Optimal adaptation strategies to face shocks on groundwater resources

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Abstract

We consider an exogeneous and irreversible shock on a groundwater resource: a decrease in the recharge rate of the aquifer. We compare optimal extraction paths and social costs for optimal adaptation in two cases: under certainty, i.e. when the date of occurrence of the shock is known and under uncertainty, when the date of occurrence of the shock is a random variable. We show that the increase of uncertainty leads to a decrease in precautionnary behaviour, in the short-run and in the long-run. Moreover, we apply our model to the particular case of the Western la Mancha aquifer. We show in this context that information aquisition may not be interesting for the manager of the resource, at least when the shock occurs later in time.

JEL classification: C61,Q25.
Key words: Groundwater resource, optimal behavior, exogeneous shock, uncertainty.

1 Introduction

Our study relates to the situation of a common groundwater resource, used by several farmers for irrigation, which is subject to droughts (see Amigues [1], Zilberman and al. [20]). The aquifer is managed by a social planner, the water agency, which seeks to adapt optimally to these episodes of low precipitation. In a near future, the problem of low precipitations may become more important, with the phenomenon of global warming. In this context, the evolution of the natural system may be subject to abrupt changes which can be qualified as "regime shifts". In this paper, we model this type of problem through shocks in the dynamics of the resource, and we characterize the impact of the shock on the optimal management of the water agency. In particular, we study the optimal path of extraction, and we estimate the consequences of these adaptation strategies from an ecological point of view, via the study of the aquifer level, and from an economic point of view, via the calculation of social welfare.

To do so, we use a simple groundwater model, the Gisser and Sanchez [11] model (for similar model frameworks, see for example Cummings [5] or Roseta-Palma (2002, 2003) [15] [16]), in which we introduce two types of shocks: a deterministic shock at a given date and a random shock which may occur with a certain probability. This shock may correspond to a decrease in mean precipitations which leads to a decrease in the recharge of the aquifer, or it may correspond to an abstraction of a certain amount of water which is dedicated to other uses in case of a drought, such as the filling of drinking water reservoirs.

First, when the date of the shock is known, the intuition may be that the water agency could prepare herself, with a strategy of more careful extraction. We will see that our results contradict our initial intuitions and that extractions are more important in the short-term. We can find this result in the existing literature (see Di Maria et al. 2012 [8]), in the context of polluting resources, where the phenomenon is known as the "announcement effect" or the "abundance effect". Second, when the date of the shock is a random variable, we may derive some intuitions on the type of solutions from the existing literature on catastrophic events, in the context of groundwater resource management, (see Tsur and Zemel (1995,2004) [17], [19]), and pollution control (see Clarke and Reed (1994) [4], Brozobic and Schlenker (2011) [2], Tsur and Zemel (1996) [18], and de Zeeuw and Zemel (2012) [7]). Indeed, there is an extensive literature on the relationship between precautionary behaviour and uncertainty: there are papers that show that an increase in uncertainty leads to non-monotonic changes in precautionary behaviour (see Clarke and Reed 1994 [4], Brozobic and Schlenker (2011) [2]), other papers show that an increase in uncertainty leads to a decrease in precautionary behaviour (see Tsur and Zemel (1995, 2004) [17], [19]), and again other papers conclude that more uncertainty leads to an increase in precautionary behaviour (see Zeuw and Zemel 2012 [7], Tsur and Zemel [19]). Clarke and Reed find this non-monotonic relation for an irreversible event, which is exogenous, Brozovic and Schlenker for a reversible event which is endogenous, i.e. occurs when a threshold level is reached. Tsur et Zemel proved in [19], that the increase in uncertainty leads to more intensive extractions, and thus lower levels of the long-term tablecloth of the aquifer in the case of exogenous irreversible events. However, they also showed that a more precautionary behavior can happen in the long-run when the event is exogenous and reversible or when the event is endogenous. In contrast, in [7], Zeeuw et Zemel proved that the introduction of a random jump in the damage function of a pollution control model leads to more precautionary behaviour, both for endogenous events and irreversible exogenous events.

In this article, we study irreversible exogenous events and analyse analytically the relation between the caracteristics of the shock and the adaptation behavior in the long run and the short run. We show that our results correspond to the solutions found by Tsur and Zemel in [19], but our paper differs from [19] in several ways: First, Tsur and Zemel study catastrophic events (such as saltwater intrusion) which render further use of the resource impossible (unless restoration activites are undertaken). We are interested in a shock (on the recharge rate) that does not hinder further exploitation. Second, Tsur and Zemel focus on endogenous events. In that case, when the threshold level that triggers the event is known, it is optimal to avoid the occurence of the event. In our study, the deterministic exogenous shock can not be avoided. Lastly, when Tsur and Zemel consider a case with an exogenous shock, they only compare the solution to the non-event solution. We are interested in comparing the deterministic shock to the randomly occuring shock.

Moreover, we are interessed in applying our model to the Western la Mancha aquifer, in the South of Spain. In this area, the state of the groundwater resources is preoccupying, jeopardizing the maintainance of important ecosystem services (see Esteban and Albiac (2011) [9] and Esteban and Dinar (2012) [10] for details). The average annual groundwater recharge in this aquifer is 360 Millions of cubic meters. Unfortunately, in the last decades, the aquifer has been subject to droughts. For example, in 1999, the recharge rate has decreased by aproximatively 100 Millions of cubic meters. We have decided to analyse the impact of shocks that have the magnitude of past variations in recharge rates, but we study a lower benchmark problem with one variation in the recharge rate. Even though we suppose that the water agency adapts optimally to these shocks, we show that the costs to society are important, and can reach values of the order of several millions of euros. Moreover, we want to know whether the water agency should try to forsee the date of the shock or not. We show that it may not be interesting for the water agency to aquire additional information on the occurence date of the shock, even if this information were costless. Indeed, we show that information aquisition is only interesting, when the shock takes place in the distant future. However, we confirm that it is always better for the water agency to have an adaptative behavior, with or without knowledge on the date of the shock, than not to prepare to the shock.

This paper is organized in the following way. In section 2, we remind the underlying model, the Gisser and Sanchez model. We then introduce an exogenous shock which incorporates the idea of the lack of water, and we derive some theorical results. In section 3, we make a numeric illustration where we analyze optimal adaptation behavior and the impact of the shock on social welfare, in the short run and in the long-run. Finally, in section 4, we conclude and give some perspectives for future research.

2 The model

We base our analysis on the groundwater extraction model by Gisser and Sanchez, (see [11]), where $G(t)$ and $g(t)$ are respectively the stock of the aquifer (in volume)[1] and water pumping of the aquifer as a function of time[2]. We assume that

[1] G corresponds to the volume of water, and it is calculated multiplying H, the water table elevation above sea level by, A*S, where A is the area of the aquifer and S is the storavity coeficient.

