Reduced Models (and control) of in-situ decontamination of large water resources

Antoine Rousseau, Alain Rapaport

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Antoine ROUSSEAU,
Inria, Team LEMON (Montpellier)

In honor of my ”brother in sciences” Claudius

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Joint work with Alain Rapaport (INRA)

Outline

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Applicative Framework

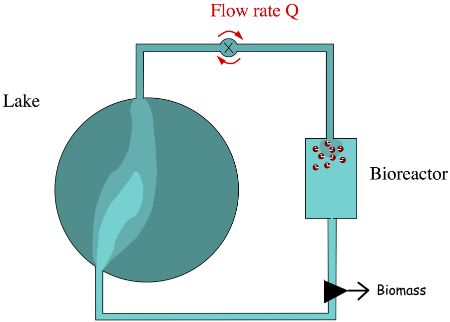
Polluted Taihu (algae), Yangtze Delta plain, Wuxi, China.

Courtesy Desert Research Institute

Objectives :

- use a bioreactor to remove pollution from the lake
- do it as efficiently as possible

Bioremediation problem

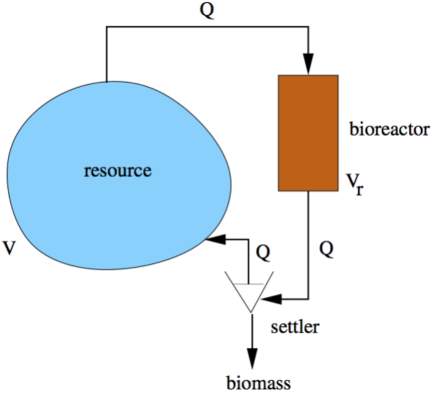


Parameters and unknowns

- V_L and V_R the volumes of the resource and bioreactor,
- X_R the biomass concentration in the bioreactor,
- S_L and S_R the pollutant concentrations,
- $\mu(\cdot)$ the biomass growth law,
- $Q = Q(t)$ the pump discharge, controlled by the user.

Outline

ODE-based model : the homogeneous case $S_L = S_L(t)$



$$\begin{cases} \dot{X}_R = \mu(S_R)\, X_R - \dfrac{Q}{V_R}\, X_R, \\ \dot{S}_R = -\mu(S_R)\, X_R + \dfrac{Q}{V_R}(S_L - S_R), \\ \dot{S}_L = \dfrac{Q}{V_L}(S_R - S_L), \end{cases}$$

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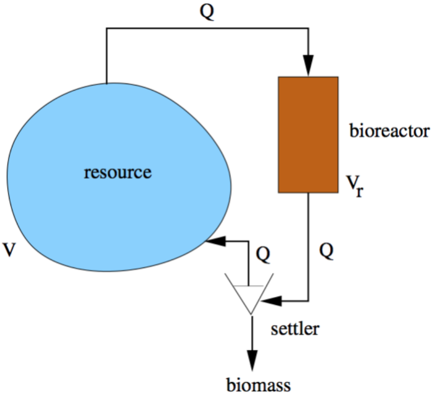

$$\begin{cases} \dot{X}_R = \mu(S_R)\,X_R - \dfrac{Q}{V_R}X_R, \\ \dot{S}_R = -\mu(S_R)\,X_R + \dfrac{Q}{V_R}(S_L - S_R), \\ \dot{S}_L = \dfrac{Q}{V_L}(S_R - S_L), \end{cases}$$

Chemostat equations :

J. Monod (1910-1976)

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Reference : Gajardo et al, Automatica 2011.

Slow-fast approximation :

$$\varepsilon = V_R/V_L \ll 1$$

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Theorem (case $\mu(X) = \mu X$)

For any $X_R(0) > 0$ and $S_R(0) = S_L > 0$, our system has a unique global solution. In addition, there exists a critical flow rate $Q_c > 0$, depending on S_L, such that asymptotically :

- if $Q > Q_c$, then $\big(S_R(s), X_R(s)\big)$ converges towards $(S_L, 0)$,
- if $0 < Q < Q_c$, then $\big(S_R(s), X_R(s)\big)$ converges towards (S_R^∞, X_R^∞) with $S_R^\infty = \dfrac{Q}{\mu V_R} < S_L$.

Optimal flow rate for the bioremediation (linear μ)

Back to the lake equations...

$$\boxed{\dot{S_L} = \frac{Q}{V_L}\left(\frac{Q}{\mu\, V_R} - S_L\right),}$$

What is the best flow rate Q?

