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ROBUST INTERVAL-BASED SISO REGULATION OF AN ANAEROBIC REACTOR

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Abstract : A robust regulation law is proposed for the stabilization of an anaerobic digester for the treatment of organic highly loaded wastewater. This process exhibits a highly nonlinear dynamic behavior. In addition, it must work under an uncertain environment with the presence of unknown inputs. Supporting some structural and operational conditions, this regulation law exponentially stabilizes the regulated variable around its nominal value in the presence of uncertainties and input disturbances. Simulations are carried out handling operational conditions close to those used in a real plant. *Copyright © 2000 IFAC*

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1. INTRODUCTION

A normally common situation in the wastewater treatment field is the partial knowledge - and even sometimes the total lack of knowledge - about the parameters and functions involved in the nonlinear reaction rates. The anaerobic digestion process is a multistep biological process in which organic matter is degraded into a gas mixture of methane (CH_4) and carbon dioxide (CO_2) involving complex ecosystems. This process reduces the Chemical Oxygen Demand (COD) of the influent while producing valuable energy (*i.e.*, methane). This biological process is a highly nonlinear time varying system in which kinetic parameters are badly or poorly known.

One of the most important problem when dealing with wastewater treatment plants is the lack of sensors. As a consequence, the systems are usually not observable nor detectable. Part of the process input vector is thus considered as unmeasured input disturbances and classical observer schemes (see for example [Bastin and Dochain, 1990; Gauthier and Kupka, 1994]) can usually not be used. However, using new observers (called *interval observers*) (see for instance [Rapaport and Harmand, 1998ª, 1998ᵇ]), it is possible to reconstruct a guaranteed interval on the unmeasured states instead of reconstructing their precise numerical values.

A recent control design approach, first introduced by [Rapaport, 1998], adopts the differential game theory [Bernhard and Rapaport, 1996] to design a "robust controlled Lyapunov function" that is capable of coping with this conjunction of lack of observability and requirement of robustness (*i.e.*, find a control law that guarantees the regulation whatever the uncertainties are). Based upon the work of [Rapaport, 1998], [Rapaport and Harmand, 1998ª, 1998ᵇ], and depending on the control objectives, the general SISO robust stabilization of an anaerobic digester for the treatment of wastewater is addressed in this paper.

The paper is organized as follows. Firstly, the general nonlinear model considered in this study is described. Secondly, a dynamical nonlinear interval observer is introduced assuming that the nonlinearities are partially known and input disturbances as well as initial conditions are unknown but belong to a prescribed bounded set. Then, the partial information provided by this interval observer is used to synthesize a robust regulation law that exponentially stabilizes the regulated variable about its set point. It should be pointed out that it is possible to asymptotically stabilize the system around a neighborhood of the reference value without having to reconstruct the controlled variable and ignoring their dynamics (see [Allgower *et al.*, 1997], [Alvarez, 1994] and [Alvarez *et al.*, 1996]). However, in the approach proposed hereafter, we use as much as possible the partial knowledge about the structure of the dynamics and its uncertainty to design a control law that has a smaller – and so a more reasonable – magnitude that guarantees exponential convergence. Simulation results using operational conditions close to those used on a real anaerobic digestion plant are provided before some conclusions and perspectives are drawn.

2. THE CONSIDERED MODEL

The following general nonlinear time varying lumped model is considered :

$$\dot{x}(t)=CK(t)f(x(t),t)+A(t)x(t)+b(t) \tag{1}$$

where $x(t)=[x_i(t)] \in \Re^n$ is the state vector (*i.e.*, concentrations in the case of mass balance models, concentrations and temperatures in the case of energy balances), $f(x(t),t) \in \Re^r$ denotes the vector of nonlinearites (including reaction rates) $C \in \Re^{n\times r}$ and $K(t) \in \Re^{r\times r}$ represents matrices of coefficients (*e.g.* stoichiometric, yield or kinetic coefficients) where C is constant and $K(t)$ is possibly time varying. The time varying matrix $A(t) \in \Re^{n\times n}$ explicits the linear dependence between the state variables while $b(t) \in \Re^n$ belongs to a vector gathering the inputs (*e.g.*, mass and/or energy feeding rate vector) and/or other possibly time varying functions (*e.g.*, the gaseous outflow rate vector if any).