[2] We omit the time indicator in all following equations, whenever this is possible without causing misunderstandings, in order to make equations more easily readable.

$$g = a - bp \tag{1}$$

is a linear function that represents the demand for irrigation water, where p is the price of water, and a,b are coefficients of the demand function, with a>0, b>0.

Consider a linear cost function for extractions costs:

$$\bar{C} = z - cG,$$

where z are fixed costs, c marginal pumping costs and z,c are coefficients of the linear cost function, with z>0, c>0.

The dynamics of the aquifer,

$$\dot{G} = -(1 - \alpha)g + r \tag{2}$$

depend on hydrological characteristics of the aquifer, where r is the recharge rate and α is the return flow coefficient.

The total revenu of farmers, using the agricultural surface characterized by the demand function of water, equation (1)), is then:

$$\int p(g)dg = \int \frac{a-g}{b} dg = \frac{a}{b}g - \frac{1}{2b}g^2.$$

The problem of the social planner is to maximise the social welfare, that is the present value of private revenus of farmers, with ρ, the discount rate, taking into account the dynamics of the aquifer (see equation (2)), and subject to initial conditions and positivity contraints:

$$\max_{g(.)} \int_0^\infty \quad F(G, g) \quad \mathrm{e}^{-\rho \mathrm{t}} \quad \mathrm{dt},$$

where,

$$F(G, g) = \frac{a}{b}g - \frac{1}{2b}g^2 - (z - cG)g,$$

$$\dot{G} = -(1 - \alpha)g + r,$$

$$G(0) = G_0 \quad \text{given},$$

$$g \geq 0 \qquad G \geq 0.$$

The full resolution of the problem is described in the Appendix (A.2).

In the following sections, we are going to introduce an exogeneous shock to our initial model, the decrease in the recharge rate of the aquifer at the moment t_a. First, we solve the deterministic case where the moment of the shock t_a is known, and second, we solve the stochastic case where t_a, is a random variable that follows an exponential distribution.

2.1 The deterministic case

We assume that there is a decrease in the recharge rate from r_1 to r_2 at the known instant t_a. Indeed, the main problem is to model the fact that the availability of water for irrigation decreases from t_a. This can happen because there is an estimation of a decrease in mean precipitations or because of a specific extraction of water for other uses from t_a on. In theory, these ideas are equivalent and we can describe them as a decrease of the recharge rate (see proof on appendix A.1).

We are interested in the optimal path of extractions in presence of this "dry period". The problem of the social planner is now :

$$\max_{g(.)} \int_0^{\infty} \quad F(G,g) \quad \mathrm{e}^{-\rho t} \quad \mathrm{dt},$$

where,

$$F(G,g) = \frac{a}{b}g - \frac{1}{2b}g^2 - (z - cG)g,$$

$$\dot{G} = \begin{cases} -(1-\alpha)g + r_1 & \text{if} \quad t \leq t_a \\ -(1-\alpha)g + r_2 & \text{if} \quad t > t_a, \end{cases} \tag{3}$$

$$G(0) = G_0, t_a \quad \text{given}, \qquad r_1 > r_2,$$

$$g \geq 0 \qquad G \geq 0.$$

We can solve the problem in two steps. First, we find $\phi(ta, G_{ta})$, the scrap value function that represents the maximisation between t_a and ∞, that is:

$$\phi(ta, G_{ta}) = \max_{g(.)} \int_{t_a}^{\infty} \quad F(G,g)\mathrm{e}^{-\rho(\mathrm{t}-\mathrm{t_a})}\mathrm{dt}.$$

After this, our problem is to find G,g and $G(t_a)$ that maximise:

$$\int_0^{t_a} \quad F(G,g)\mathrm{e}^{-\rho \mathrm{t}}\mathrm{dt} + \mathrm{e}^{-\rho \mathrm{t_a}}\phi(\mathrm{t_a}, \mathrm{G_{ta}}).$$

We may write the Hamiltonian of this last problem:

$$H = F(G,g) + \pi(-(1-\alpha)g + r_1),$$

where π is the adjoint variable. We are now in a free-endpoint problem, with t_a known and need an additional transversality condition (see for example Léonard and Ngo van Long [14]):

$$\pi(t_a) = \frac{\partial \phi(ta, G_{ta})}{\partial G_{ta}}.$$

The full resolution of this modified extraction problem is deferred to the Appendix, (A.3).

Now, we present some theorical results proved by studying analytical solutions of the previous problems.

Let

- $g^*_{SP}(t)$(and $G^*_{SP}(t)$) be optimal extractions and the stock of the simple problem $(r_1 = r_2 > 0)$;

- $g^*_{r_2^1}(t)$(and $G^*_{r_2^1}(t)$) be optimal extractions and the stock of the deterministic shock when $(r_2 = r_2^1)$, where $r_1 > r_2 = r_2^1 > 0$;

- $g^*_{r_2^2}(t)$(and $G^*_{r_2^2}(t)$) be optimal extractions and the stock of the deterministic shock when $(r_2 = r_2^2)$, where $r_1 > r_2 = r_2^2 > 0$.

Proposition 2.1 *If $r_1 > r_2 > 0$, $G^*_{SP}(\infty) > G^*_{r_2}(\infty)$.*

Proof:

As we can see in appendix A.2 and A.3, the steady state of the simple problem and the steady state of the modified problem are given by equation (22) and (29) respectively, with $r = r_1$,

$$G^*_{SP}(\infty) = \frac{r_1}{(1-\alpha)cb} + \frac{r_1}{\rho} - \frac{a}{bc} + \frac{z}{c},$$

$$G^*_{r_2}(\infty) = \frac{r_2}{(1-\alpha)cb} + \frac{r_2}{\rho} - \frac{a}{bc} + \frac{z}{c},$$

so,

$$G^*_{SP}(\infty) - G^*_{r_2}(\infty) = (r_1 - r_2)\left(\frac{1}{(1-\alpha)cb} + \frac{1}{\rho}\right) > 0, \tag{4}$$

and then, $G^*_{SP}(\infty) > G^*_{r_2}(\infty)$.

According to proposition 2.1, the steady-state of the stock of the modified problem is smaller than the steady-state of the stock of the simple problem. Indeed, when the shock takes place, the resource is more exploited in the long term.

Proposition 2.2 *$G^*_{r_2}(t_a)$ is a decreasing monotonous function of r_2, i.e., the optimal value of the stock in t_a decreases when the value of the shock increases, (when r_2 decreases).*

We prove this analytically but expressions are too long to be given here. The proof is available from the authors.

Proposition 2.2 states that the more important the value of the shock, the more exploited the resource in t_a.

