Way of doing matters! Contract design and resource allocation

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VERY PRELIMINARY

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Abstract

We consider a contract design problem where an agent runs a project that requires two kinds of actions/inputs. We consider a new type of asymmetric information: whereas the principal can control the global amount of resources (working time, effort, money, etc) that the agent puts into the project, he has no information (or no direct control) regarding the 'way" the agent produces, i.e. he has no information on the allocation of the resources between the two actions/inputs. This model differs from the multitask problem (see Holmstrom and Milgrom 1987). We show that the optimal incentive contract cannot reach the first best although the principal can contract upon the total amount of resources spent by the agent. Moreover, we show that the second best level of resources is lower (or equal) to its first best level. We also discuss how the principal chooses to make or buy depending on the substitutability/complementarity of the actions. Our model applies to optimal production contract design with asymmetric information on the production process and to optimal budget design in federations when the central government has no control on the allocation of local governments' spendings.

Keywords: Contract Theory, Principal-Agent Model, Production Process, Resource Allocation.

JEL codes: D82, D86

1 Introduction

In modern societies, products and services as well as production processes are increasingly sophisticated. This trend is reinforced by the demand for quality and technological goods and by increasingly global economic relationships (through international trade and political and economic integration). As a consequence, the control of end-users (clients, consumers, voters, etc.) over production/decision processes is getting more difficult.

In this paper, we consider a contract design problem where an agent is expected to run a project requiring the combination of two different kinds of actions. We highlight a new type of information asymmetry: whereas the principal can control the global amount of resources (working time, effort, money, etc) that the agent puts into the project, he has no information (or no direct control) regarding the way the agent produces. Then, the principal cannot contract directly upon how the agent chooses to allocate the resources between the two actions of the project. While principal-agent models have studied in depth the conflict of interest between one principal and one agent related to the level of resources spent by the agent (moral hazard) and to the selection of the agent (adverse selection), the way the agent chooses to allocate his resources between the various tasks he has to perform (i.e. his production practices) has received little attention. And yet, in many economic situations, there can be a conflict of interest between the principal and the agent coming from the information asymmetry (or lack of control) over the allocation of resources within the production process that may induce economic inefficiency.

Asymmetries of information over a production process concern goods for which it is difficult to know from consumption experience or observation how it has been produced (e.g. wine, fine food, high technology goods, etc). A typical example of is wine production. Vineyard owners (principals) often hire wine makers (agents) to produce a wine with specific characteristics. The final quality of the wine depends on the involvment and effort of the wine maker during the different steps of vinification: selection of the grapes, harvesting time and method, fermentation, etc. The vineyard owner may be able to evaluate the final quality of the wine and he cannot monitor and control the wine maker's work at each step of vinification. Moreover, it is very costly to hire experts to assert which step of the production process has been managed properly or not. Another example is manager labor contract: consider an employer who hires a manager to improve the value of sales.

In order to do so, the manager has to boost relationships with former customers and to look for new ones by sending e-mails. The firm can fix the working time of the manager ex-ante and control the value of sales ex-post. However, according to individual data protection legislations, it is often forbidden to check managers' e-mails. In this context, if the manager neglects one of thess two activities, even if he invests a lot in the other one, this may be detrimental to the firm. The 'way of doing"' problem is also present in federal political systems. The central government may want to control the budgets of its federal states, even if the allocation of the local budgets is left to local authorities.

Although multiple-activity principal-agent models focuses on how an agent splits his time between several activities (see Holmstrom and Milgrom 1987, 1991, 1994 and Itoh 1991), they differ from our model because they consider that the principal can get signals (albeit imperfect) on individual actions and that he cannot control the total level of effort exerted by the agent. Multi-tasking problems typically consider an agent who has to exert some effort in two (or more) tasks with the principal receiving one "performance" signal for *each* task. The main problem for the principal is then that task-targeted incentives may offset across tasks. In contrast, in our paper, we consider the case where the principal receives only one "global performance" signal, which is the source of the asymmetric information regarding the production process.

Our model shares with team production models (Holmstrom 1982, McAfee and McMillan 1991, Che and Yoo 2001, Winter 2009) the assumption that the principal receives only one signal for the level of production and not one signal for each task as in the multi-task literature. However, this litterature considers that the tasks can be clearly affected to different agents. The closest model to ours is MacDonald and Marx (2001), but they do not consider that the principal has control over the total amout of resource.

We show that our problem turns out to be a trade-off between incentives and resource spending: high powered incentives align the objective of the agent with the objective of the principal but reduces the (risk averse) agent payoff. Participation of the agent may be maintained with a decrease in the expected level of resource invested. However, we show that the optimal second best contract does not always distort resource spending compared to the first best situation. The paper focuses on the two polar cases of perfect substitutability and perfect complementarity between the tasks. We discuss how this affect the results aformentionned and how this affects the principal choice to make or buy depending on the substitutability/complementarity of the actions.

The paper is organized as follows. Section 1 presents the model. Section 2 sets out our main assumptions. In Section 3, we show our results for the case of perfect substitutes and in Section 4, our results for the case of perfect complements. Section 5 concludes.

2 Production and Contract

A principal wants to hire an agent to carry out a project. Completion of the project requires that the agent invests resources in two different kinds of action A and E. Let a denote the amount of resources allocated to action A and e the amount of resources allocated to action E. Typically a and e may be interpreted as the agent's time or money spent, or effort exerted, on action A and E respectively. To carry out the project, the agent is required to invest a total amount of resource T that can be allocated between action A and action E. Thus,

$$T = a + e. \tag{1}$$

This constraint means that, for the agent, spendings on action A and spendings on action E are perfect substitute uses of the total amount of resource T that he is required to invest.

Whereas the contract specifies how much the agent must spend for the project (T), it does not specify how the agent must allocate resources between action A and action E. Indeed, we assume that the principal is able to observe/verify/monitor the agent's total investment for the project but that he has no way/mean/permission to verify how the resources have been shared out amongst the different actions. However, the agent's resource allocation choice is of importance for the principal since the project's observable outcome $\widetilde{f}$ – which is observed by the principal – is affected by the (unobservable) levels of resources a and e that the agent puts into (resp.) action A and E. We assume that the project's outcome is also affected by a normally distributed alea $\widetilde{\varepsilon}$, with mean 0 and variance σ^2, whose realization reflects a state of nature. The final outcome can then be formally written

$$\widetilde{f}(a,e) = f(a,e) + \widetilde{\varepsilon},$$

with $f(a,e)$ being observed by the principal.

Investments in actions A and E positively influence the project's outcome but also create a cost for the agent. The agent's total cost $C(a,e)$ of providing a and e is assumed to be

$$C(a,e) = \frac{1}{2}a^2 + \frac{1}{2}\delta e^2$$

with $0 < \delta < 1$.

The total (expected) project's profit is then given by

$$\pi(a,e) = E\left[\widetilde{f}(a,e) - C(a,e)\right] = f(a,e) - C(a,e), \tag{2}$$

where $E[.]$ is the expectation operator.

The principal determines the agent's payment. As the agent's resource allocation choice is not observable, the principal rewards the agents according to a (linear) contract based on the project's (observable) outcome and the total amount of resource invested. In other words, a contract is defined by a triplet $c \equiv (\beta, F, T)$ that prescribes the (expected) transfert that the agent receives, $\beta E\left[\widetilde{f}(a,e)\right] + F$ and the amount of resource invested T such that $a + e = T$. We assume that the risk-averse agent's preferences over wealth is described by a constant absolute risk aversion utility function. Hence, using the Arrow-Pratt approximation and given a contract $c = (\beta, F, T)$ and resource allocation (a,e), the agent's final (expected) utility is:

$$U(a,e,c) \simeq \beta E\left[\widetilde{f}(a,e)\right] + F - C(a,e) - \frac{1}{2}r\sigma^2\beta^2$$

or

$$U(a,e,c) \simeq \beta f(a,e) + F - C(a,e) - \frac{1}{2}r\sigma^2\beta^2 \tag{3}$$

where r is the (constant) relative risk aversion of the agent.

We assume, for simplicity, that the principal is risk neutral and, for a given contract c, his (expected) payoff is:

$$V(a,e,c) = (1-\beta) E\left[\widetilde{f}(a,e)\right] - F,$$

or

$$V(a,e,c) = (1-\beta) f(a,e) - F, \tag{4}$$

The principal's problem is to determine the contract $c^* = (\beta^*, F^*, T^*)$ that maximizes the principal's payoff $V(a,e,c)$ under the condition (i) that the agent maximizes his expected utility when choosing the allocation of T between A and E (i.e. under the agent's *incentive constraint*) and (ii) that the agent attains a certain minimum utility corresponding to his outside option (i.e. under the agent's *participation constraint*). Let us normalize the outside option to 0, the participation constraint then writes:

$$U(a,e,c) \simeq \beta f(a,e) + F - C(a,e) - \frac{1}{2} r\sigma^2 \beta^2 \geq 0. \tag{5}$$

Given a contract $c = (\beta, F, T)$, the resources invested by the agent in actions A and E, namely a and e, are characterized by the following *incentive constraint*:

$$(a,e) \in \arg\max U(a,e,c), \tag{6}$$

with $T = a + e$.