Notice that the structure of the model (1) can be used by a large number of chemical and biochemical processes.

The following hypotheses are introduced :

Hypotheses H1 :
a) $A(t)$ is known for each $t \geq 0$.
b) m states are measured on-line and one of them is the state that one wants to regulate.
c) C is known.
d) $A(t)$ is bounded, that is, there exist matrices A^- and A^+ such as $A_{ij}^- \leq A_{ij}(t) \leq A_{ij}^+$.
e) Guaranteed bounds on the initial conditions of the state vector are known with $x_i^-(0) \leq x(0) \leq x_i^+(0)$.
f) Guaranteed bounds on the unknown inputs are given as $b^-(t) \leq b(t) \leq b^+(t)$.
g) Guaranteed bounds on the unknown matrix K are given as $K_{ij}^-(t) \leq K_{ij}(t) \leq K_{ij}^+(t)\, \forall i,j$ (*e.g.*, uncertainty on the kinetic parameters).

Note : The operator $\leq$ applied between vectors and between matrices should be understood as a collection of inequalities between components.

Using the hypotheses *H1b* and *H1c*, it is assumed that the state space can be split in such a way that (1) can be rewritten as :

$$\begin{aligned}\dot{x}_1(t)&=C_1K(t)f(x(t),t)+A_{11}(t)x_1(t)\\&\quad+A_{12}(t)x_2(t)+b_1(t)\end{aligned} \tag{2a}$$

$$\begin{aligned}\dot{x}_2(t)&=C_2K(t)f(x(t),t)+A_{21}(t)x_1(t)\\&\quad+A_{22}(t)x_2(t)+b_2(t)\end{aligned} \tag{2b}$$

where the m measured states have been grouped in the $x_2(t)$ vector and the variables that have to be estimated are represented by $x_1(t)$. Denoting $dim\ x_1(t)=s$, notice that $m=dim x_2(t)=n-s$. Matrices $A_{11}(t) \in \Re^{s\times s}$, $A_{12}(t) \in \Re^{s\times m}$, $A_{21}(t) \in \Re^{m\times s}$, $A_{22}(t) \in \Re^{m\times m}$, $C_1 \in \Re^{s\times r}$, and $C_2 \in \Re^{m\times r}$ are the corresponding partitions of $A(t)$ and C respectively.

3. THE INTERVAL OBSERVER

When dealing with interval observers, first introduced by [Rapaport, 1998], it is established that a necessary condition for designing such interval observers is that an observer – in fact any observer that can be derived if $b(t)$ is known – exists. If it exists and if $b(t)$ is unknown (only lower and upper bounds are known), the structure of this observer can be used to build an interval observer. Therefore, the following hypothesis is introduced :

Hypotheses H2 [Alcaraz *et al.*, 2000[a]].:
Under hypotheses *H1a-H1d*, when the vector $b(t)$ is known, there exists a matrix N such that the following system :

$$\begin{cases}\dot{\hat{w}}(t)=W(t)\hat{w}(t)+X(t)x_2(t)+Nb(t)\\ \hat{w}(0)=N\hat{x}(0)\\ \hat{x}_1(t)=N_1^{-1}(\hat{w}(t)-N_2x_2(t))\end{cases} \tag{3}$$

where $N=[N_1 \vdots N_2]$, $W(t)=(N_1A_{11}(t)+N_2A_{21}(t))N_1^{-1}$, $X(t)=N_1A_{12}(t)+N_2A_{22}(t)-W(t)N_2$, $N_2=-N_1C_1C_2^{-1}$ and $N_1 \in \Re^{s\times s}$ is an invertible matrix, is an asymptotic nonlinear observer for the nonlinear time varying model (1), (*i.e.*, $\hat{x}_1(t)$ converges asymptotically towards $x_1(t)$ for any initial conditions).

Remark 1 : Until now, besides the existence of N_1^{-1}, no other restriction on N_1 has been introduced. However, without loss of generality, it will be assumed for simplicity that $N_1=\pi I_s$ where π is an arbitrary, real and positive constant parameter.