Proposition 2.3 *If $r_1 > r_2$, $g^*_{SP}(\infty) > g^*_{r_2}(\infty)$, i.e., extractions are more conservative in the long run, when the shock takes place.*

Proof:

$$g^*_{SP}(\infty) = \frac{r_1}{(1-\alpha)}$$

$$g^*_{r_2}(\infty) = \frac{r_2}{(1-\alpha)}$$

As $r_1 > r_2$, then, $g^*_{SP}(\infty) > g^*_{r_2}(\infty)$.

Proposition 2.4 *$g^*_{r_2}(0)$ is a decreasing monotonous function of $r_2 \Longrightarrow g^*_{SP}(0) < g^*_{r_2^1}(0) < g^*_{r_2^2}(0)$, with, $r_1 > r_2^1 > r_2^2$, that is, optimal extractions in t=0, increase the more important the shock.*

The proof is available from the autors.

Proposition 2.5 *$g^*_{r_2}(t_a)$ is a decreasing monotonous function of $r_2 \Longrightarrow g_{ta} < g^*_{r_2^1}(t_a) < g^*_{r_2^2}(t_a)$, with, $r_1 > r_2^1 > r_2^2$, that is, optimal extractions in $t = t_a$, increase the more important the shock.*

The proof is available from the autors.

Proposition 2.6 $\frac{r_1 - r_2^1}{G^*_{SP}(t_a) - G^*_{r_2^1}(t_a)} = \frac{r_1 - r_2^2}{G^*_{SP}(t_a) - G^*_{r_2^2}(t_a)}$.

The proof is available from the autors.

Corolaire 1 *$G^*_{SP}(t_a) - G^*_{r_2^2}(t_a) = \frac{r_1 - r_2^2}{r_2^2 - r_2^1}(G^*_{r_2^1}(t_a) - G^*_{r_2^2}(t_a))$, i.e., the distance between optimal values of the stock in t_a, G^*_{ta}, for two different shocks, depends only on the values of different shocks.*

This is the result of proposition 2.6.

Logically, as the moment of the shock, t_a, is known, the difference (or distance) between optimal solutions for different shocks in t_a, depends only on the value of shocks.

2.2 The stochastic case

Now, the moment when the shock takes places (T_a), that is, the moment of the decrease in the recharge rate, is a random variable.[3] Let $f_T(t)$ be the density function, and $\psi_T(t)$ the distribution function of T

with,

$$\psi_T(t) = \int_0^t f_X(x)dx,$$

and

$$\Omega_T(t) = 1 - \psi_T(t).$$

The problem of the social planner is now to maximise the expected value of total revenus of farmers,

$$\max_{g(.)} E_T(\int_0^\infty \mathrm{e}^{-\rho \mathrm{t}} \mathrm{F(G,g)} \quad \mathrm{dt}) \tag{5}$$

$$\dot{G} = \begin{cases} -(1-\alpha)g + r_1 & \text{if} \qquad t \leq T \\ -(1-\alpha)g + r_2 & \text{if} \qquad t > T \end{cases}$$

$$G(0) = G_0 \quad \text{given},$$

$$g \geq 0 \qquad G \geq 0.$$

with the profit function F(G,g), as in the previous problem,

$$F(G,g) = \frac{a}{b}g - \frac{1}{2b}g^2 - (z - cG)g. \tag{6}$$

Following the procedure used in Dasgupta and Heal (see [6]), let $\phi(t, G(t))$ be the scrap value function,

$$\phi(t, G(t)) = \max_{g(t)} \int_T^\infty \mathrm{e}^{-\rho(\mathrm{t-T})} \mathrm{F(G(t), g(t))dt}.$$

Equation (5) is equal to

$$\max_{g(.)} \int_0^\infty f_T(T)[\int_0^T \mathrm{e}^{-\rho \mathrm{t}} \mathrm{F(G,g)dt} + \mathrm{e}^{-\rho \mathrm{T}} \phi(\mathrm{T}, \mathrm{G(T)})] \quad \mathrm{dT}, \tag{7}$$

so:

[3] In what follows, we note $T_a = T$ for simplicity.

$$\max_{g(.)} \int_0^\infty f_T(T)[\int_0^T \mathrm{e}^{-\rho \mathrm{t}}\mathrm{F(G,g)dt}] \quad \mathrm{dT} + \int_0^\infty \mathrm{e}^{-\rho \mathrm{T}}\mathrm{f_T(T)}\phi(\mathrm{T,G(T)}) \quad \mathrm{dT}. \tag{8}$$

Solving by parts the first integral in the previous equation (8), we find that:

$$\int_0^\infty f_T(T)[\int_0^T \mathrm{e}^{-\rho \mathrm{t}}\mathrm{F(G,g)dt}] \quad \mathrm{dT} = \int_0^\infty \mathrm{e}^{-\rho \mathrm{T}}\Omega_{\mathrm{T}}(\mathrm{T})\mathrm{F(G(T),g(T))} \quad \mathrm{dT}.$$

Equation (8) and hence equation (5) can be written as:

$$\max_{g(.)} \int_0^\infty \mathrm{e}^{-\rho \mathrm{t}}[\Omega_{\mathrm{T}}(\mathrm{t})\mathrm{F(G,g)} + \mathrm{f_T(t)}\phi(\mathrm{t,G})] \quad \mathrm{dt}, \tag{9}$$

Next, we want to estimate the distribution of the random shock. Meteorological studies showed that the gamma distribution gives a good estimation of annual recharges[4], (see for example Leizarowitz and Tsur [13]). To simplify the resolution of our stochastic problem and to obtain analytical solutions, we choose an exponential distribution to estimate the moment of the shock, that is a particular case of the law of gamma, $\gamma(1,\theta)$.

T follows now an exponential distribution, where $f_T(t)$, is the density fonction:

$$f_T(t) = \theta \mathrm{e}^{-\theta \mathrm{t}} \qquad \mathrm{t} = 0..\infty$$

and, $\Omega_T(t)$ is the inverse of the distribution function,

$$\Omega_T(t) = \mathrm{e}^{-\theta \mathrm{t}} \qquad \mathrm{t} = 0..\infty$$

The problem of the social planner becomes:

$$\max_{g(.)} \int_0^\infty \mathrm{e}^{-(\rho+\theta)\mathrm{t}}[\mathrm{F(G,g)} + \theta\phi(\mathrm{t,G})] \quad \mathrm{dt} \tag{10}$$

$$\dot{G} = -(1-\alpha)g + r_1 \tag{11}$$

$$G(0) = G_0 \quad \text{given}, \tag{12}$$

$$g \geq 0 \qquad G \geq 0. \tag{13}$$

The full resolution of this modified extraction problem is deferred to the Appendix A.4.

[4]It is important to know that the filling of the aquifer is made from seasonal rains.