We define social utility as the joint surplus J, i.e. the sum of the principal's and the agent's payoffs. It is given by

$$J = f - C - \frac{1}{2} r\sigma^2 \left(\beta^{SB}\right)^2.$$

To analyze this principal-agent problem, we successively consider two polar cases: we first assume that actions are perfect substitutes in the project's production function and then that actions are perfect complements.

3 Perfect Substitutes

In this section, we suppose that the two actions A and E are perfect substitutes in the project's production function. We then consider the following simple production function:

$$\widetilde{f}(a,e) = a + \gamma e + \widetilde{\varepsilon},$$

with γ the marginal productivity of task E. The marginal productivity of task A is normalized to 1. We assume that $0 < \gamma < \delta < 1$.

3.1 First best

We begin our analysis by looking at the contract design at the first-best situation. The first best is characterized by the resource levels $\left(a^{FB}, e^{FB}\right)$ that maximize the total (expected) profit defined by equation (2):

$$\left(a^{FB}, e^{FB}\right) \in \arg\max \pi(a,e) = a + \gamma e - C(a,e),$$

Proposition 1: The first-best resource allocation is given by:

$$e^{FB} = \frac{\gamma}{\delta} \text{ and } a^{FB} = 1.$$

and then the first best total amount of resources invested is

$$T^{FB} = \frac{\delta + \gamma}{\delta}.$$

At the first-best, the resource allocation is such that the marginal rate of technical substitution of action A for action E is equal to the ratio of the marginal productivity of action A over action E,

$$\frac{\frac{\partial f(a,e)}{\partial a}}{\frac{\partial f(a,e)}{\partial e}} = \frac{\frac{\partial C(a,e)}{\partial a}}{\frac{\partial C(a,e)}{\partial e}} > \frac{\frac{\partial T(a,e)}{\partial a}}{\frac{\partial T(a,e)}{\partial e}},$$

which is equivalent to

$$\frac{1}{\gamma} = \frac{a}{\delta e} > 1. \tag{7}$$

The rate bove measures the additionnal amount of resources that must be invested in action E to keep out at project's production level $\bar{f}$ when the amount of resources invested in action A is decreased from one unit. Equation (7) tells us that resources put into action E must be increased (resp. decreased) from more that one unit when resources into action A are decreased (resp. increased) from one unit.

The first-best output is

$$f^{FB} = \frac{\delta + \gamma^2}{\delta},$$

the first-bes cost is

$$C^{FB} = \frac{\delta + \gamma^2}{2\delta},$$

and the first-best profit – equal to the first-best joint surplus – is

$$\pi^{FB} = \frac{\delta + \gamma^2}{2\delta}.$$

3.2 Second best

We now suppose that the resource allocation chosen by the agent to whom the project has been delegated cannot be contracted upon by the principal. The principal then designs a contract c based on the final expected project's outcome $E\left[\widetilde{f}(a,e)\right] = f(a,e)$ taking into account the fact that under the implemented incentive scheme the agent will desire to choose a resource allocation that can maximize his own (*positive* – as we have fixed the agent's outside alternative expected utility to 0) utility.

Given the principal expected payoff (4), the agent's incentive constraint (6), the agent's participation constraint (5), and the constraint on the total amount of resource invested (1) defined above, and using production function (3), the principal-agent problem can be written as

$$\underset{c}{Max}\left\{V(a,e,c) = (1-\beta)(a+\gamma e) - F\right\} \tag{8}$$

subject to incentive constraint,

$$(a,e) \in \arg\max\left\{U(a,e,c) = \beta(a+\gamma e) + F - \frac{1}{2}a^2 - \frac{1}{2}\delta e^2 - \frac{1}{2}r\sigma^2\beta^2\right\}, \tag{9}$$

participation constraint,

$$U(a,e,c) = \beta(a+\gamma e) + F - \frac{1}{2}a^2 - \frac{1}{2}\delta e^2 - \frac{1}{2}r\sigma^2\beta^2 \geq 0 \tag{10}$$

and resource investment constraint,

$$a + e = T. \tag{11}$$

We study this problem by first determining the agent's resource allocation choice under a given contract $c = (\beta, F, T)$ and then by characterizing the principal's incentive contract choice according to the agent's response.

3.2.1 Agent's resource allocation choice

For a given contract design $c = (\beta, F, T)$, the agent's incentive constraint can be stated as the following utility maximization problem

$$\underset{a\geq 0, e\geq 0}{Max} \left\{ U(a,e) = \beta(a+\gamma e) + F - \frac{1}{2}a^2 - \frac{1}{2}\delta e^2 - \frac{1}{2}r\sigma^2\beta^2 \right\}$$

s.t.

$$a + e = T$$

Substituting the resource investment constraint in the agent's utility function, the first-order condition for the case of an interior optimum $(\widetilde{a}, \widetilde{e})$ is

$$\frac{\partial}{\partial e}\left[\beta(T-(1-\gamma)e) + F - \frac{1}{2}(T-e)^2 - \frac{1}{2}\delta e^2 - \frac{1}{2}r\sigma^2\beta^2\right] = 0$$

which is equivalent to

$$\frac{\beta - \widetilde{a}}{\beta\gamma - \delta\widetilde{e}} = 1. \tag{12}$$

The numerator of the left-hand side of equation (12) corresponds to the marginal change in the agent's utility arising from a marginal increase in a, i.e. the marginal utility of action A ($\frac{\partial U(a,e)}{\partial a}$). This is positive if and only if a is lower than β. The denominator represent the marginal utility of action E ($\frac{\partial U(a,e)}{\partial e}$), which is positive if and only if $e < \beta\gamma/\delta$. Thus, the left-hand side of equation (12) represents the agent's marginal rate of substitution of action A for action E at $(\widetilde{a},\widetilde{e})$ $(\frac{\partial T(a,e)}{\partial a}/\frac{\partial T(a,e)}{\partial e})$. It tells us the reduction in resources allocated to action E that the agent must make to compensate for a one-unit marginal increase in his investment in action A. The right-hand side in (12) accounts for the fact that, for a given T, and increase in one-unit of resource invested in action A must be compensated by a one-unit decrease in resources invested in action E.

Finally, the first-order condition (12) means that at an interior optimum $(\widetilde{a},\widetilde{e})$, the agent's marginal rate of substitution of action A for action E must be equal to the marginal rate of exchange between them in the total resource investment constraint imposed by the principal $(T = a + e)$. Here, the agent reduces resources invested in action E from one unit while increasing resources invested in action A from one unit to maintain his utility level. This (second-best) marginal rate of substitution is greater than the marginal rate of substitution prevailing at the first-best situation. Indeed, at the second-best situation the agent reduces investment in action E from *one-unit* resource while increasing investment in action A from one-unit resource to maintain his utility level whereas at the first-best situation a one-unit increase in resources invested in action A is compensated by a *more-than-one-unit* decrease in resources invested in action E to maintain the project's output level.

Proposition 2: For a given contract design $c = (\beta, F, T)$, the agent's resource allocation that respect his incentive constraint subject to the resource investment constraint imposed by the principal is such that

$$\widetilde{a}(\beta, T) = \frac{\delta T + \beta(1-\gamma)}{1+\delta}, \tag{13}$$

and,

$$\widetilde{e}(\beta, T) = \frac{T - \beta(1-\gamma)}{1+\delta}. \tag{14}$$

Let remark that if the principal chooses to fix the total investment to its first best level and does not provide incentives to the agent, i.e. $T = T^{FB}$ and $\beta = 0$, then the resource allocation chosen by the agent is

$$\begin{aligned}\widetilde{e}\left(0,T^{FB}\right) &= \frac{\delta+\gamma}{\delta(1+\delta)} > e^{FB} = \frac{\gamma}{\delta} \\ \text{and } \widetilde{a}\left(0,T^{FB}\right) &= \frac{\delta+\gamma}{(1+\delta)} < a^{FB} = 1,\end{aligned}$$

In other words, if the remuneration of the agent does not depend on the output level, the resource allocation that he chooses is distorted compared to the first best. The agent prefers to allocate more resources to the less costly action, i.e. the agent puts more resources into action E and fewer resources into action A compared to the first best.

