

Genomic evaluation

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Whole Genome Sequence

- After Human (2001) and mice, WGS of the chicken (2004), the dog (2005), bovine (2006), horse (2007), pig (2009), ...
- Entirely in the public domain
- Sequencing of different individuals => **polymorphisms**
 - 3.2 million bovine polymorphisms in db**SNP**
 - >30 millions known today
- New technologies for genotyping and sequencing

SNP : Single Nucleotide Polymorphism

DNA Variation of one base

..GAATCTTATGCTATACATAATTATATACTAAT**C**GGGTATTGTTCTTAT.

..GAATCTTATGCTATACATAATTATATACTAAT**A**GGGTATTGTTCTTAT..

↑
SNP

Genotyping chips

- Simultaneous genotyping of many SNP
- From few dozens up to several million SNP
- Two main technology providers, Illumina and Affymetrix
- Illumina products in cattle
 - 3000 (□ 7000 □ 1000 □ 20000 = « LD »)
 - **54 000 = « 50k »**
 - 777 000 = « HD »

FIGURE 1: BOVINESNP50 BEADCHIP



The BovineSNP50 BeadChip features more than 54,000 evenly-spaced SNPs across the entire bovine genome.

Chips of different densities:

IMPUTATION

i_p_ta_io_c_nsi_t_i_pr_di_t_n_t_e m__s__g_l_t_er__i_h__
w__d_o__a_s__t__c_ + a dictionary

- imputation consists in predicting the missing letters within a word or a sentence

7K

T				T				G						C				A			T			C
T				T				C						A				C			T			A

+ an imputation population

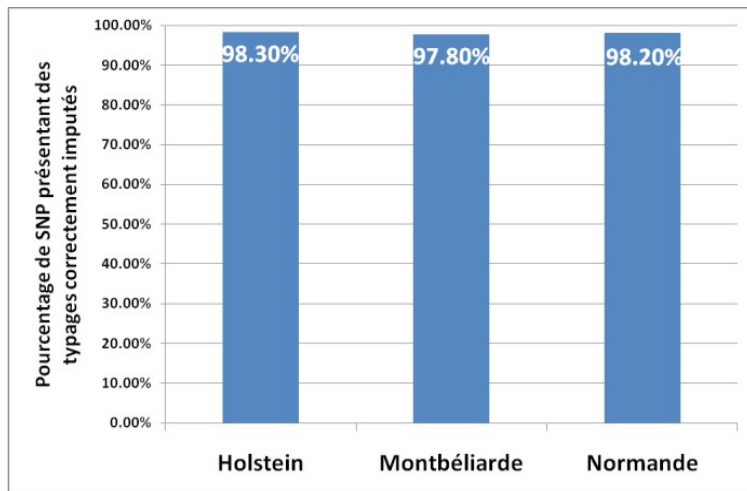
54K

T	C	A	G	T	T	G	T	G	A	A	T	G	A	C	T	G	G	A	C	G	T	G	C	C
T	A	A	G	T	G	G	T	C	A	C	T	G	A	A	T	G	G	C	C	G	T	T	C	A

Works well ! (~1% error in Holstein if sire genotyped with 54k)

Imputation

Beagle/DagPhase



(Dassonneville et al.,
2011)

