Faustmann rotation and aquaculture in the presence of an epidemic risk

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1 Introduction

For the management of natural resources, the first question that arises is : what is the optimal duration of cycle production. This is the case both in forestry, aquaculture, production of renewable resources. In the case where a calculation method to predict earnings for various terms of the cycle is available, Faustmann [5] proposed a formalism based on the expected discounted yield. Many authors have successively improved or reformulated the method, Ohlin [11], Pearse [12]. Clark [4] has applied this method to natural resources. Bjorndal [2] analyzes the optimal duration of farmed fish. Arneson [1] studied optimal feeding and harvest time with respect to fish-growth function. Heaps [8] analysed optimal feeding schedules and harvesting policies for farmed fish. The preceeding works are characterized by the absence of risk of destructive events. The risk of destruction has been introduced to forest stands by Martell [10] and Routledge [14] in discrete time. Thereafter, Reed [13] has studied the optimal forest rotation in continuous time with the risk of fire. Thorsen and Helles [15] maximized a not discounted criterion taking into account the risk and using a population model. Most of the work on Faustmann rotation and in particular the study of Reed [13] are developed in the context of forestry but are not specific to forestry and can be applied to the production of renewable resources. In the context of random prices in aquaculture Guttormsen [6] studies a method based on dynamic programming.

For the absence of risk of epidemic events all the production cycles are carried out to the same term. When the risk of destructive event exists and is taken into account, the authors cited above assume that the operator systematically decides to interrupt the current cycle and begin a new cycle. This is fully justified in the case of severe epidemic. In the case of an

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epidemic of limited impact on mortality, to the first question about the optimal term a second question is added : should we interrupt the current cycle and begin a new cycle or is it better to continue the current cycle ? If there are alternatives, what is the criterion to choose ? To fulfill this goal, we define a criterion for choice. We consider an averaged fish model, inspired by the models of Hannesson [7] and Heaps [9]. For specific decisions and thus specific criteria, we study two particular cases. In the first case, the operator systematically interrupts the cycle in case of a epidemic event, we generalize the results obtained by Reed [13] under less restrictive assumptions. We show that the results obtained in [13] are valid under the assumption that the operator does not harvest during the production cycle. The proposed method takes into account intermediate harvesting. In the second case, the operator continues the cycle even in case of epidemic events (which makes sense if the epidemic is minor) and we deduce the corresponding expected discounted yield. In a first part, we determine the Faustmann value without or with the presence of risk. In a second part, for a fixed rotation period. we compare the results of the optimization with respect the intermediate culling in the absence and presence of risk. Then we deduce the impact of the presence of a risk for aquacultural system. In a third part, we optimize the rotation period and the culling.

2 The Faustmann value

In the first part we study the Faustmann value without the risk of epidemic event through a model of population dynamics. Then we study the same Faustmann value with the risk of epidemic event.

2.1 Without the risk of epidemic event

We first consider an aquacultural system without the presence of risk of epidemic event. The study of this case will allow us to define a benchmark management of the system.

For a rotation period T, a culling $h(.)$ and a feeding $f(.)$ the Faustmann value W_0 (up to a constant c_1) is the discounted value of cutting incomes net cost of start-up :

$$W_0 = \sum_{i=1}^{+\infty} (\mathcal{V}(h(.), f(.), T) - c_1) e^{-i\delta T} = \frac{(\mathcal{V}(h(.), f(.), T) - c_1) e^{-\delta T}}{1 - e^{-\delta T}} = \frac{\mathcal{V}(h(.), f(.), T) - c_1}{e^{\delta T} - 1}$$

where $\mathcal{V}(h(.), f(.), T)$ is the income generated by the cutting at time T and c_1 is the cost of start-up. $\mathcal{V}(h(.), f(.), T)$ is by definition the sum of the culling income on period $[0, T]$ and the income at final rotation period T.

The Faustmann value can also be interpreted as, W_0 is the instantaneous value of income in time T discounted at the initial time and is solution of :

$$W_0 = (W_0 + \mathcal{V}(h(.), f(.), T) - c_1)e^{-\delta T}$$

The population dynamic model

To take into account the culling in the calculation of the Faustmann value, i.e. to express $\mathcal{V}(h(.), f(.), T)$, we introduce a model of population dynamics. The considered model is an average fish model : the state variables are the number n of fishes and the weight w. The evolution of these two variables is governed by the system of ordinary differential equations :

$$\frac{dn(t)}{dt} = -(m(t) + h(t))n(t)$$
$$\frac{dw(t)}{dt} = G(f(t), n(t), w(t))$$

where $m(.)$ is the natural mortality, $G(., ., .)$ is the possibly density dependent growth function : individual fish-growth may depends on the fish density $n(t)$. $f(t)$ is the amount of feed per fish per week.