3 Numerical application

In this section, we apply our problem to the Western la Mancha aquifer. We use real values of parameters from several sources (e.g. Esteban and Albiac(2011) [9], Esteban and Dinar(2012) [10]). This leads us to use the parameters values listed in table 1.
The Western la Mancha aquifer is located in Southeastern Spain. The development of intensive irrigation agriculture in recent decades has led to an increase in groundwater extractions and, as a consequence, the decrease in the water table. This problem has caused significant damages to aquatic ecosystems and also to human uses downstream. Moreover, the Western la Mancha aquifer has suffered from inefficient management regimes, (see for details [9]). In presence of a dry period, this problem may become more acute.

Parameters	Description	Units	Value
b	Water demand slope	Euros/Millions of m^3	0.097
a	Water demand intercept	Euros/Millions of m^3	4403.3
z	Pumping costs intercept	Euros/Millions of m^3	266000
c	Pumping costs slope	Euros/Millions of m^3. Millions of m^3	3.162
α	Return flow coefficient		0.2
r	Natural recharge	Millions of m^3	360
G_0	Current water table (in volume)	Millions of m^3	80960
ρ	Social discount rate	%	0.05

Figure 1: Values of parameters of the Western la Mancha aquifer.

3.1 Optimal solutions for the deterministic case

We first give a numerical example of the deterministic case. Figure 2 depicts optimal solutions of stock $G^*(t)$ (left-hand side) and water pumping $g^*(t)$ (right-hand side) in millions of cubic meters, for the initial problem described in section 2 (in green) and for different values of the deterministic shock after the five first years of resource use, (i.e in $t_a = 5$). More specifically, we can see optimal solutions for a decrease in the recharge from an initial level of r_1= 360 to a level of r_2= 330 (in red), 300 (in magenta) and 290 (in black) millions of cubic meters (Mm^3).[5].

First, we note that the steady state of the stock and the water pumping is smaller when r_2 decreases. Hence the resource is driven to a lower level and extractions are more conservative in the long-run. For example, the level of the resource decreases by 8000 Mm^3 (and extractions by 400 Mm^3 respectively) for a shock of $70 Mm^3$. Second, we can observe that the resource is more exploited at the moment of the shock when the shock

[5]We have chosen $t_a = 5$ years and the different values of the shock following the observations made in the area in the 90s, (see [10] for details)

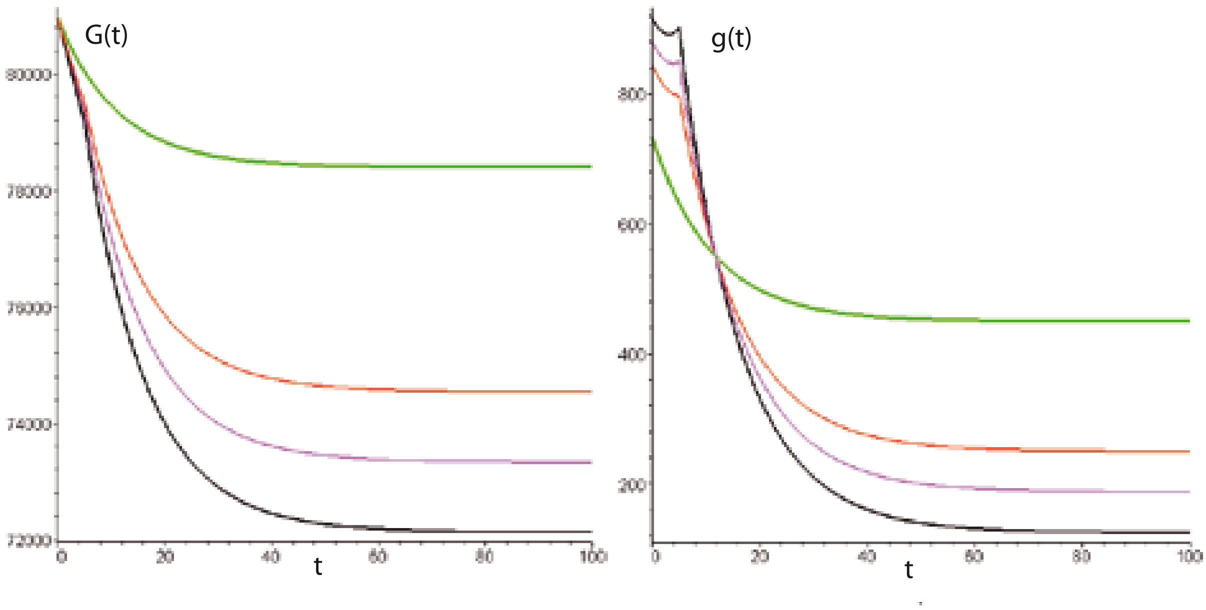

Figure 2: $G^*(t)$(left-hand side) et $g^*(t)$(right hand side) in ta=5 years for different values of r_2 ,in millions of cubic meters.

is more important, i.e. $G^*(t_a)$ decreases and $g^*(t_a)$ increases by about 300 Mm^3, when r_2 decreases by about 70 Mm^3. We can also confirm that the resource is more exploited in the beginning of the exercice (i.e. for $t = 0$), when the shock is more important: $g^*(0)$ increases from 750 to 950 Mm^3, when r_2 decreases by 70 Mm^3. These results can illustrate the above propositions 2.1, 2.2, 2.3, 2.4 and 2.5, proved in section 2.1.

We can also observe a surprising behaviour of optimal water pumping: when the shock is more important (in black), there is an increase of extractions just before the occurence of the shock. This result can be explained intuitively by the fact that when the shock is important, the water agency does not have time to adapt to it quickly enough.

Next, in table 3, we calculated the social welfare for our numerical example. We notice that social welfare increases in the first period (before t_a, column 1) and decreases in the second period (after t_a, column 2) the more important the shock. Logically, the total social welfare (column 3) decreases by about 67 Millions of euros, when the value of the shock increases by 70 Mm^3.

As a result, the theorical and numerical solutions make clear that in the first period, there is an increase in extractions when the shock takes place. This happens in order to accumulate gains and compensate the losses of the second period. In the second period,

Social Welfare (in millions of euros)	[0, ta] (1)	[ta, ∞] (2)	[0, ∞] (3)	(1)/(3) (in %)	(2)/(3) (in %)
$r_2 = 360$	92	215	307	30	70
$r_2 = 330$	102	174	276	37	63
$r_2 = 300$	112	137	249	45	55
$r_2 = 290$	115	126	240	48	52

Figure 3: Social welfare for different values of r_2 in $t_a = 5$.

there is a decrease in extractions. However, the level of the resource is lowered in the long-run, because of the behavior adopted in the first period.

3.2 Optimal solutions for the stochastic case

We now construct a numerical example for the stochastic case. T_a, the moment of the shock, is a random variable which follows an exponential distribution, that is the occurence of shock is less probable in later time periods.

