Reduced Models (and control) of in-situ decontamination of large water resources
Antoine Rousseau, Alain Rapaport

To cite this version:

Antoine Rousseau, Alain Rapaport. Reduced Models (and control) of in-situ decontamination of large water resources. Sino-French Conference on Modeling, Mathematical Analysis and Computation, Chuanju Xu; Alain Miranville, Jun 2017, Xiamen, China. pp.27. hal-02785037

HAL Id: hal-02785037
https://hal.inrae.fr/hal-02785037
Submitted on 4 Jun 2020

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Distributed under a Creative Commons Attribution - ShareAlike 4.0 International License
Reduced Models (and control) of in-situ decontamination of large water resources

Antoine ROUSSEAU,
Inria, Team LEMON (Montpellier)

In honor of my "brother in sciences" Claudius

June 9th, 2017, SFC2MAC, Xiamen

Joint work with Alain Rapaport (INRA)
Outline

1. Introduction: the Taihu problem
2. A simple ODE model
3. A PDE-based model for the lake
4. Back to ODEs
Outline

1. Introduction: the Taihu problem
2. A simple ODE model
3. A PDE-based model for the lake
4. Back to ODEs
Applicative Framework

Polluted Taihu (algae), Yangtze Delta plain, Wuxi, China.  
Courtesy Desert Research Institute

Objectives :
- use a bioreactor to remove pollution from the lake
- do it as efficiently as possible
Bioremediation problem

Parameters and unknowns

- $V_L$ and $V_R$ the volumes of the resource and bioreactor,
- $X_R$ the biomass concentration in the bioreactor,
- $S_L$ and $S_R$ the pollutant concentrations,
- $\mu(\cdot)$ the biomass growth law,
- $Q = Q(t)$ the pump discharge, controlled by the user.
Outline

1. Introduction: the Taihu problem
2. A simple ODE model
3. A PDE-based model for the lake
4. Back to ODEs
ODE-based model: the homogeneous case \( S_L = S_L(t) \)

\[
\begin{align*}
\dot{X}_R &= \mu(S_R) X_R - \frac{Q}{V_R} X_R, \\
\dot{S}_R &= -\mu(S_R) X_R + \frac{Q}{V_R} (S_L - S_R), \\
\dot{S}_L &= \frac{Q}{V_L} (S_R - S_L),
\end{align*}
\]
ODE-based model: the homogeneous case $S_L = S_L(t)$

$$
\begin{align*}
\dot{X}_R &= \mu(S_R) X_R - \frac{Q}{V_R} X_R, \\
\dot{S}_R &= -\mu(S_R) X_R + \frac{Q}{V_R} (S_L - S_R), \\
\dot{S}_L &= \frac{Q}{V_L} (S_R - S_L),
\end{align*}
$$

Chemostat equations: J. Monod (1910-1976)
ODE-based model: the homogeneous case \( S_L = S_L(t) \)

\[
\begin{align*}
\dot{X}_R &= \mu(S_R)X_R - \frac{Q}{V_R}X_R, \\
\dot{S}_R &= -\mu(S_R)X_R + \frac{Q}{V_R}(S_L - S_R), \\
\dot{S}_L &= \frac{Q}{V_L}(S^\infty_R - S_L),
\end{align*}
\]

Slow-fast approximation:

\[ \varepsilon = \frac{V_R}{V_L} \ll 1 \]

ODE-based model: the homogeneous case $S_L = S_L(t)$

\[
\begin{align*}
\dot{X}_R &= \mu(S_R) X_R - \frac{Q}{V_R} X_R, \\
\dot{S}_R &= -\mu(S_R) X_R + \frac{Q}{V_R} (S_L - S_R), \\
\dot{S}_L &= \frac{Q}{V_L} (S_R^\infty - S_L),
\end{align*}
\]

Slow-fast approximation:
\[\varepsilon = \frac{V_R}{V_L} \ll 1\]


**Theorem (case $\mu(X) = \mu X$)**

For any $X_R(0) > 0$ and $S_R(0) = S_L > 0$, our system has a unique global solution. In addition, there exists a critical flow rate $Q_c > 0$, depending on $S_L$, such that asymptotically:

- if $Q > Q_c$, then $(S_R(s), X_R(s))$ converges towards $(S_L, 0)$,
- if $0 < Q < Q_c$, then $(S_R(s), X_R(s))$ converges towards $(S_R^\infty, X_R^\infty)$ with $S_R^\infty = \frac{Q}{\mu V_R} < S_L$. 

