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## Statistical analysis of forest genetic experiments. Some key points.

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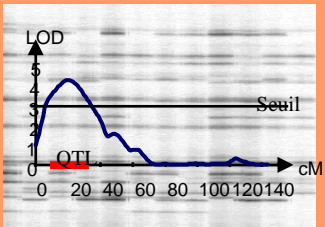
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# Statistical analysis of forest genetic experiments

## Some key points

*Catherine Bastien  
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**PROFOREST Workshop, Warsaw, 24-27 August 2004**



# Objectives of forest genetic field experiments

- Comparison of **different populations** of a given species for *quantitative* and *qualitative* traits expressed in **forest conditions**: **provenance tests**
- Genetic evaluation in **forest conditions** of **phenotypical selections** : **progeny tests** (« + » trees open-pollinated progenies, polymix progenies, controlled crosses), **clonal tests**, **multisite experiments**
- **Backward** selection in clonal seed orchard on multitrait evaluation in **forest conditions** of **phenotypical selections** : **progeny tests** (« + » trees open-pollinated progenies, polymix progenies, controlled crosses)
- **Forward** selection on multitrait evaluation in **forest conditions** for long-term breeding strategies : **progeny tests** (« + » trees open-pollinated progenies, polymix progenies, controlled crosses)
- Evaluation of **genetic variability** of natural and artificial populations for *quantitative* and *qualitative* traits expressed in forest conditions : **progeny tests**

INTRODUCTION

# forest genetic field experiments

Genotype : provenance, progeny-family, clone

A basic common model : Fisher (1918)



*Fixed situation*

- Precise estimation of genotypic values and genotype stability over a given set of environmental conditions

*Random situation*

- Precise estimation of genetic and GxE variances in a multitrait context

# forest genetic field experiments

Prediction of  $G_i$  values in a given experiment

$$P_{ij} = G_i + B_{lock}^* + R_{ij}$$

Controlled  
experimental variation



*To maximize for a better control of environmental variation*

Residual  
Uncontrolled  
variation

*To minimize for maximum precision (experimental designs)*

\* Complete or incomplete block design with single or multitree plots



# forest genetic field experiments

- 1 - Test and adjustment for local environmental effects:
  - Efficiency of block designs
  - Correction with **spatial analysis** : Papadakis iterative method

*PLAN*



# forest genetic field experiments



Prediction of Breeding Values  $A_i$   
before genetic thinning in clonal seed orchards

**Fisher's key insights** : Each individual pass to its offspring a fraction of its genetic value which at a minimum is equal to  $\frac{1}{2}$  genetic additive value  $A$

Evaluation criteria

Breeding objective

Own performance  $P_1$

Performance of offspring  $P_2$

Correlated Traits  $P_n$

Molecular markers  $M_n$

Breeding value  $A_i$



Multiple linear regression

$$A = b_1P_1 + b_2P_2 + \dots + b_nP_n + \dots + c_nM_m$$

INTRODUCTION

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# forest genetic field experiments

- 1 - Test and adjustment for local environmental effects:
  - Efficiency of block designs
  - Correction with spatial analysis : Papadakis iterative method
- 2 – Estimation of breeding values
  - BLUP's
  - variance components estimation

PLAN



# forest genetic field experiments

Multitrait selection  
and economic weights of the different selection objectives

## INTRODUCTION

### Adaptation

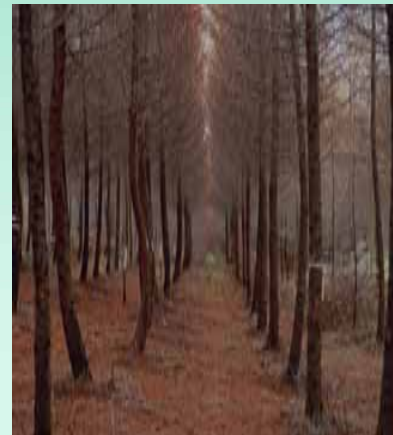
(biotic & abiotic factors)



### Volume production



### Stem quality



### Wood quality



$$\text{Selection Index : } I = a_1G_1 + a_2G_2 + \dots + a_nG_n$$



# forest genetic field experiments

## PLAN

- 1 - Test and adjustment for local environmental effects:
  - Efficiency of block designs
  - Correction with spatial analysis : Papadakis iterative method
- 2 – Estimation of breeding values
  - BLUP's
  - variance components estimation
- 3 – Multi-trait selection
  - Prediction of response to selection
  - Independent Culling vs. Index
  - Economic vs technical weights in selection index



*Adjustment for local environmental effects*

# forest genetic field experiments

## Control of environmental variation





# forest genetic field experiments

Control of environmental variation by block effects

*Example : analysis of total height of a clonal test  
in a 6 complete block design*