Now, let the hypotheses *H1f-g* be verified. In other words, in one hand, some bounds are now available on the initial conditions and, in the other hand, the vector $b(t)$ is considered in the following as unmeasured, but some lower and upper bounds – possibly varying with time – are known. In such a situation, notice that the model (1) can be no longer detectable. Consequently, it is not possible to design an asymptotic observer like (3). Nevertheless, its basic exponentially stable structure can be used. The idea developed in the following is to design a set-valued observer in order to build guaranteed intervals for the unmeasured variables instead of estimating them precisely. First, let us introduce the following hypothesis :

Hypothesis H3 : $W_{e.ij}(t)\geq 0, \forall i\neq j$ where $W_e(t)=N_1W(t)N_1^{-1}$.

Now, the following result is recalled :

Lemma 1 [Smith, 1995] :

Let $\dot{\zeta}=f(\zeta,t)$. This system is said to be a *cooperative system* if $\frac{\partial f_i}{\partial \zeta_j}(\zeta,t)\geq 0, \forall i\neq j$. It implies that if $\zeta(0)\geq 0$, then $\zeta(t)\geq 0,\ \forall t\geq 0$. In addition, it is known that cooperative systems generate a monotone semiflow in the forward time direction.

Thus, with reference to the previous lemma, the hypothesis *H3* guarantees the cooperativity for the system under interest. Therefore, under hypotheses *H1-H3*, the following interval observer guarantees that $x_1^-(t)\leq x_1(t)\leq x_1^+(t), \forall t\geq 0$ given that $x^-(0)\leq x(0)\leq x^+(0)$ [Alcaraz *et al.*, 2000[b]] (for the upper and for the lower bounds respectively) :

$$\begin{cases}\dot{w}^+(t)=W(t)w^+(t)+X(t)x_2(t)+Mz^+(t)\\ w^+(0)=Nx^+(0)\\ \hat{x}_1^+(t)=N_1^{-1}(w^+(t)-N_2x_2(t))\end{cases} \tag{4a}$$

$$\begin{cases}\dot{w}^-(t)=W(t)w^-(t)+X(t)x_2(t)+Mz^-(t)\\ w^-(0)=Nx^-(0)\\ \hat{x}_1^-(t)=N_1^{-1}(w^-(t)-N_2x_2(t))\end{cases} \tag{4b}$$

with

$$z^+(t)=\left[b_1^+(t)\ \ \frac{1}{2}(b_2^+(t)+b_2^-(t))\ \ \frac{1}{2}(b_2^+(t)-b_2^-(t))\right]^T,$$

$$z^-(t)=\left[b_1^-(t)\ \ \frac{1}{2}(b_2^+(t)+b_2^-(t))\ \ \frac{-1}{2}(b_2^+(t)-b_2^-(t))\right]^T$$

$$.M=\left[N_1 \vdots N_2 \vdots \tilde{N}_2\right],\ \tilde{N}_2=\left[|N_{2.ij}|\right],$$

4. ROBUST FEEDBACK REGULATION

This section is devoted to develop the main idea of this study. The goal is to regulate one of the measured states around a certain set point. Notice that, in a first regard, the hypotheses *H1* about the model suggest a highly uncertain environment. Furthermore, since the system is non-detectable, only the guaranteed bounds provided by the set-valued observer (3) on the unmeasured state are available. However, under additional hypotheses stated hereafter, even using this partial and uncertain information, it is possible to guarantee the feedback regulation of the state variable under interest. First let us introduce the following notation :

Notation :

$y(t)=x_{2.p}(t)$ is the regulated variable where $x_{2.p}(t)$ is the p^{th} element of $x_2(t)$ that it is desired to regulate. The vector $q(t)$ is the p^{th} row of $A_{22}(t)$ and thus $q_p(t)$ is the p^{th} element of $q(t)$. The vector c is the p^{th} row of C. $\bar{c}=\left[|c_i|\right]\ \ \forall i$ and $b_{2.p}(t)$ is the p^{th} element of $b_2(t)$. y_n is the set point around of which is desired to regulate $y(t)$. $u(t)=|q_p(t)|$. $g(K(t),x_1(t),x_2(t),t)=K(t)f(x(t),t)$. The symbol σ denotes "+" or "-". $\beta=-q_p(t)/|q_p(t)|$.