As β is positive, the agent reduces the amount of resource allocated to action E and increases the amount of resource allocated to action A.

3.2.2 Principal's contract choice

Substituting the agent's (saturated) participation constraint (10) in the principal's payoff function (8) and according to the agent's resource allocation choice $(\widetilde{a}(\beta,T), \widetilde{e}(\beta,T))$ defined by (13) and (14), the principal's problem can be stated as

$$\underset{\beta,F,T}{Max}\left\{\widetilde{a}(\beta,T)+\gamma\widetilde{e}(\beta,T)-\frac{1}{2}\widetilde{a}(\beta,T)^2-\frac{1}{2}\delta\widetilde{e}(\beta,T)^2-\frac{1}{2}r\sigma^2\beta^2\right\}$$

with

$$\widetilde{a}(\beta,T)+\gamma\widetilde{e}(\beta,T)-\frac{1}{2}\widetilde{a}(\beta,T)^2-\frac{1}{2}\delta\widetilde{e}(\beta,T)^2-\frac{1}{2}r\sigma^2\beta^2=-F$$

Proposition 3: When actions are substitutes, the optimal contract design is

$$\begin{aligned}T^{SB} &= \frac{\delta+\gamma}{\delta} = T^{FB} \\ \beta^{SB} &= \frac{(1-\gamma)^2}{(1-\gamma)^2+r\sigma^2(1+\delta)} \\ F^{SB} &= \frac{\left((\gamma-1)^2\left(\delta+\gamma^2\right)+(\gamma+\delta)^2 r\sigma^2\right)\left(-(\gamma-1)^2+(1+\delta)r\sigma^2\right)}{2\delta\left((\gamma-1)^2+(1+\delta)r\sigma^2\right)^2}\end{aligned}$$

When the agent is risk averse, the principal gives him positive incentive. The level of incentive given is deacresing with respect to the agent's risk aversion. The total investment expected from the agent is fixed to the first-best level. The optimal contract induces a better resource allocation from the agent. But the first-best outcome cannot be implemented because of risk aversion. These results show that resource allocation is an issue even if the principal is able to control the agent's total investment. The conflict of interest between the principal and the agent then does not concern the volume of resources that must be put into the project but how these resources are to be splitted between the different actions needed to carry out the project. This conflict of interest is at the origin of the agency cost which prevents an efficient design of contract.

3.2.3 Second-best resource allocation

Substituting β^{SB} and T^{SB} in $\widetilde{e}$ and $\widetilde{a}$ we can find the second-best resource allocation.

Proposition 4: The second-best resource allocation is such that:

$$\begin{aligned} e^{SB} &= \frac{1}{(1+\delta)}\left[\frac{\delta+\gamma}{\delta} - \frac{(1-\gamma)^3}{(1-\gamma)^2 + r\sigma^2(1+\delta)}\right] \\ a^{SB} &= \frac{1}{(1+\delta)}\left[\delta+\gamma + \frac{(1-\gamma)^3}{(1-\gamma)^2 + r\sigma^2(1+\delta)}\right]. \end{aligned}$$

with $a^{FB} > a^{SB}$ and $e^{FB} < e^{SB}$.

We can easily check that $e(0, T^{FB}) > e^{SB} > e^{FB}$ and that $a(0, T^{FB}) < a^{SB} < a^{FB}$. When the agent receive no incentive, but is asked to invest the first-best total amount of resource, the resource allocation is distorted in favour of action E, which is less costly to perform. Giving the agent positive incentives allows to correct (imperfectly) this distortion by making the agent spend more resource on action A and less resource on action E.

The second-best output is

$$f^{SB} = \frac{1}{1+\delta}\left[\frac{(\delta+\gamma)^2}{\delta} + \frac{(1-\gamma)^4}{(1-\gamma)^2 + r\sigma^2(1+\delta)}\right],$$

the second-best cost is

$$C^{SB} = \frac{1}{2(1+\delta)}\left[\frac{(\delta+\gamma)^2}{\delta} + \left(\frac{(1-\gamma)^3}{(1-\gamma)^2 + r\sigma^2(1+\delta)}\right)^2\right],$$

and the second-best profit is thus

$$\pi^{SB} = \frac{1}{(1+\delta)}\left[\delta+\gamma+\frac{(1-\gamma)^3}{(1-\gamma)^2+r\sigma^2(1+\delta)}\right]+\frac{\gamma}{(1+\delta)}\left[\frac{\delta+\gamma}{\delta}-\frac{(1-\gamma)^3}{(1-\gamma)^2+r\sigma^2(1+\delta)}\right]$$
$$-\frac{1}{2(1+\delta)^2}\left[\delta+\gamma+\frac{(1-\gamma)^3}{(1-\gamma)^2+r\sigma^2(1+\delta)}\right]^2-\frac{\delta}{2(1+\delta)^2}\left[\frac{\delta+\gamma}{\delta}-\frac{(1-\gamma)^3}{(1-\gamma)^2+r\sigma^2(1+\delta)}\right]^2$$
$$\pi^{SB} = \frac{(1-\gamma)^4}{2(1+\delta)}\frac{\left[(1-\gamma)^2+2r\sigma^2(1+\delta)\right]}{\left[(1-\gamma)^2+r\sigma^2(1+\delta)\right]^2}.$$

The joint surplus is

$$J = \pi^{SB}-\frac{1}{2}r\sigma^2\beta^{SB}$$

4 Perfect Complements

We now suppose that the two actions A and E are perfect complements. The project's production function can then be written as:

$$\widetilde{f}(a,e) = \min\{a,\gamma e\}+\widetilde{\varepsilon}.$$

We still assume that $0<\gamma<\delta<1$.

4.1 First best

The first best is characterized by the effort levels $\left(a^{FB},e^{FB}\right)$ that maximize the total (expected) profit:

$$\left(a^{FB},e^{FB}\right)\in\arg\max\pi(a,e) = \min\{a,\gamma e\}-C(a,e),$$

Proposition 5: The first best resource allocation is such that

$$e^{FB} = \frac{\gamma}{\gamma^2+\delta} \text{ and } a^{FB} = \frac{\gamma^2}{\gamma^2+\delta}.$$

Let remark that $a^{FB} = \gamma e^{FB}$.

The first-best total amount of resources invested is

$$T^{FB} = \frac{\gamma^2 + \gamma}{\gamma^2 + \delta}.$$

The first-best output is

$$f^{SB} = \frac{\sigma^2}{\sigma^2 + \delta}.$$

The first-best cost is

$$C^{FB} = \frac{\gamma^2}{2(\gamma^2 + \delta)}.$$

And the first-best profit/joint surplus is

$$\pi^{FB} = \frac{\gamma^2}{2(\gamma^2 + \delta)}.$$

4.2 Second best

When the principal delegates the realization of the project to an agent whose resource allocation is not monitored, the principal-agent problem can be written

$$\underset{c}{Max} \left\{ V(a,e,c) = (1-\beta) \min\{a, \gamma e\} - F \right\} \tag{15}$$

subject to incentive constraint,

$$(a,e) \in \arg\max \left\{ U(a,e,c) = \beta \min\{a, \gamma e\} + F - \frac{1}{2}a^2 - \frac{1}{2}\delta e^2 - \frac{1}{2} r \sigma^2 \beta^2 \right\}, \tag{16}$$

participation constraint,

$$U(a,e,c) = \beta \min\{a, \gamma e\} + F - \frac{1}{2}a^2 - \frac{1}{2}\delta e^2 - \frac{1}{2} r \sigma^2 \beta^2 \geq 0 \tag{17}$$

and resource investment constraint,

$$a + e = T. \tag{18}$$

4.2.1 Agent's resource allocation choice

Using the resource investment constraint and substituting, the agent's problem is:

$$\underset{e\geq 0}{Max}\left\{U(a,e)=\beta\min\{T-e,\gamma e\}+F-\frac{1}{2}(T-e)^2-\frac{1}{2}\delta e^2-\frac{1}{2}r\sigma^2\beta^2\right\},$$

with $T=a+e$.