**ANOVA Table 2003 Total Height**

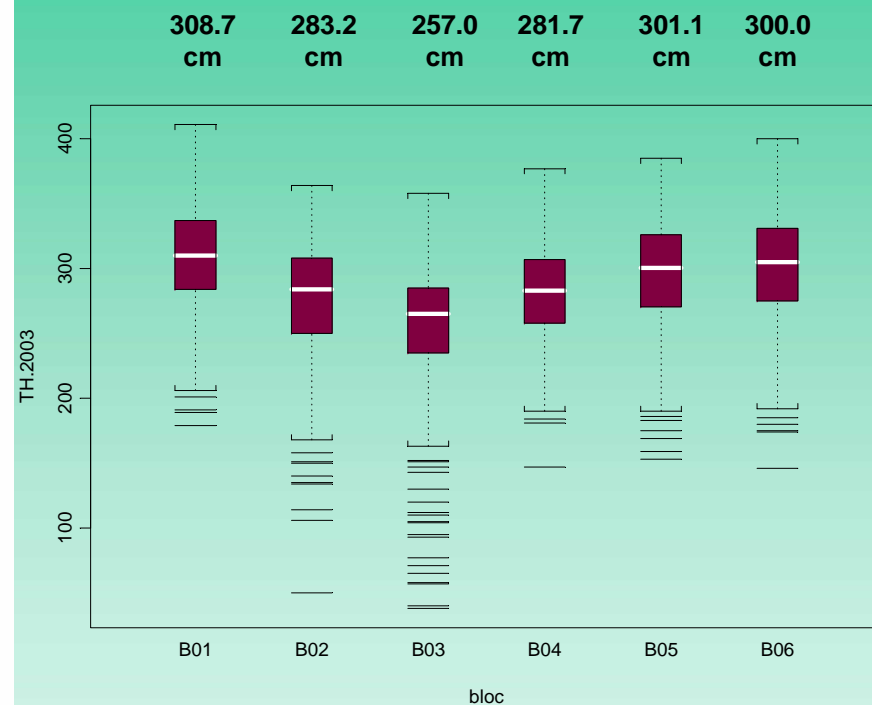
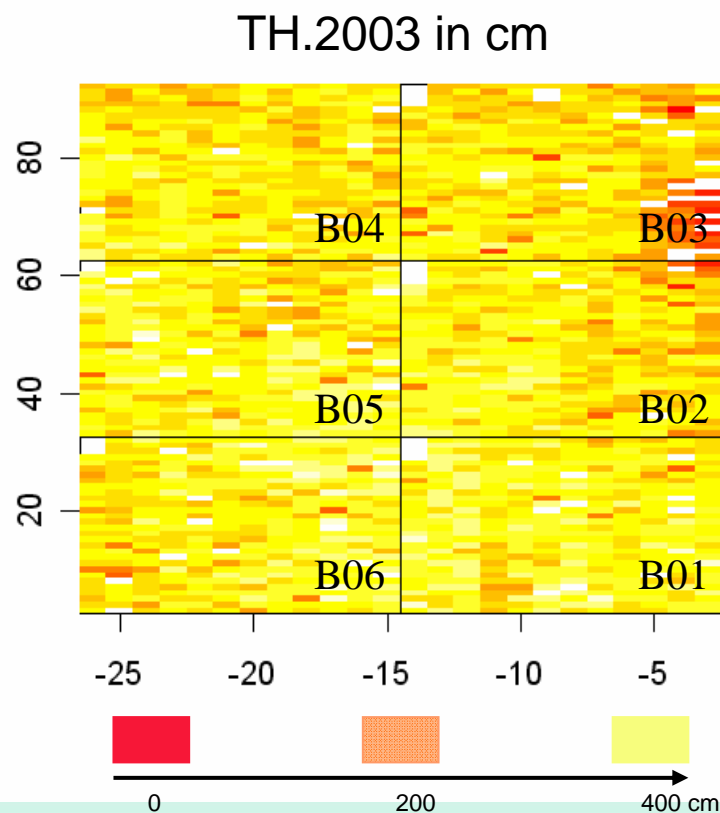
	Df	SSE	MS	F-test	P-value
<b>Bloc</b>	5	603119	120624	72.36	0.000
<b>Genotype</b>	354	1988304	5617	3.3692	0.000
<b>Residuals</b>	1718	2864033	1667		

***Strong block effects !  
 $CV_r = 14.1\%$***



# Control of environmental variation by block effects

*Example : analysis of total height of a clonal test  
in a 6 complete block design*

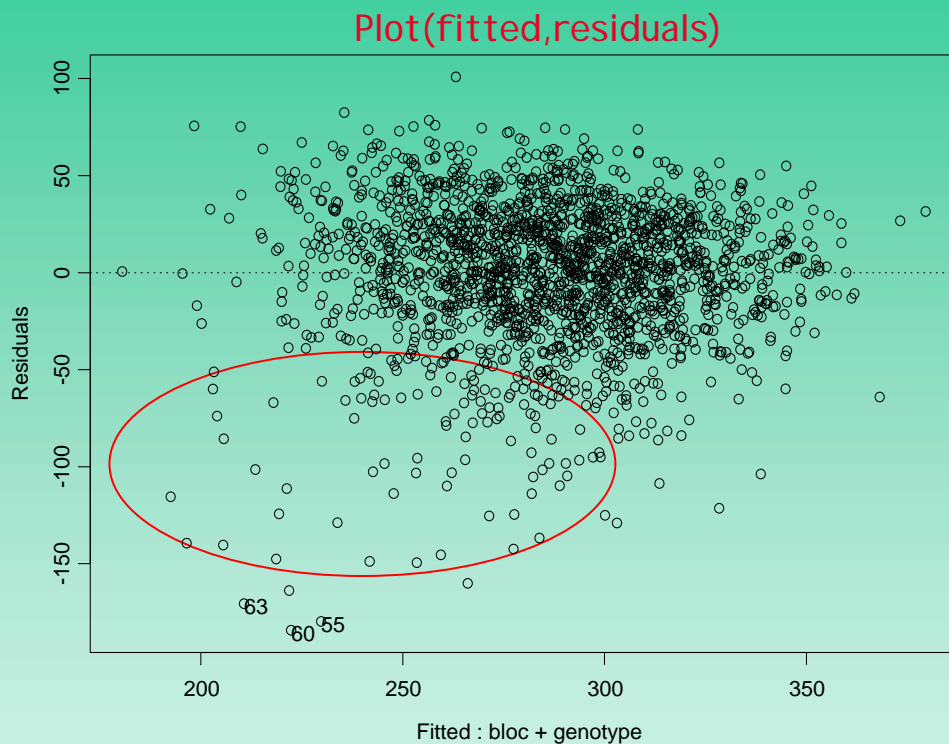


*block effects will control part of environmental  
variation. What does remain ?*



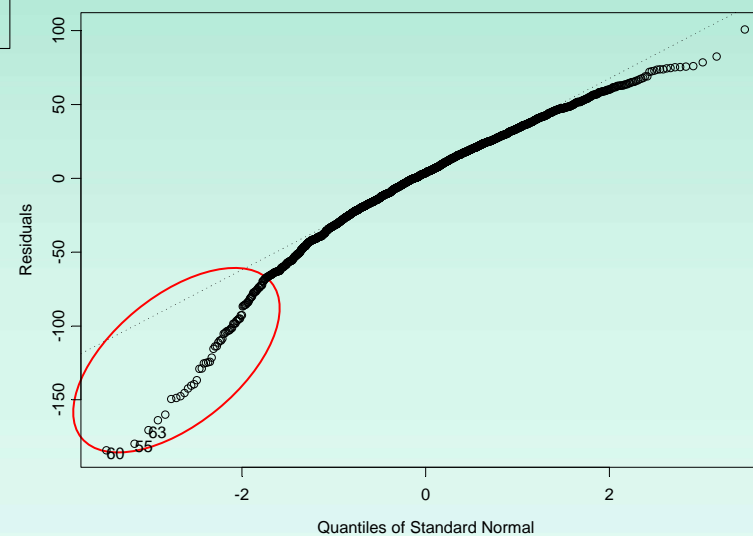
# Control of environmental variation by block effects

