Reduced Models (and control) of in-situ decontamination of large water resources

Antoine Rousseau, Alain Rapaport

Reduced Models (and control) of in-situ decontamination of large water resources

Antoine ROUSSEAU,
Inria, Team LEMON (Montpellier)

In honor of my ”brother in sciences” Claudius

June 9th, 2017, SFC2MAC, Xiamen

Joint work with Alain Rapaport (INRA)

Outline

Outline

Applicative Framework

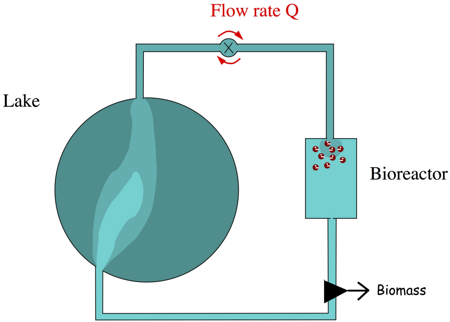
Polluted Taihu (algae), Yangtze Delta plain, Wuxi, China.

Courtesy Desert Research Institute

Objectives :

- use a bioreactor to remove pollution from the lake
- do it as efficiently as possible

Bioremediation problem

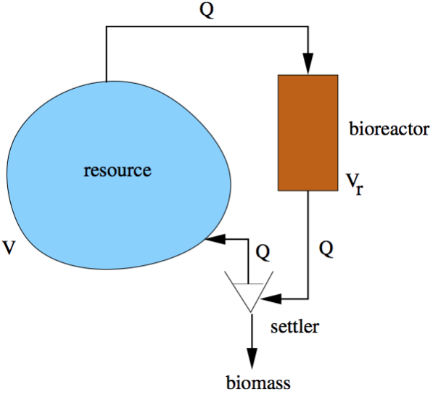


Parameters and unknowns

- V_L and V_R the volumes of the resource and bioreactor,
- X_R the biomass concentration in the bioreactor,
- S_L and S_R the pollutant concentrations,
- $\mu(\cdot)$ the biomass growth law,
- $Q = Q(t)$ the pump discharge, controlled by the user.

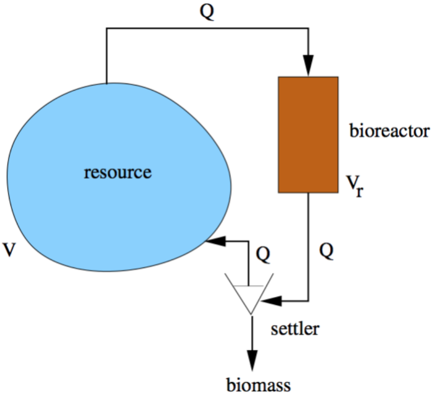
Outline

ODE-based model : the homogeneous case $S_L = S_L(t)$



$$\begin{cases} \dot{X}_R = \mu(S_R)\,X_R - \dfrac{Q}{V_R}\,X_R, \\ \dot{S}_R = -\mu(S_R)\,X_R + \dfrac{Q}{V_R}(S_L - S_R), \\ \dot{S}_L = \dfrac{Q}{V_L}(S_R - S_L), \end{cases}$$

ODE-based model : the homogeneous case $S_L = S_L(t)$



$$\begin{cases} \dot{X}_R = \mu(S_R)\,X_R - \dfrac{Q}{V_R}\,X_R, \\ \dot{S}_R = -\mu(S_R)\,X_R + \dfrac{Q}{V_R}(S_L - S_R), \\ \dot{S}_L = \dfrac{Q}{V_L}(S_R - S_L), \end{cases}$$

Chemostat equations :

J. Monod (1910-1976)

ODE-based model : the homogeneous case $S_L = S_L(t)$

$$\begin{cases} \dot{X}_R = \mu(S_R)\, X_R - \dfrac{Q}{V_R} X_R, \\ \dot{S}_R = -\mu(S_R)\, X_R + \dfrac{Q}{V_R}(S_L - S_R), \\ \dot{S}_L = \dfrac{Q}{V_L}(S_R^\infty - S_L), \end{cases}$$

Reference : Gajardo et al, Automatica 2011.

Slow-fast approximation :

$$\varepsilon = V_R/V_L \ll 1$$

ODE-based model : the homogeneous case $S_L = S_L(t)$

$$\begin{cases} \dot{X}_R = \mu(S_R)\,X_R - \dfrac{Q}{V_R}\,X_R, \\ \dot{S}_R = -\mu(S_R)\,X_R + \dfrac{Q}{V_R}(S_L - S_R), \\ \dot{S}_L = \dfrac{Q}{V_L}(S_R^\infty - S_L), \end{cases}$$

Slow-fast approximation :

$$\varepsilon = V_R/V_L \ll 1$$

Reference : Gajardo et al, Automatica 2011.