Now the following hypotheses are introduced :

Hypotheses H4 :

a. $A_{21}=0$,

b. $q_i(t)=0\quad \forall i\neq p$

c. There exists a function $h(t)$ not depending on $q_i(t)\,\forall i$ such that $b_{2.p}(t)=-u(t)h(t)$.

d. At least for some l and j $\forall t$, g verifies $g(K(t),x_1(t),x_2(t),t)\geq 0$ and $g_{lj}(K(t),x_1(t),x_2(t),t)>0$,

e. $g(K(t),x_1(t),x_2(t),t)$ is monotonic with respect to each component of x.

Remark 2 : Notice that the hypotheses *H4a-b* imply that only one term of $q(t)$ has one effect on $y(t)$. From the hypotheses *H1f* and *H4c*, guaranteed bounds on the unknown function $h(t)$ are given as $h^-(t)\leq h(t)\leq h^+(t)$. The hypothesis *H4d* means that at least one component of $g(K(t),x_1(t),x_2(t),t)$ is positive and the rest can be positive or zero.

Then, from the model (1), and under the hypotheses *H4a-c*, $y(t)$ has the following dynamics :

$$\dot{y}(t)=cg(K(t),x_1(t),x_2(t),t)-u(t)(h(t)+\beta y(t)) \tag{5}$$

Two cases may arise in practice : either the desired βy_n is below $h(t)$ for any time (i.e., the reactor operates in a consumption mode) or the desired βy_n is above $h(t)$ for any time (i.e., the reactor operates in a production mode). Then, the following hypotheses are introduced :

Hypotheses H5 :

(case a : consumption mode)

$$\begin{Bmatrix}\text{a) } c_i\leq 0\quad \forall i \quad \text{and at least one}\quad c_i<0\\ \text{b) } \min\limits_{\sigma}(h^\sigma(t)+\beta y(0))>0\quad \forall t\\ \text{c) } \min\limits_{\sigma}(h^\sigma(t)+\beta y_n)>0\quad \forall t\end{Bmatrix}$$

or

(case b : production mode)

$$\begin{cases} \text{a) } c_i \geq 0 \quad \forall i \quad \text{and at least one} \quad c_i > 0 \\ \text{b) } \max_\sigma\left(h^\sigma(t) + \beta y(t)\right) < 0 \quad \forall t \\ \text{c) } \max_\sigma\left(h^\sigma(t) + \beta y_n\right) < 0 \quad \forall t \end{cases}$$

This hypothesis means that when the dilution rate equals zero, the output y is always decreasing (case a) or always increasing (case b). As a consequence, the constraint $D \geq 0$ is feasible. Notice again that a bang-bang or a proportional control law could give satisfactory regulation results without using any estimator (see [Alvarez *et al.*, 1996]). However, the use of an interval observer hereafter will allow a smaller range of variation for the dilution rate. In addition, let us suppose that the output feedback law proposed hereafter must be higher than a certain nonnegative value δ, (i.e., because of physical constraints, this is a possibly necessary condition for guarantying the hypothesis *H2* ; see more details in [Alcaraz *et al.*, 2000[a]]). Therefore, using the guaranteed dynamic interval $\{x_1^- \; x_1^+\}$ of the previous section and the guaranteed interval information given by hypotheses *H1e-g*, the following proposition gives an output feedback law forcing $y(t)$ to converge exponentially towards y_n.

Proposition 1 :

If the dynamics of y can be represented by (5) (thanks to hypotheses *H4*) and if the hypotheses *H5a* or *H5b* are verified, then the following output feedback law :

$$u^*(t) = \frac{\rho^*(t) - \lambda(y(t) - y_n)}{\psi^*(t)} \tag{6}$$

with

$$\rho^*(t) = \begin{cases} \min_{\sigma',\sigma'}\left(\bar{c}g\left(\left[K_{ij}^{\sigma'}(t)\right], \left[x_{1,i}^{\sigma'}(t)\right], x_2(t), t\right)\right) & \text{if } y(t) > y_n \\ \max_{\sigma',\sigma'}\left(\bar{c}g\left(\left[K_{ij}^{\sigma'}(t)\right], \left[x_{1,i}^{\sigma'}(t)\right], x_2(t), t\right)\right) & \text{if } y(t) < y_n \end{cases}$$