Antoine ROUSSEAU
Decontamination of large water resources
SFC2MAC, Xiamen
Optimal flow rate for the bioremediation (linear $\mu$)

Back to the lake equations...

$$\dot{S}_L = \frac{Q}{V_L} \left( \frac{Q}{\mu V_R} - S_L \right),$$

What is the best flow rate $Q$?
Optimal flow rate for the bioremediation (linear $\mu$)

Back to the lake equations...

$$\dot{S}_L = \frac{Q}{V_L} \left( \frac{Q}{\mu V_R} - S_L \right),$$

What is the best flow rate $Q$?

$$Q(t) = \frac{\mu V_R}{2} S_L(t)$$
Optimal flow rate for the bioremediation (linear $\mu$)

Back to the lake equations...

$$\dot{S}_L = \frac{Q}{V_L} \left( \frac{Q}{\mu V_R} - S_L \right),$$

What is the best flow rate $Q$?

$$Q(t) = \frac{\mu V_R}{2} S_L(t)$$

![Graph showing concentration over time with a best constant $Q^*$ and $Q(t)$]
Outline

1. Introduction: the Taihu problem
2. A simple ODE model
3. A PDE-based model for the lake
4. Back to ODEs
The non-homogeneous case: \( S_L = S_L(t, x, y) \)

Navier-Stokes & transport equations in the resource

\[
\begin{align*}
\frac{\partial u}{\partial t} + u \cdot \nabla u - \nu_u \Delta u + \nabla p &= 0, \\
\nabla \cdot u &= 0, \\
\frac{\partial S_L}{\partial t} + u \cdot \nabla S_L - \nu_S \Delta S_L &= 0.
\end{align*}
\]

Boundary conditions

\[
\begin{align*}
\quad u_{in}(t) &= A \times Q(t), \\
u_{out}(t) &= A \times Q(t), \\
\text{B.C on } S_L.
\end{align*}
\]
Simulations

Circular lake : 1 pump
Comparing models

Pollution concentration in the lake with ODE (black) and PDE (red)
Outline

1. Introduction: the Taihu problem
2. A simple ODE model
3. A PDE-based model for the lake
4. Back to ODEs
Active and dead zones model

New ODE model

\[
\begin{align*}
\dot{X} &= \mu(S_R)X - \frac{Q}{V_R}X, \\
\dot{S}_R &= -\mu(S_R)X + \frac{Q}{V_R}(S_1 - S_R), \\
\dot{S}_1 &= \frac{Q}{V_1}(S_R - S_1) + \frac{d}{V_1}(S_2 - S_1), \\
\dot{S}_2 &= \frac{d}{V_2}(S_1 - S_2).
\end{align*}
\]
Active and dead zones model

New ODE model

\[\begin{align*}
\dot{X} &= \mu(S_R)X - \frac{Q}{V_R} X, \\
\dot{S}_R &= -\mu(S_R)X + \frac{Q}{V_R} (S_1 - S_R), \\
\dot{S}_1 &= \frac{Q}{V_1} (S_R - S_1) + \frac{d}{V_1} (S_2 - S_1), \\
\dot{S}_2 &= \frac{d}{V_2} (S_1 - S_2).
\end{align*}\]

Existence, uniqueness, optimal control for two zones


Two parameters we can play with : \(\frac{V_1}{V_1 + V_2}\) and \(d\).
Comparing models (with A. Rapaport and S. Barbier)

Optimization on the parameters $\frac{V_1}{V_1 + V_2}$ and $d$

Pollution concentration in the lake
1 ODE (black)
2 ODEs (blue)
PDE (red)

Best parameter $d$ vs viscosity $\nu_S$ (dashed blue)
Best meansquare fit (plain red)
In summary...

\[ d = 0, \ r = 1 \]

\[ V \gg V_R \]

H1

H2, H3
Conclusion

The “take home” message is...

- we have designed a first model and questioned it,
- we proposed a more complicated (too much ?) one,
- we took an intermediary model, which seems satisfactory, on which optimal control can be done.

BUT: is the reference (PDE) model a realistic model?
Certainly not...
Conclusion

The “take home” message is...

- we have designed a first model and questioned it,
- we proposed a more complicated (too much ?) one,
- we took an intermediary model, which seems satisfactory, on which optimal control can be done.

BUT : is the reference (PDE) model a realistic model ?
Conclusion

The “take home” message is...

- we have designed a first model and questioned it,
- we proposed a more complicated (too much?) one,
- we took an intermediary model, which seems satisfactory, on which optimal control can be done.

**BUT : is the reference (PDE) model a realistic model?**

Certainly not...

**Sino-French project ANSWER:**

Analysis and Numerical Simulation of Water Ecosystems in Response to anthropogenic environmental changes
Conclusion

The “take home” message is...

- we have designed a first model and questioned it,
- we proposed a more complicated (too much ?) one,
- we took an intermediary model, which seems satisfactory, on which optimal control can be done.

BUT : is the reference (PDE) model a realistic model ?
Certainly not...

**Sino-French project**

**ANSWER:**

Analysis and Numerical Simulation of Water Ecosystems in Response to anthropogenic environmental changes

*The more we fail, the more chances we have to succeed*
Thank you for your attention

A few references...

- P. Gajardo, J. Harmand, H. Ramírez, A. Rapaport and V. Riquelme
  Minimal time bioremediation of natural water resources. Automatica 2011

- P. Gajardo, H. Ramírez, A. Rapaport and V. Riquelme
  Bioremediation of natural water resources via optimal control techniques.
  BIOMAT 2012

- A. Rousseau
  Texte public de l’agrégation externe de mathématiques.

  Procédé de traitement d’une ressource fluide, programme d’ordinateur et module

- S. Barbier, A. Rapaport et A. Rousseau.
  Modelling of biological decontamination strategies of a water resource in natural

- A. Rousseau, A. Rapaport, A. Pacholik and C. Leininger
  Action Dépollution. Serious game. https://depollution.inria.fr/