and demand curves in a static approach. After calibrating the model, we compare our results to Ballesteros'. Having the opportunity to invest in capacity of usable water reduces the recipient's dependency on the water transfer and, hence, less water is transferred. The price of the water transfer initially starts below Ballestero's and decreases through time towards an even lower long-run value.

The paper is organized as follows. Section 2 presents the welfare functions for the donor and the recipient as well as the dynamics of the capacity of usable water. Section 3 solves the differential game under open-loop and feedback information structures. Results are compared for the decentralized solutions with or without commitment. In Section 4 we prove that the two players are better off when they can bargain on price and quantity of the water transfer, than under the case of no transfer. Section 5 characterizes the cooperative solution. And in section 6 we prove that although the static non-cooperative solution is Pareto efficient, this is not the case for the dynamic non-cooperative solution under feedback strategies. Numerical simulations in Section 7 compare our results with those in Ballestero (2004). Finally, Section 8 concludes.

2 The model

This section describes the dynamic interaction between region with higher precipitation (donor) to a region with higher productivity of water (recipient). A water transfer from the river-basin in the donor to the river-basin in the recipient increases overall productivity. However, it worsens the environmental constraint in the donor. The recipient can rely on the water transfer or he can invest in infrastructure to increase alternative water supplies.

2.1 The Donor

The river-basin in the donor region is characterized by relatively high precipitation rates and relatively low productive uses. An inherent characteristic of water inflows and outflows is their randomness. The intensity and the duration of rainfalls or the snow melting are irregularly distributed over the year. Likewise, water needs for irrigation or household water use differ throughout the year or even at different hours of the same day. Nevertheless, developed countries have managed to reduce the impact of drought periods by building dams in the head of the rivers, which help to maintain stability of the water level of the river. Merely all Interbasin Water Transfer projects include such infrastructure (UNESCO (1999)) which allows the river authority to release more water when the demand is higher and inflows are lower (and vice versa). Here we make the extreme assumption that the water surplus, or the water level in the river after covering demands remains constant through time. Thus, we define R as the maximum possible water surplus in the river without risking the exhaustion of the reservoir.

The water stored in the dam can be released in the local river, or transferred through the aqueduct to the recipient. Henceforth, the water transfer reduces the surplus or the water level in the river. Because the water demand is already covered, the donor's agriculture, industry or households are not directly affected by the transfer. However, the reduction in the water level in the river causes a degradation of the quality of the water. (For example the Tagus basin in Spain collects water from the city of Madrid which renders minimum water levels important to maintain the dilution capacity of the river). We assume in the following that the associated

environmental problem grows more than linearly² with water transferred. The environmental amenities or environmental services from the water level in the river can be represented by:

$$E(\tau(t)) = c\left(R - \frac{1}{2}\frac{\tau(t)^2}{R}\right), \qquad R, c > 0.$$

Notice that the effect of the water surplus is twofold. Firstly, in the absence of water transfer, environmental amenities increase linearly with the water surplus in the river: cR. Secondly, for increments in the water transfer, the marginal reduction in environmental amenities is inversely proportional to this surplus, (while proportional the share of water transferred): $c\tau(t)/R$. We suppose in the following that environmental amenities and damage are measured in monetary terms.

The donor receives a monetary payment, p(t), from the recipient for each unit of water transferred. The instantaneous welfare function for the donor is then expressed as:

$$F^{d}(p(t),\tau(t)) = E(\tau(t)) + p(t)\tau(t) = c\left(R - \frac{1}{2}\frac{\tau(t)^{2}}{R}\right) + p(t)\tau(t).$$
(1)

2.2 The Recipient

If precipitations are low in the river-basin of the recipient region, but the productivity of water is high, then this region will be willing to pay for the water transferred from the donor. Productive activities in this region (like agriculture or tourism activities) do depend on other inputs like labor or capital. Nevertheless we consider them as fixed inputs and focus on the effect of available water on production activities. The water used in the recipient region is diverse: a fixed amount from the river-basin in the recipient region, a variable quantity transferred by the donor, $\tau(t)$. Additionally, the recipient possesses an alternative source to increase the available water volume. He/she may invest in the necessary equipment to either save water by reducing the use of water or the water leakage in the distribution network, increase recycling (use of grey water) or produce water (for example through desalination plants) (Merett (1997)). We gather together these three possibilities in a single variable, x(t), defined as the capacity to produce, recycle or save water with the current equipment (for shortness capacity of usable water). It is measured in cubic meters. Capacity increases with new investments and decreases with depreciation:

$$\dot{x}(t) = s(t) - \delta x(t), \qquad x(0) = x_0 \ge 0,$$
(2)

²In fact, although not included in the model, an excessively low level might represent an irreversible catastrophe (see for example Tsur and Zemel (1995), (2004))

with $\delta > 0$ the depreciation rate and s(t) the investment to increase the current capacity, i.e. the cubic meters additionally saved, produced or recycled above the current capacity.

The welfare in the recipient region comes from the amount of available water: either water transferred, τ , or the capacity of usable water, x(t).³ Welfare increases with the amount of available water at a decreasing rate. Moreover, investments in new capacity are costly at an increasing rate, so we consider quadratic investments costs. This can reflect both the existence of increasing transaction costs and the incremental cost of successive projects to produce, save or recycle. Finally, instantaneous welfare decreases with transfer payments made to the donor. Hence, the welfare function of the receipient is then expressed by:

$$F^{r}(p(t),\tau(t),x(t),s(t)) = Q(\tau(t),x(t)) - p(t)\tau(t) - C(s(t))$$

= $d\left(\tau(t) + x(t) - \alpha \frac{(\tau(t) + x(t))^{2}}{2}\right) - p(t)\tau(t) - \beta \frac{s(t)^{2}}{2},$ (3)

with $d, \alpha, \beta > 0$.

3 The Nash game

We present in this section the dynamic interaction between the donor and the recipient regions. The amount and the price of the water transfer are determined from the supply and demand decisions taken by the donor and the recipient respectively. The donor determines the supply of water in order to maximize the stream of welfare discounted at a constant rate throughout an infinite time horizon:⁴

$$\max_{\tau} \int_0^\infty \left[c \left(R - \frac{\tau^2}{2R} \right) + p\tau \right] e^{-\rho t} dt.$$
(4)

Correspondingly, the recipient must decide on the demand for water from the donor and on the investment in capacity of usable water, to maximize discounted welfare:

$$\max_{\tau,s} \int_0^\infty \left[d\left(\tau + x - \alpha \frac{(\tau+x)^2}{2}\right) - p\tau - \beta \frac{s}{2} \right] e^{-\rho t} dt.$$
(5)

The recipient is a farsighted player whose maximization problem is subject to the evolution of the capacity of usable water in (2). By contrast, the donor behaves as a static or myopic player⁵

$$\max_{\tau} \left[c \left(R - \frac{\tau^2}{2R} \right) + p\tau \right].$$

 $^{^{3}}$ We neglect here the water from the recipient's river-basin (assumed constant) and focus on these two alternative sources of water.