Fimpute

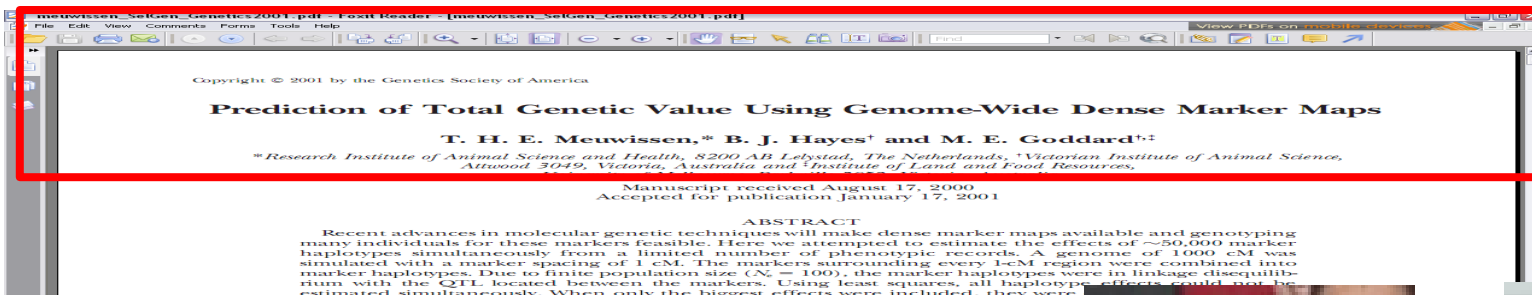


(Saintilan et al., 2014)

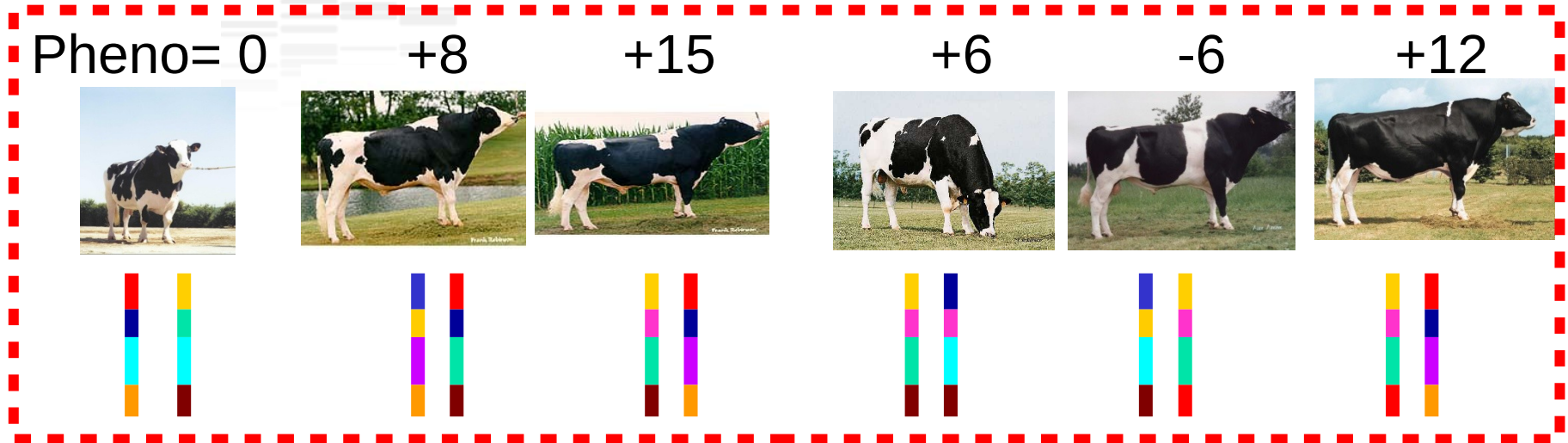
Computation time divided by 3 – 200-300 SNP/animal corrected

Genomic selection

selection based on the estimation of the genetic value of candidates using information on dense markers covering the whole genome
= selection based on results from a **genomic evaluation**