Theorem (case $\mu(X) = \mu X$)

For any $X_R(0) > 0$ and $S_R(0) = S_L > 0$, our system has a unique global solution. In addition, there exists a critical flow rate $Q_c > 0$, depending on S_L, such that asymptotically :

- if $Q > Q_c$, then $\big(S_R(s), X_R(s)\big)$ converges towards $(S_L, 0)$,
- if $0 < Q < Q_c$, then $\big(S_R(s), X_R(s)\big)$ converges towards (S_R^∞, X_R^∞) with $S_R^\infty = \dfrac{Q}{\mu V_R} < S_L$.

Optimal flow rate for the bioremediation (linear μ)

Back to the lake equations...

$$\boxed{\dot{S}_L = \frac{Q}{V_L}\left(\frac{Q}{\mu\, V_R} - S_L\right),}$$

What is the best flow rate Q?

Optimal flow rate for the bioremediation (linear μ)

Back to the lake equations...

$$\boxed{\dot{S_L} = \frac{Q}{V_L}\left(\frac{Q}{\mu\, V_R} - S_L\right),}$$

What is the best flow rate Q?

$$\boxed{Q(t) = \frac{\mu\, V_R}{2} S_L(t)}$$

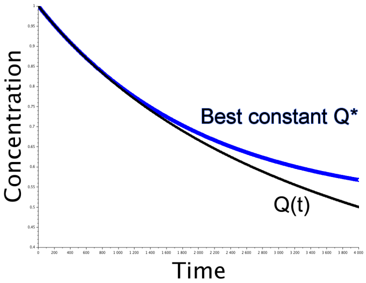
Optimal flow rate for the bioremediation (linear μ)

Back to the lake equations...

$$\boxed{\dot{S_L} = \frac{Q}{V_L}\left(\frac{Q}{\mu\, V_R} - S_L\right),}$$

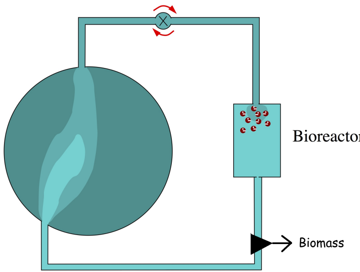
What is the best flow rate Q?

$$\boxed{Q(t) = \frac{\mu\, V_R}{2} S_L(t)}$$



Outline

The non-homogeneous case : $S_L = S_L(t,x,y)$



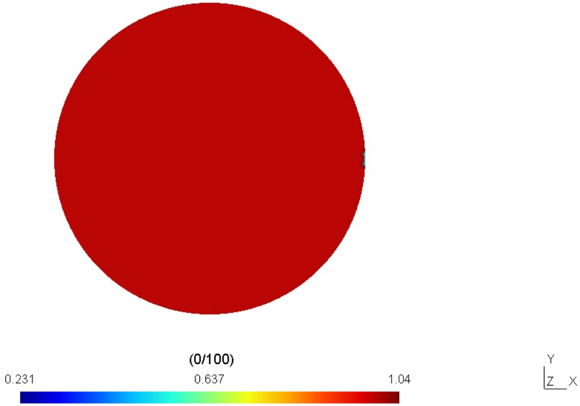
Navier Stokes & transport equations in the resource

$$\begin{cases} \dfrac{\partial u}{\partial t} + u \cdot \nabla u - \nu_u \Delta u + \nabla p &= 0, \\ \nabla \cdot u &= 0, \\ \dfrac{\partial S_L}{\partial t} + u \cdot \nabla S_L - \nu_S \Delta S_L &= 0. \end{cases}$$

Boundary conditions

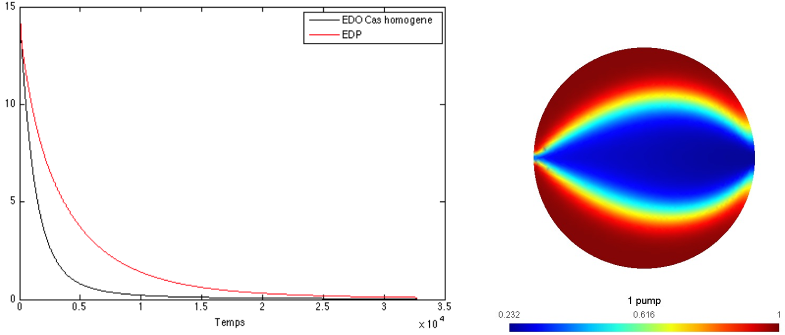
$$\begin{cases} u_{in}(t) = \mathcal{A} \times Q(t), \\ u_{out}(t) = \mathcal{A} \times Q(t), \\ \text{B.C on } S_L \end{cases}$$