⁴Here and henceforth time argument is omitted when no confusion can arise.

⁵Problem (4) could be written as:

because the amount or the price of water transfer has no effect on the dynamics of the capacity of usable water, and because neither the stock nor the investments in capacity of usable water influence the donor's welfare.

Contrary to standard differential games, where control variables differ among players, here we consider that the two players take decisions on the amount of water transfer from the donor to the recipient. Thus, optimality conditions for problems (4) and (5) give the supply and the demand of water as a function of its price:

$$\tau^{S}(p) = \frac{pR}{c}, \qquad \tau^{D}(p, x) = \frac{d-p}{d\alpha} - x.$$
(6)

The amount and the price of the water transfer at the equilibrium is obtained by equating supply and demand, $\tau^{S}(p) = \tau^{D}(p, x)$. We denote this a Nash equilibrium.

$$\left(p^{N}(x),\tau^{N}(x)\right) = \left(\frac{cd}{Y+c}(1-\alpha x),\frac{dR}{Y+c}(1-\alpha x)\right),\tag{7}$$

with $Y = Rd\alpha$. From this definition immediately follows that the water transfer would only take place if the capacity of usable water remains below $1/\alpha$. From (6) it follows that $1/\alpha$ would be the demand for water transfer at a zero price and for a null capacity of usable water. Hence, this amount can be understood as the maximum water requirements in the recipient.

The solution to this problem depends on the information structure of the players. Thus, we distinguish between the open-loop and the feedback or Markov perfect equilibria. An open-loop information structure assumes that the players only know the initial state of the game, and commit to follow an optimal path from this starting time on. Conversely, in a Markov perfect solution, players adapt their decisions taking into account the state of the game at each point in time. In the present paper, the donor is a myopic agent, and hence, its optimal decision given by $\tau^{S}(p)$ in (6). Conversely, the farsighted recipient would act differently in the two solution concepts considered.

3.1 Open-loop Nash equilibrium

This section computes the commitment solution for the game described by the optimization problems in (4) and (5), subject to the dynamics of the capacity of usable water in (2) and the equilibrium condition $\tau^{S}(p) = \tau^{D}(p, x)$. The optimal open-loop solution is described by the Nash equilibrium price and quantity in (7), and the optimal investment in capacity of usable water given by:

$$s^{\rm OL}\left(\lambda_r^{\rm OL}\right) = \frac{\lambda_r^{\rm OL}}{\beta}.$$
(8)

with λ_r^{OL} the costate variable for the recipient associated with x.

Further, the system dynamics is described by:

$$\dot{x} = \frac{\lambda_r^{\text{OL}}}{\beta} - \delta x,\tag{9}$$

$$\dot{\lambda}_r^{\text{OL}} = (\rho + \delta)\lambda_r^{\text{OL}} - \frac{cd}{c+Y}(1 - \alpha x).$$
(10)

The optimal time paths for the capacity of usable water and its shadow price read:

$$x^{\rm OL}(t) = (x_0 - \bar{x}^{\rm OL})e^{\phi^{\rm OL}t} + \bar{x}^{\rm OL},$$
(11)

$$\lambda_r^{\rm OL}(t) = \beta \left[(\phi^{\rm OL} + \delta) x^{\rm OL}(t) - \phi^{\rm OL} \bar{x}^{\rm OL} \right].$$
(12)

with

$$\bar{x}^{\rm OL} = \frac{dc}{(c+Y)\Lambda + dc\alpha} < \frac{1}{\alpha},\tag{13}$$

$$\phi^{\rm OL} = \frac{1}{2\beta} \left[\rho\beta - \sqrt{\Delta^{\rm OL}} \right] < -\delta, \quad \text{with} \quad \Delta^{\rm OL} = (\rho + 2\delta)^2 \beta^2 + 4 \frac{dc\alpha\beta}{(c+Y)}. \tag{14}$$

and $\Lambda = \beta \delta(\rho + \delta)$.

Here and henceforth, we make the assumption that the initial stock of the capacity of usable water, x_0 , is lower than its long-run value (for instance, it might be initially zero). Then, since the capacity of usable water is lower than $1/\alpha$ at the steady state, we can guarantee that it remains always below this $1/\alpha$. Consequently, a positive amount of water is transferred at a positive price.

Given the optimal time paths for the state and the costate variables, the time paths for τ , p and s can be immediately obtained from (7) and (8). Their steady-state values are given by:

$$\bar{p}^{\rm OL} = \frac{cd\Lambda}{(c+Y)\Lambda + d\alpha c}, \quad \bar{\tau}^{\rm OL} = \frac{Rd\Lambda}{(c+Y)\Lambda + d\alpha c}, \quad \bar{s}^{\rm OL} = \frac{\delta dc}{(c+Y)\Lambda + d\alpha c}.$$
 (15)

3.2 Feedback Nash equilibrium

Open-loop solutions present a problem of credibility. With an OL information structure, the recipient, who is the sole farsighted agent, commits to an optimal demand path from the beginning. Given the supply settled by the static donor, the amount and the price of the water transfer are known as functions of time. Why would he stick to the committed strategy in the light of ulterior information (the state of the system at any time)?

Conversely, when the recipient plays feedback, he determines the demand taking into account the current capacity of usable water. Thus the demand of water is not a function of time, but a function of the state of the system. In consequence the equilibrium price and amount of water transfer are state-dependent. We consider this as the more realistic solution, while the open-loop solution can be regarded as a benchmark scenario more difficult to attain in practice.

While the static donor behaves as in the open-loop scenario, the maximization problem for the farsighted recipient can be written as the Hamilton-Jacobi-Bellman(HJB) equation:

$$\rho V_r^{\rm F}(x) = \max_{\tau,s} \left\{ d \left[(x+\tau) - \frac{\alpha (x+\tau)^2}{2} \right] - p\tau - \beta \frac{s^2}{2} + (V_r^{\rm F})'_x(x)(s-\delta x) \right\},\,$$

with $V_r^{\rm F}(x)$ the value function for the recipient. Again, the price and quantity of water transfer at the equilibrium are determined from the equation $\tau^S(p) = \tau^D(p, x)$.