$$\psi^*(t) = \begin{cases} \max_\sigma \left|h^\sigma(t) + \beta y(t)\right| & \text{if } y(t) > y_n \\ \min_\sigma \left|h^\sigma(t) + \beta y(t)\right| & \text{if } y(t) < y_n \end{cases}$$

$$0 < \lambda \leq \frac{\min_\sigma \left(\bar{c}g(\bullet^\sigma(0), y(0)), \;\; \bar{c}g(\bullet^\sigma(0), y_n)\right) - \delta}{\max_\sigma \left(\left|h^\sigma(0) + \beta y(0)\right|, \;\; \left|h^\sigma(0) + \beta y_n\right|\right)}$$

$$0 < \delta < \min_\sigma \left(zg(\bullet^\sigma(0), y(0)), \;\; zg(\bullet^\sigma(0), y_n)\right)$$

exponentially stabilizes $y(t)$ towards y_n with a guaranteed decay rate and it is ensured that $u(t) > \delta > 0$ where the symbol $\bullet(t)$ stands for "the rest of the arguments".

Proof : Fix any constant $\lambda > 0$ sufficiently small and consider the candidate partial Lyapunov function $V(y(t)) = (y(t)-y_n)^2/2$. Its time derivative is :

$$\dot{V} = (y(t) - y_n)\left(cg(K(t), x_1(t), x_2(t), t) - u^*(t)(h(t) + \beta y(t))\right)$$

Under hypotheses *H5* it is possible to check the following :

$$\dot{V} \leq (y(t) - y_n)\begin{pmatrix} \rho^*(t) - \lambda(y(t) - y_n) \\ - cg(K(t), x_1(t), x_2(t), t) \end{pmatrix} \tag{7a}$$

$$\leq -\lambda(y(t) - y_n)^2 \tag{7b}$$

So, $y(t)$ to converge exponentially towards y_n for any constant $\lambda > 0$ sufficiently small. Now, under hypotheses *H4d-e* and regarding (7) it is easy to verify that $\lambda > 0$ and $u(t) > \delta > 0$. Thus the claimed result is proved. ■

Remark 3 : The proposed formula (6) for $u^*(t)$ is not continuous at $y(t)=y_n$ and so, depending on the disturbances $K_{ij}(t)$ and $h(t)$, may leads to a "chattering" control along the arc $y(t)=y_n$ between the two values $u^{*1}(t)$ (expression of $u^*(t)$ when $y(t)>y_n$) and $u^{*2}(t)$ (expression of $u^*(t)$ when $y(t) < y_n$). Nevertheless, one can modify this formula to obtain a continuous feedback that "practically" stabilizes the system. For any $\varepsilon > 0$, the output feedback law :

$$u_\varepsilon^*(t) = \begin{cases} u^{*1}(t) \text{if } y(t) > y_n + \varepsilon \\ u^{*1}(t)\dfrac{(y(t) - y_n + \varepsilon)}{2\varepsilon} \\ + u^{*2}(t)\dfrac{(y_n + \varepsilon - y(t))}{2\varepsilon} \text{if } |y(t) - y_n| \leq \varepsilon \\ u^{*2}(t) \text{if } y(t) < y_n - \varepsilon \end{cases} \tag{8}$$

stabilizes the system between $(y_n-\varepsilon)$ and $(y_n+\varepsilon)$.

5. EXAMPLE : APPLICATION TO A WASTEWATER TREATMENT PROCESS

5.1 - The anaerobic digester model

In the following, a model of an anaerobic digestion process carried out in a continuous fixed bed reactor for the treatment of industrial wine distillery vinasses is considered [Bernard *et al.*, 1998] :