Proposition 6: For a given contract design $\{\beta,F,T\}$, the agent's resource allocation that respect (16) and (18) is:

$$\widetilde{a}(\beta,T)=\begin{cases}\frac{\delta T-\gamma\beta}{1+\delta}\text{ if }\beta<\frac{\delta-\gamma}{\gamma(1+\gamma)}T\\ \frac{\gamma T}{1+\gamma}\text{ if }\frac{\delta-\gamma}{\gamma(1+\gamma)}T\leq\beta\end{cases}$$

and

$$\widetilde{e}(\beta,T)=\begin{cases}\frac{\gamma\beta+T}{1+\delta}\text{ if }\beta<\frac{\delta-\gamma}{\gamma(1+\gamma)}T\\ \frac{T}{1+\gamma}\text{ if }\frac{\delta-\gamma}{\gamma(1+\gamma)}T\leq\beta\end{cases}$$

When β is large ($>\frac{\delta-\gamma}{\gamma(1+\gamma)}T$), a change in the level of incentive given to the agent does not change his efforts on action A and E. An increase in T increases both effort on action A and on action E. We always have $\widetilde{a}=\gamma\widetilde{e}$ like in the first-best, but if $T<T^{FB}$, $\widetilde{a}$ and $\widetilde{e}$ are smaller than a^{FB} and e^{FB} respectively. Conversely, if $T>T^{FB}$, $\widetilde{a}$ and $\widetilde{e}$ are greater than a^{FB} and e^{FB} respectively.

When $\beta=0$ (i.e. when the agent bears the cost of the production without benefiting from the output) the agent chooses his efforts on action A and action E so that $a=\delta e$ (that is, a multiplied by its marginal cost is equal to e multiplied by its marginal cost). We have shown above that the first best effort levels are such that $a=\gamma e$. Since we have suppose that γ is lower than δ, this means that when the agent receives no incentives, he tends to invest relatively too much in action A and not enough in action E. [But $e>a$.]

As long as β is not too much high ($<\frac{\delta-\gamma}{\gamma(1+\gamma)}T$), increasing the level of incentive given to the agent makes him spend less on A and more on E (for a given T).

4.2.2 Principal's contract choice

Proposition 7: When actions are complements, the contract maximizing the principal's payoff subject to the agent's response is such that

(i) When the agent is not too much risk averse ($r \leq \frac{\gamma^3}{(\delta-\gamma)\sigma^2}$),

$$\begin{aligned} \beta^{SB} &= \frac{\delta-\gamma}{\delta+\gamma^2}\ , \\ T^{SB} &= \left(\frac{\gamma+\gamma^2}{\delta+\gamma^2}\right) = T^{FB} \text{ and} \\ F^{SB} &= \frac{r\sigma^2\left(\gamma^2+\delta\right)\left(\delta-\gamma\right)+\gamma^2\left(2\gamma-\delta+\gamma^2\right)}{2\left(\gamma^2+\delta\right)^2} \end{aligned}$$

(ii) and, when the agent is sufficiently risk averse (when $\frac{\gamma^3}{(\delta-\gamma)\sigma^2} < r$),

$$\begin{aligned} \beta^{SB} &= \frac{\gamma^2}{\gamma^2+(1+\delta)\,r\sigma^2}\ , \\ T^{SB} &= \frac{\gamma}{\delta} < T^{FB} \text{ and} \\ F^{SB} &= \frac{1}{2}\frac{\gamma^2}{\delta}\frac{\left(r\sigma^2+\gamma^2\right)\left((1+\delta)\,r\sigma^2-\gamma^2\right)}{\left(\gamma^2+(1+\delta)\,r\sigma^2\right)^2}. \end{aligned}$$

Corrolary 2: The more the agent is risk averse, the less incentive he receives (i.e. the lower β^{SB}), the lower the amount of total resources he is expected to spend (i.e. the lower T^{SB}), and the higher the fixed remuneration he receives (i.e. the higher F^{SB}).

4.2.3 Second-best resource allocation

Proposition 8: When the principal chooses a contract $c^{SB} = (\beta^{SB}, T^{SB}, F^{SB})$, the agent's resource allocation choice is such that

(i) When the agent is not too much risk averse, $r \leq \frac{\gamma^3}{(\delta-\gamma)\sigma^2}$

$$e^{SB} = \frac{\gamma}{\delta+\gamma^2} = e^{FB} \text{ and } a^{SB} = \frac{\gamma^2}{\delta+\gamma^2} = a^{FB},$$

and (ii) when *the agent is sufficiently risk averse,* $\frac{\gamma^3}{(\delta-\gamma)\sigma^2} < r$

$$e^{SB} = \frac{\gamma}{\delta}\frac{\gamma^2+\sigma^2 r}{\gamma^2+(1+\delta)\,\sigma^2 r} < e^{FB} \text{ and } a^{SB} = \frac{\gamma\sigma^2 r}{\gamma^2+(1+\delta)\,\sigma^2 r},$$

with $a^{SB} < a^{FB}$ *if and only if* $r < \frac{1}{1-\gamma}\frac{\gamma^3}{(\delta-\gamma)\sigma^2}$.

When the agent is not too much risk averse, the second best efforts are identical to the first best levels.

The effort on task E decreases with the agents' risk aversion and the effort on task A is a non-monotonic function of the agent's risk aversion.

At the first-best situation and when the agent is not too much risk averse ($r > \frac{\gamma^3}{(\delta-\gamma)\sigma^2}$), we have $a = \gamma e$. One can check that when $r > \frac{\gamma^3}{(\delta-\gamma)\sigma^2}$, a^{SB} is equal to $\frac{r\sigma^2\delta}{r\sigma^2+\gamma^2}e^{SB}$ with $\frac{r\sigma^2\delta}{r\sigma^2+\gamma^2}$ greater than γ. Thus, when $r > \frac{\gamma^3}{(\delta-\gamma)\sigma^2}$, a^{SB} is relatively too high compared with e^{SB} (even when its level is lower than its first-best level). We verify that $\frac{r\sigma^2\delta}{r\sigma^2+\gamma^2}$ is increasing with r so that a^{SB} is increasingly too high relatively to e^{SB} as the agent's risk aversion increases.

Corrolary 3: *(i) When the agent is not too much risk averse,* $r \leq \frac{\gamma^3}{(\delta-\gamma)\sigma^2}$, the second-best production is identical to the first best one:

$$f(r) = \frac{\gamma^2}{\delta+\gamma^2} = a^{SB} = \gamma e^{SB} = a^{FB} = \gamma e^{FB}$$

and (ii) when the agent is sufficiently risk averse, $\frac{\gamma^3}{(\delta-\gamma)\sigma^2} < r$*, the second best production is*

$$f(r) = \frac{\gamma^2}{\delta}\frac{\gamma^2+\sigma^2 r}{\gamma^2+(1+\delta)\,\sigma^2 r} = \gamma e^{SB} < a^{SB}.$$

When r tends to infinity, $f(r)$ tends to $\frac{\gamma^2}{\delta(1+\delta)}$.

Corrolary 4: *(i) When the agent is not too much risk averse,* $r \leq \frac{\gamma^3}{(\delta-\gamma)\sigma^2}$, the second-best cost is identical to the first best one:

$$C(r) = \frac{\gamma^2}{2(\gamma^2+\delta)}$$

and (ii) when the agent is sufficiently risk averse, $\frac{\gamma^3}{(\delta-\gamma)\sigma^2} < r$*, the second-best cost is*

$$C(r) = \frac{\gamma^2\left(\gamma^2+\sigma^2 r\right)^2+\delta\left(\gamma\sigma^2 r\right)^2}{2\delta\left(\gamma^2+(1+\delta)\,\sigma^2 r\right)^2}.$$

Corrolary 5: *(i) When the agent is not too much risk averse,* $r \leq \frac{\gamma^3}{(\delta-\gamma)\sigma^2}$, the second-best profit is identical to the first best one:

$$\pi(r) = \frac{\gamma^2}{2(\gamma^2+\delta)}$$

and (ii) when the agent is sufficiently risk averse, $\frac{\gamma^3}{(\delta-\gamma)\sigma^2} < r$*, the second-best profit is*

$$\pi(r) = \frac{\gamma^2}{2\delta\left(\gamma^2+(1+\delta)\,\sigma^2 r\right)^2}\left[\gamma^4+\sigma^2 r\,(1+\delta)\left(2\gamma^2+\sigma^2 r\right)\right]$$

Corrolary 6: *(i) When the agent is not too much risk averse,* $r \leq \frac{\gamma^3}{(\delta-\gamma)\sigma^2}$, the joint surplus is:

$$J^+ = \frac{\gamma^2\left(\gamma^2+\delta\right)-(\delta-\gamma)^2 r\sigma^2}{2\left(\delta+\gamma^2\right)^2} \text{ if } r \leq \frac{\gamma^3}{(\delta-\gamma)\,\sigma^2}$$

and (ii) when the agent is sufficiently risk averse, $\frac{\gamma^3}{(\delta-\gamma)\sigma^2} < r$*, the joint surplus is*

$$J^- = \frac{1}{2}\frac{\gamma^2}{\delta}\frac{\gamma^2+r\sigma^2}{\gamma^2+(1+\delta)\,r\sigma^2} \text{ if } r > \frac{\gamma^3}{(\delta-\gamma)\,\sigma^2}$$