Proposition 1 For any differential game between a static and a farsighted player described by:

$$\max_{\tau} \left[E(\tau) + p\tau \right], \quad \max_{\tau,s} \int_0^\infty \left[Q(\tau, x) - p\tau - C(s) \right] e^{-\rho t} dt,$$

s.a.: $\dot{x} = f(s, x), \quad \tau^D(p, x) = \tau^S(p),$

The optimal price and quantity can be written as the same function of the state under the feedback or the open-loop Nash equilibria.

Proof. The optimal demand of water chosen by the static player is characterized by equation $p = -E'(\tau)$ regardless of the information structure considered. Moreover, because the amount of water transfer does not affect the dynamics of the capacity of usable water, the supply of water transfer is also the same for the open-loop or the feedback solution, and it is given by equation: $p = Q'_{\tau}(\tau, x)$. Equating supply and demand the price and the optimal amount of water transfer can be written as the same function of the state.

Thus, the price and the optimal quantity in the feedback Nash equilibrium are again given by (7), while the optimal investment in capacity of usable water reads:

$$s^{\rm F}(x) = \frac{(V_r^{\rm F})'_x(x)}{\beta},$$
 (16)

with $(V_r^{\rm F})'_x(x)$ the marginal value of additional units of the capacity of usable water, x.

Given the linear-quadratic structure of the problem, we conjecture a quadratic value function: $V_r^{\rm F}(x) = c_r^{\rm F} x^2 + b_r^{\rm F} x + a_r^{\rm F}$. Taking into account the optimal values of τ , p and s, the dynamics of the capacity of usable water, (2), and solving the associated Ricati system of equations, we obtain the time path of the capacity of usable water as:

$$x^{\rm F}(t) = (x_0 - \bar{x}^{\rm F})e^{\phi^{\rm F}t} + \bar{x}^{\rm F}, \qquad (17)$$

with

$$\bar{x}^{\rm OL} < \bar{x}^{\rm F} \equiv \frac{cd(c+2Y)}{\Lambda(c+Y)^2 + dc\alpha(c+2Y)} < \frac{1}{\alpha},\tag{18}$$

$$\phi^{\rm F} \equiv \frac{1}{2\beta} \left[\rho\beta - \sqrt{\Delta^{\rm F}} \right] < \phi^{\rm OL} < -\delta, \quad \text{with} \quad \Delta^{\rm F} = (\rho + 2\delta)^2 \beta^2 + 4 \frac{dc\alpha\beta(2Y+c)}{(c+Y)^2}. \tag{19}$$

Again, because we are assuming $x_0 < \bar{x}^{\text{OL}}$ then $x(t) < \bar{x}^{\text{OL}} < \bar{x}^{\text{F}} < 1/\alpha$ and as a consequence $\tau^{\text{F}}(t), p^{\text{F}}(t) > 0, \forall t > 0$. Knowing the time path for the capacity of usable water allows to compute the amount and the price of water transfer, while coefficients of the value function are:

$$c_r^{\rm F} = \frac{\beta(\delta + \phi^{\rm F})}{2} = \frac{(\rho + 2\delta)\beta - \sqrt{\Delta^{\rm F}}}{4}, \quad b_r^{\rm F} = \frac{dc(c + 2Y)}{(c + Y)^2 (\rho - \phi^{\rm F})} = \frac{2dc\beta(c + 2Y)}{(c + Y)^2 (\rho\beta + \sqrt{\Delta^{\rm F}})},$$
$$a_r^{\rm F} = \frac{(b_r^{\rm F})^2}{2\beta\rho} + \frac{d}{2\alpha\rho} \frac{Y^2}{(c + Y)^2}.$$

and their steady-state values are given by:

$$\bar{p}^{\rm F} = \frac{dc\Lambda}{\Lambda(c+Y) + dc\alpha\frac{c+2Y}{c+Y}}, \quad \bar{\tau}^{\rm F} = \frac{dR\Lambda}{\Lambda(c+Y) + dc\alpha\frac{c+2Y}{c+Y}}, \quad \bar{s}^{\rm F} = \frac{cd\delta}{\Lambda\frac{(c+Y)^2}{c+2Y} + dc\alpha}.$$
 (20)

3.3 Comparison between the Open-loop and the Feedback Nash equilibria

In this section, we compare the optimal solutions under OL and feedback information structures. When the recipient does not commit from the beginning, but determines the demand for water transfer as a function of the existing capacity of usable water, this stock attains a higher value, not only in the long run, but also at any point in time. To attain a larger stock, investments must be generally higher. This comes clear in the long run ($\bar{s}^{\rm F} > \bar{s}^{\rm OL}$ immediately follows from (15) and (20)), though we have not proved it at each point in time.

Proposition 2 If $x_0 < \bar{x}^{OL}$, then $x^{OL}(t) < x^F(t) \quad \forall t > 0$.

Proof. Because $\bar{x}^{\text{OL}} < \bar{x}^{\text{F}}$, then $x_0 < \bar{x}^{\text{OL}}$ implies $x_0 < \bar{x}^{\text{F}}$. Further, since $\phi^{\text{F}} < \phi^{\text{OL}} < 0$, then $x^{\text{F}}(t)$ converges faster towards a higher steady state value than $x^{\text{OL}}(t)$. Therefore, proposition follows.

Because the capacity of usable water in the recipient is greater without commitment, the amount of the water transfer and the price paid for it will be lower.

Proposition 3 If $x_0 < \bar{x}^{OL}$, then $\tau^{OL}(t) > \tau^F(t)$, and $p^{OL}(t) > p^F(t)$ $\forall t > 0$.

Proof. From Proposition 2, $x^{\text{OL}}(t) < x^{\text{F}}(t) \quad \forall t > 0$. And the expressions for the amount of water transfer and its price in (7), as functions of x, are valid both under open-loop or feedback information structures. Then proposition follows.

Hence, in OL information, the recipient invests less and accepts a higher water transfer at a higher price. In feedback information, the recipient invests more and induces a lower transfer amount at a lower price. We can thus conjecture that the OL situation generates a higher payoff for the donor and the feedback situation a higher payoff for the recipient.

Proposition 4 The donor's welfare is higher under open-loop than under feedback: $V_d^{OL} > V_d^F$.

Proof. Using (7) we can compute $V_d(p^N(x), \tau^N(x))$. The derivative of this function in x is negative:

$$\frac{dF^d(p^N(x),\tau^N(x))}{dx} = \frac{cRd^2\alpha(-1+\alpha x)}{(Y+c)^2} < 0,$$

because $x < 1/\alpha$. As by proposition 2 $x^{OL} < x^F$ we have

$$V_d^{OL} > V_d^F$$

The donor is thus better off playing open-loop.