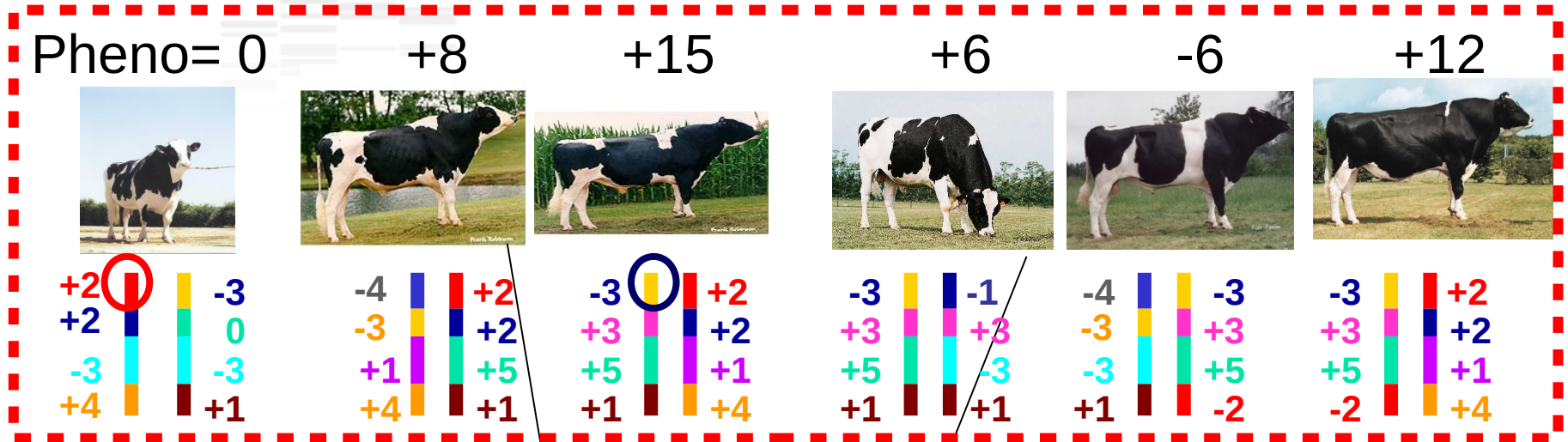


What is genomic selection?



Genotyped and
phenotyped animals ↔ Reference
population

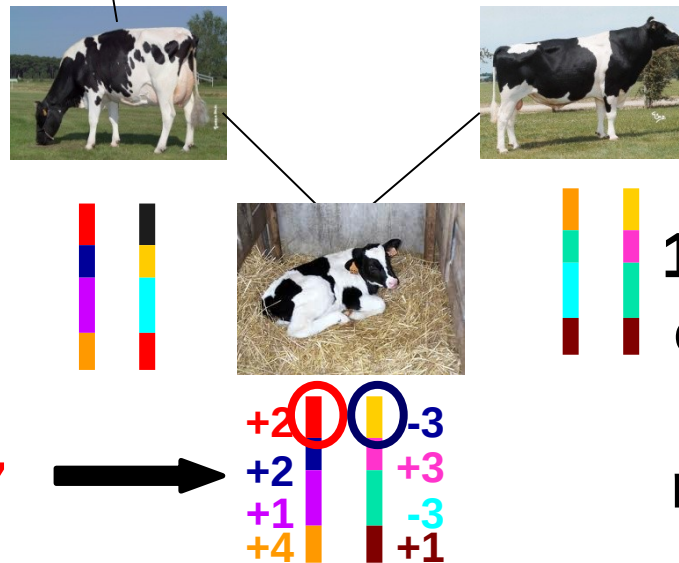
What is genomic selection?



2/ Genotype
at birth

3/ Apply!