Corollary 7: *An increase in the degree of risk aversion* r *may increase the joint surplus. Indeed, the second best joint surplus* J^{SB} *decreases with* r *for* $r < \frac{\gamma^3}{(\delta-\gamma)\sigma^2}$*, jumps upward at* $r = \frac{\gamma^3}{(\delta-\gamma)\sigma^2}$ *and then decreases after.*

4.3 Discussion

4.3.1 Inefficiency

Remember that the second best joint surplus is

$$J^{SB} = \begin{cases} J^+ = \frac{\gamma^2\left(\gamma^2+\delta\right)-(\delta-\gamma)^2 r\sigma^2}{2\left(\delta+\gamma^2\right)^2} \text{ if } r \leq \frac{\gamma^3}{(\delta-\gamma)\sigma^2} \\ J^- = \frac{1}{2}\frac{\gamma^2}{\delta}\frac{\gamma^2+r\sigma^2}{\gamma^2+(1+\delta)r\sigma^2} \text{ if } r > \frac{\gamma^3}{(\delta-\gamma)\sigma^2}. \end{cases}$$

$$
\begin{aligned}
J^{FB} - J^{+} &= \frac{\gamma^2}{2(\gamma^2+\delta)} - \frac{\gamma^2\left(\gamma^2+\delta\right) - \left(\delta-\gamma\right)^2 r\sigma^2}{2\left(\delta+\gamma^2\right)^2} \\
&= \frac{(\delta-\gamma)^2 r\sigma^2}{2(\gamma^2+\delta)^2} > 0
\end{aligned}
$$

$$
\begin{aligned}
J^{FB} - J^{-} &= \frac{\gamma^2}{2(\gamma^2+\delta)} - \frac{1}{2}\frac{\gamma^2}{\delta}\frac{\gamma^2 + r\sigma^2}{\gamma^2 + (1+\delta)\, r\sigma^2} \\
&= \frac{\gamma^2\left(r\sigma^2(\delta^2-\gamma^2) - \gamma^4\right)}{2\delta\left(\gamma^2+\delta\right)\left(\gamma^2+(1+\delta)r\sigma^2\right)}
\end{aligned}
$$

This is positive if and only if $r > \frac{\gamma^4}{\sigma^2(\delta^2-\gamma^2)}$. We check that $\frac{\gamma^4}{\sigma^2(\delta^2-\gamma^2)} < \frac{\gamma^3}{\sigma^2(\delta-\gamma)}$, so that $J^{FB} - J^{-}$ is always positive over $r > \frac{\gamma^3}{(\delta-\gamma)\sigma^2}$.

Inefficiency due to the agent's resource *misallocation* can also be measured by $I = \gamma e^{SB} - a^{SB}$. The closer I to 0, the lower inefficiency. When $I > 0$, decreasing investment in action E would not reduce the output $f\,(.)$ whereas it would decrease the total cost. When $I < 0$, decreasing investment in action A would decrease the total cost without reducing the output.

Corollary 8: *(i) When the agent is not too much risk averse,* $r \leq \frac{\gamma^3}{(\delta-\gamma)\sigma^2}$, there is no production inefficiency,

$$
I = 0,
$$

and (ii) when *the agent is sufficiently risk averse,* $\frac{\gamma^3}{(\delta-\gamma)\sigma^2} < r$*, the production inefficiency is negative,*

$$
I = \frac{\gamma}{\delta}\frac{\gamma^3 - (\delta-\gamma)\, r\sigma^2}{\gamma^2 + (1+\delta)\, r\sigma^2} < 0,
$$

which means that the effort in task A is inefficiently high.

4.3.2 Substitution between expected resource spending and incentives

Corrolary 9: The variable part of the payment scheme and the total amount of resources act as substitutes.

If the total amount of resources is fixed at its first best level, $T = T^{FB} = \frac{\gamma+\gamma^2}{\delta+\gamma^2}$, the optimal (second best) variable payment, β^{SB}, is:

(i) When the agent is not too much risk averse ($r \leq \frac{\gamma^3}{(\delta-\gamma)\sigma^2}\frac{1+\gamma}{1+\delta}$),

$$\beta^{SB} = \frac{\delta - \gamma}{\delta + \gamma^2}$$

(ii) and, when the agent is sufficiently risk averse (when $\frac{\gamma^3}{(\delta-\gamma)\sigma^2}\frac{1+\gamma}{1+\delta} < r$),

$$\beta^{SB} = \frac{\gamma^2}{\gamma^2 + (1+\delta)\, r\sigma^2}$$

When the total amount of resource is not an instrument in the contract scheme, the incentives fall down from $\frac{\delta-\gamma}{\delta+\gamma^2}$ to $\frac{\gamma^2}{\gamma^2+(1+\delta)r\sigma^2}$ for a smaller value of the agent risk aversion than in the case where T is chosen by the principal.

When the principal can use the total amout of resources as an instrument, he chooses $T^{SB} < T^{FB}$ for larger values of r compared to the present situation where the total amount of resources is fixed to its first best level. Hence, the principal uses the reduction in the total amount of resources to be invested by the agent in order to maintain strong incentives ($\beta^{SB} = \frac{\delta-\gamma}{\delta+\gamma^2}$) for larger agent's risk aversion. For any r such that $\frac{\gamma^3}{(\delta-\gamma)\sigma^2} > r > \frac{\gamma^3}{(\delta-\gamma)\sigma^2}\frac{1+\gamma}{1+\delta}$, the incentives are reduced here ($\beta^{SB} = \frac{\gamma^2}{\gamma^2+(1+\delta)r\sigma^2}$) compared to the situation where the principal is allowed to reduce the total amount of resources to $T^{SB} < T^{FB}$.

Appendix

Proof of Proposition 1 : The maximization program of the surplus writes as follows:

$$\underset{a,e}{Max}\left\{J = a + \gamma e - \frac{1}{2}a^2 - \frac{1}{2}\delta e^2\right\}$$

The first order conditions for an interior solution are given by:

$$\begin{aligned} 1-a &= 0, \\ \gamma-\delta e &= 0. \end{aligned}$$

And the solution is $a^{FB}=1$ and $e^{FB}=\frac{\gamma}{\delta}$. One can check that J is concave with respect to (a,e) and then the first order conditions are also sufficient.$\square$

Proof of Proposition 2: The first order condition for an interior effort e is:

$$\frac{\partial U}{\partial e}=-\beta\left(1-\gamma\right)+\left(T-e\right)-\delta e=0.$$

And then $\widetilde{e}\left(\beta,T\right)=\frac{T-\beta(1-\gamma)}{1+\delta}$. Using the time constraint, $\widetilde{a}$ follows.$\square$

Proof of Proposition 3: The problem of the principal can be written as:

$$\underset{\beta,T}{Max}\left\{J=\frac{\delta T+\beta(1-\gamma)}{(1+\delta)}+\gamma\left[\frac{T-\beta(1-\gamma)}{(1+\delta)}\right]-\frac{1}{2}\left[\frac{\delta T+\beta(1-\gamma)}{(1+\delta)}\right]^2-\frac{\delta}{2}\left[\frac{T-\beta(1-\gamma)}{(1+\delta)}\right]^2-\frac{1}{2}r\beta^2\sigma^2\right\} \tag{19}$$

We have

$$\frac{\partial J}{\partial \beta}=\frac{(1-\gamma)^2}{(1+\delta)}-(1-\gamma)\left[\frac{\delta T+\beta(1-\gamma)}{(1+\delta)^2}\right]+\delta(1-\gamma)\left[\frac{T-\beta(1-\gamma)}{(1+\delta)^2}\right]-r\sigma^2\beta \tag{20}$$

and

$$\frac{\partial J}{\partial T}=\frac{\delta+\gamma}{1+\delta}-\frac{\delta}{(1+\delta)^2}\left(\delta T+\beta(1-\gamma)\right)-\frac{\delta}{(1+\delta)^2}\left(T-\beta(1-\gamma)\right). \tag{21}$$

Hence, the first order conditions (FOC) can be written as

$$\frac{\partial J}{\partial \beta}=0\Leftrightarrow\frac{(1-\gamma)^2}{(1+\delta)}(1-\beta)-r\sigma^2\beta=0 \tag{22}$$

and

$$\frac{\partial J}{\partial T}=0\Leftrightarrow\frac{\delta+\gamma}{1+\delta}-\frac{\delta T}{1+\delta}=0. \tag{23}$$

We check that J is concave with respect to β and T so that the first order conditions are also sufficient.