The net income

Once chosen the model of population dynamics we can express the total income. Total income $\mathcal{V}(h(.), f(.), T)$ for fixed culling $h(.)$ and fixed feeding $f(.)$ is the sum of the culling income net the feeding and fixed costs $\mathcal{H}(h(.), f(.), T)$ on the period $[0, T]$ and the final income $\mathcal{V}_0(T)$:

$$\mathcal{V}(h(.), f(.), T) = \mathcal{H}(h(.), f(.), T) + \mathcal{V}_0(T)$$

The culling income net the feeding and the fixed costs $\mathcal{H}(h(.), f(.), t)$ on $[0, t]$ actualized to time t are :

$$\mathcal{H}(h(.), f(.), t) = \int_0^t [p(w(u))h(u)n(u) - sf(u)n(u) - k]e^{\delta(t-u)}du$$

where $n(.)$ and $w(.)$ are solutions of the dynamic model, s is the feeding costs and k fixed costs.

Calculation of the Faustmann value

Finally the Faustmann value is given by :

$$W_0 = \frac{\mathcal{V}(h(.), f(.), T) - c_1}{e^{\delta T} - 1}$$

with : $\mathcal{V}(h(.), f(.), T) = \int_0^T [(p(w(u))h(u) - sf(u))n(u) - k]e^{\delta(T-u)}du + \mathcal{V}_0(T)$

The maximal value of the Faustmann value for a fixed $f(.)$ is obtained by solving the problem :

$$(\mathcal{P}_{F_0}) : \qquad \max_{h(.),T} \frac{\mathcal{V}(h(.), f(.), T) - c_1}{e^{\delta T} - 1}$$

The maximization of the Faustmann value W_0 with respect to the culling $h(.)$, the feeding $f(.)$ and the rotation period T can be decomposed in two steps : first we maximize $\mathcal{V}(h(.), f(.), T)$ with respect to $h(.)$: and $f(.)$: $\mathcal{U}_0(T) = \max_{h(.),f(.)} \mathcal{V}(h(.), f(.), T)$ then we maximize $\frac{\mathcal{U}_0(T) - c_1}{e^{\delta T} - 1}$ with respect to T with $\mathcal{U}_0(T)$ resulting from the first step.

2.2 In the presence of risk of epidemic event

We suppose that epidemic events occur in a Poisson process i.e. that epidemic events occur independently of one another, and ramdomly in time.

The distribution of the epidemic event time is an exponential with mean $\frac{1}{\lambda}$: $F(x) = 1 - e^{-\lambda x}$ where λ is the expected number of epidemic events per unit time. No assumption is made on the type of epidemic events.

We assume that θ_t is the proportion of survival fishes following a epidemic event. We define the expectations $\alpha(t) = E(\theta_t) \leq 1$.

When the risk of epidemic event exists and is taken into account, in the literature, the authors classically assume that the operator systematically decides to interrupt the current cycle and begin a new cycle. This is fully justified in the case of severe epidemic. In the case of an epidemic of limited impact on mortality, to the first question about the optimal term a second question is added : should we interrupt the current cycle and begin a new cycle or is it better to continue the current cycle ? If there are alternatives, what is the criterion to choose ? To fulfill this goal, we define a possible criterion for choice based on a function z of θ_t ? We assume z increasing with $0 \leq z(\theta) \leq 1$. We choose a level z_0 such that, in the case of epidemic event :

- if $z(\theta) < z_0$ (severe epidemic), we decide to interrupt the current cycle and begin a new cycle.

- if $z(\theta) > z_0$ (minor epidemic), we decide to continue the cycle.

It is also possible to suppose that z depends on the survival fish at the date of the epidemic event.

Here, we study two particular cases. In the first case, the operator systematically interrupts the cycle in case of a epidemic event ($z_0 > 1$). In the second case, the operator continues the cycle even in case of epidemic events (which makes sense if the epidemic is minor) ($z_0 < 0$).