In figure 4, we can see the optimal solutions of stock $G^*(t)$ and extractions $g^*(t)$ in millions of cubic meters, for the initial problem described in section 2 (in green) and for different values of the stochastic shock (r_2= 330 (in red), 290 (in magenta) and 100 (in black) Mm^3), when $\theta = 0.01$. More specifically, a shock that occurs before the end of the fifth year would for example have a probability of 95 pourcent. In the right corner of the figure, we can see a zoom of optimal extractions between t=0 and t=2 years.

First, we observe that as before, the steady state of the stock is smaller (of around 400 Mm^3) but water pumping is the same in the long-run, when the shock is more important (r_2 decreases). In the right corner of the figure, we can note that there is an increase in extractions in $t = 0$ if we compare the stochastic shock (in black) with the simple problem without shock (in green). Furthermore, it is important to notice that for the stochastic shock, the steady states of $G(t)$ and $g(t)$ are reached earlier that in the simple problem. Thus, extractions are more intensive in the first years for the stochastic shock.

Second, we study optimal solutions for different values of θ, the parameter of the distribution function. In figure 5, we observe similar behaviour than in figure 4. When it is more likely that the shock takes place in the first few years (, i.e. when θ increases), there is an increase in extractions (aproximatively 200 Mm^3) in $t = 0$, and for this reason the level of the resource is lowered in the long run.

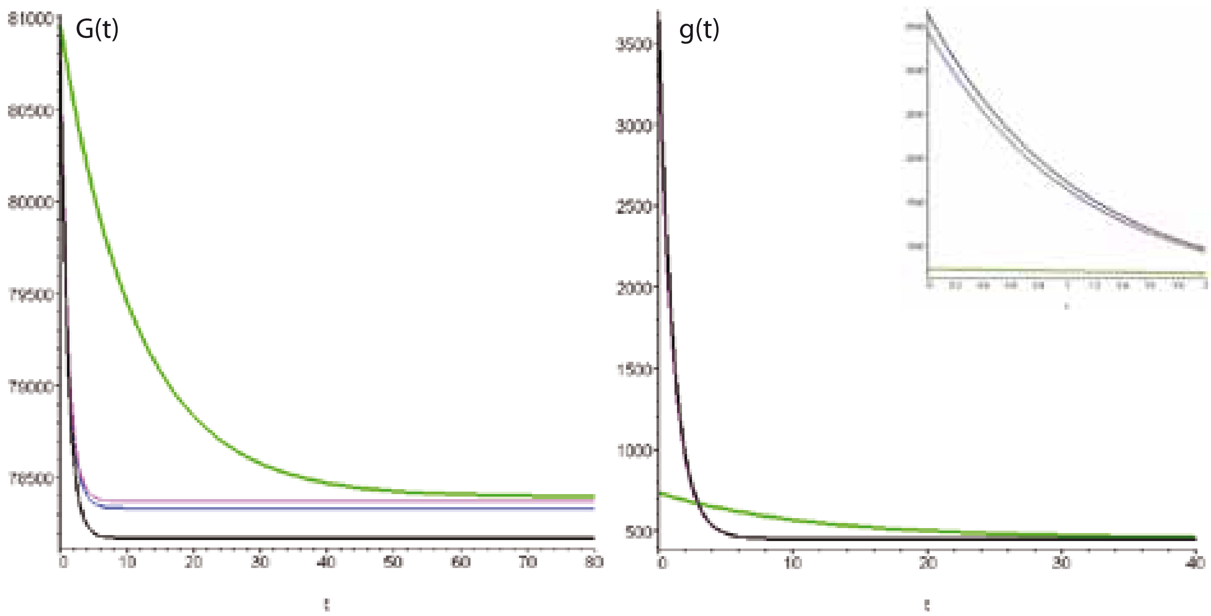

Figure 4: $G^*(t)$(left-hand side) et $g^*(t)$(right hand side) for different values of r_2 and $\theta = 0.01$. Upper Right-hand corner: zoom of $g^*(t)$ between $t = 0$ and $t = 2$ years.

Finally, we calculate social welfare for different scenarios of occurrence probability (θ) and values of the shock (r_2). When the occurrence probability (θ) is fixed, social welfare decreases the more important the shock (, i.e. the higher r_2). Likewise, when the value of the shock (r_2) is fixed, social welfare decreases the greater the probability that the shock takes place in the first few years(, i.e. the higher θ).

3.3 Deterministic case vs. Stochastic case

In this section, we compare optimal solutions of the simple problem, the deterministic case and the stochastic case.

The differences can be seen in Figure 7. We compare optimal solutions of the stock $G^*(t)$ and extractions $g^*(t)$ for the initial problem described in section 2 (in green) for the deterministic shock (in black) and for the stochastic shock (in magenta), when the value of the shock is fixed ($r_2 = 300$). Focusing on the left-hand side of the figure, we note that the level of the resource is (about 1000 Mm^3) smaller in the long run in case of the deterministic shock than in the the case of the stochastic shock. In the short run, however, the inverse holds: the stock is lower in the stochastic case than in the deterministic case. On the other hand, in the right hand side of the figure, we can see that extractions at the initial time are much bigger (of about 2500 Mm^3) in the stochastic case than in the