$$
\begin{cases}
\dot{X}_1 = (\mu_{max_1}\mu_1 - \alpha D)X_1 \\
\dot{X}_2 = (\mu_0\mu_2 - \alpha D)X_2 \\
\dot{Z} = D(Z^i - Z) \\
\dot{S}_1 = D(S_1^i - S_1) - k_1\mu_{max_1}\mu_1 X_1 \\
\dot{S}_2 = D(S_2^i - S_2) + k_2\mu_{max_1}\mu_1 X_1 - k_3\mu_0\mu_2 X_2 \\
\dot{C}_{TI} = D(C_{TI}^i - C_{TI}) + k_7(k_8 P_{CO_2} + Z - C_{TI} - S_2) + k_4\mu_{max_1}\mu_1 X_1 + \mu_0 k_5 \mu_2 X_2
\end{cases} \tag{9}
$$

where X_1, X_2, Z, S_1, S_2 and C_{TI} are respectively the concentrations of acidogenic bacteria, methanogenic bacteria, strong ions, chemical oxygen demand, volatile fatty acids and total inorganic carbon and they are supposed to be positive for any time. The parameter α represents a proportionality parameter of experimental determination. In all cases, the upper index i indicates "influent concentration". The variable $D=D(t)\geq 0$ is the dilution rate and is supposed to be a persisting input, *i.e.*, $\int_0^\infty D(\tau)d\tau > 0$.

Detailed definition of the different functions, parameters and their values can be found in [Bernard *et al.*, 1998]. The nonlinear interval observer and the robust feedback law developed in previous sections is then applied to the dynamic process model (9) defining the state vector $\xi_1 = X_1$; $\xi_2 = X_2$, $\xi_3 = C_{TI}$, $\xi_4 = Z$; $\xi_5 = S_1$, $\xi_6 = S_2$.

The model (9) can easily be written under the following form with appropriate matrix definition :

$$
\dot{\xi} = CKf(\xi(t),t) + A(t)\xi(t) + b(t) \tag{10}
$$

The observer (3) has been already successfully applied for the process model (9) in [Alcaraz *et al.*, 1999, 2000[a-b]]. Indeed, by using D, P_{CO_2} and the two substrate concentrations S_1, S_2, as measurements, an interval observer can easily be derived to estimate guaranteed intervals on X_1, X_2, C_{TI} and Z. Now the robust regulation method depicted in the previous section will be applied to regulate S_1 around $S_{1n} < S_1^{i-}$ with $S_1^{i-} > S_1(0)$ and $S_1(0) > S_{1n}$. It is straightforward to verify that the hypotheses *H4-5* are completely fulfilled. Then, in agreement with the proposition 1, the following regulation law :

$$
D(t) \equiv D^*(t) = \frac{k_1\mu_{max_1}^* \mu_1(S_1(t))X_1^*(t) - \lambda(S_1(t) - S_{1n})}{S_1^{i*}(t) - S_1(t)} \tag{11}
$$

$$
\left(\mu_{max_1}^*, X_1^*, S_1^{i*}(t)\right) = \begin{cases} \left(\mu_{max_1}^-, X_1^-, S_1^{i+}(t)\right) & \text{if } S_1(t) > S_{1n} \\ \left(\mu_{max_1}^+, X_1^+, S_1^{i-}(t)\right) & \text{if } S_1(t) < S_{1n} \end{cases}
$$

exponentially stabilizes $S_1(t)$ around S_{1n} for any $\lambda > 0$ sufficiently small and, the following λ and δ:

$$
0 < \lambda \leq \frac{k_1\mu_{max_1}\mu_1(S_{1n})X_1^-(0) - \delta}{S_1^{i+}(0) - S_{1n}}, \tag{12}
$$

$$
\delta < k_1\mu_{max_1}\mu_1(S_{1n})X_1^-(0)
$$

ensure that $D(t) > \delta > 0$ $\forall t$.

5.2 Simulation results

Simulations were carried out using the parameter values reported in the Tables 1 for the model (9). They were carried out over a 100 days period at different dilution rates and at different input substrate concentrations and it was considered that input concentrations was unknown and only guaranteed intervals on these inputs was known. In order to add some realism to these simulations, small fluctuations as well as drastic step perturbations were alternatively introduced in the input concentrations : see Figures 1 (input S_2 and Z concentrations are not shown because of lack of space). The variables C_{TI}^i, X_1^i and X_2^i are supposed to be negligible in the model (9). The "real input concentrations" shown in these graphics were only used to simulate the model (9) from which the measurements S_1 and S_2 and the partial CO_2 pressure P_{CO_2} were taken directly. Uncertainties upon μ_{max_1} were taken as $1.125 \leq \mu_{max_1} \leq 1.375$. Estimation results for the unmeasured state X_1 are presented in the Figure 4. "Predictions of the model" values in this graphic were also directly obtained from the model. In agreement with (11)-(12), the dilution rate D was used for the regulation of S_1 about the nominal value S_{1n} = 0.5 g/l. The time evolution of D and S_1 are shown in Figures 3 and 4.