2.2.1 In case of epidemic event, the cycle is interrupt

For a rotation period T, a fixed culling $h(.)$ and a fixed feeding $f(.)$, the Faustmann value W_0 is the actualized value at initial time of the sum of two terms. The first one is the sum of the Faustmann value and the expectation $\mathcal{V}_1(h(.), f(.), t)$ of the total income $\mathcal{V}(h(.), f(.), \theta, t)$ for the period $[0, t]$ in case of a epidemic event at time t. The second one is the sum of the Faustmann value and the total income $\mathcal{V}(h(.), f(.), T)$ for the period $[0, T]$ in case of no epidemic event.

$\mathcal{V}(h(.), f(.), \theta, t)$ is the sum of the culling income net the feeding and fixed costs $\mathcal{H}(h(.), f(.), t)$ during $[0, t]$ actualized at time t and the final income $\mathcal{V}_F(\theta, t)$ at time t. The final income $\mathcal{V}_F(\theta, t)$ is assumed proportional to the final income without risk of epidemic event $\mathcal{V}_0(T)$ and is given by : $\mathcal{V}_F(\theta, t) = \theta\mathcal{V}_0(t)$. Then the total income is : $\mathcal{V}(h(.), f(.), \theta, t) = \mathcal{H}(h(.), f(.), t) + \theta\mathcal{V}_0(t)$. From definition of α we deduce the final income expectation :

$$\mathcal{V}_1(h(.), f(.), t) = E(\mathcal{V}(h(.), f(.), \theta, t)) = \mathcal{H}(h(.), f(.), t) + \alpha(t)\mathcal{V}_0(t)$$

Calculation of the Faustmann value

From (1) and expression of $\mathcal{V}_1(h(.), f(.), t)$ we deduce that, in the presence of the risk of epidemic event the Faustmann value is given by :

$$W_0 = \frac{\delta+\lambda}{\delta}\frac{\mathcal{V}_\lambda(h(.),f(.),T)-c_1}{e^{(\delta+\lambda)T}-1} - \frac{\lambda}{\delta}(c_1+c_2)$$

As in the case without risk, we find that the Faustmann value can be deduced from the income $\mathcal{V}_\lambda(h(.),f(.),T)$. The parameter δ is replaced by $\delta+\lambda$. The major difference is the following : $\mathcal{V}_1(T)$ is not as in the case without risk the total income but a modified expression of the income $\mathcal{V}_\lambda(h(.),f(.),T)$. This difference is reflected by the substitution in the cases without risk of term $\mathcal{H}(h(.),f(.),T)$ by a term $\mathcal{H}_\lambda(h(.),f(.),T)$, or even more precise replacing $p(w(t))h(t)n(t)-sf(t)n(t)-k$ by $[p(w(t))h(t)n(t)-sf(t)n(t)-k+\lambda\alpha(t)\mathcal{V}_0(t)]e^{\lambda(T-t)}$.

The maximal value of the Faustmann value is obtained by solving :

$$(\mathcal{P}_{F_1}): \quad \max_{h(.),T} W_0 = \frac{\delta+\lambda}{\delta}\frac{\mathcal{V}_\lambda(h(.),f(.),T)-c_1}{e^{(\delta+\lambda)T}-1} - \frac{\lambda}{\delta}(c_1+c_2)$$

As in the case without risk, the maximal value of the Faustmann value with respect to the culling $h(.)$, the feeding $f(.)$ and the rotation period T can be decomposed in two steps : first we maximize $\mathcal{V}_\lambda(h(.),f(.),T)$ with respect to $h(.)$ and $f(.)$: $\mathcal{U}_\lambda(T) = \max_{h(.),f(.)} \mathcal{V}_\lambda(h(.),f(.),T)$ then we maximize $\frac{\delta+\lambda}{\delta}\frac{\mathcal{U}_\lambda(T)-c_1}{e^{(\delta+\lambda)T}-1} - \frac{\lambda}{\delta}(c_1+c_2)$ with respect to T with $\mathcal{U}_\lambda(T)$ resulting from the first step.

2.2.2 In case of epidemic event, the cycle is not interrupt

The Faustmann value is obtained by considering the value without risk but calculated with the expected number of survival fish.

3 Optimisation for a fixed rotation period T and a fixed feeding f

We first consider the limiting case of no density dependent growth which will be used as a reference in the case study of density dependent growth.

3.1 Without risk of epidemic event

Let us consider the case with fixed feeding f. In this case the evolution of the fish-weight w does not depend of a control.