Optimal flow rate for the bioremediation (linear μ)

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$$\boxed{Q(t) = \frac{\mu\, V_R}{2} S_L(t)}$$

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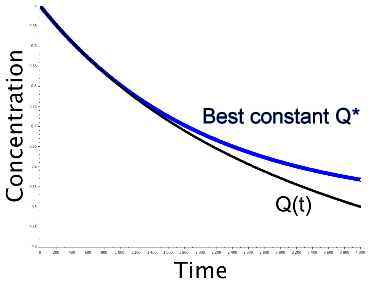
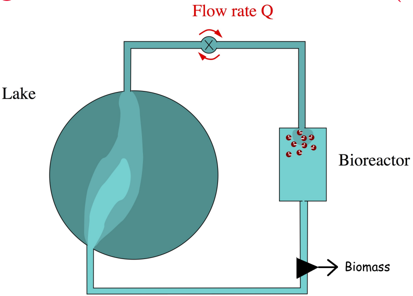


Outline

The non-homogeneous case : $S_L = S_L(t, x, y)$



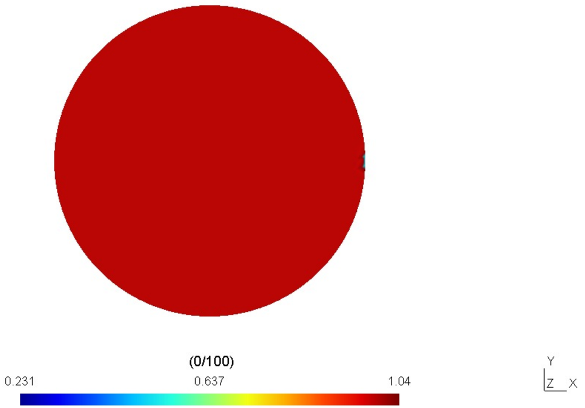
Navier Stokes & transport equations in the resource

$$\begin{cases} \dfrac{\partial u}{\partial t} + u \cdot \nabla u - \nu_u \Delta u + \nabla p & = & 0, \\ \nabla \cdot u & = & 0, \\ \dfrac{\partial S_L}{\partial t} + u \cdot \nabla S_L - \nu_S \Delta S_L & = & 0. \end{cases}$$

Boundary conditions

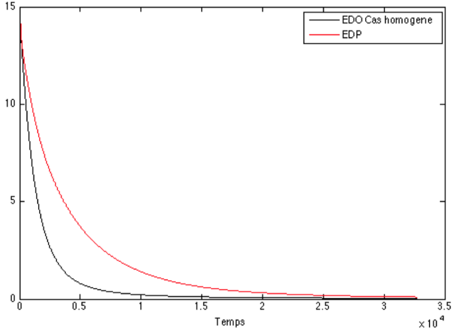
$$\begin{cases} u_{in}(t) = \mathcal{A} \times Q(t), \\ u_{out}(t) = \mathcal{A} \times Q(t), \\ \text{B.C on } S_L \end{cases}$$

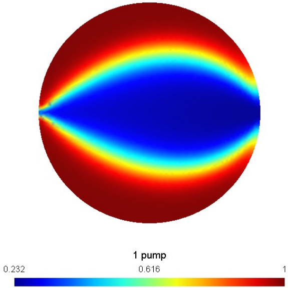
Simulations



Circular lake : 1 pump

Comparing models





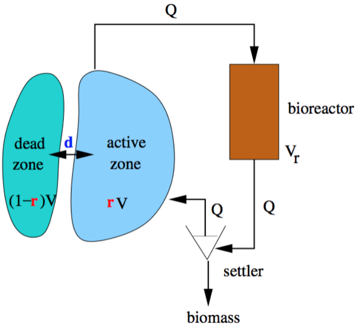
Pollution concentration in the lake
with ODE (black) and PDE (red)

Outline

Active and dead zones model

New ODE model

$$
\begin{aligned}
\dot{X} &= \mu(S_R)X - \frac{Q}{V_R}X, \\
\dot{S}_R &= -\mu(S_R)X + \frac{Q}{V_R}(S_1 - S_R), \\
\dot{S}_1 &= \frac{Q}{V_1}(S_R - S_1) + \frac{d}{V_1}(S_2 - S_1), \\
\dot{S}_2 &= \frac{d}{V_2}(S_1 - S_2).
\end{aligned}
$$



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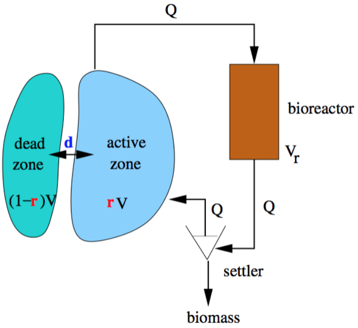


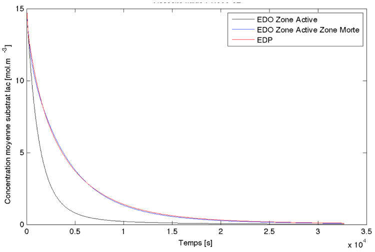
Existence, uniqueness, optimal control for two zones

Gajardo, Ramírez, Rapaport, Riquelme. Bioremediation of natural water resources via optimal control techniques.
BIOMAT 2011, 178-190, World Sci. Publ., Hackensack, NJ, 2012.