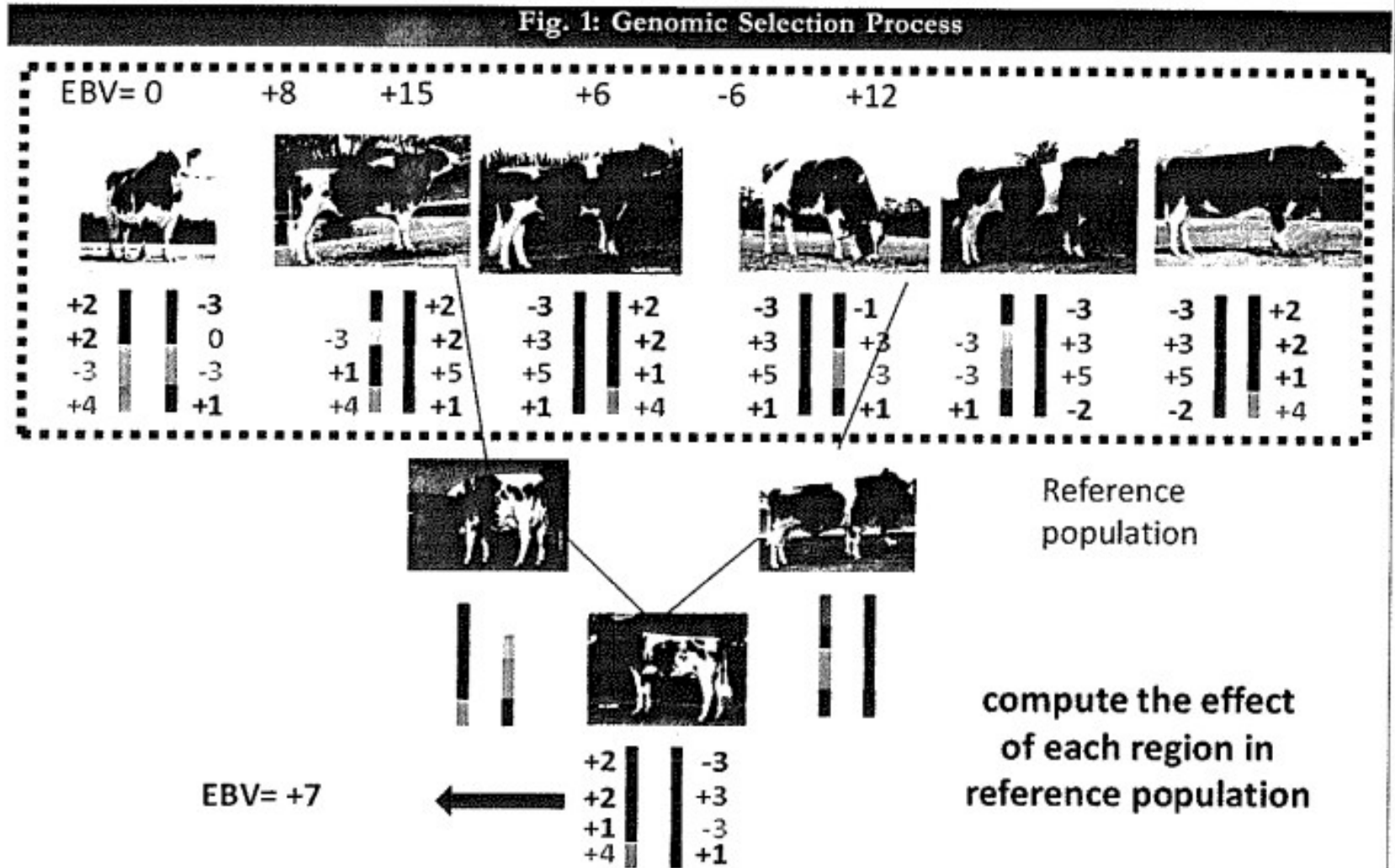
(G)EBV= +7



Reference
population

1/ Compute the effect
of each chromosome
region in the
reference population

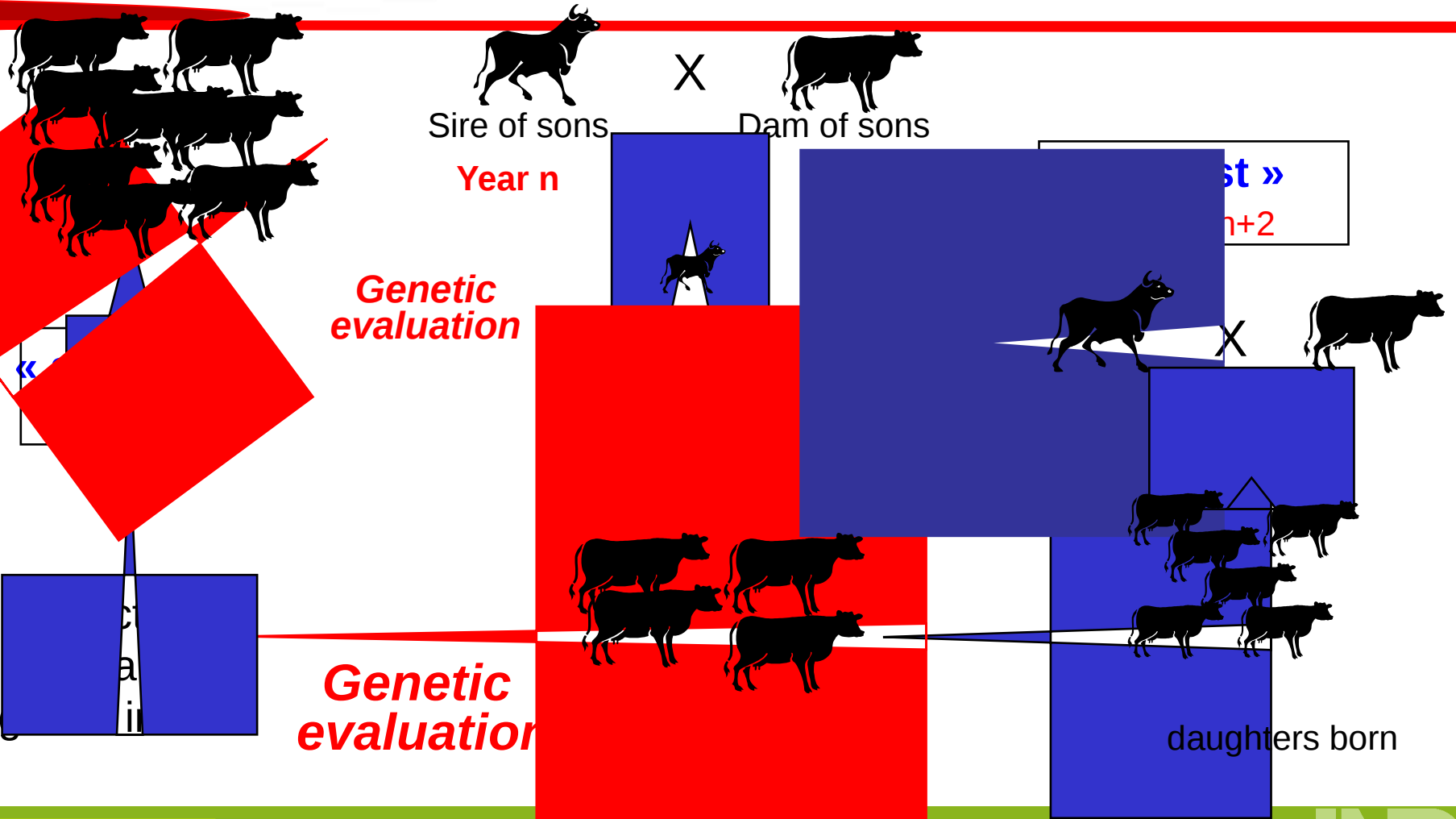
Digression: don't do this !



and another set of validation bulls; using stored semen | the Netherlands, Nordic countries, Poland and Spain

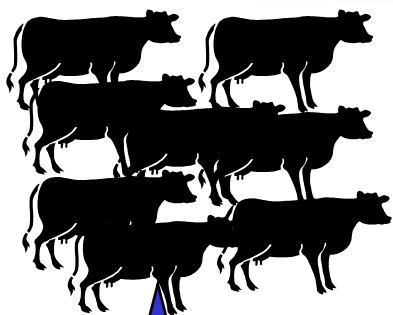
Remember: “Classical” progeny test

Genetic evaluation

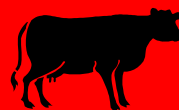


A typical genomic breeding scheme

**Genetic
evaluation**



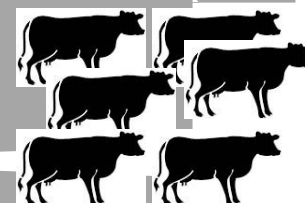
Sire of so
year n



« service »
year n+2

**Genomic
evaluation**

**Genetic
evaluation**



Daughters bon

In practice ...