In the feedback case, contrary to the open-loop solution, the recipient does not regard the price as fixed, but it is aware of the negative relationship between the price and the capacity of usable water, p'(x) < 0 (assuming an affine decreasing function p'(x) would be constant). For a Hamiltonian that considers this feedback effect, the shadow price of the capacity of usable water would decrease faster and towards a higher long-run value.⁶

With a Feedback information structure the recipient is thus aware of the fact that by increasing the capacity of usable water it reduces the price of the water transfer. This gives the investment in capacity of usable water an extra-value. We can interpret this result by arguing that the recipient has market power since he knows how the price decreases with the capacity of usable water. This market power allows him to buy water at a lower price. Necessarily, less water is supplied at a lower price. The recipient is better off when his/her decisions are linked to the stock of capacity of usable water.

We can illustrate this with a numerical example. We have run 10^5 iterations where parameters are drawn randomly from a uniform distribution, with values varying as follows: from 0.001 to 0.5 for δ , from 0 to 0.5 for ρ , from 0.001 to 1 for R, d, and α , from 0 to 1 for all other

$$H^{\rm F} = d\left[(x+\tau) - \alpha \frac{(x+\tau)^2}{2} \right] - p(x)\tau - \beta \frac{s^2}{2} + \mu(as - \delta x)$$

⁶Indeed, the Feedback Nash solution could be alternatively computed using the Hamiltonian. It would read:

parameters. In all cases, we assume x(0) = 0. In all cases where the transfer were feasible, i.e. smaller than the maximum possible surplus in the water ($\tau < R$), it is better for the recipient to play feedback than open-loop: $V_r^{\text{OL}}(x_0) < V_r^{\text{F}}(x_0)$.

4 No water transfer

This section analyzes the optimal investment in the capacity of usable water either in absence of an aqueduct to conduct the water, or assuming that players decide not to transfer water. With no water transfer the donor would not face any optimization problem, while the recipient would choose investments in capacity of usable water to maximize:

$$\max_{s} \int_{0}^{\infty} \left[d\left(x - \alpha \frac{x^2}{2}\right) - \beta \frac{s}{2} \right] e^{-\rho t} dt,$$

subject to (2).

Proposition 5 With no water transfer, the investment in capacity of usable water, its stock and its shadow price are given by:

$$s^{NT}(\lambda_r^{NT}) = \frac{\lambda_r^{NT}}{\beta},\tag{21}$$

$$x^{NT} = (t) = (x_0 - \bar{x}^{NT})e^{\phi^{NT}t} + \bar{x}^{NT},$$

$$\lambda_r^{NT}(t) = \beta \left[(\phi^{NT} + \delta) x^{NT}(t) - \phi^{NT} \bar{x}^{NT} \right],$$
(22)

with

$$\begin{split} &\frac{1}{\alpha} > \bar{x}^{\scriptscriptstyle NT} \equiv \frac{d}{\Lambda + d\alpha} > \bar{x}^{\scriptscriptstyle F} > \bar{x}^{\scriptscriptstyle OL}, \\ &\phi^{\scriptscriptstyle NT} \equiv \frac{1}{2\beta} \left[\rho\beta - \sqrt{\Delta^{\scriptscriptstyle NT}} \right] < \phi^{\scriptscriptstyle F} < \phi^{\scriptscriptstyle OL} < 0, \qquad \Delta^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 \beta^2 + 4\alpha\beta d \delta^{\scriptscriptstyle OL} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 \beta^2 + 4\alpha\beta d \delta^{\scriptscriptstyle OL} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 \beta^2 + 4\alpha\beta d \delta^{\scriptscriptstyle OL} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 \beta^2 + 4\alpha\beta d \delta^{\scriptscriptstyle OL} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 \beta^2 + 4\alpha\beta d \delta^{\scriptscriptstyle OL} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 \beta^2 + 4\alpha\beta d \delta^{\scriptscriptstyle OL} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 \beta^2 + 4\alpha\beta d \delta^{\scriptscriptstyle OL} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 \beta^2 + 4\alpha\beta d \delta^{\scriptscriptstyle OL} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 \beta^2 + 4\alpha\beta d \delta^{\scriptscriptstyle OL} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 \beta^2 + 4\alpha\beta d \delta^{\scriptscriptstyle OL} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 \beta^2 + 4\alpha\beta d \delta^{\scriptscriptstyle OL} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 \beta^2 + 4\alpha\beta d \delta^{\scriptscriptstyle OL} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 \beta^2 + 4\alpha\beta d \delta^{\scriptscriptstyle OL} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 \beta^2 + 4\alpha\beta d \delta^{\scriptscriptstyle OL} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 \beta^2 + 4\alpha\beta d \delta^{\scriptscriptstyle OL} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 \beta^2 + 4\alpha\beta d \delta^{\scriptscriptstyle OL} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 \beta^2 + 4\alpha\beta d \delta^{\scriptscriptstyle OL} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 \beta^2 + 4\alpha\beta d \delta^{\scriptscriptstyle OL} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 \beta^2 + 4\alpha\beta d \delta^{\scriptscriptstyle OL} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 \beta^2 + 4\alpha\beta d \delta^{\scriptscriptstyle OL} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 \beta^2 + 4\alpha\beta d \delta^{\scriptscriptstyle OL} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 \beta^2 + 4\alpha\beta d \delta^{\scriptscriptstyle OL} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 \beta^2 + 4\alpha\beta d \delta^{\scriptscriptstyle OL} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 \beta^2 + 4\alpha\beta d \delta^{\scriptscriptstyle OL} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 \beta^2 + 4\alpha\beta d \delta^{\scriptscriptstyle OL} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 \beta^2 + 4\alpha\beta d \delta^{\scriptscriptstyle OL} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 \beta^2 + 4\alpha\beta d \delta^{\scriptscriptstyle OL} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 + 4\alpha\beta d \delta^{\scriptscriptstyle OL} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 + 4\alpha\beta d \delta^{\scriptscriptstyle OL} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 + 4\alpha\beta d \delta^{\scriptscriptstyle NT} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 + 4\alpha\beta d \delta^{\scriptscriptstyle NT} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 + 4\alpha\beta d \delta^{\scriptscriptstyle NT} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 + 4\alpha\beta d \delta^{\scriptscriptstyle NT} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 + 4\alpha\beta d \delta^{\scriptscriptstyle NT} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 + 4\alpha\beta d \delta^{\scriptscriptstyle NT} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 + 4\alpha\beta d \delta^{\scriptscriptstyle NT} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 + 4\alpha\beta d \delta^{\scriptscriptstyle NT} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 + 4\alpha\beta d \delta^{\scriptscriptstyle NT} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 + 4\alpha\beta d \delta^{\scriptscriptstyle NT} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 + 4\alpha\beta d \delta^{\scriptscriptstyle NT} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 + 4\alpha\beta d \delta^{\scriptscriptstyle NT} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 + 4\alpha\beta d \delta^{\scriptscriptstyle NT} \\ &\phi^{\scriptscriptstyle NT} = (\rho + 2\delta)^2 + 4\alpha\beta d$$

being λ_r^{NT} the shadow price of the capacity of usable water when no transfer is possible.⁷

