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► **To cite this version:**

Antoine Rousseau, Alain Rapaport. Reduced Models (and control) of in-situ decontamination of large water resources. Sino-French Conference on Modeling, Mathematical Analysis and Computation, Chuanju Xu; Alain Miranville, Jun 2017, Xiamen, China. pp.27. hal-02785037

HAL Id: hal-02785037

<https://hal.inrae.fr/hal-02785037>

Submitted on 4 Jun 2020

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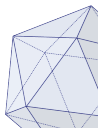
Reduced Models (and control) of in-situ decontamination of large water resources

Antoine ROUSSEAU,
Inria, Team LEMON (Montpellier)

In honor of my "brother in sciences" Claudius



IMAG
INSTITUT MONTELLIERAIN
ALEXANDER GROTHENDIECK



June 9th, 2017, SFC2MAC, Xiamen

Joint work with Alain Rapaport (INRA)

Outline

- 1 Introduction : the Taihu problem
- 2 A simple ODE model
- 3 A PDE-based model for the lake
- 4 Back to ODEs

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Applicative Framework



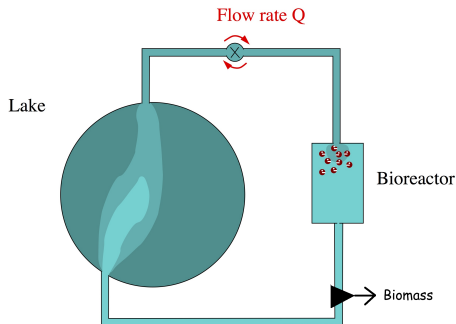
Polluted Taihu (algae), Yangtze Delta plain, Wuxi, China.

Courtesy Desert Research Institute

Objectives :

- use a bioreactor to remove pollution from the lake
- do it as efficiently as possible

Bioremediation problem



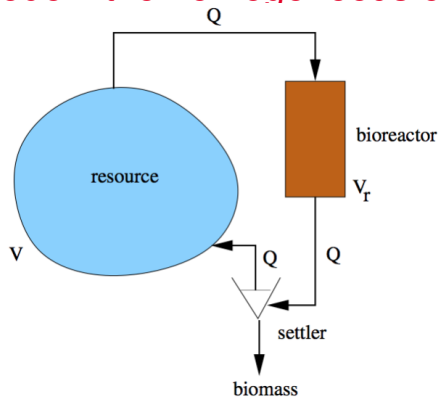
Parameters and unknowns

- V_L and V_R the volumes of the resource and bioreactor,
- X_R the biomass concentration in the bioreactor,
- S_L and S_R the pollutant concentrations,
- $\mu(\cdot)$ the biomass growth law,
- $Q = Q(t)$ the pump discharge, controlled by the user.

Outline

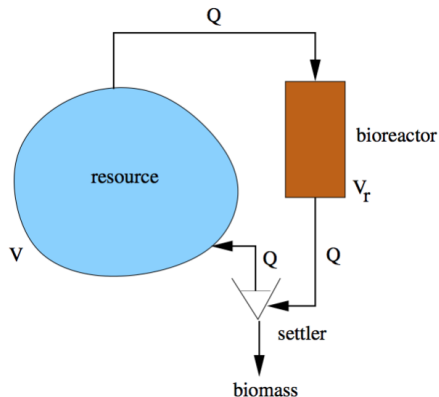
- 1 Introduction : the Taihu problem
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ODE-based model : the homogeneous case $S_L = S_L(t)$



$$\begin{cases} \dot{X}_R &= \mu(S_R) X_R - \frac{Q}{V_R} X_R, \\ \dot{S}_R &= -\mu(S_R) X_R + \frac{Q}{V_R} (S_L - S_R), \\ \dot{S}_L &= \frac{Q}{V_L} (S_R - S_L), \end{cases}$$

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Chemostat equations :
J. Monod (1910-1976)

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Slow-fast
approximation :

$$\varepsilon = V_R/V_L \ll 1$$

Reference : Gajardo et al, Automatica 2011.

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Slow-fast
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Theorem (case $\mu(X) = \mu X$)

For any $X_R(0) > 0$ and $S_R(0) = S_L > 0$, our system has a unique global solution. In addition, there exists a critical flow rate $Q_c > 0$, depending on S_L , such that asymptotically :

- if $Q > Q_c$, then $(S_R(s), X_R(s))$ converges towards $(S_L, 0)$,
- if $0 < Q < Q_c$, then $(S_R(s), X_R(s))$ converges towards (S_R^∞, X_R^∞) with

$$S_R^\infty = \frac{Q}{\mu V_R} < S_L.$$

Optimal flow rate for the bioremediation (linear μ)

Back to the lake equations...

$$\dot{S}_L = \frac{Q}{V_L} \left(\frac{Q}{\mu V_R} - S_L \right),$$

What is the best flow rate Q ?