- Genomic selection
 - Requires **phenotypes** and **genotypes**
 - To minimize costs, genotype bulls in countries with large existing progeny test programs
 - But bulls don't have phenotypes (of interest)
 - summarize daughters' phenotypes at bull level
 - ✓ *Advantage* : more precise phenotypes

Basic principle for implementation

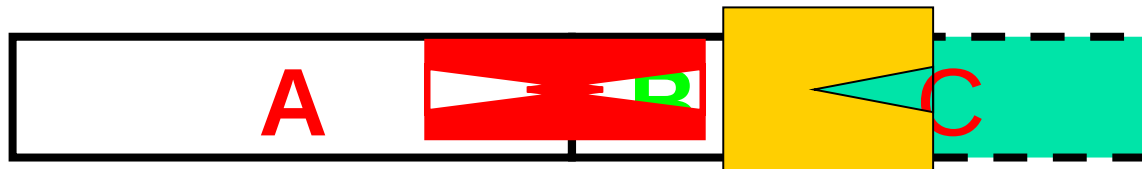
A **reference population A+B** : several thousands of genotyped animals with phenotype / performances

- From population A (**training population**), find the relationship between the phenotypes and the markers (=a prediction equation)
- **purely statistical approach** (no/very few genetic assumptions)



Basic principle for implementation

- then **validate** this prediction equation in **population B** (**validation population**)
- If satisfactory, redo the analysis on A+B and **apply** the resulting prediction equation to obtain a « genomic breeding value » for selection candidates C, for example at birth