From equation (22) we get

$$\beta^{SB} = \frac{(1-\gamma)^2}{(1-\gamma)^2 + r\sigma^2(1+\delta)} \tag{24}$$

and equation (23) gives

$$T^{SB} = \frac{\delta+\gamma}{\delta} = T^{FB}. \tag{25}$$

Proof of Proposition 5: The maximization program of the surplus writes as follows:

$$\underset{a,e}{Max}\left\{J = \min\{a, \gamma e\} - \frac{1}{2}a^2 - \frac{1}{2}\delta e^2\right\}$$

If $a \leq \gamma e$, let λ be the corresponding Lagrangian parameter and the first order condition write:

$$\begin{aligned} 1 - a - \lambda &= 0, \\ -\delta e + \lambda\gamma &= 0, \\ \lambda(\gamma e - a) &= 0, a \leq \gamma e \end{aligned}$$

One can easely check that one must have $\lambda > 0$ and then

$$\begin{aligned} a &= \frac{\gamma^2}{\gamma^2+\delta} \\ e &= \frac{\gamma}{\gamma^2+\delta}, \\ \lambda &= \delta e/\gamma. \end{aligned}$$

One can easely check that there is no possible solution such that $\gamma e < a$. □

Proof of Proposition 6: One can first check that the case where $T - e < \gamma e$ is not possible since it would imply $\beta < 0$.

The case $T - e < \gamma e$ implies that

$$U=\beta\gamma e-\frac{1}{2}\left(T-e\right)^{2}-\frac{1}{2}\delta e^{2}-\frac{1}{2}r\sigma^{2}\beta^{2}.$$

The first order condition for an interior solution is

$$\gamma\beta+T-e-\delta e=0.$$

Then

$$\widetilde{e}_1=\frac{\gamma\beta+T}{1+\delta}>0$$

and

$$\widetilde{a}_1=\frac{\delta T-\gamma\beta}{1+\delta}<T.$$

One can check that $\widetilde{a}_1>0$ and $\widetilde{e}_1<T$ if and only if $\beta<\frac{\delta T}{\gamma}$. Moreover, one verifies that $T-e>\gamma e$ if and only if $\beta<\frac{(\delta-\gamma)}{\gamma(1+\gamma)}T$. One can easily check that $\frac{\delta T}{\gamma}>\frac{(\delta-\gamma)}{\gamma(1+\gamma)}T$.

Hence, the payoff of the agent when $\beta<\frac{(\delta-\gamma)}{\gamma(1+\gamma)}T$ is

$$U_1=U(\widetilde{a}_1,\widetilde{e}_1)=\frac{\gamma^2\beta^2+2\gamma T\beta-\delta T^2}{2(1+\delta)}-\frac{1}{2}r\sigma^2\beta^2.$$

Now, when $T-e=\gamma e$ it is immediate that

$$\widetilde{e}_2=\frac{T}{1+\gamma}$$

and

$$\widetilde{a}_2=\frac{\gamma T}{1+\gamma}.$$

The payoff of the agent is then

$$U_2=U(\widetilde{a}_2,\widetilde{e}_2)=\frac{1}{2(1+\gamma)}\left(2\gamma T\beta-\frac{\gamma^2+\delta}{1+\gamma}T^2\right)-\frac{1}{2}r\sigma^2\beta^2$$

Finally, when $\beta\geq\frac{(\delta-\gamma)}{\gamma(1+\gamma)}T$, $(\widetilde{a}_2,\widetilde{e}_2)$ is the only possible solution to the maximization of the agent's utility function. Then, when $\beta < \frac{(\delta-\gamma)}{\gamma(1+\gamma)}T$ we verify that $U_1 > U_2$

$$\begin{aligned}
U_1 - U_2 &= \frac{1}{2(1+\delta)}\left(\gamma^2\beta^2 + 2T\gamma\beta - \delta T^2\right) - \frac{1}{2(1+\gamma)}\left(2T\gamma\beta - \frac{\gamma^2+\delta}{1+\gamma}T^2\right) \\
&\propto \beta^2\gamma^2(1+\gamma)^2 + T^2(\delta-\gamma)^2 - 2\gamma T\beta(1+\gamma)(\delta-\gamma) = \gamma^2\left(1+\gamma\right)^2\left(\beta - \frac{T\left(\delta-\gamma\right)}{\gamma\left(1+\gamma\right)}\right)^2 > 0,
\end{aligned}$$

so that $(\widetilde{a}_1, \widetilde{e}_1)$ maximizes the payoff of the agent. □

Proof of Proposition 7: Using the participation constraint and substituting, the problem of the principal can be rewritten as follows:

$$\underset{(\beta,T)}{Max}\left\{J\left(\beta,T\right) = \gamma\widetilde{e}\left(\beta,T\right) - \frac{1}{2}\left(\widetilde{a}\left(\beta,T\right)\right)^2 - \frac{1}{2}\delta\left(\widetilde{e}\left(\beta,T\right)\right)^2 - \frac{1}{2}r\sigma^2\beta^2\right\}$$

$J\left(\beta,T\right)$ is a continuous function because $\widetilde{a}$ and $\widetilde{e}$ are continuous. One can also check that J is a concave function over $\beta < \frac{\delta-\gamma}{\gamma(1+\gamma)}T$ and over $\frac{\delta-\gamma}{\gamma(1+\gamma)}T \leq \beta$. One now study J over each of these domains.

First consider $\beta < \frac{\delta-\gamma}{\gamma(1+\gamma)}T$. The joint surplus writes:

$$J\left(\beta,T\right) = \gamma\frac{\gamma\beta+T}{1+\delta} - \frac{1}{2}\left(\frac{\delta T-\gamma\beta}{1+\delta}\right)^2 - \frac{1}{2}\delta\left(\frac{\gamma\beta+T}{1+\delta}\right)^2 - \frac{1}{2}r\sigma^2\beta^2,$$

and thus the FOCs for an interior solution are:

$$\begin{aligned}
\frac{\partial J}{\partial\beta} &= \frac{\gamma^2 - \left(\left(1+\delta\right)r\sigma^2 + \gamma^2\right)\beta}{1+\delta} = 0 \\
\frac{\partial J}{\partial T} &= \frac{1}{\delta+1}\left(\gamma - T\delta\right) = 0
\end{aligned}$$

Hence, the solution is

$$T^- = \frac{\gamma}{\delta} \text{ and } \beta^- = \frac{\gamma^2}{\gamma^2 + \left(1+\delta\right)r\sigma^2}.$$

And the condition $\beta^- < \frac{\delta-\gamma}{\gamma(1+\gamma)}T^-$ is equivalent to

$$\frac{\gamma^3}{\left(\delta-\gamma\right)\sigma^2} < r.$$

Let J^{-} be the joint surplus levels corresponding to (β^{-}, T^{-}):

$$J^{-} = \frac{1}{2}\frac{\gamma^2}{\delta}\frac{\gamma^2 + r\sigma^2}{\gamma^2 + (1+\delta)\, r\sigma^2} > 0.$$

Finally, (β^{-}, T^{-}) corresponds to a local maximum when $\frac{\gamma^3}{(\delta-\gamma)\sigma^2} < r$ and if $\frac{\gamma^3}{(\delta-\gamma)\sigma^2} \geq r$, since J is continuous, there is no local maximum over $\beta < \frac{\delta-\gamma}{\gamma(1+\gamma)}T$.

Second consider $\frac{\delta-\gamma}{\gamma(1+\gamma)}T \leq \beta$. The joint surplus writes:

$$J(\beta, T) = \gamma\frac{T}{1+\gamma} - \frac{1}{2}\left(\frac{\gamma T}{1+\gamma}\right)^2 - \frac{1}{2}\delta\left(\frac{T}{1+\gamma}\right)^2 - \frac{1}{2}r\sigma^2\beta^2,$$

and thus we have:

$$\begin{aligned}
\frac{\partial J}{\partial \beta} &= -r\sigma^2\beta \leq 0 \\
\frac{\partial J}{\partial T} &= \frac{\gamma + \gamma^2 - T\left(\delta + \gamma^2\right)}{(1+\gamma)^2},
\end{aligned}$$

then, the maximum is for

$$\begin{aligned}
\beta &= \frac{\delta - \gamma}{\gamma(1+\gamma)}T \\
\frac{\gamma + \gamma^2 - T\left(\delta + \gamma^2\right)}{(1+\gamma)^2} &= 0
\end{aligned}$$

and the solution is

$$T^{+} = \frac{\gamma + \gamma^2}{\delta + \gamma^2} = T^{FB} \text{ and } \beta^{+} = \frac{\delta - \gamma}{\delta + \gamma^2}.$$

Let J^{+} be the joint surplus levels corresponding to (β^{+}, T^{+}):

$$J^{+} = \frac{\gamma^2\left(\gamma^2 + \delta\right) - (\delta - \gamma)^2 r\sigma^2}{2\left(\delta + \gamma^2\right)^2}.$$

This level of joint surplus is positive only if

$$\frac{\gamma^2\left(\gamma^2 + \delta\right)}{(\delta - \gamma)^2\sigma^2} \geq r.$$

Otherwise, the principal will choose $T^{+} = 0$ and $\beta^{+} = 0$, and then $\widetilde{a} = \widetilde{e} = 0$ and the joint surplus is 0.