Example : analysis of total height of a clonal test in a 6 complete block design



Analysis of residual variation

Quantile-Quantile plot of residuals



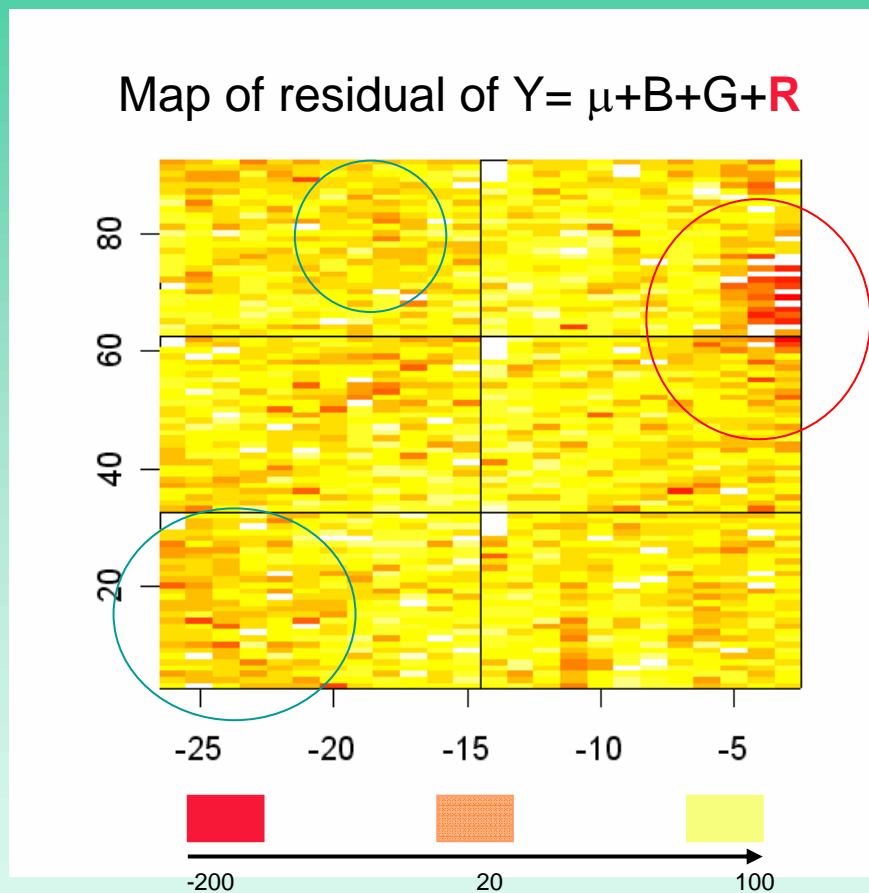
- A lot of plants with relative low height  
transplantation effect ?  
local environmental effects ?



# Control of environmental variation by block effects

*Example : analysis of total height of a clonal test in a 6 complete block design*

*spatial distribution of residual variation*



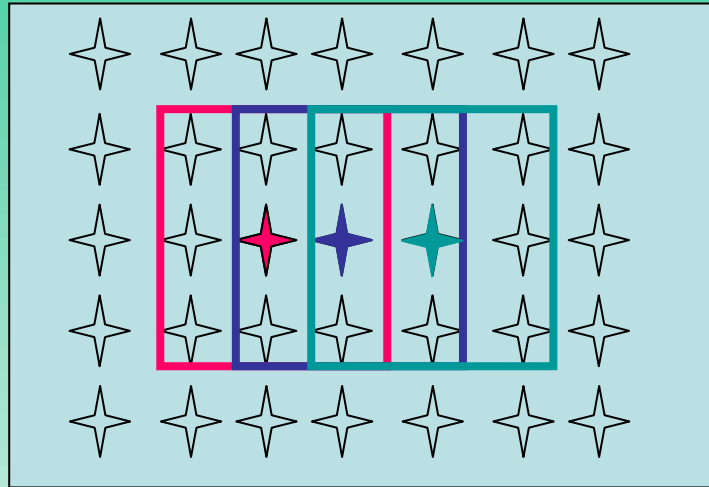
*Environmental variation still exist within block !*



# Control of environmental variation by spatial analysis

*Example : analysis of total height of a clonal test in a 6 complete block design*

## *Papadakis iterative method*



*Environmental variation is measured by the neighborhood residual information ( $\Psi r$ )*

$$\mathbf{P}_{ij} = \mu + \mathbf{G}_i + \mathbf{b} \mathbf{E}(\Psi r) + \mathbf{R}'_{ij}$$

$$\mathbf{E}(\Psi r) = \sum_{i'j'} \mathbf{R}'_{i'j'} / \mathbf{n}_{(r)}$$

$$\mathbf{P}'_{ij} = \mathbf{P}_{ij} - \mathbf{b} \mathbf{E}(\Psi r)$$



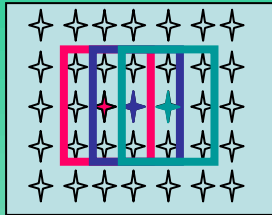
*Iterative procedure*





# Control of environmental variation by spatial analysis

*Example : analysis of total height of a clonal test in a 6 complete block design*



## *Papadakis iterative method*

Neighborhood : 5 trees x 9 trees

***ANOVA Table 2003 on Total Height corrected by Papadakis***

	<b>Df</b>	<b>SSE</b>	<b>MS</b>	<b>F-test</b>	<b>P-value</b>
<b>Bloc</b>	5	7023	1405	1.0711	0.3745
<b>Genotype</b>	354	1861064	5257	4.0089	<0.0001
<b>Residuals</b>	1718	2252986	1311		

*Reduced residual variation*

$$CV_r = 12.8\%$$

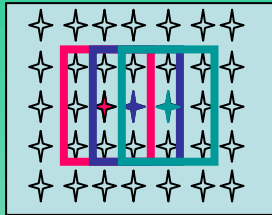
Adjustment for local environmental effects

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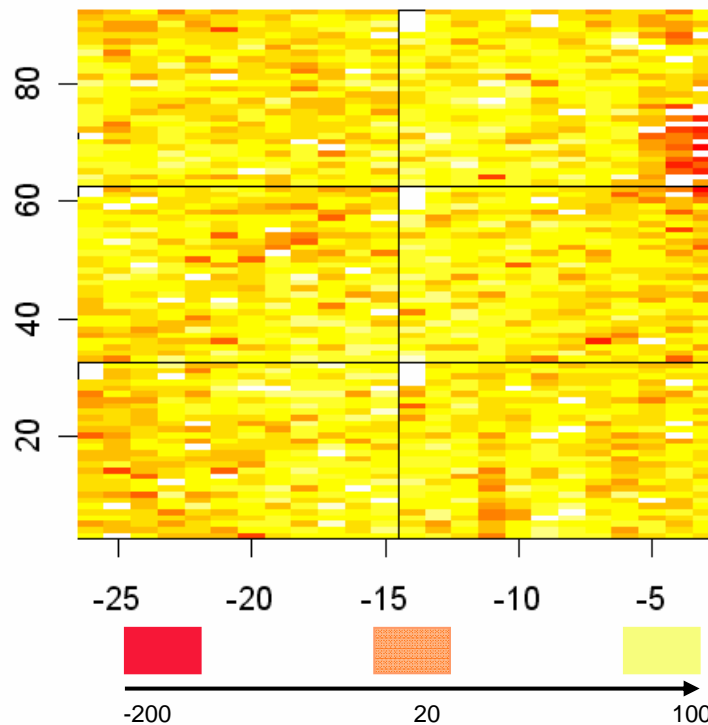
# Control of environmental variation by spatial analysis