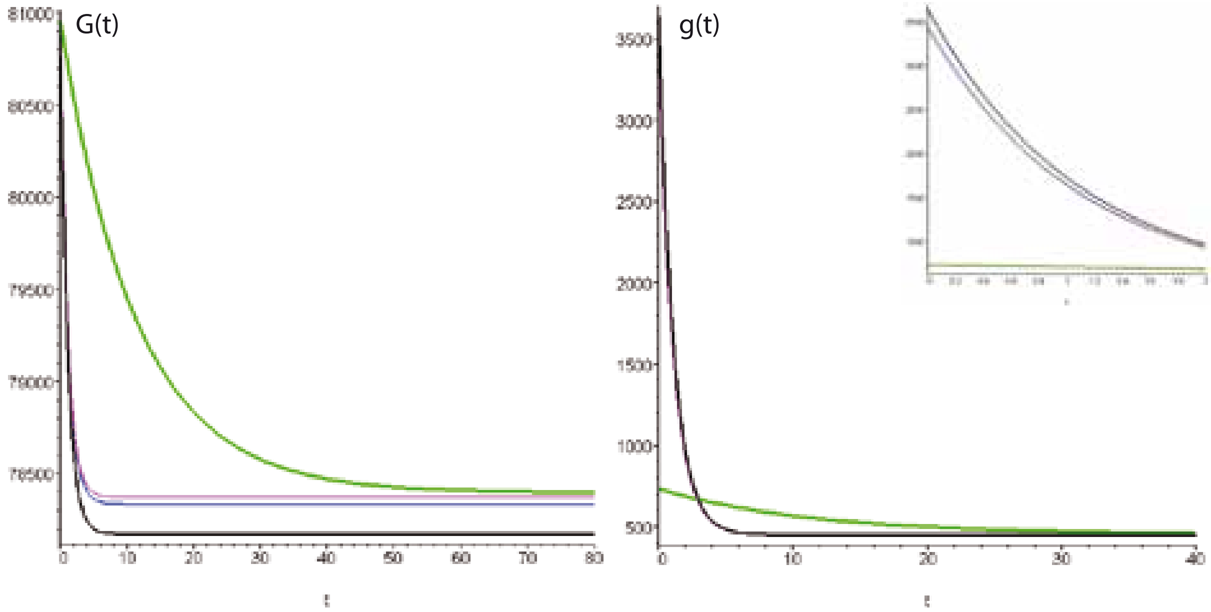

Figure 5: $G^*(t)$(left-hand side) et $g^*(t)$(right hand side) for different values of θ and $r_2 =$ 300 millions of cubic meters. Upper Right-hand corner: zoom of $g^*(t)$ between $t = 0$ and $t = 2$ years.

Social Welfare (in millions of euros)	**Θ=0.001**	**Θ=0.01**	**Θ=0.05**
$r_1 = r_2 = 360$	288	288	286
$r_2 = 330$	288	285	275
$r_2 = 300$	288	281	265
$r_2 = 290$	287	280	262

Figure 6: Social welfare for different values of θ when $r_2 = 300$.

deterministic case, although extractions are similar and rather conservative in the long-run.

Next, we determine the difference (D) of the social welfare between the deterministic case and the stochastic case, (see table 8). In absence of any shock and for a given value of θ, we note that there is no difference between social welfares that occur at different moments in time, e.g. for $t_a = 5$ and $t_a = 50$ years.[6]. However, if a shock occurs, we see

[6]Remember when $\theta = 0.01$, a shock that occurs before the end of the fifth year (or before the end of the fiftieth year) would have a probability of 95 pourcent (or 60 pourcent respectively)

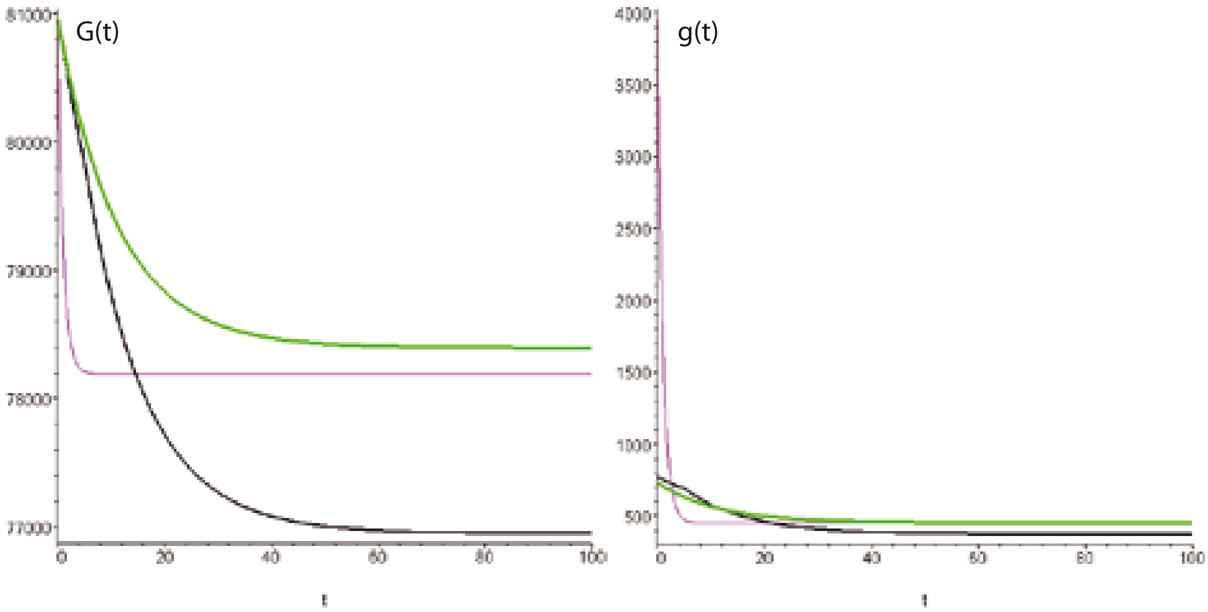

Figure 7: $G^*(t)$(left-hand side) et $g^*(t)$(right hand side) for the simple problem(in green), the deterministic shock(in black) and the stochastic shock(in magenta), when $r_2 = 300$ millions of cubic meters.

that D is almost always negative when $ta = 5$ and always positive when $ta = 50$. This means that, when the shock takes place earlier (for example in $t_a = 5$), the stochastic shock is more profitable for the society, that is, it is more favorable to have some information on the moment of the shock. In contrast, when the shock takes place later (for example in $t_a = 50$), the deterministic shock is more profitable for the society, that is, it is better to know the moment of the shock. Moreover, the sign of D does not depend on the probability of occurence of the shock.

$SW_{DC} - SW_{SC}$ (in millions of euros)	**Θ=0.001**		**Θ=0.01**		**Θ=0.05**	
	Ta=5	**Ta=50**	**Ta=5**	**Ta=50**	**Ta=5**	**Ta=50**
$r_1 = r_2 = 360$	19	19	19	19	21	21
$r_2 = 330$	-12	18	-9	21	1	31
$r_2 = 300$	-39	16	-32	23	-16	39
$r_2 = 290$	-47	17	-40	24	-22	42

Figure 8: Difference between Social welfare of the deterministic case and the stochastic case for different values of θ, $r_2 = 300$ and t_a.

For the water agency in the La Mancha basin, this means that it is not always interesting to acquire more information on the occurence date of the shock, even if this information was available. Indeed, in some cases, it is less costly for the water agency to adapt to a randomly occurring shock rather than to a deterministic shock.