Finally,

$$T^{+}=\begin{cases}\frac{\gamma+\gamma^{2}}{\delta+\gamma^{2}} \text{ if } r\leq\frac{\gamma^{2}\left(\gamma^{2}+\delta\right)}{(\delta-\gamma)^{2}\sigma^{2}}\\ 0 \text{ else}\end{cases}$$

and,

$$J^{+}=\begin{cases}\frac{\gamma^{2}\left(\gamma^{2}+\delta\right)-(\delta-\gamma)^{2}r\sigma^{2}}{2(\delta+\gamma^{2})^{2}} \text{ if } r\leq\frac{\gamma^{2}\left(\gamma^{2}+\delta\right)}{(\delta-\gamma)^{2}\sigma^{2}}\\ 0 \text{ else}\end{cases}.$$

Notice that $\frac{\gamma^{3}}{(\delta-\gamma)\sigma^{2}}<\frac{\gamma^{2}\left(\gamma^{2}+\delta\right)}{(\delta-\gamma)^{2}\sigma^{2}}$.

Finally, there is one local maximum over $r\leq\frac{\gamma^{3}}{(\delta-\gamma)\sigma^{2}}$, which is (β^{+},T^{+}). For $\frac{\gamma^{3}}{(\delta-\gamma)\sigma^{2}}<r\leq\frac{\gamma^{2}\left(\gamma^{2}+\delta\right)}{(\delta-\gamma)^{2}\sigma^{2}}$, there are two local maxima, (β^{+},T^{+}) and (β^{-},T^{-}).

$$J^{+}-J^{-}=\frac{\gamma^{2}\left(\gamma^{2}+\delta\right)-(\delta-\gamma)^{2}r\sigma^{2}}{2\left(\delta+\gamma^{2}\right)^{2}}-\frac{1}{2}\frac{\gamma^{2}}{\delta}\frac{\gamma^{2}+r\sigma^{2}}{\gamma^{2}+(1+\delta)\,r\sigma^{2}},$$

and the difference is positive if and only if

$$\sigma^{4}\delta\left(1+\delta\right)\left(\delta-\gamma\right)^{2}r^{2}-\sigma^{2}\gamma^{3}\left(2\delta+\gamma^{2}+\gamma\delta\right)\left(\delta-\gamma\right)r+\gamma^{6}\left(\gamma^{2}+\delta\right)\leq0.$$

The LHS is a degree 2 polynomial and its determinant is $-\left(\delta-\gamma\right)^{3}\left(\gamma^{3}+4\delta^{2}+3\gamma^{2}\delta\right)<0$. Hence the polynomial is always positive and then

$$J^{+}<J^{-}.$$

Then (β^{-},T^{-}) is a global maximum over $\frac{\gamma^{3}}{(\delta-\gamma)\sigma^{2}}<r\leq\frac{\gamma^{2}\left(\gamma^{2}+\delta\right)}{(\delta-\gamma)^{2}\sigma^{2}}$. For $r>\frac{\gamma^{2}\left(\gamma^{2}+\delta\right)}{(\delta-\gamma)^{2}\sigma^{2}}$, there are two local maxima, $(0,0)$ and (β^{-},T^{-}). We have $J(0,0)=0$ and $J^{-}>0$ so that (β^{-},T^{-}) is a global maximum over $r>\frac{\gamma^{2}\left(\gamma^{2}+\delta\right)}{(\delta-\gamma)^{2}\sigma^{2}}$.

Finally (β^{+},T^{+}) corresponds to the optimal (second-best) contract over $r\leq\frac{\gamma^{3}}{(\delta-\gamma)\sigma^{2}}$ and (β^{-},T^{-}) corresponds to the optimal (second-best) contract over $r>\frac{\gamma^{3}}{(\delta-\gamma)\sigma^{2}}$ and (β^{-},T^{-}).□

Proof of Corrolary 2:

β is decreasing with r

When $\frac{\gamma^3}{(\delta-\gamma)\sigma^2} < r$, one can check that $\frac{\gamma^2}{\gamma^2+(1+\delta)r\sigma^2} < \frac{\delta-\gamma}{\delta+\gamma^2}$. The agent faces less incentives when he his sufficiently risk averse.

T **decreasing with** r is immediate.

F **is increasing with** r

The participation constraint is saturated for the optimal contract. It characterizes the fixed part of the payment scheme and then it is such that:

$$U^{SB} = \beta^{SB} \min\left\{a^{SB}, \gamma e^{SB}\right\} - \frac{1}{2}\left(a^{SB}\right)^2 - \frac{1}{2}\delta\left(e^{SB}\right)^2 - \frac{1}{2}r\sigma^2\left(\beta^{SB}\right)^2 + F^{SB} = 0,$$

or,

$$F^{SB} = -\beta^{SB}\gamma e^{SB} + \frac{1}{2}\left(a^{SB}\right)^2 + \frac{1}{2}\delta\left(e^{SB}\right)^2 + \frac{1}{2}r\sigma^2\left(\beta^{SB}\right),$$

Hence, for $r \leq \frac{\gamma^3}{(\delta-\gamma)\sigma^2}$, we have

$$\begin{aligned} F^{SB} &= -\frac{\delta-\gamma}{\delta+\gamma^2}\gamma\frac{\gamma}{\delta+\gamma^2} + \frac{1}{2}\left(\frac{\gamma^2}{\delta+\gamma^2}\right)^2 + \frac{1}{2}\delta\left(\frac{\gamma}{\delta+\gamma^2}\right)^2 + \frac{1}{2}r\sigma^2\left(\frac{\delta-\gamma}{\delta+\gamma^2}\right), \\ &= \frac{r\sigma^2\left(\gamma^2+\delta\right)(\delta-\gamma) + \gamma^2\left(2\gamma-\delta+\gamma^2\right)}{2\left(\gamma^2+\delta\right)^2} \end{aligned}$$

which is increasing with respect to r.

For $\frac{\gamma^3}{(\delta-\gamma)\sigma^2} < r$, we have:

$$\begin{aligned} F^{SB} &= -\frac{\gamma^2}{\gamma^2+(1+\delta)r\sigma^2}\gamma\frac{\gamma}{\delta}\frac{\gamma^2+\sigma^2 r}{\gamma^2+(1+\delta)\sigma^2 r} + \frac{1}{2}\left(\frac{\gamma\sigma^2 r}{\gamma^2+(1+\delta)\sigma^2 r}\right)^2 \\ &\quad + \frac{1}{2}\delta\left(\frac{\gamma}{\delta}\frac{\gamma^2+\sigma^2 r}{\gamma^2+(1+\delta)\sigma^2 r}\right)^2 + \frac{1}{2}r\sigma^2\left(\frac{\gamma^2}{\gamma^2+(1+\delta)r\sigma^2}\right)^2 \\ &= \frac{1}{2}\frac{\gamma^2}{\delta}\frac{\left(r\sigma^2+\gamma^2\right)\left((1+\delta)r\sigma^2-\gamma^2\right)}{\left(\gamma^2+(1+\delta)r\sigma^2\right)^2}. \end{aligned}$$

The derivative with respect to r is given by:

$$\frac{\partial F^{SB}}{\partial r}=\frac{1}{2}\sigma^2\gamma^4\frac{\left(\delta\left(1-\delta\right)+2\right)r\sigma^2+\left(3\delta+2\right)\gamma^2}{\delta\left(\gamma^2+\left(1+\delta\right)r\sigma^2\right)^3}>0,$$

and this concludes the proof.□

Proof of Corrolary 9:

$J\left(\beta,T^{FB}\right)$ is a continuous function because $\widetilde{a}$ and $\widetilde{e}$ are continuous. One can also check that J is a concave function over $\beta<\frac{\delta-\gamma}{\gamma(1+\gamma)}T^{FB}$ and over $\frac{\delta-\gamma}{\gamma(1+\gamma)}T^{FB}\leq\beta$. One now study J over each of these domains.