Example : analysis of total height of a clonal test in a 6 complete block design

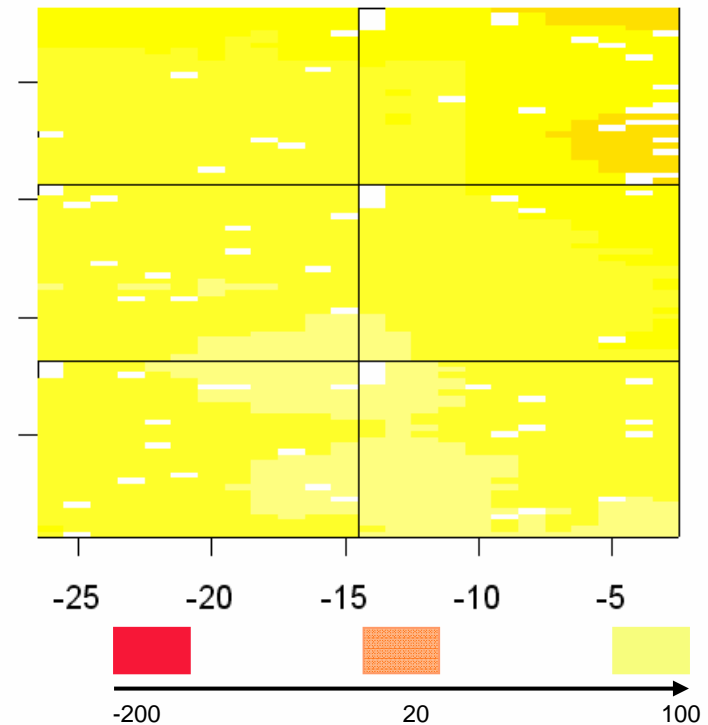


*Papadakis iterative method*

Adjustment to block effects



Adjustment by spatial analysis



Final choice : *adjustment by spatial analysis and elimination of five rows in block 03*

Adjustment for local environmental effects

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# Control of environmental variation by spatial analysis

## *Papadakis iterative method*

Kempton RA and Howes CW 1981. The use of neighbouring plot values in the analysis of variety trials. *Applied Statistics* 30 (1), 59-70

Dagnélie P. 1989. The method of Papadakis in Agricultural Experimentations. An overview *Buletyn Oceny Odmian*, 21-22, 111-122.

Besag J and Kempton R. 1986. Statistical Analysis of Field Experiments Using neighbouring plots. *Biometrics* 42, 231-251.

Bartlett MS. 1978. Nearest neighbour models in the Analysis of Field Experiments. *J.R. Statist. Soc.* 2, 147-174.

# forest genetic field experiments

Estimation of breeding values and phenotypic variance components



VG

VR

VA

$h^2$

rG

*Estimation of breeding values*

# forest genetic field experiments



Prediction of Breeding Values  $A_i$   
before genetic thinning in clonal seed orchards

Finding the optimal regression coefficients  $b_n$

$$A = b_1P_1 + b_2P_2 + \dots + b_nP_n$$

$$Y = f(X) = b X \quad b = \frac{\text{cov}(X, Y)}{\text{var}(X)} \quad \text{BLUP} = \text{Best linear unbiased prediction}$$

Evaluation criteria

Own performance  $P_1$

$$b_1 = \text{Cov}(P_1, A) / \text{var}(P_1)$$

$$b_1 = \text{Cov}(A + \cancel{D} + \cancel{I} + \cancel{E}, A) / \text{var}(P_1)$$

$$b_1 = \text{Cov}(A, A) / \text{var}(P_1)$$

$$b_i = h^2$$



# Prediction of Breeding Values $A_i$ before genetic thinning in clonal seed orchards

Evaluation criteria

Open pollinated progeny performance  $P_2$

$$A = b_2 P_2$$

$n = \text{nb. ind in progeny}$

$\text{Var}(\text{mean}) = \text{common variance} + \text{specific} / n$

$$b_2 = \text{Cov}(P_2, A) / \text{var}(P_2)$$

$$b_2 = 1/2 V_A / \text{var}(P_2)$$

$$b_2 = 1/2 V_A / (V_{\text{Fam}} + V_{\text{resid}} / n)$$

Heritability of  
progeny test

$$b_2 = h^2_{\text{Fam}} = \frac{2n}{n + \frac{4 - h^2}{h^2}}$$

$b_2$  depends on the  
number of progeny  
and on the  
heritability



# Prediction of Breeding Values $A_i$ before genetic thinning in clonal seed orchards

Evaluation criteria

Open pollinated progeny performance  $P_2$

*k traits measured*

$$A = b_2^1 P_2^1 + b_2^2 P_2^2 + \dots + b_2^k P_2^k$$

$M_P$  = matrix of phenotypic variances-covariances

$M_A$  = matrix of additive genetic variances-covariances

$$b_2 = \frac{1}{2} M_A M_P^{-1}$$

$b_2$  for  $A_{\text{Total height age 15}}$

Total height age 15

0.562

Total height age 15  
Total height age 10  
Girth age 15  
Branch angle age 10



0.714

From Bastien 1999, unpublished data

Estimation of breeding values

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## Prediction of Breeding Values $A_i$ before genetic thinning in clonal seed orchards

### Estimation of breeding values

- *Efficiency* of **BLUP** estimation proved in many animal and plant breeding programs
- **BLUP** estimation is always superior to phenotypical selection on progeny means
- Measuring **correlated traits** could increase significantly precision of breeding values estimation
- **BLUP** could be easily calculated with all softwares including linear model predictions [SAS, ASREML, Splus,....]
- **BLUP** needs only accurate estimation of  $M_A$  (heritabilities and additive genetic correlations)



# Estimation of variance components

- Two key statistical ANOVA identities
  - Total variance = between-group variance ( $V_{Fam}$ ) + within-group variance ( $V_W$ )
  - Variance(between groups) = covariance (within groups)
- One key genetic property of Fisher model (Kempthorne 1957)  
 X and Y , two individuals

$$\text{Cov}( X, Y ) = 2 r_{XY} V_A + u_{XY} V_D$$

*In practice*

**Open-pollinated progenies** collected **randomly** in most Scots pine stands could be considered as a **random** sample of **half-sib progenies**

$$V_{Fam} = \text{Cov} (HS)$$

$$V_{Fam} = V_A / 4$$



4  $V_{Fam}$  gives an estimation of  $V_A$



Estimation of variance components according to the experimental design

# Estimation of variance components

## Two methods

Expected means squares of  
Analysis of Variance  
(ANOVA)  
Henderson III

- Independent estimation of **fixed** and **random** effects
- **Biased** estimation in case of **non-orthogonal** (unbalanced) designs
- difficulty to analyze jointly variety of relatives

Restricted maximum  
likelihood estimation  
(REML)

- Simultaneous estimation of **fixed** and **random** effects
- no demand on design or balance of data
- no demand on design or balance of data
- **now available** in most statistical softwares