3.4 Adaptation vs. Non Adaptation

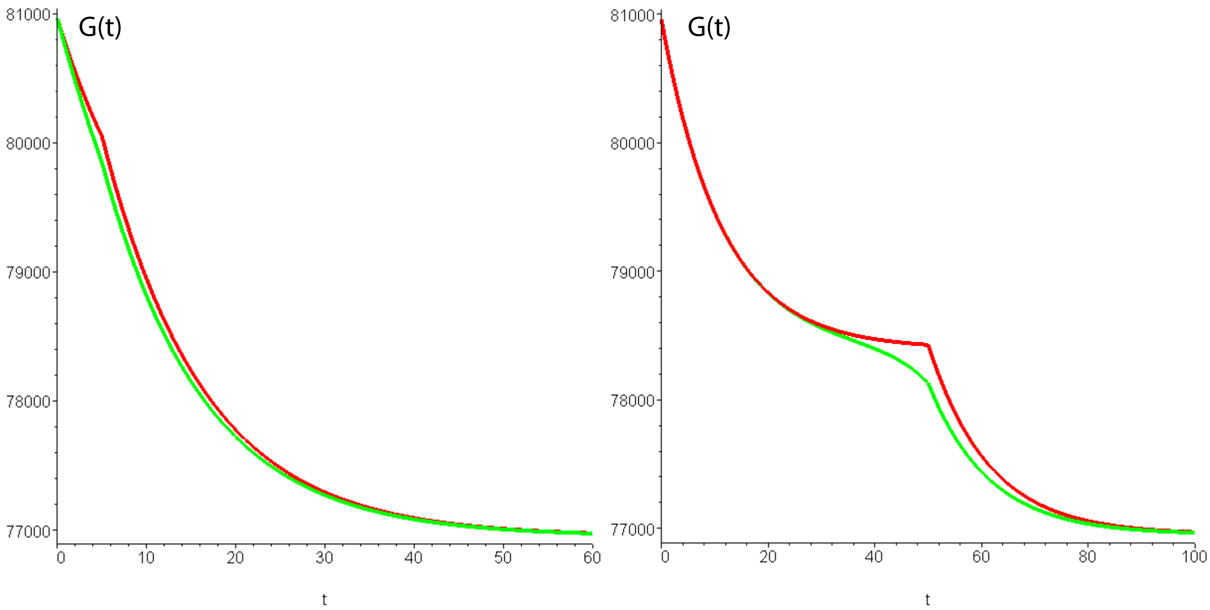

Figure 9: $G^{*}(t)$ for the problem of non-adaptation (in red) and adaptation (in green) in $t_a = 5$ (left-hand side) and $t_a = 50$ (right-hand side).

Finally, we computed the optimal solutions of our problem when the water agency ignores the information about the occurrence of the shock, that is, the water agency does not adapt to the shock. In this paper, we don't write in details the analytical resolution of this case. It is just necessary to say that the resolution of the problem consists in considering two simple and infinity problems for the two different periods of the problem. Indeed, as the water agency does not have any information about the shock until it happens, optimal behaviour corresponds to optimal solutions of the simple problem (appendix A.2) with $r = r_1$, before the shock. In contrast, from the moment of occurrence of the shock (t_a), optimal behaviour corresponds to optimal solutions of the simple problem, but now taking into account the value of the shock (i.e $r = r_2$).

Next, we compare the situation of non-adaptation (in red) to the situation with adaptation (in green), that is a case in which the water agency does adapt to the shock and it corresponds to the problem of the deterministic shock descrived previously in section

2.1. As depicted in figure 9, the resource is more heavily used in the short term when the shock is anticipated (in green). Indeed, the level of the aquifer is bigger in the total unertainty case than in the adaptation case: when $t_a = 5$ ($t_a = 50$), the long-term stock is higher in about 100 Mm^3 (in about 400 Mm^3). We observe in figure 10 that losses due to non-adaptation are important in the short and long term. For example, we see that total welfare losses from inadaptation are around 42 Millions of euros (respectively 15 M. of euros), when the shock correspond to a decrease of the recharge in about 60 Mm^3, and takes place early in time , $t_a = 5$,(respectively when it takes place later, $t_a = 50$).

As a conclusion, it is interesting for the agency to take into account information about the occurrence of the shock, although this information entails more intensive extraction before the occurrence of the shock.

Social welfare (in millions of euros)	[0,ta]		[ta, ∞]		Total	
	ta=5	ta=50	ta=5	ta=50	ta=5	ta=50
non-adaptation	92	288	147	1	239	289
adaptation	97	289	184	15	281	304

Figure 10: Social welfare for the problem of non-adaptation and adaptation in $t_a = 5$ and $t_a = 50$, for a shock of 60 Mm^3.

4 Conclusions and extensions

We conclude that from an ecological point of view, the increase of uncertainty leads to more intensive extractions before the shock and more conservative extractions after the shock, but a lowered level of the aquifer in the long run. Moreover, from an economic point of view, this loss of information may lead to an increase or a decrease in social costs, depending on the date of occurence of the shock.

There are various possible extensions to our paper. We can introduce a subsidy (or more generally a taxation policy) in the second period of the problem to avoid the over-exploitation of the resource, and then, calculate social costs of this environmental policy. We can also study the impact of several successive changes in recharge rates: for example the recharge could decrease because of decreased precipitations, as discussed above, but it could then increase again, for example as a result of investments in desalinisation plants. Finally, uncertainty about the extend of climate change may diminish over time: if new information is acquired during the considered time period, we would need a new optimization method, taking into account rolling horizons.

A The model

A.1 Proof of equivalently of problems

If you want to introduce a decrease in the recharge rate in the dynamics of the resource, we have to write equation (3), so:

$$\dot{G}(t) = \begin{cases} -(1-\alpha)g + r_1 & \text{if} \quad t \leq t_a \\ -(1-\alpha)g + r_2 & \text{if} \quad t > t_a \end{cases}$$

If there is an specific extraction of value A, from t_a, caused by a need for other uses (for example: water consumption), we can introduce a different dynamic of the aquifer,

$$\dot{G}(t) = \begin{cases} -(1-\alpha)g + r & \text{if} \quad t \leq t_a \\ -(1-\alpha)g - (1-\alpha)A + r & \text{if} \quad t > t_a \end{cases} \tag{14}$$

If we analyze the right-hand side of equation(14) from t_a,

$$\dot{G} = -(1-\alpha)g - (1-\alpha)A + r \Longrightarrow \tag{15}$$

$$\dot{G}(t) = -(1-\alpha)g - (1-\alpha)A + r \Longrightarrow \tag{16}$$

$$\dot{G}(t) = (r - (1-\alpha)A) - (1-\alpha)g \tag{17}$$

We can observe that the right-hand side of equation (17) is equivalent of the right-hand side of equation (14) from t_a, if,

$$r_2 = r - (1-\alpha)A = r_1 - (1-\alpha)A \Longrightarrow r_1 - r_2 = (1-\alpha)A, \tag{18}$$

that is, if the decrease in the recharge rate is equal to the value of water extracted of the aquifer for other uses, multiplied by the inverse of the return flow coefficient, so the real "lack" of water. After this, we present only results for the problem of the decrease on the recharge rate, caused by the equivalence of problems.