The capacity of usable water is the highest when no transfer occurs between the two riverbasins. Furthermore, because the speed of convergence is the fastest at this scenario, the stock of capacity of usable water is also the highest at any time, t. Solving this problem using HJB equation, with value function $V_r^{\text{NT}}(x) = c_r^{\text{NT}}x^2 + b_r^{\text{NT}}x + a_r^{\text{NT}}$, the coefficients are given by:

$$c_r^{\rm \scriptscriptstyle NT} = \frac{\beta(\delta + \phi^{\rm \scriptscriptstyle NT})}{2} = \frac{(\rho + 2\delta)\beta - \sqrt{\Delta^{\rm \scriptscriptstyle NT}}}{4}, \quad b_r^{\rm \scriptscriptstyle NT} = \frac{d}{\rho - \phi^{\rm \scriptscriptstyle NT}} = \frac{2d\beta}{\rho\beta + \sqrt{\Delta^{\rm \scriptscriptstyle NT}}}, \quad a_r^{\rm \scriptscriptstyle NT} = \frac{(b_r^{\rm \scriptscriptstyle NT})^2}{2\beta\rho}.$$

In what follows it is proved that both regions are better off when an aqueduct exists and is utilized to transfer water from the supplier to the recipient.

⁷Superscript NT will refer to the no transfer case.

Proposition 6 If $x_0 < \bar{x}^{OL}$ the donor would be better off playing feedback Nash than under the no water transfer scenario, $V_d^{NT}(x) < V_d^F(x)$.

Proof. The donor's instantaneous welfare playing Nash surpasses its welfare without water transfer if and only if:

$$F^{d}(p^{N}(x^{F}(t))), \tau^{N}(x^{F}(t))) > F^{d}(0,0) \equiv cR$$

That is,

$$-c\frac{\tau^{\scriptscriptstyle \mathrm{N}}(x^{\scriptscriptstyle \mathrm{F}})^2}{2R}+p^{\scriptscriptstyle \mathrm{N}}(x^{\scriptscriptstyle \mathrm{F}})\tau^{\scriptscriptstyle \mathrm{N}}(x^{\scriptscriptstyle \mathrm{F}})>0.$$

Under assumption $x_0 < \bar{x}^{\text{OL}}$, it holds that $x^{\text{F}} < 1/\alpha$ and then $\tau^{\text{N}} > 0$. Hence condition above can be written as:

$$p^{\mathrm{N}}(x^{\mathrm{F}}) > c \frac{\tau^{\mathrm{N}}(x^{\mathrm{F}})}{2R}.$$

That is:

$$\frac{cd}{Y+c}(1-\alpha x^{\mathrm{F}}) > \frac{c}{2R}\frac{dR}{Y+c}(1-\alpha x^{\mathrm{F}}) \Leftrightarrow 1 > \frac{1}{2}.$$

If the instantaneous welfare for the donor is greater at any time, then the aggregate discounted welfare would also be greater. We hence have $F^d(p^{\mathbb{N}}(x^{\mathbb{F}}(t))), \tau^{\mathbb{N}}(x^{\mathbb{F}}(t))) > F^d(0,0) \implies V_d^{\mathbb{F}}(x) > V_d^{\mathbb{NT}}(x).$

The monetary payment to the donor for the water transfer more than offsets the reduction in the environmental amenities caused by the decrement of the water level in the donor's river. The only requirement is that the capacity of usable water in the recipient initially lies below its long run value.

Proposition 7 If $x_0 < \bar{x}^{OL}$ the recipient is better off playing feedback Nash than under the no water transfer scenario, $V_r^{NT}(x) < V_r^F(x)$.

Proof. The instantaneous payoff for the recipient if it behaves optimally in absence of an aqueduct is: $F^r(0, 0, x^{NT}(t), s^{NT}(t))$.

Let us assume that the recipient continues with the investment $s^{NT}(t)$, and hence with the capacity of usable water $x^{NT}(t)$, but now an aqueduct allows a water transfer, for which the price is settled playing à la Nash. Then, the variation in its instantaneous utility with a marginal increment in the amount of water transfer would be:

$$F_{\tau}^{r} = d(1 - \alpha(x^{NT} + \tau)) - p(\tau) - p'(\tau)\tau, \text{ where } p(\tau) = p^{S}(\tau) = \frac{c\tau}{R}.$$

And it can be easily derived that:

$$F_{\tau}^{r} > 0 \Leftrightarrow \tau < \frac{dR}{2c+Y} (1 - \alpha x^{\rm NT})$$

Assumption $x_0 < \bar{x}^{\text{OL}}$ implies $x^{\text{NT}} < 1/\alpha$, and starting with no water transfer ($\tau = 0$), then condition above always holds for initial increments in τ . The instantaneous welfare increases with a marginal increment in water transfer at any point in time. In consequence, playing the optimal strategy that the recipient would choose if an aqueduct did not exist, $s^{NT}(t)$, and setting $\tau = 0$ is not optimal. The recipient can do better by marginally increasing the amount of water transfer.

$$\exists \hat{\tau} > 0 \mid W(x_0, 0, s^{\text{NT}}(t)) < W(x_0, \hat{\tau}, s^{\text{NT}}(t)),$$

where $W(x_0, \tau(t), s(t)) = \int_0^\infty F^r(\tau(t), p(t), x(t), s(t)) e^{-\rho t} dt$, with $\dot{x}(t) = s(t) - \delta x(t), x(0) = x_0$.

When the recipient plays Nash, it chooses the demand of water transfer and the investment to maximize $W(x_0, \tau(t), s(t))$ subject to (2) and the equilibrium condition between supply and demand. Therefore, it must hold:

$$W(x_0, \tau^{\mathrm{F}}(t), s^{\mathrm{F}}(t)) \ge W(x_0, \hat{\tau}, s^{\mathrm{NT}}(t)) > W(x_0, 0, s^{\mathrm{NT}}(t)).$$

Playing Nash gives a higher payoff to the recipient than not to play.