We study the maximization of the Faustmann value with respect to culling $h(.)$ for a fixed rotation period T :

$$(\mathcal{S}_0) \quad \max_{h(.)} \mathcal{V}(h(.), f(.), T) = \int_0^T [(p(w(t))h(t) - sf(t))n(t) - k]e^{\delta(T-t)}dt + \mathcal{V}_0(T)$$

with the constraint $0 \leq h(t) \leq \overline{h}$.

Assume that the final income is given by : $V_0(n, w) = p(w)n$. From the no density dependence of the individual growth, $p(w(t))$ is independent of the cullings and only depends on t. Then we define $R(t) = p(w(t))$. We denote the function π_0 : $\pi_0(t) = R'(t) - (\delta + m(t))R(t) - sf(t)$. Applying the maximum Pontryagin Principle to the problem $\mathcal{P}_{m,0}$ we can deduce the optimal culling :

- if $\pi_0(T) \geq 0$ then $h_* \equiv 0$ in the vicinity of T

- if $\pi_0(T) < 0$ then $h(t) = \overline{h}$ in the vicinity of T.

3.2 In the presence of risk of epidemic event

3.2.1 In case of epidemic event, the cycle is interrupt

We study the maximization of the Faustmann value with respect to culling $h(.)$ for a fixed rotation period T and a fixed feeding $f(.)$:

$$(\mathcal{S}_1) \quad \mathcal{V}_\lambda(h(.), f(.), T) = \max_{h(.)} \mathcal{H}_\lambda(h(.), f(.), T) + \mathcal{V}_0(T)$$

with the constraint $0 \leq h \leq \overline{h}$.

Using the expression of $V_0(T)$ we deduce that, compared to the case without risk, $p(w(t))h(t) - sf(t)$ is replaced by $[p(w(t))h(t) - sf(t) + \lambda\alpha(t)p(w(t))]e^{\lambda(T-t)}$ in presence of risk of epidemic event. Thus we deduce that, for the fixed rotation period T, the optimization differs and depends explicitly on λ and α. So we will pay attention to the consequence for the aquacultural practice.

We consider, as in the case without risk, a non density dependent growth for the fishes to facilitate the comparaison.

Let denote the function $\pi_1 : \pi_1(t) = R'(t) - (\lambda(1-\alpha(t)) + \delta + m(t))R(t) - sf(t)$. Applying the maximum Pontryagin Principle to the problem $\mathcal{P}_{m,\lambda}$, we deduce the optimal culling :

- if $\pi_1(T) \geq 0$ then $h_* \equiv 0$ in the vicinity of T

- if $\pi_1(T) < 0$ then $h(t) = \overline{h}$ in the vicinity of T

3.2.2 In case of epidemic event, the cycle is not interrupt

We study the maximization of the Faustmann value with respect to culling $h(.)$ for a fixed rotation period T.

From the expression of the expected of survival fish $N(t) : N(t) = n(0)e^{-\int_0^t (m(u)+h(u)+\lambda(1-\alpha(u)))du}$, applying the maximum Pontryagin Principle to the problem $\mathcal{P}_{m+\lambda(1-\alpha),0}$ we can deduce the optimal culling :

- if $\pi_1(T) \geq 0$ then $h_* \equiv 0$ in the vicinity of T

- if $\pi_1(T) < 0$ then $h(t) = \overline{h}$ in the vicinity of T.

3.3 Comparaison : without and with presence of risk

By comparing the three propositions, for the optimal solution in the vicinity of rotation period, the natural mortality $m(t)$ in the case without risk is replaced by the mortality due to events $m(t) + \lambda(1 - \alpha(t))$ in the presence of risk. It is equivalent also, from a mathematical point of view, to replace the fixed discount rate δ by the variable discount rate $\delta + \lambda(1 - \alpha(t))$ in the previous problem.

By comparing the results of the two propositions we deduce that, for a fixed rotation period T, it is usually best to do culling at least at the end of the period in the presence of risk. From $\pi_1(t) = \pi_0(t) - \lambda(1-\alpha(t))R(t)$, we deduce that even if $h \equiv 0$ in the unrisky case, for sufficently large value of $\lambda(1 - \alpha(T))$, $h = \overline{h}$ in the vicinity of T.

Comparing the results of the two proposals is permitted if the rotation periods are identical. If we consider the maximization problem, with respect to the rotation period, the rotation periods have no reason to be the same. In that case the comparison is not permitted and only simulations can allow us to compare the respective culling. We will therefore perform simulations.