Optimal flow rate for the bioremediation (linear μ)

Back to the lake equations...

$$\dot{S}_L = \frac{Q}{V_L} \left(\frac{Q}{\mu V_R} - S_L \right),$$

What is the best flow rate Q ?

$$Q(t) = \frac{\mu V_R}{2} S_L(t)$$

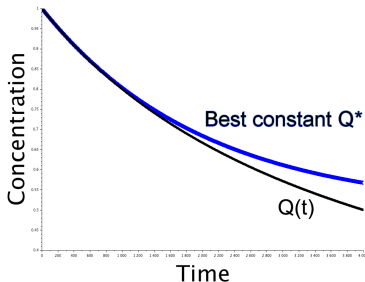
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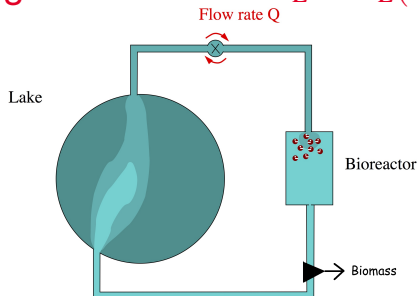
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The non-homogeneous case : $S_L = S_L(t, x, y)$



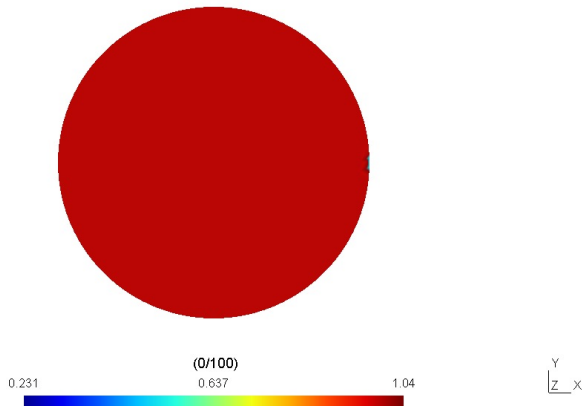
Navier Stokes & transport equations in the resource

$$\left\{ \begin{array}{l} \frac{\partial u}{\partial t} + u \cdot \nabla u - \nu_u \Delta u + \nabla p = 0, \\ \nabla \cdot u = 0, \\ \frac{\partial S_L}{\partial t} + u \cdot \nabla S_L - \nu_S \Delta S_L = 0. \end{array} \right.$$

Boundary conditions

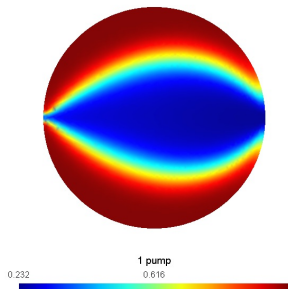
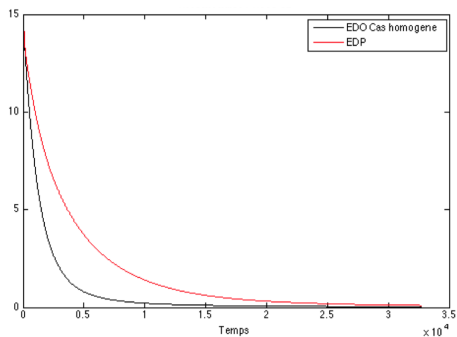
$$\left\{ \begin{array}{l} u_{in}(t) = \mathcal{A} \times Q(t), \\ u_{out}(t) = \mathcal{A} \times Q(t), \\ \text{B.C on } S_L \end{array} \right.$$

Simulations



Circular lake : 1 pump

Comparing models



Pollution concentration in the lake
with ODE (black) and PDE (red)

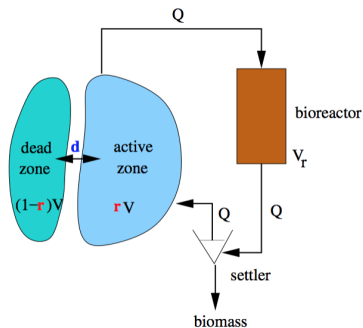
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Active and dead zones model