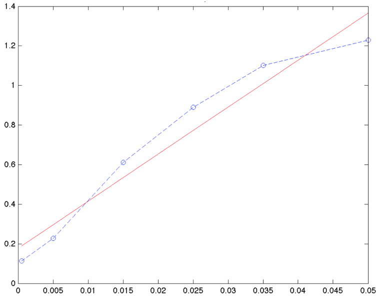
Two parameters we can play with : $\dfrac{V_1}{V_1 + V_2}$ and d.

Comparing models (with A. Rapaport and S. Barbier)

Optimization on the parameters $\dfrac{V_1}{V_1 + V_2}$ and d

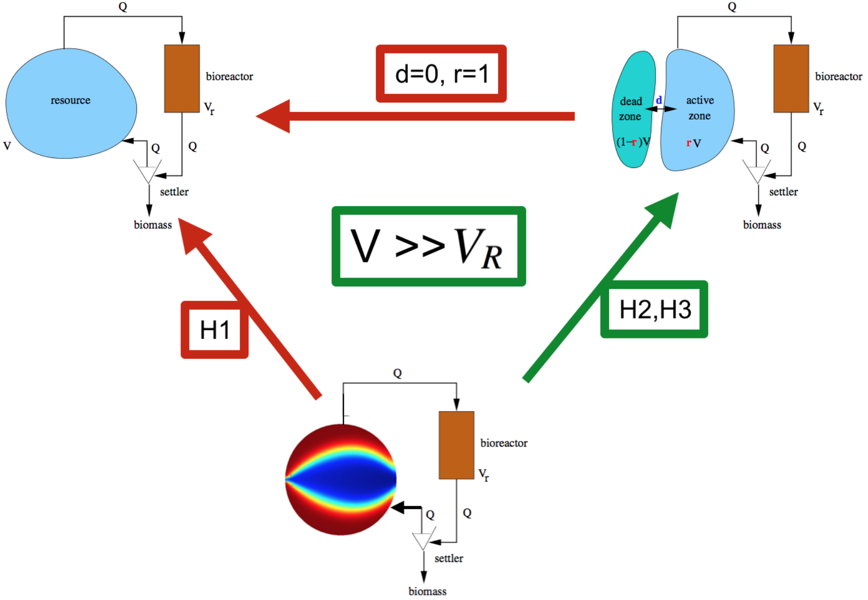


Pollution concentration in the lake
1 ODE (black)
2 ODEs (blue)
PDE (red)



Best parameter d vs viscosity ν_S (dashed blue)
Best meansquare fit (plain red)

In summary...



Q
resource
bioreactor
V
V_r
Q
Q
settler
biomass
d=0, r=1
Q
dead zone
d
active zone
(1−r)V
rV
bioreactor
V_r
Q
Q
settler
biomass
$V >> V_R$
H1
H2,H3
Q
bioreactor
V_r
Q
Q
settler
biomass

Conclusion

The “take home” message is...

- we have designed a first model and questioned it,
- we proposed a more complicated (too much ?) one,
- we took an intermediary model, which seems satisfactory, on which optimal control can be done.

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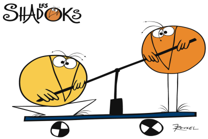
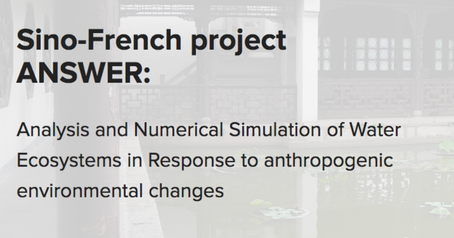
Certainly not...





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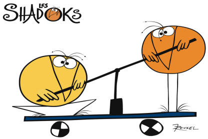

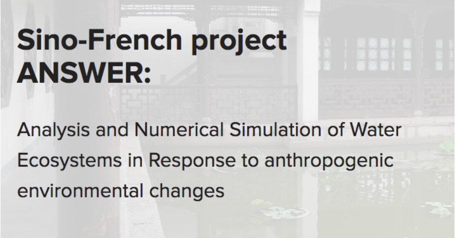

The more we fail, the more chances we have to succeed

Thank you for your attention

A few references...

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