Making a numerical analysis identical to the one carried out to compare the value functions under open-loop and feedback strategies, for 10^5 iterations we can confirm that in all cases where the transfer is feasible, it is better for the recipient to play rather than to stick to the no water transfer strategy, $V_r^{\text{NT}}(x_0) < V_r^{\text{F}}(x_0)$. In fact, we find that the value function for the recipient and for the leader without water transfers is never greater than their value function either under open-loop or feedback information structures: $V_i^{\text{NT}}(x_0) < \min\{V_i^{\text{OL}}(x_0), V_i^{\text{F}}(x_0)\}, i \in \{r, d\}$.

5 Cooperation

In the particular case where the donor and the recipient are located in the same country, a central government might decide on the amount of water transfer and the investment in capacity of usable water in the donor region. The maximization problem in the cooperative case reads:

$$\begin{split} \max_{\tau,s} \int_0^\infty e^{-\rho t} \left[c \left(R - \frac{\tau^2}{2R} \right) + d \left[(x+\tau) - \alpha \frac{(x+\tau)^2}{2} \right] - \beta \frac{s^2}{2} \right] dt, \\ \text{s.a.:} \quad \dot{x} = s - \delta x, \quad x(0) = x_0. \end{split}$$

The amount of water transferred and the investment in capacity of usable water in the cooperative equilibrium are:

$$\tau^{\rm C}(x) = \frac{dR}{c+Y}(1-\alpha x), \quad s^{\rm C}(\lambda^{\rm C}) = \frac{\lambda^{\rm C}}{\beta},$$

where λ^{c} is the shadow price of x for the central planner. And the system dynamics is described by the system of differential equations:

$$\dot{x} = \frac{\lambda^{\rm C}}{\beta} - \delta x,$$

$$\dot{\lambda}^{\rm C} = (\rho + \delta)\lambda^{\rm C} - \frac{cd}{c+Y}(1 - \alpha x).$$

The cooperative equilibrium for τ^{C} and s^{C} coincide with the expressions in (7) and (8) under open-loop. Likewise, the dynamics of the state and the costate variables are identical to their dynamics in (9) and (10) under open-loop. In consequence, $\lambda^{C}(t) = \lambda^{OL}(t)$, $x^{C}(t) = x^{OL}(t)$, $\tau^{C}(t) = \tau^{OL}(t)$ and $s^{C}(t) = s^{OL}(t)$, $\forall t \geq 0$. The investment in, the stock of and the shadow price of the capacity of usable water, as well as the amount of water transfer, are identical under a central authority or decentralized players following open-loop strategies. The cooperative solution coincides with the Nash open-loop solution, except that there is no monetary payment for the water transfer under cooperation. The commitment Nash solution is then Pareto efficient.

There are two main characteristics of the game that lead to the equivalence between openloop Nash and cooperation:

- The optimal decision on water transfer does not influence the dynamics of the state of the system (the capacity of usable water). There is neither a direct effect nor an indirect effect through s (i.e. f(s, x) and s^{OL} are independent of τ).
- The stock or the investments in capacity of usable water do not directly affect the donor's welfare $(F^d(p, \tau)$ is independent of x and s).

Proposition 8 For any differential game between a static and a farsighted player described by:

$$\max_{\tau} \left[E(\tau) + p\tau \right], \quad \max_{\tau,s} \int_0^\infty \left[Q(\tau, x) - p\tau - C(s) \right] e^{-\rho t} dt,$$

s.a.: $\dot{x} = f(s, x), \quad \tau^D(p, x) = \tau^S(p),$

the non-cooperative open-loop solution coincides with the solution to the cooperative game:

$$\max_{\tau,s} \int_0^\infty \left[E(\tau) + Q(\tau,x) - p\tau - C(s) \right] e^{-\rho t} dt,$$

s.a.: $\dot{x} = f(s,x).$

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Proof. Under open-loop Nash, water transfer (market clearance) and investments are characterized by:

$$p = -E'(\tau) = p = Q'_{\tau}(\tau, x)$$
 $s = \frac{\lambda_r^{\text{OL}}}{\beta}.$

Correspondingly, water transfer and investments under cooperation come from the equations:

$$E'(\tau) + Q'_{\tau}(\tau, x) = 0 \quad s = \frac{\lambda_r^{\rm C}}{\beta}.$$

The amount of water transfer and the investments are therefore described by the same functions of x under the two scenarios.

The system dynamics in the open-loop Nash game also matches the cooperative dynamics:

$$\dot{\lambda}^i_r = \rho \lambda^i_r - \left[Q'_x(\tau, x) + \lambda^i_r f'_x(\lambda^i_r, x) \right], \quad \dot{x}^i = f(\lambda^i_r, x), \quad i \in \{\text{ol}, \text{c}\}$$

Therefore, $\lambda_r^{\text{OL}}(t) = \lambda_r^{\text{C}}(t)$ and $x^{\text{OL}}(t) = x^{\text{C}}(t)$. Consequently, $s^{\text{OL}}(t) = s^{\text{C}}(t)$ and $\tau^{\text{OL}}(t) = \tau^{\text{C}}(t)$.

Open-loop solutions oblige players to follow the optimal strategy settled at the beginning of the game. These type of equilibria are not credible because players might have an incentive to deviate at any point in time from the committed strategy (especially for an infinite time horizon). Unless this commitment could be enforced, rational players would adjust strategies at any time, knowing the state of the game. The recipient would play feedback strategies, linking the demand for water transfer and the investments in the capacity of usable water to the actual level of this stock. Its valuation of the capacity of usable water being higher, this would lead to greater investments and lower water transfer. In consequence, without a mechanism to enforce commitment, the non-cooperative solution deviates from Pareto efficiency with over-investment in capacity and a shortfall in water transfer.

6 Static versus dynamic game

In the model presented so far, the recipient is facing a trade-off. He can buy water from the donor at a market price or he can invest in the creation and maintenance of an equipment to save, produce or recycle water, the stock of which we call the capacity of usable water. As we have seen, the capacity of usable water is strategic for the recipient because it allows him to influence the market price in the transfer market. Until now, we supposed that building up this water capacity required time. In this section, we make the simplifying assumption that capacity of usable water is to the optimal level, once the investment decision is taken.

We also consider that this capacity, once created, never declines, that is that the depreciation rate is zero. Of course, this are two unrealistic assumptions but they allow us to transform our dynamic model into a static model and to compare the results. If both models give the same insights, one might be tempted to rely on the more simple static model, rather than the more complicated dynamic one.