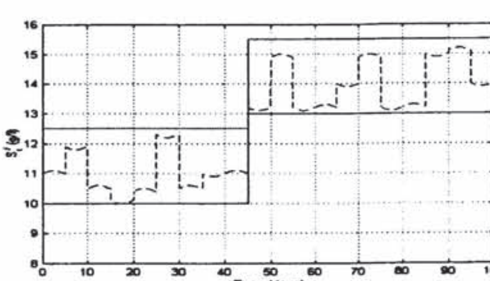

Figure 1 : Influent S_1 concentration
(– : upper and lower bounds, -- : input concentration).

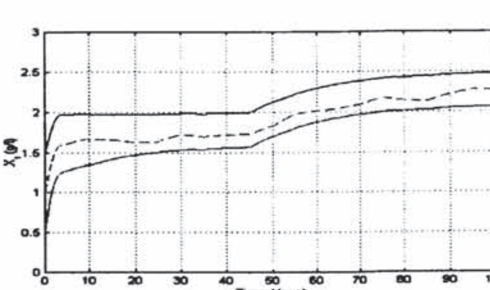

Figure 2 : Estimation of the acidogenic bacteria concentration
(– : upper and lower estimated states, -- : predictions of the model).

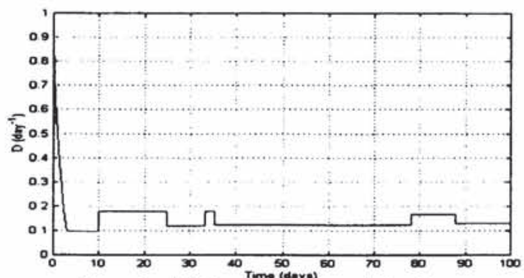

Figure 3 : Dilution rate (the regulation law).

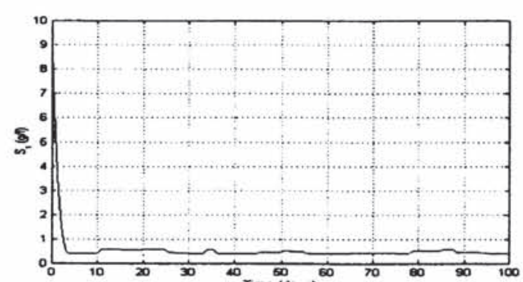

Figure 4 : Effluent concentration of S_1 (the regulated variable).

In Figures 3 and 4 it is shown how the dilution rate D operates quickly to drive the regulated variable S_1 towards the nominal value S_{1n} = 0.5 g/l. Also, in these Figures it is shown how, once the regulation goal is achieved, the dilution rate D performs adequately to keep S_1 around its nominal value despite the highly uncertain environment (*e.g.*, fluctuations and drastic step perturbations on the unknown input concentrations, uncertainties on μ_{max_i} and uncertainties on X_1).

6. CONCLUSIONS AND PERSPECTIVES

In this paper, a robust set-valued SISO regulation law has been proposed for an anaerobic digester for the wastewater treatment whose behavior is described by a highly nonlinear dynamic system. Simulations were carried out handling operational conditions close to those used in a real plant. This regulation law presented an excellent performance keeping the regulated variable towards its nominal value even under a highly uncertain environment (*e.g.*, fluctuations and drastic step perturbations on the unknown input concentrations, uncertainties on the kinetic parameters and uncertainties on state variables that play in the kinetic rates). Logical extensions of this approach, now under study, are the SIMO and MIMO cases based upon the same philosophy. Because of the large interest of this approach at the experimental scale, the authors are actually working in the experimental validation on the aforementioned real plant.

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