# Estimation of variance components

Example : analysis of total height and branch angle in a Scots pine progeny test

## Expected means squares of Analysis of Variance (ANOVA)

ANOVA Table on Total Height adjusted to block effects  $Y' = Y - \text{Block}$

		Df	SSE	MS	E(MS)
Fixed	<b>Bloc</b>	41	100978	2463	$V_R + k\phi_{\text{bloc}}$
Random	<b>Genotype</b>	64	1039806	16247	$V_R + nV_{\text{Fam}}$
Random	<b>Residuals</b>	1935	7260365	3752	$V_R$

Average  $n = 31.4$  trees per progeny

$$\hat{V}_R = 3752$$

$$\hat{V}_{\text{Fam}} = (16247 - 3752) / 31.4 = 397.9$$

$$\hat{V}_A = 4 * \hat{V}_F = 1591.7$$

$$\hat{h}^2 = 1591.7 / (397.9 + 3752) = 0.383$$



# Estimation of variance components

Example : analysis of total height and branch angle in a Scots pine progeny test

## Expected means squares of Analysis of Variance (ANOVA)

MANOVA Total Height , Branch angle adjusted to block effects  $Y' = Y - \text{Block}$

		Df	SCPE	MCP	E(MCP)
Fixed	<b>Bloc</b>	41	-232.27	-5.66	$\text{Cov}_R + k\phi_{\text{bloc}}$
Random	<b>Genotype</b>	64	722.97	11.30	$\text{Cov}_R + n\text{Cov}_{\text{Fam}}$
Random	<b>Residuals</b>	1911	2423.75	1.27	$\text{Cov}_R$

Average  $n = 31.2$  trees per progeny

$$\hat{\text{Cov}}_R = 1.27$$

$$\hat{\text{Cov}}_{\text{Fam}} = (11.30 - 1.27) / 31.2 = 0.32$$

$$\hat{\text{Cov}}_A = 4 * \hat{\text{Cov}}_F = 1.28$$

$$\hat{r}_A = 1.28 / \sqrt{(1591.7 * 0.45)} = 0.047$$



# Estimation of variance components

*Example : analysis of total height and branch angle in a Scots pine progeny test*

## Restricted maximum likelihood estimation (REML)

$$\text{Model : } Y_{\text{adj}} = \mu + \mathbf{B}_{\text{lock}} + \mathbf{F}_{\text{amily}} + \mathbf{R}_{\text{esidual}}$$

Fixed          Random    Random

	Total Height	Branch angle
$V_{\text{Fam}}$	411.86	0.116
$Sd(V_{\text{Fam}})$	95.21	0.024
$V_{\text{R}}$	3753	0.609
$Sd(V_{\text{R}})$	29.33	0.020
$h^2$	0.395	0.640
$Sd(h^2)$	0.085	0.051

***Precision of variance component estimations depends on nb. of progenies***



# Estimation of variance components

*Example : analysis of total height and branch angle in a Scots pine progeny test*

## Restricted maximum likelihood estimation (REML)

$$\text{Model : } Y_{\text{adj}} = \mu + \mathbf{B}_{\text{lock}} + \mathbf{F}_{\text{amily}} + \mathbf{R}_{\text{esidual}}$$

Fixed          Random    Random

### *Estimation of Covariance components*

$$\text{Cov}( X+Y) = V_X + V_Y + 2 \text{Cov}( X,Y)$$

$$\text{Cov}( X,Y) = \frac{1}{2} (\text{Cov}( X+Y) - (V_X + V_Y))$$

	Total Height-Branch angle
$\text{Cov}_{\text{Fam}}$	-0.120
$\text{Cov}_{\text{R}}$	3753
$r_A$	-0.017

# 3- Multitrait selection and economic weights of the different selection objectives

Multi-trait selection

**Adaptation**  
(biotic & abiotic factors)



**Volume  
production**



**Stem quality**



**Wood quality**



**Selection Index :  $I = a_1G_1 + a_2G_2 + \dots + a_nG_n$**



# forest genetic field experiments

## Prediction of response to selection

« + » tree selection  
(natural stands, provenance tests)



Seed collection



Multisite evaluation  
progeny testing



Grafting



Clonal Collection for  
recombination

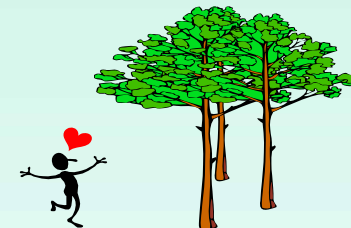
Clonal seed orchard



Forward selection  
Genetic thinning in seed orchard



Realized  
genetic gain



Multitrait selection

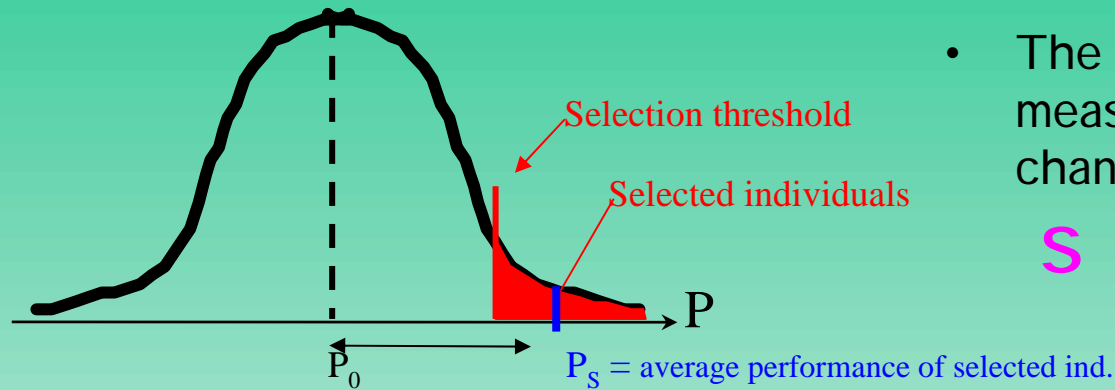
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# forest genetic field experiments

## Prediction of response to selection

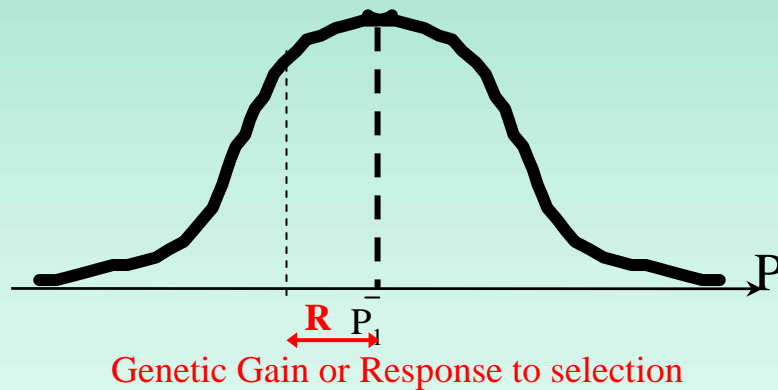
Response to selection



- The **selection differential S** measures the within-generation change in the mean

$$S = P_s - P_0$$

↓  
Recombination of selected individuals



- The **response R** is the between-generation change in the mean

$$R = P_1 - P_0$$



# forest genetic field experiments

## Prediction of response to selection

- Selection can change the distribution of phenotypes. We typically measure this by changes in mean.