The payoff functions for the two players in the static setting read:

$$F^{d}(p,\tau) = c\left(R - \frac{1}{2}\frac{\tau^{2}}{R}\right) + p\tau,$$
(23)

$$F^{r}(p,\tau,s) = d\left[x + \tau - \frac{\alpha}{2}(s+\tau)^{2}\right] - p\tau - \frac{1}{2}\beta s^{2}, \quad x = x_{0} + s.$$
(24)

where s is the investment decision and x the capacity of usable water. As before, let's suppose initial investments to be zero, i.e. $x_0 = 0$, which implies x = s and the recipients payoff function can be written as:

$$F^{r}(p,\tau,s) = d\left[s + \tau - \frac{\alpha}{2}(s+\tau)^{2}\right] - p\tau - \frac{1}{2}\beta s^{2}.$$
(25)

The Nash equilibrium for this static game is obtained by maximizing $F^d(p,\tau)$ w.r.t τ , $F^r(p,\tau,s)$ w.r.t τ and s, and taking into account the market clearing condition to determine the price of the water transfer, $\tau^S(p) = \tau^D(p,s)$. The optimal solution is given by:

$$\tau^{N} = \frac{\beta dR}{\alpha dc + \beta (c+Y)}, \quad p^{N} = \frac{\beta dc}{\alpha dc + \beta (c+Y)}, \quad s^{N} = \frac{dc}{\alpha dc + \beta (c+Y)}.$$
 (26)

It can be easily seen that the payoffs for the two players at this equilibrium are greater than under the assumption of no water transfer between the regions.

Proposition 9 $F^{d}(p^{N}, \tau^{N}) > F^{d}(0, 0)$ and $F^{r}(p^{N}, \tau^{N}, s^{N}) > F^{r}(0, 0, s^{NT}).$

Proof. We can compute $s^{NT} = \frac{d}{\beta + d\alpha}$ and we can easily see that

$$F^{d}(p^{N},\tau^{N}) - F^{d}(0,0) = (1/2) \frac{cR\beta^{2}d^{2}}{(\beta Rd\alpha + \beta c + d\alpha c)^{2}} > 0.$$

$$F^{r}(p^{N},\tau^{N},s^{N}) - F^{r}(0,0,s^{NT}) = (1/2) \frac{\alpha R^{2}\beta^{3}d^{3}}{(\beta + d\alpha)(\beta Rd\alpha + \beta c + d\alpha c)^{2}} > 0.$$

• Hence, under the above conditions, the water transfer will take place. If the two regions decide to cooperate, they will fix the amount of water transfer, τ , and the alternative source of water, s, in order to maximize the joint welfare:

$$\max_{\tau,s} \left\{ F^d(p,\tau) + F^r(p,\tau,s) \right\}.$$

The cooperative solution τ^{CS} , s^{CS} disregards the price of the water transfer. Apart from that, the transfer and the alternative source of water match their non-cooperative values:

$$\tau^{CS} = \frac{\beta dR}{\alpha dc + \beta (c+Y)}, \quad p^{CS} = \frac{\beta dc}{\alpha dc + \beta (c+Y)}, \quad s^{CS} = \frac{dc}{\alpha dc + \beta (c+Y)}.$$
 (27)

Proposition 10 $\tau^N = \tau^{CS}$ and $s^N = s^{CS}$.

Proof. The proof is obvious, comparing equations (26) and (27). \blacksquare

Remark 11 In the static game the Nash equilibrium is Pareto efficient. In the dynamic game, the lack of an enforcement mechanism leads the recipient to deviate from the open-loop solution which coincided with the cooperative solution. The Nash feedback dynamic equilibrium is not Pareto efficient.

If we consider adequate to treat the interaction between the donor and the recipient as a static game, the non-cooperative solution is Pareto efficient. Conversely, if we consider the more realistic dynamic setting with an accumulative capacity of usable water which depreciates at a given rate, we have seen that the non-cooperative solution will be the feedback solution, because the donor has a higher payoff function and because commitment is generally not easy to impose. The feedback solution is not equal to the cooperative solution and hence the non-cooperative solution in the dynamic game will not be Pareto efficient.

Therefore, analyzing this problem in a static setting, has to major caveats: first, the model fails to properly reflect reality, in particular the dynamic nature of investments and depreciation. Second, the model fails to capture the most salient feature of the solution: the possibility to use a feedback strategy in which the recpient takes into account the impact of his investments on the price path of the water transfer. Hence, we may mistakingly conclude that the non-cooperative solution is Pareto efficient when it is not. The market equilibrium that establishes in the dynamic game will result in a lower transfer and a higher investment than in the cooperative case (and with a lower price than in the open-loop case). We can illustrate this point in the following numerical case.

7 Numerical illustration: the Tagus-Segura transfer

We may relate our results to the data of the Tagus-Segura water transfer described in Ballestero (2004), in particular concerning the information on total supply and demand, the transfer amount and the transfer price.

Ballestero reports the following mean amounts over the period 1979-1996 (page 84 Table IV). Surplus⁸ in the Tagus donor basin: 593.67 million m³. Agricultural water needs in the Lorca recipient region: 74 million m³ (of which some parts can be covered with available groundwater). He also computes the market clearing transfer amount, τ^B and the associated price, p^B in his static model as: $\tau^B = 58$ million m³ and $p^B = 0.46$ dollar US/m³.

We define R as being the maximum possible water surplus in the river, hence R = 593.67million m³. We also define $1/\alpha$ as being the maximum water requirements in the recipient. We can therefore set $\alpha = 1/74 = 0.0135$. We next suppose the discount rate is $\rho = 0.001$ and the equipment depreciation rate $\delta = 0.1$. Initial investments are set to be zero: x(0) = 0. We can find an optimal dynamic solution in which the long-term market-clearing transfer corresponds to the static market-clearing transfer in Ballestero.

Let us compare Ballestero's results to our open-loop solution. Using the parameter values $\beta = 3$ c = 4 d = 10, we get a long-term transfer of $\bar{\tau}^{OL} = 58.22$ million m³ and a long-term transfer price of $\bar{p}^{OL} = 0.39$ dollar US/m³. This price is slightly smaller than the 0.46 dollar US/m³ reported in Ballestero but still much higher than the average cost-based price of 0.082 dollar US/m³ which Ballestero met in the field. Moreover, the initial value of the price path, for t = 0, is $p_0 = 0.47$ dollar US/m³. Our price path hence lies around the values reported in Ballestero. The optimal long-term capacity in the OL case is $\bar{x}^{OL} = 12.95$ million m³ and optimal long-term investments are equivalent to $\bar{s}^{OL} = 1.29$ million m³, a variable that did not exist in Ballestero's paper.