Simulations



Circular lake : 1 pump

Comparing models



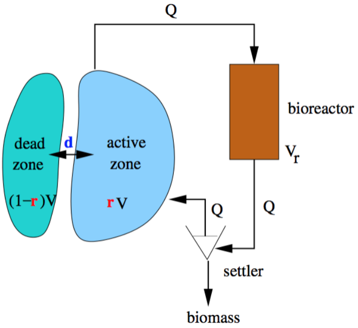
Pollution concentration in the lake
with ODE (black) and PDE (red)

Outline

Active and dead zones model

New ODE model

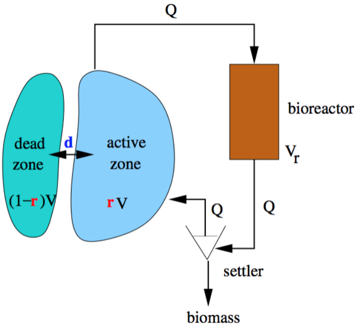
$$
\begin{aligned}
\dot{X} &= \mu(S_R)X - \frac{Q}{V_R}X, \\
\dot{S}_R &= -\mu(S_R)X + \frac{Q}{V_R}(S_1 - S_R), \\
\dot{S}_1 &= \frac{Q}{V_1}(S_R - S_1) + \frac{d}{V_1}(S_2 - S_1), \\
\dot{S}_2 &= \frac{d}{V_2}(S_1 - S_2).
\end{aligned}
$$



Active and dead zones model

New ODE model

$$
\begin{aligned}
\dot{X} &= \mu(S_R)X - \frac{Q}{V_R}X, \\
\dot{S_R} &= -\mu(S_R)X + \frac{Q}{V_R}(S_1 - S_R), \\
\dot{S_1} &= \frac{Q}{V_1}(S_R - S_1) + \frac{d}{V_1}(S_2 - S_1), \\
\dot{S_2} &= \frac{d}{V_2}(S_1 - S_2).
\end{aligned}
$$



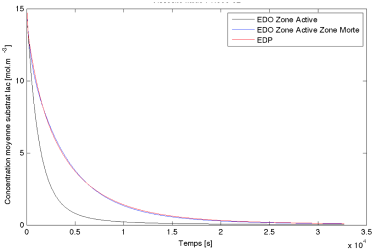
Existence, uniqueness, optimal control for two zones

Gajardo, Ramírez, Rapaport, Riquelme. Bioremediation of natural water resources via optimal control techniques.
BIOMAT 2011, 178-190, World Sci. Publ., Hackensack, NJ, 2012.

Two parameters we can play with : $\frac{V_1}{V_1 + V_2}$ and d.

Comparing models (with A. Rapaport and S. Barbier)