This is a within-generation change measured by  $S = P_s - P_0$

- Selection can also change the distribution of breeding values (changes in allele frequencies).

This is a the response to selection, the change in the trait in the next generation (between-generation change) measured by

$$R = P_1 - P_0$$

# Prediction of response to selection

## The Breeder's Equation

$$R = h^2 S$$

- Note that no matter how strong **S**, if **h<sup>2</sup>** is small, the response is small
- **S** is a measure of selection, **R** the actual response. One can get lots of selection but no response

## Applications

- In **agriculture** and **forestry breeding**
- Construction of divergent pedigree for QTL mapping and gene expression (microarray) analysis : inferences about nb. Of loci, effects and frequencies
- **Evolutionary inferences** : correlated characters, effects on fitness, long-term response

## Prediction of response to selection

### The Selection Intensity, $i$

Populations with the same selection differential ( $S$ ) may experience very different amounts of selection  
The **selection intensity**  $i$  provided a suitable measure for comparisons between populations,

$$i = \frac{S}{\sigma_p}$$

$$R = h^2 S = i h^2 \sigma_p = i h \sigma_A$$

- Since  $h$  = correlation between phenotypic and breeding values

$$\text{Response} = \text{Intensity} * \text{Accuracy} * \text{Spread in } V_A$$





## The correlated response

*Selection on Trait 1, predicting response of Trait 2*

$$R_2 = i_1 r_{A1,2} h_1 h_2 \sigma_{p2}$$

*Response to selection*

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## Prediction of response to selection

### A general formulation

X = trait selected

Y = trait measured

$$R = i \rho \sigma_{Ax}$$

$$\rho = 2 \cdot r \cdot r_{Ax,Y} \cdot h'_Y \cdot \sqrt{(n/[1+(n-1)t])}$$

Ollivier, 2002

$r$  = coancestry coefficient between candidate and ind. measured  
(OP progeny  $\rightarrow$  parent-offspring  $\rightarrow r=1/4$ )

$r_{Ax,Y}$  = genetic correlation between X and Y if different

$h'_Y$  = heritability of the selection criterion  
(ind. Values, progeny means)

$n$  = nb. of measures on the candidate (nb. offspring per parent)

$t$  = correlation between observations on the same candidate  
(OP progeny  $\rightarrow h^2 / 4$ )



# Response to selection with progeny testing

## *Forward selection*

$$\rho = 2 \cdot r \cdot r_{AX,Y} \cdot h'_Y \cdot \sqrt{n / [1 + (n-1)t]}$$

Ollivier, 2002

Response to selection

	<u>Response X</u>	<u>Response Y</u>
$r$	$1/4$	$1/4$
$r_{AX,Y}$	$1$	$r_{AX,Y}$
$h'_Y$	$h_X$	$h_X$
$n$	$n = \text{nb. of measures on the candidate (nb. offspring per parent)}$	
$t$	$h^2_X/4$	$h^2_X/4$



# Response to selection with progeny testing

## Forward selection

$$R_X = 0.5 i h_X \cdot \sqrt{(n/[1+(n-1)h_X^2/4])} \sigma_{Ax}$$

$$R_{Y/X} = 0.5 i r_{Ax,Y} h_X \cdot \sqrt{(n/[1+(n-1)h_X^2/4])} \sigma_{AY}$$

	<u>Response X</u> Total height	<u>Response Y</u> Branch angle
$h^2$	0.395	0.640
$\sigma^2_A = 4 * \sigma^2_{Fam}$	1647.44	0.464

$r_{Ax,Y} = -0.210$   
 $n = 30$   
 $i = 1.755$  (10%)

### Forward selection on X

$$R_X = 62.4 \text{ cm}$$

$$R_{Y/X} = -0.43$$

### Phenotypic selection on X

$$R_X = 44.8 \text{ cm}$$

$$R_{Y/X} = -0.16$$

Response to selection

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# Response to selection with progeny testing

## Forward selection

$$R = 2 h^2_f S_f = 2 i h^2_f \sigma_{Pf}$$

Selection on 2 parents (male and female)

$$h^2_f = \frac{\sigma^2_f}{\sigma^2_f + \sigma^2_R/n}$$

$$\sigma^2_{Pf} = \sigma^2_f + \sigma^2_R/n$$

Response X  
Total height

$h^2$  0.395

$\sigma^2_{Fam}$  411.86

$\sigma^2_R$  3753

$h^2_f = 0.767$

$\sigma^2_{Pf} = 537$

$R_x = 62.4$

Response to selection

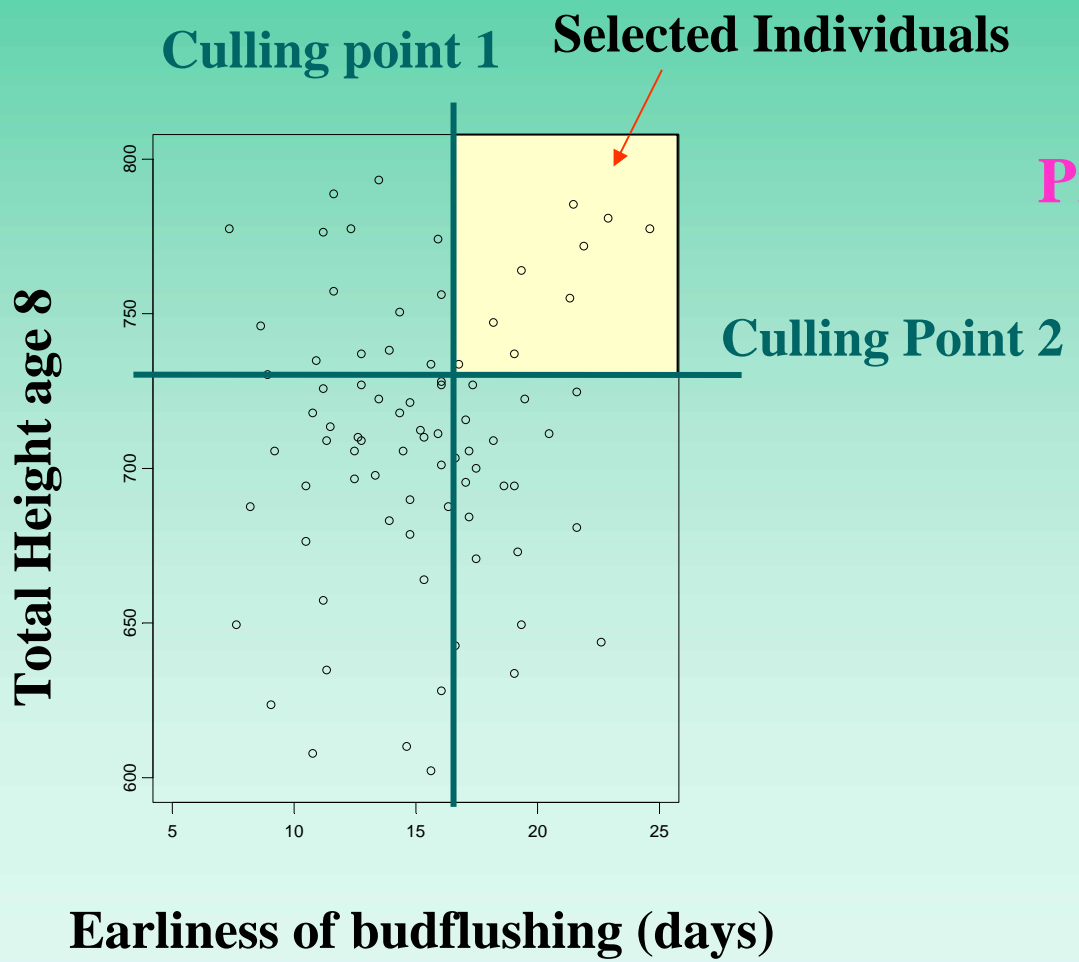
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Multitrait selection : Independent Culling vs. Index