4 Optimization with respect to the rotation period T

We now consider the case where individual growth is density dependent. If the growth is weakly density dependent, by continuity with the case of no density dependent growth, the obtained results are still valid at fixed rotation period T. If this is not the case, we cannot obtain analytical results for the solutions, then simulations are required.

The function of individual growth is given by : $G(f,n,w) = w(0.04 - 0.01w - 10^{-9}\frac{w^2 n}{f})$ [9]. The structure of the growth function is generic and can be used for other species. The price is assumed to depend on the weight : $p(w) = p_0 w - c_0$.

Instead of estimating the feeding function $f(.)$ as in [9], to simplify the optimization problem, we decide to search the function $f(.)$ in a parametrized family : $f(t) = aw(t)^b$ with a and b unknown coefficients. The determination of the rotation period T is important because of its impact on optimization. To better describe the optimization in the presence of random risk, it is wiser to look at the effective cutting age $\mathcal{T}$ and the effective final fish-weight $\mathcal{S}$. Thus we calculate the respective expectations and variances :

$$E(\mathcal{T}) = \int_0^T t dF(t) + T(1 - F(T)) = \frac{F(T)}{\lambda}$$

$$Var(\mathcal{T}) = \int_0^T (t - \frac{F(T)}{\lambda})^2 dF(t) + (T - \frac{F(T)}{\lambda})^2(1 - F(T)) = \frac{2}{\lambda^2}(1 - F(T))(F(T) - \lambda T) + \frac{F^2(T)}{\lambda^2}$$

$E(\mathcal{W}) = \int_0^T w(t)dF(t) + w(T)(1 - F(T))$ and $Var(\mathcal{W}) = \int_0^T w^2(t)dF(t) + w^2(T)(1 - F(T))$ can be derived from the simulations.

4.1 Results and Discussion

The unit of time for the rotation period is the week. We suppose : $m = 0.0042$ week^{-1}, $\lambda = 0.005$ week^{-1}, $\delta = 0.001$ week^{-1}, $\overline{h} = 0.06$ week^{-1}.

Risk	$h(.)$	T	W_0
No	$h(t) = \overline{h}, 91.9 \leq t \geq 104.5$	117.6	$16.4n_0$
No*	$h(t) = \overline{h}, 85.5 \leq t \geq 98.3$	110.8(85.1)	
Yes	$h(t) = \overline{h}, 88.6 \leq t \geq 101.4$	115.2 (87.6)	$12.1n_0$

Table 1 : Optimal culling for $\alpha = 0.5$, $n(0) = 1500000$ fish.

The results are included in Table 1, to complete, the parameter of optimal feeding function are $a = .5748, b = 1.1907$ for the unrisky case and $a = .5739, b = 1.1698$ in presence of risk. The standard deviation of the effective rotation period was calculated.

We assume that the expected proportion of safe fish after an epidemic event is equal to $.5$. Without risk and with the presence of risk, there is culling for the optimal solution (Table 1).

The second line of table 1 corresponds to an unrisky case with δ replaced by $\delta + \lambda$, the result of the line must compared to the last line. Usually the risky case is assimilated to the unrisky case with δ replaced by $\delta + \lambda$ and we can see that the results are significantly different.

The presence of risk involves earlier culling. The earlier culling provides a kind of self-insurance against risk.

By observing the curves of the Faustmann value W_0 depending on the rotation problem T with optimal culling, we find that the Faustmann value least varies in the vicinity of the optimal rotation period with risk than without risk. This is another consequence of the fact that, with risk the optimal rotation period T is achieved with a relatively low probability.

5 Conclusion

We have studied the management of an aquacultural system in the presence of risk of epidemic event. In order to determine the optimal culling relative to the Faustmann criterion, we have considered a model of population dynamics, the choosen model is of average fish type. This model has allowed us to make the comparison without and with risk and highlighted the influence of the presence of risk of epidemic event on optimal culling.

Specifically, the obtained Faustmann values, without or with the risk of epidemic event, highlighted differences in the criteria to be maximized. In the case of no density dependent individual growth, we have highlighted the impact of the presence of risk on the strategies, generically regardless of the considered species.

In the case of density dependent growth, the calculations for an aquacultural system have shown that the presence of risk of destruction event involves earlier culling for the optimal strategy.

The obtained results are conditioned by the choice of an individual fish growth model and by the specification of a weight model and a price model of fishes. Other studies using models adapted for other species would make the obtained results more generic.

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