New ODE model

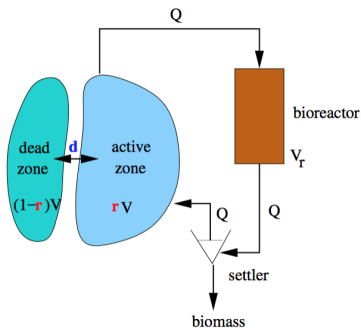
$$\begin{aligned}\dot{X} &= \mu(S_R)X - \frac{Q}{V_R}X, \\ \dot{S}_R &= -\mu(S_R)X + \frac{Q}{V_R}(S_1 - S_R), \\ \dot{S}_1 &= \frac{Q}{V_1}(S_R - S_1) + \frac{d}{V_1}(S_2 - S_1), \\ \dot{S}_2 &= \frac{d}{V_2}(S_1 - S_2).\end{aligned}$$



Active and dead zones model

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Existence, uniqueness, optimal control for two zones

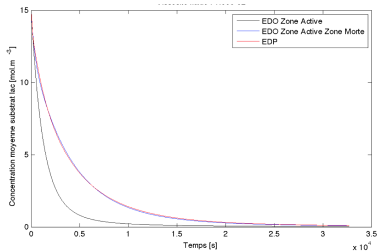
Gajardo, Ramírez, Rapaport, Riquelme. Bioremediation of natural water resources via optimal control techniques.

BIOMAT 2011, 178-190, World Sci. Publ., Hackensack, NJ, 2012.

Two parameters we can play with : $\frac{V_1}{V_1 + V_2}$ and d .

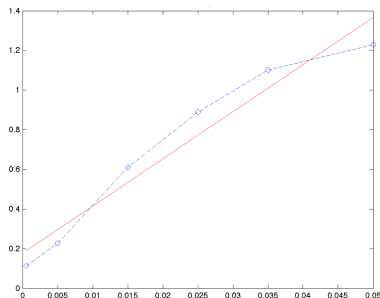
Comparing models (with A. Rapaport and S. Barbier)

Optimization on the parameters $\frac{V_1}{V_1 + V_2}$ and d



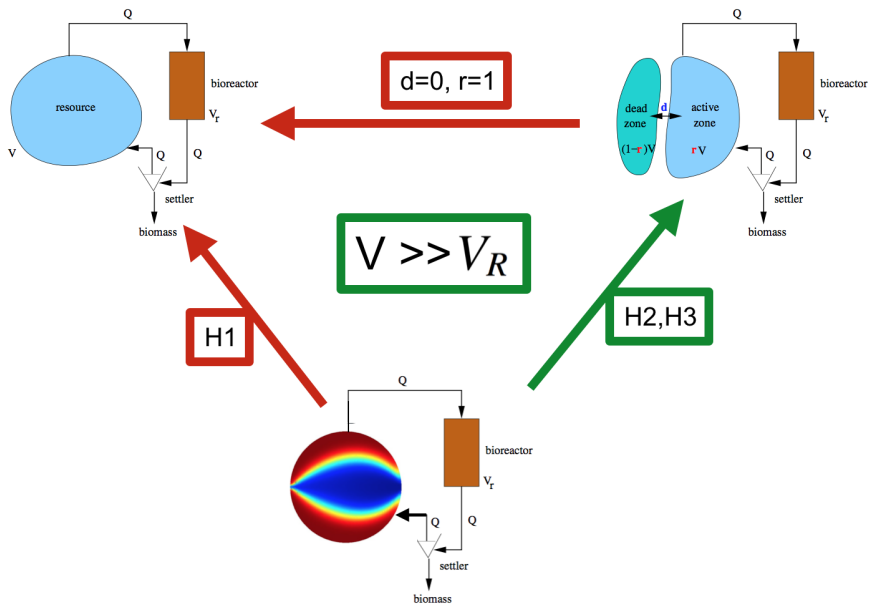
Pollution concentration in the lake

- 1 ODE (black)
- 2 ODEs (blue)
- PDE (red)



Best parameter d vs viscosity ν_S (dashed blue)
Best meansquare fit (plain red)

In summary...



Conclusion

The “take home” message is...

- we have designed a first model and questioned it,
- we proposed a more complicated (too much ?) one,
- we took an intermediary model, which seems satisfactory, on which optimal control can be done.

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BUT : is the reference (PDE) model a realistic model ?

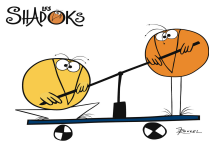
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Sino-French project ANSWER:

Analysis and Numerical Simulation of Water Ecosystems in Response to anthropogenic environmental changes

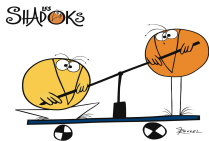
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Sino-French project ANSWER:

Analysis and Numerical Simulation of Water Ecosystems in Response to anthropogenic environmental changes

The more we fail, the more chances we have to succeed

Thank you for your attention

A few references...

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Bioremediation of natural water resources via optimal control techniques. BIOMAT 2012
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