Multi-trait selection

Independent Culling



Problem if unfavourable correlation

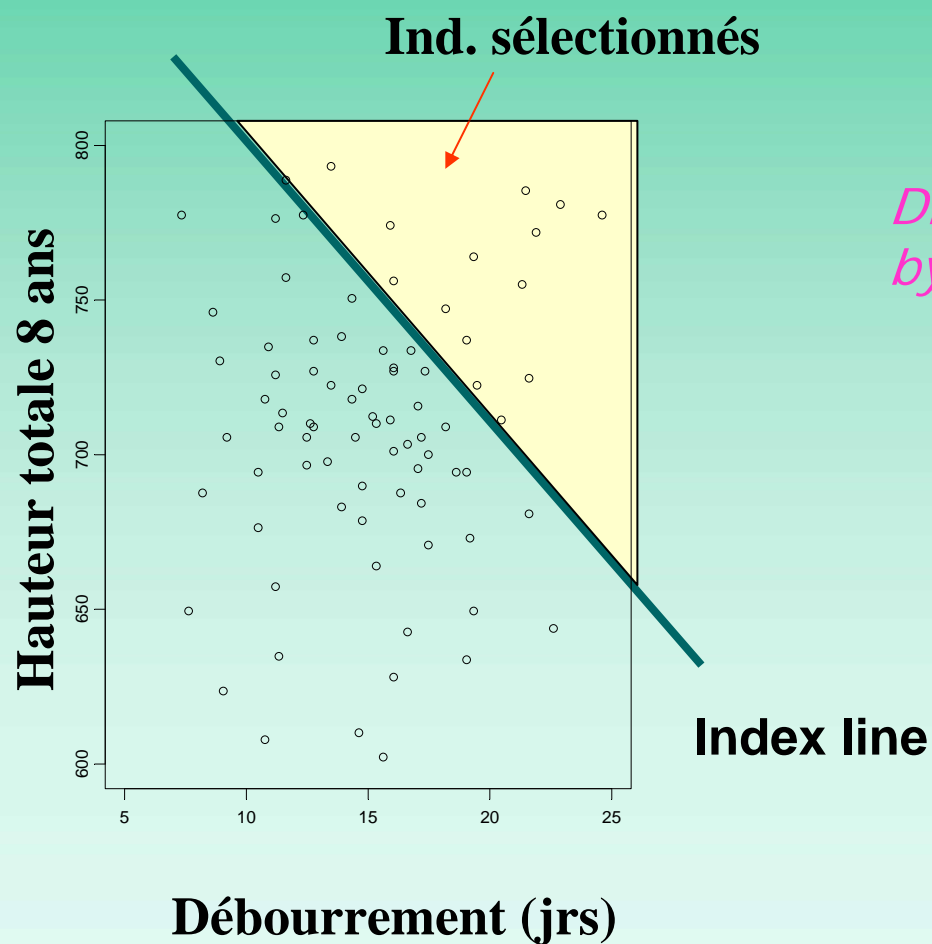




Multitrait selection : Independent Culling vs.  
Index

## Multi-trait selection Index Selection

$$I = w_1 A_1 + w_2 A_2 + \dots + w_q A_q$$



*Disadvantage in one trait off set  
by advantage in the other*



# Index Selection vs. Independent Culling

## Theoretical comparisons

If same total of nb. of individuals measured on all traits:

genetic gain

Index S > Independent Culling > Tandem selection

## Practical considerations

- **Index selection:**
  - must keep all individuals until all traits measured
  - cull in one stage
- Traits differ greatly in **costs** to measure
- Traits differ greatly in **age of evaluation**
- **Selection intensity** may be greater for multistage (culling) selection

# Index Selection

$$I = w_1 A_1 + w_2 A_2 + \dots w_q A_q = [w' A]$$

w = vector of technical or economical weights

$$I = b_1 P_1 + b_2 P_2 + \dots b_q P_q = [b' P]$$

b = vector of weights for phenotypic predictors

BLUP properties :  $A = M_P^{-1} M_A Z_{\text{centered}}$



$$b = [M_P^{-1} M_A w]$$

<u>Example</u>	$\sigma_P$	$h^2$	$r_A$	w	→	b
Wood density	0.4	0.3	0.5	<b>5</b>		<b>0.53</b>
Volume	0.2	0.5	0.5	<b>-1</b>		<b>-0.31</b>

In general, weights on phenotypic information sources are not easy to « recognize »

## Response to Index Selection

$$I = w_1 A_1 + w_2 A_2 + \dots w_q A_q = [w' A]$$

$w$  = vector of technical or economical weights

$$R = \begin{matrix} R_1 \\ R_2 \\ \cdot \\ R_k \end{matrix} = \frac{i w' M_A}{\sqrt{w' M_P w}}$$



# Response to Index Selection

## Forward selection

Example : HUMPTULIPS Population

$$I = w_1 \text{ BudFlush} + w_2 \text{ TH} + w_3 \text{ Ang} + w_4 \text{ Br} + w_5 \text{ Def}$$

Estimation of maximum relative genetic expected gains

W	BudFlush	TH	Ang	Br	Def
(-1,0,0,0,0)	<b>-55%</b>	8.9%	1.8%	12%	-19%
(0,1,0,0,0)	-24.4%	<b>20%</b>	-0.5%	-0.4%	9.9%
(0,0,1,0,0)	-4.4%	-0.4%	<b>22.5%</b>	11.0%	-3.9%
(0,0,0,1,0)	-32.2%	-0.4%	12.1%	<b>20.5%</b>	-16.3%
(0,0,0,0,-1)	-34%	-6.4%	2.8%	10.8%	<b>-31%</b>