Optimization on the parameters $\dfrac{V_1}{V_1 + V_2}$ and d

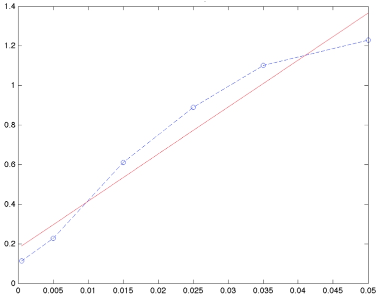


Pollution concentration in the lake

1 ODE (black)

2 ODEs (blue)

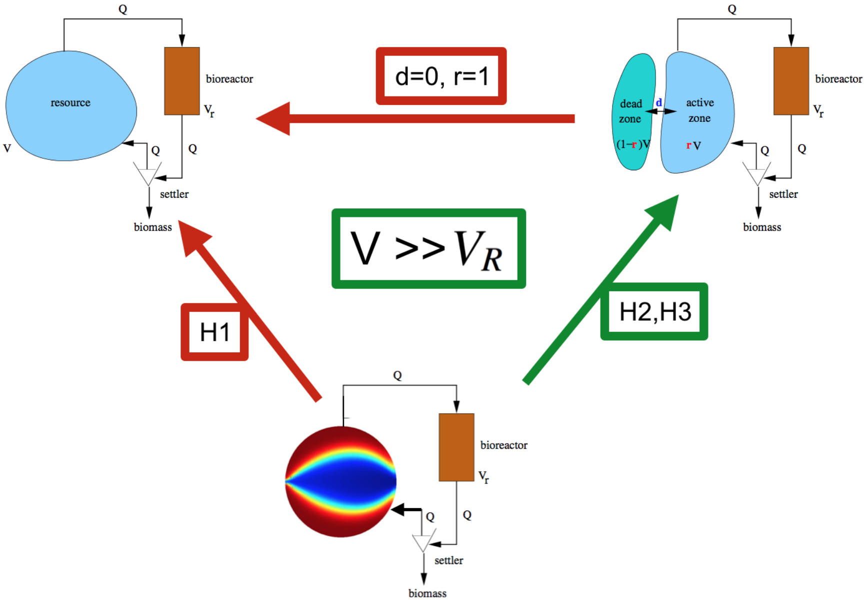
PDE (red)



Best parameter d vs viscosity ν_S (dashed blue)

Best meansquare fit (plain red)

In summary...



Q
resource
bioreactor
V_r
V
Q
Q
settler
biomass
d=0, r=1
Q
dead
zone
d
active
zone
(1−r)V
r V
bioreactor
V_r
Q
Q
settler
biomass
V >> V_R
H1
H2,H3
Q
bioreactor
V_r
Q
Q
settler
biomass

Conclusion

The “take home” message is...

- we have designed a first model and questioned it,
- we proposed a more complicated (too much ?) one,
- we took an intermediary model, which seems satisfactory, on which optimal control can be done.

Conclusion

The “take home” message is...

- we have designed a first model and questioned it,
- we proposed a more complicated (too much ?) one,
- we took an intermediary model, which seems satisfactory, on which optimal control can be done.

$\boxed{\text{BUT : is the reference (PDE) model a realistic model ?}}$

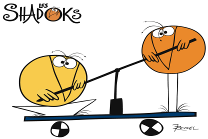
Conclusion

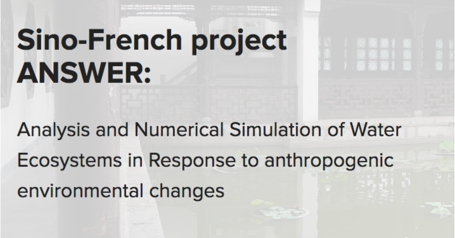
The “take home” message is...

- we have designed a first model and questioned it,
- we proposed a more complicated (too much ?) one,
- we took an intermediary model, which seems satisfactory, on which optimal control can be done.

BUT : is the reference (PDE) model a realistic model ?

Certainly not...





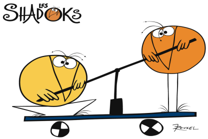
Conclusion

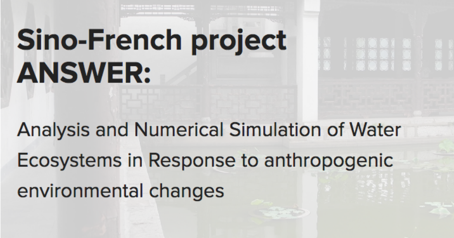
The “take home” message is...

- we have designed a first model and questioned it,
- we proposed a more complicated (too much ?) one,
- we took an intermediary model, which seems satisfactory, on which optimal control can be done.

BUT : is the reference (PDE) model a realistic model ?

Certainly not...





The more we fail, the more chances we have to succeed

Thank you for your attention

A few references...

- P. Gajardo, J. Harmand, H. Ramírez, A. Rapaport and V. Riquelme
  Minimal time bioremediation of natural water resources. Automatica 2011
- P. Gajardo, H. Ramírez, A. Rapaport and V. Riquelme
  Bioremediation of natural water resources via optimal control techniques. BIOMAT 2012
- A. Rousseau
  Texte public de l'agrégation externe de mathématiques. `http://agreg.org/Textes/public2016-B4.pdf`, 2012.
- A. Rapaport, A. Rousseau and J. Harmand.
  Procédé de traitement d'une ressource fluide, programme d'ordinateur et module de traitement associés. Patent No : FA 78 4546 - FR 13 55129, 2014.
- S. Barbier, A. Rapaport et A. Rousseau.
  Modelling of biological decontamination strategies of a water resource in natural environment. Journ. Sci. Comp, 2016.
- A. Rousseau, A. Rapaport, A. Pacholik and C. Leininger
  Action Dépollution. Serious game. `https://depollution.inria.fr/`