Now turn to the feedback case. Using the same parameter values, we get: $\bar{\tau}^F = 49.91$ million m³ and $\bar{p}^F = 0.34$ dollar US/m³, with the same initial price as before⁹: $p_0 = 0.47$ dollar US/m³. The optimal long-term capacity in the feedback case is $\bar{x}^F = 21.67$ million m³ and the optimal long-term investment is equivalent to $\bar{s}^F = 2.17$ million m³, which is higher than in the OL case. Hence, combining investments and transfers, the recpient would use 49.91+21.67 = 71.58 million m³ which is slightly below his/her agricultural needs of 74 million m³. Having the opportunity to invest and to exert his/her market power hence reduces the recipient's dependence on the donor: roughtly one third of the water needs are provisioned by capacity investments and the transfer price is reduced in consequence. This possibility was not given in Ballestero's paper. Our simulations depend of course on the parameter values adopted for the welfare functions. In this example, the water in the recipient region is valued higher than in the donor region, i.e.

⁸Surplus is given by water stored (833.67 million m³) minus annual needs in the Tagus basin (240 million m³). ⁹Indeed, for x(0) = 0, the initial price in open-loop and feedback information structure is the same.

d > c and environmental amenities in the donor's region are valued at 4 dollar US/m³. Hence, an optimal transfer of about 8% (τ/R), would generate a marginal reduction in environmental amenities of 0.03 dollar US/m³ and hence reduce environmental services in the donor's region by an equivalent of 2 million dollar US. These assumptions seem plausible.

8 Conclusions

The paper analyzes the water transfer between two river-basins interconnected through an aqueduct. The transfer of the water surplus might not affect the donor's economy, but it deteriorates the environment in this region. The recipient region who is benefited from the transfer has to pay the donor for forgone benefits from holding to the water resource. This latter is facing a dynamic dilemma. It can pay the price of the water transfer or it can invest in alternative water supplies to enhance the capacity of usable water. The donor is a myopic agent whose supply decisions ignore this accumulation process. Our main objective is the optimal determination of the water transfer price and quantity considering a bilateral monopoly with a sole supplier and a sole demander.

The dynamic interaction between the two regions is analyzed as a differential game with two particularities. Firstly, the two players do not take decisions on distinct variables, by contrast, they both decide on the amount of water transfer. The donor fixes the supply and the recipient the demand, both as a function of the price. The market clearance condition will determine price and quantity at the equilibrium. The second characteristic is the coexistence of a farsighted buyer with a myopic seller. We a have two main findings which are valid for a rather general differential game with one myopic and one farsighted player taking supply and demand decisions in a bilateral monopoly market. The first result states that supply and demand decisions are the same regardless of whether the farsighted player commits to an optimal path from the start of the game, or he adjusts his decisions taking into account the state of the system. Thus, optimal price and quantity are the same function of the state under the feedback or the open-loop Nash equilibria. The second and main result of the paper advises that treating the interaction between the players as a static game the non-cooperative solution would match the cooperative solution, being then Pareto efficient. Conversely, in a dynamic setting with no mechanism to enforce commitment, the non-cooperative solution would be characterized by feedback strategies. The feedback Nash equilibrium does not coincide with the cooperative solution and therefore it is not Pareto efficient. Only if the players have no information on the state of the system or their

commitment is somehow exogenously enforced, would it be adequate to consider open-loop information structures. And only in that case, the laisser-faire policy would be Pareto efficient.

In the game described in the paper, a feedback recipient who does not commits from the beginning, is aware of the fact that a greater capacity of usable water would reduce the price of the water transfer. This knowledge gives the recipient a market power and the investments in the capacity of usable water an extra-value. In consequence, the recipient invests more in the infrastructure to produce, save or recycle water. And thus, the capacity of usable water grows higher under feedback than under open-loop information structures. This implies that less water is transferred and at a lower price. Due to this market power, we prove (analytically) that the recipient is better off under feedback strategies, while (numerically) the donor is worse off. Since the open-loop solution is Pareto efficient, the aggregate welfare would be higher under commitment. It is further proved that when price and quantity of the water transfer are optimally chosen, either with or without commitment, both players are better off than in the case of no transfer.

We finally, compare the price and quantity of water transfer in the two non-cooperative solutions (open-loop and feedback), with the supply-demand scheme in Ballestero (2004), who takes into account uncertainty, but not the recipient's opportunity to invest in the capacity of usable water. After calibrating the model, we observe that the opportunity to invest in alternative water supplies reduces the dependency on the water transfer. Less water is transferred at a lower price, which decreases through time.

In this paper we analyze the interaction between the players assuming that the aqueduct already exists. Either a central government decided or the two parts agreed to build this infrastructure. Once the transfer is physically feasible, our analysis was on the optimal price and quantity. An immediate natural extension would be to analyze how to share the fix cost of building the aqueduct. A second interesting extension, specially when the two river-basins are located in different countries, would be to study the model considering hierarchical modes of play. Depending on the circumstances, either the donor or the recipient could be regarded as the Stackelberg leader and its opponent the follower.

References

 Ambec, S., and L. Ehlers (2008) "Sharing a river among satiable agents." Games and Economic Behavior, Vol. 64, 35-50

- [2] Ambec, S., and Y. Sprumont (2002) "Sharing a river." Journal of Economic Theory, Vol. 107, 453-462
- [3] Ballestero, E., 2004, Inter-Basin Water Transfer Agreements: A decision Approach to Quantity and Price, Water Resources Management 18: 75-88.
- [4] Bhaduri, A., Barbier, E.B., 2008, International Water Transfer and Sharing: The Case of the Ganges River, Environment and Development Economics, 13, 29-51.
- [5] Bravo, M., González, I., García-Bernabeu, A., 2010, Ranking supply oriented policies of water management policies from institutional stakeholders' political views, European Water, 31, 43-58.
- [6] Kumar, D., 2006, Environmental impact of inter-basin water transfer projects: some evidence from Canada, Economic and Political Weekly, 17, 1703-1707.
- [7] Lekakis, J.N., 1998, Bilateral Monopoly: a market for intercountry river water allocation, Environmental Management, 22, 1-8.
- [8] Merrett, S., Introduction to the Economics of Water Resources an international perspective, UCL Press, London 1997, 211p.
- [9] Tsur, Y., Zemel, A., 1995, Uncertainty and irreversibility in groundwater resource management, Journal of Environmental Economics and Management, 29,149-161.
- [10] Tsur, Y., Zemel, A., 2004, Endangered aquifers: Groundwater management under threats of catastrophic events, Water Resources Research 40, 1-10.
- [11] Interbasin Water Transfer, Proceedings of the International Workshop, IHP-V, Technical Documents in Hydrology No. 28, UNESCO, Paris, 1999.