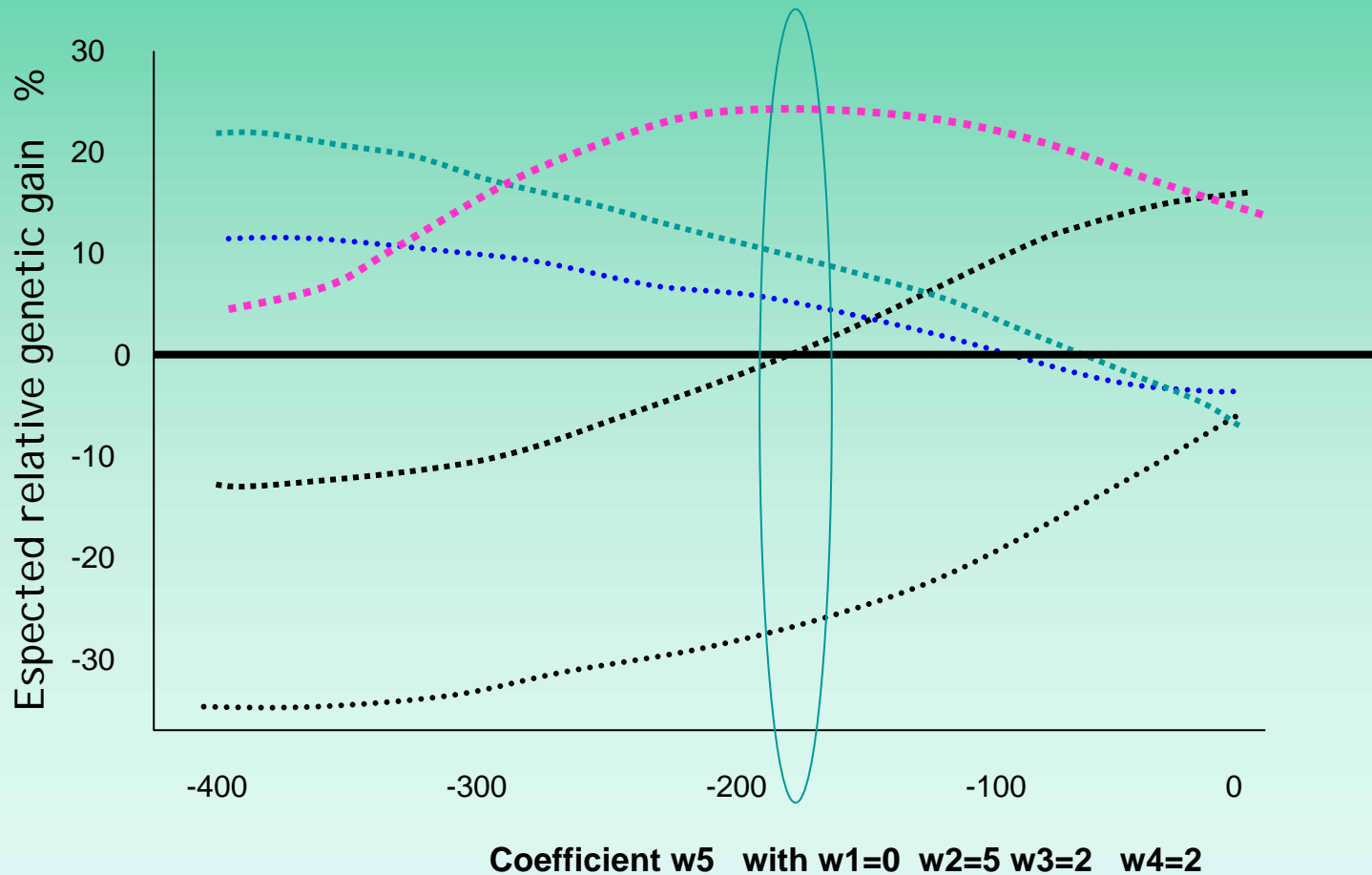


# Response to Index Selection

Simulation of expected genetic gains with varying w

Example : HUMPTULIPS Population

$$I = w_1 \text{ BudFlush} + w_2 \text{ TH} + w_3 \text{ Ang} + w_4 \text{ Br} + w_5 \text{ Def}$$







# Response to Index Selection

## Forward selection

$$I = w_1 A_1 + w_2 A_2 + \dots + w_q A_q = [w' A]$$

w = vector of technical or economical weights

$$\text{BLUP : } A = \frac{1}{2} M_A M_{Pf}^{-1} P$$

$$I = \frac{1}{2} w' M_A M_{Pf}^{-1} P$$

$$R = \frac{2i w' M_{Fam}}{\sqrt{w' M_{PFam} w}}$$

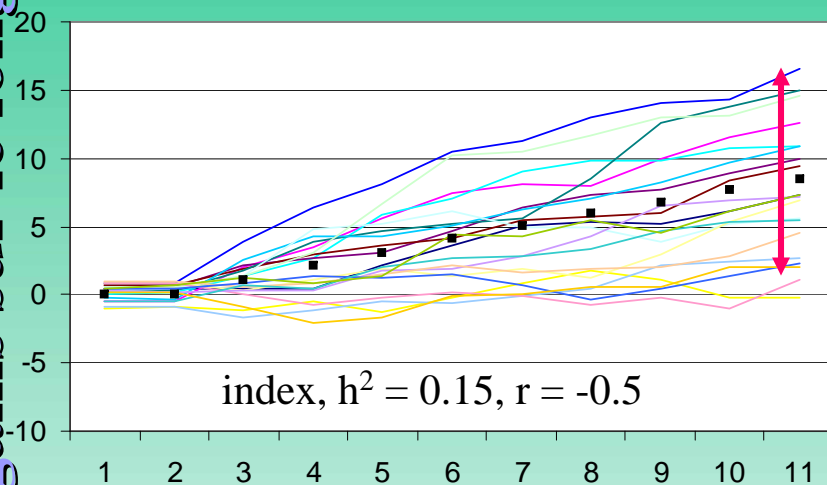
$$\sigma_{PFam}^2 = \sigma_{Fam}^2 + \sigma_R^2/n$$

	Total height	Branch angle
$M_{Fam} =$	411.86	-1.451
	-1.451	0.116

	Total height	Branch angle
$M_{PFam} =$	537	-1.528
	-1.528	0.136

## Sélection multi-caractères et Liaisons génétiques défavorables

*Quels sont les effets?*



- Augmentation de la variation du progrès génétique (imprévisibilité)
- Perte du mérite général

*Comment diminuer ces effets?*

- Choix de la méthode de sélection