First consider $\beta<\frac{\delta-\gamma}{\gamma(1+\gamma)}T^{FB}$. The joint surplus writes:

$$J\left(\beta,T^{FB}\right)=\gamma\frac{\gamma\beta+T^{FB}}{1+\delta}-\frac{1}{2}\left(\frac{\delta T^{FB}-\gamma\beta}{1+\delta}\right)^2-\frac{1}{2}\delta\left(\frac{\gamma\beta+T^{FB}}{1+\delta}\right)^2-\frac{1}{2}r\sigma^2\beta^2,$$

and thus the FOCs for an interior solution are:

$$\frac{\partial J}{\partial\beta}=\frac{\gamma^2-\left(\left(1+\delta\right)r\sigma^2+\gamma^2\right)\beta}{1+\delta}=0$$

Hence, the solution is

$$\beta^-=\frac{\gamma^2}{\gamma^2+\left(1+\delta\right)r\sigma^2}.$$

And the condition $\beta^-<\frac{\delta-\gamma}{\gamma(1+\gamma)}T^-$ is equivalent to

$$\frac{\gamma^3}{\left(\delta-\gamma\right)\sigma^2}\frac{1+\gamma}{1+\delta}<r.$$

Let J^- be the joint surplus levels corresponding to β^-:

$$J^-=\frac{1}{2}\frac{\gamma^2\left(r\sigma^2\left(\gamma+1\right)\left(\delta+1\right)\left(\delta\left(1-\gamma\right)+2\gamma^2\right)+\gamma^2\left(\delta+2\gamma^2+2\gamma^3+\delta^2+\gamma^4+\gamma^2\delta\right)\right)}{\left(\delta+1\right)\left(\gamma^2+\delta\right)^2\left(\gamma^2+\left(\delta+1\right)r\sigma^2\right)}>0$$

Finally, β^- corresponds to a local maximum when $\frac{\gamma^3}{(\delta-\gamma)\sigma^2}\frac{1+\gamma}{1+\delta}<r$ and if $\frac{\gamma^3}{(\delta-\gamma)\sigma^2}\frac{1+\gamma}{1+\delta}\geq r$, since J is continuous, there is no local maximum over $\beta<\frac{\delta-\gamma}{\gamma(1+\gamma)}\frac{1+\gamma}{1+\delta}$.

Second consider $\frac{\delta-\gamma}{\gamma(1+\gamma)}T^{FB} \leq \beta$. The joint surplus writes:

$$J\left(\beta, T^{FB}\right) = \gamma\frac{T^{FB}}{1+\gamma} - \frac{1}{2}\left(\frac{\gamma T^{FB}}{1+\gamma}\right)^2 - \frac{1}{2}\delta\left(\frac{T^{FB}}{1+\gamma}\right)^2 - \frac{1}{2}r\sigma^2\beta^2,$$

and thus we have:

$$\frac{\partial J}{\partial \beta} = -r\sigma^2\beta \leq 0$$

then, the maximum is for

$$\beta^+ = \frac{\delta-\gamma}{\gamma\,(1+\gamma)}T^{FB} = \frac{\delta-\gamma}{\delta+\gamma^2}.$$

Let J^+ be the joint surplus level corresponding to β^+:

$$J^+ = \frac{\gamma^2}{2\,(\gamma^2+\delta)} - \frac{(\delta-\gamma)^2\,\sigma^2}{2\,(\gamma^2+\delta)^2}r.$$

This level of joint surplus is positive only if

$$\frac{\gamma^2\left(\gamma^2+\delta\right)}{(\delta-\gamma)^2\,\sigma^2} \geq r.$$

with $\frac{\gamma^2\left(\gamma^2+\delta\right)}{(\delta-\gamma)^2\sigma^2} > \frac{\gamma^3}{(\delta-\gamma)\sigma^2}\frac{1+\gamma}{1+\delta}$

The difference in the joint surplus is given by:

$$\begin{aligned} J^+ - J^- &= \frac{\gamma^2}{2\,(\gamma^2+\delta)} - \frac{(\delta-\gamma)^2\,\sigma^2}{2\,(\gamma^2+\delta)^2}r - \frac{1}{2}\frac{\gamma^2 \big(r\sigma^2\,(\gamma+1)\,(\delta+1)\,\big(\delta\,(1-\gamma)+2\gamma^2\big)+\gamma^2\,\big(\delta+2\gamma^2+2\gamma^3+\delta^2+\gamma^4}{(\delta+1)\,(\gamma^2+\delta)^2\,(\gamma^2+(\delta+1)\,r\sigma^2)} \\ &= -\frac{\left(r\sigma^2\gamma\delta+r\sigma^2\gamma-r\sigma^2\delta^2-r\sigma^2\delta+\gamma^4+\gamma^3\right)^2}{2\,(\delta+1)\,(\gamma^2+\delta)^2\,(\gamma^2+r\sigma^2+r\sigma^2\delta)} < 0 \end{aligned}$$

Hence,

$$J^+ < J^-.$$

□

Proof of Proposition 8: Point (i) is straightforward. In case (ii), the difference of effort in task

E relatively to the first best is:

$$\begin{aligned} e^{FB} - e^{SB} &= \frac{\gamma}{\delta + \gamma^2} - \frac{\gamma}{\delta} \frac{\gamma^2 + \sigma^2 r}{\gamma^2 + (1+\delta)\sigma^2 r} \\ &= \frac{\gamma\left(\left(\delta^2 - \gamma^2\right) r\sigma^2 - \gamma^4\right)}{\delta\left(\gamma^2 + \delta\right)\left(\gamma^2 + r\sigma^2 + r\sigma^2\delta\right)} \\ &> \frac{\gamma^4 \delta}{\delta\left(\gamma^2 + \delta\right)\left(\gamma^2 + r\sigma^2 + r\sigma^2\delta\right)} > 0 \end{aligned}$$

The difference of effort in task A relatively to the first best is:

$$a^{FB} - a^{SB} = \frac{\gamma\left(\gamma^3 - r\sigma^2(1-\gamma)(\delta-\gamma)\right)}{\left(\gamma^2+\delta\right)\left(\gamma^2 + (1+\delta) r\sigma^2\right)}$$

which is positive if and only if $r < \frac{1}{1-\gamma}\frac{\gamma^3}{(\delta-\gamma)\sigma^2}$.□

Proof of Corrolary 8: Point (i) is straightforward. In case (ii), the difference of the share of time spend in task E is:

$$\begin{aligned} \frac{e^{FB}}{T^{FB}} - \frac{e^{SB}}{T^{SB}} &= \frac{1}{\gamma+1} - \frac{\gamma^2 + \sigma^2 r}{\gamma^2 + (1+\delta)\sigma^2 r} \\ &= \frac{r\sigma^2(\delta-\gamma) - \gamma^3}{(\gamma+1)\left(\gamma^2 + (1+\delta) r\sigma^2\right)} > 0, \end{aligned}$$

□

Proof of Corollary 3: Let us compute the second best level of joint surplus. In case (i) when the agent is not too much risk averse ($r \leq \frac{\gamma^3}{(\delta-\gamma)\sigma^2}$), the joint surplus is:

$$J^{SB} = J^{+} = \frac{\gamma^2\left(\gamma^2+\delta\right) - (\delta-\gamma)^2 r\sigma^2}{2\left(\delta+\gamma^2\right)^2} \ (>0),$$

In case (ii), when the agent is sufficiently risk averse (when $\frac{\gamma^3}{(\delta-\gamma)\sigma^2} < r$) it is

$$J^{SB} = J^{-} = \frac{1}{2}\frac{\gamma^2}{\delta}\frac{\gamma^2 + r\sigma^2}{\gamma^2 + (1+\delta) r\sigma^2},$$

We now from the proof of the proposition that $J^{+} < J^{-}$ for any r. It is sufficient to notice that

$$\frac{\partial J^{+}}{\partial r} = -\frac{(\delta-\gamma)^2\sigma^2}{2\left(\delta+\gamma^2\right)^2} < 0,$$

and,

$$\frac{\partial J^{-}}{\partial r}=-\frac{1}{2}\sigma^{2}\frac{\gamma^{4}}{\left(\gamma^{2}+r\sigma^{2}+r\sigma^{2}\delta\right)^{2}}<0,$$

and this concludes the proof.□

Proof of Corollary 4: In case (i), the result is straightforward, as the efforts are the first best ones. In case (ii), we have

$$\begin{aligned} I &= \gamma\frac{\gamma}{\delta}\frac{\gamma^{2}+\sigma^{2}r}{\gamma^{2}+(1+\delta)\,\sigma^{2}r}-\frac{\gamma\sigma^{2}r}{\gamma^{2}+(1+\delta)\,\sigma^{2}r} \\ &= \frac{\gamma}{\delta}\frac{\gamma^{3}-(\delta-\gamma)\,r\sigma^{2}}{\gamma^{2}+(1+\delta)\,r\sigma^{2}} \end{aligned}$$

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