A.2 Resolution of the simple problem

The Hamiltonian (for interior solution) of this problem is given by:

$$H = F(G, g) + \lambda(-(1-\alpha)g + r),$$

where λ is the adjoint variable. Applying the maximum principle and supposing interior solutions, we have the usual first order conditions:

$$\frac{\partial H}{\partial g} = 0 \quad \Rightarrow \quad \frac{a}{b} - \frac{1}{b}g - (z - cG) - \lambda(1-\alpha) = 0, \tag{19}$$

$$\dot{\lambda} = -\frac{\partial H}{\partial G} + \rho\lambda \quad \Rightarrow \quad \dot{\lambda} = -\frac{\partial c}{\partial G}g + \rho\lambda. \tag{20}$$

From (19), we find the optimal extraction volume as a function of the resource stock and the shadow price:

$$g = a - zb + cbG - \lambda b(1-\alpha). \tag{21}$$

Substituting (21) into the equations of motion of the state (2) and adjoint variable (20), we have the following dynamic system:

$$\dot{G} = C1 - cbG + \lambda b(1-\alpha),$$

$$\dot{\lambda} = C2 - c^2 bG + (cb + \rho)\lambda,$$

with $C1$ and $C2$ constants, and $G(0) = G_0$, which allows us to find the roots of the characteristic polynom:

$$\rho_{1,2} = \frac{\rho \pm \sqrt{\rho^2 + 4cb\rho}}{2}.$$

We can also find the steady state of the system, for $\dot{G} = 0$ and $\dot{\lambda} = 0$:

$$G_\infty = \frac{r}{cb} + \frac{r}{\rho} - \frac{a}{cb} + \frac{z}{c}, \tag{22}$$

$$\lambda_\infty = cr/\rho. \tag{23}$$

Equation (22) results from substitution of (21) into the equation of motion of the state variable (2) and equation (23) results from substitution of (21) and (19) into (20).
Finally, we have the optimal extraction paths, with ρ_2, the negatif root:

$$G^*_{SP}(t) = \mathrm{e}^{\rho_2 \mathrm{t}}(\mathrm{G}_0 - \mathrm{G}_\infty) + \mathrm{G}_\infty, \tag{24}$$

and

$$\lambda^*_{SP}(t) = \mathrm{e}^{\rho_2 \mathrm{t}}(\lambda_0 - \frac{\mathrm{cr}}{\rho}) + \frac{\mathrm{cr}}{\rho}, \tag{25}$$

$$\lambda_0 = \frac{a}{b} - z + cG_0 - \frac{1}{b}(r - \rho_2(G_0 - G_\infty)),$$

which we find with (2) and (19).

A.3 Resolution of the modified extraction problem

To solve this problem, we will separate it in two parts and proceed by backward induction. First, we solve the maximisation between t_a and infinity, proceeding as in (A.2).
We have the functions:

$$G^{+}_{r_2}(t) = \mathrm{e}^{\rho_2(\mathrm{t}-\mathrm{t_a})}(\mathrm{G_{ta}} - \mathrm{G}_\infty) + \mathrm{G}_\infty, \tag{26}$$

$$\lambda^{+}_{r_2}(t) = \mathrm{e}^{\rho_2(\mathrm{t}-\mathrm{t_a})}(\lambda_{\mathrm{ta}} - \frac{\mathrm{cr_2}}{\rho}) + \frac{\mathrm{cr_2}}{\rho}, \tag{27}$$

$$g^{+}_{r_2}(t) = k1 - k2\mathrm{e}^{\rho_2(\mathrm{t}-\mathrm{t_a})}, \tag{28}$$

with,

$$k1 = r_2,$$

$$k2 = \rho_2(G_{ta} - G_\infty),$$

and,

$$G_\infty = \frac{r_2}{cb} + \frac{r_2}{\rho} - \frac{a}{cb} + \frac{z}{c}, \tag{29}$$

$$\lambda_{ta} = \frac{a}{b} - z + cG_{ta} - \frac{r_2}{b} + \frac{1}{b}\rho_2(G_{ta} - G_\infty), \quad G_{ta} \quad \text{unknown.} \tag{30}$$

Substituting (26),(27) and (28) into the objective function, we can compute the scrap value: $\phi(ta, G_{ta})$.

We can now turn to the second part of the problem, between 0 et t_a, considering the optimal solution of the first part. We know that the solutions of the problem are of the shape:

$$G^{-}_{r_1}(t) = \bar{A}\mathrm{e}^{\rho_1 \mathrm{t}} + \bar{\mathrm{B}}\mathrm{e}^{\rho_2 \mathrm{t}} + \bar{\mathrm{C}},$$
$$\lambda^{-}_{r_1}(t) = \bar{D}\mathrm{e}^{\rho_1 \mathrm{t}} + \bar{\mathrm{E}}\mathrm{e}^{\rho_2 \mathrm{t}} + \bar{\mathrm{F}}.$$

We have differential equations (2), (20) and the conditions

$$G^{-}_{r_1}(0) = \bar{A} + \bar{B} + \bar{C} = G_0, \tag{31}$$

$$\pi(t_a) = \bar{D}\mathrm{e}^{\rho_1 \mathrm{ta}} + \bar{\mathrm{E}}\mathrm{e}^{\rho_2 \mathrm{ta}} + \bar{\mathrm{F}} = \frac{\delta\phi(\mathrm{ta}, \mathrm{G_{ta}})}{\delta \mathrm{G_{ta}}}, \tag{32}$$

$$r = r_1, \quad \text{and} \quad G^{-}_{r_1}(t_a) = G^{+}_{r_2}(t_a) \tag{33}$$

This constitutes a system of 6 equations and 6 unknowns, which we can solve to find optimal solutions of the problem for the first period, between 0 and t_a[7].

[7]We do not detail analytical solutions because equations are too long. They are available from the authors

A.4 Resolution of the stochastic problem

To solve this problem, we can use the dynamic programing principle. The value fonction V(t,G) has to verify the hamilton-jacobi-bellmann equation (see for resolution details Kamien and Schwartz [12] and Caputo [3]):

$$(\rho+\theta)V(t,G)=\max_{g(.)}F(G,g)+\theta\phi(t,G)+V_G(t,G)(r_1-(1-\alpha)g) \tag{34}$$

with,

$$V(t,G)=AG^2+BG+C,$$

and,

$$\phi(t,G)=\sigma+\tau G+\upsilon G^2,\quad \text{proved and available from the authors.}$$

First, we find the optimal extraction path, g^*_{SC}, for the right-side of equation (34),

$$g^*_{SC}=b(\frac{a}{b}-z+cG-2(1-\alpha)AG-(1-\alpha)B). \tag{35}$$

After the substitution of g^*_{SC} in (34), by equalizing the right-side and left-side of equation 34), we find the optimal coefficients A,B,C of the value fonction $V(t,G)$. Next, substituting optimal values A,B,C in equation (35), we find the optimal path of extractions.

Finally, substituting equation (35) in the equation of the dynamic of the aquifer, we can solve the differential equation (11), with initial condition (12) and obtain the optimal value of stock, for the stochastic case[8].

[8]We do not detail analytical solutions because equations are too long. They are available from the authors

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