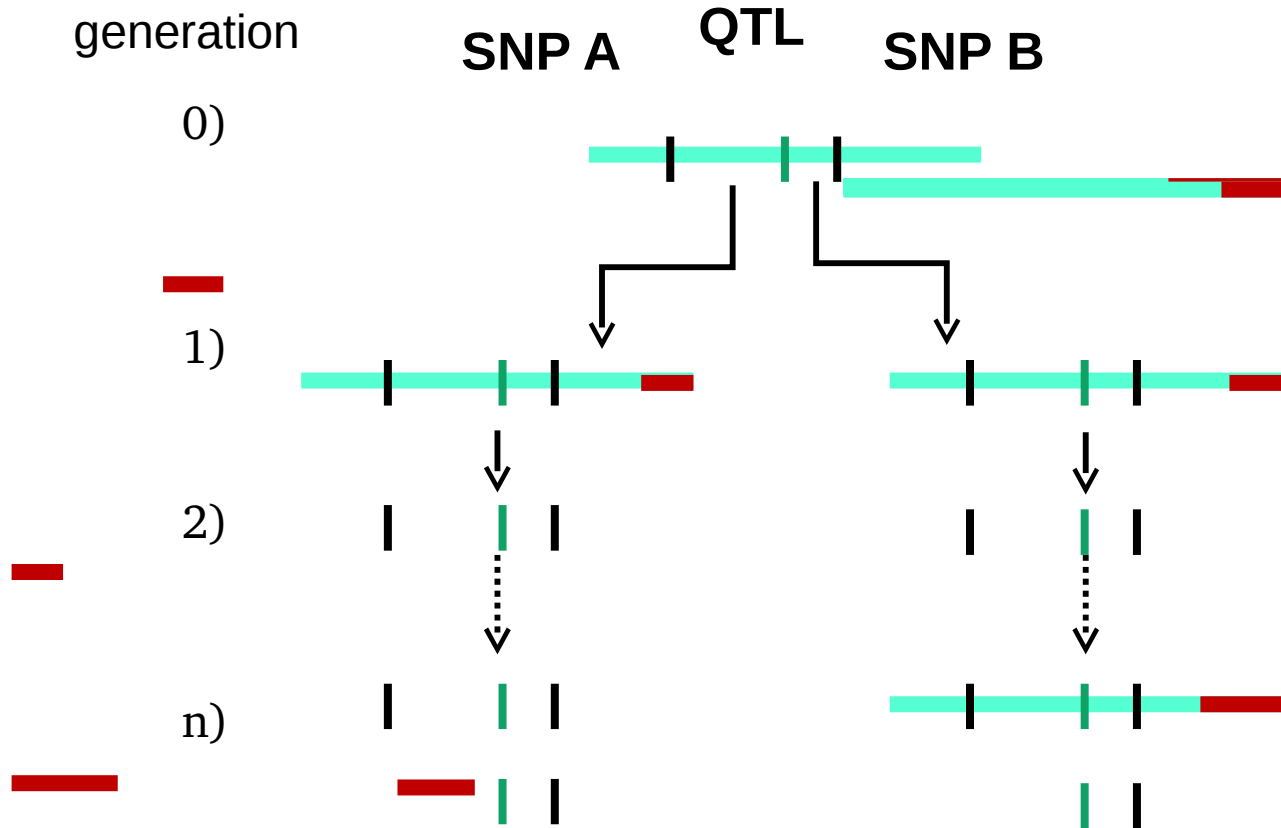
Genomic Selection: efficiency

➤ Two main factors :

- **Accuracy of SNP effect** estimation
 - size of reference population
 - heritability of the trait
 - statistical methodology used
- **Linkage Disequilibrium** (LD) between markers and QTL
 - marker density
 - effective size of the population => number of « independent » segments
 - Relationship between candidates and reference population

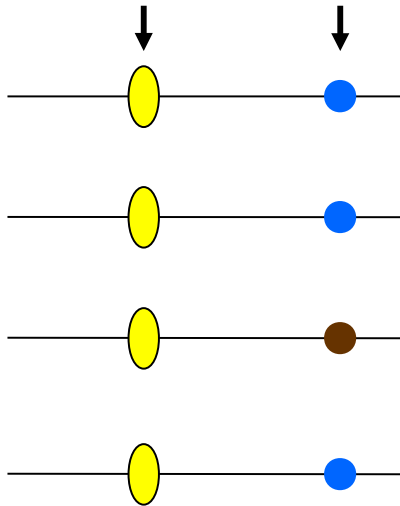
Linkage Disequilibrium

QTL-SNP association transmitted over generations



Linkage Disequilibrium

QTL frequent allele



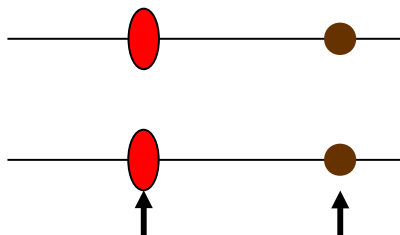
SNP ● more often associated with QTL allele

SNP ● more often associated with QTL allele



use effect of ● as proxy of effect of ●
and effect of ● as proxy of effect of ●

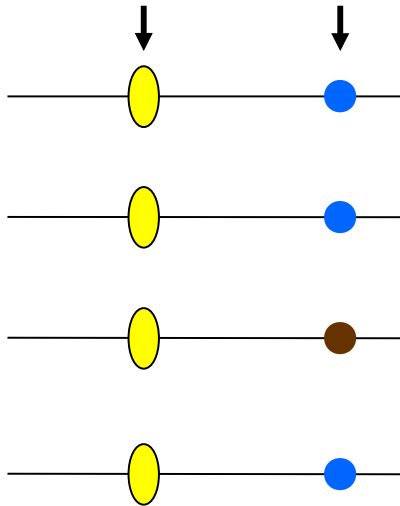
QTL rare allele



SNP

Linkage Disequilibrium

QTL frequent allele



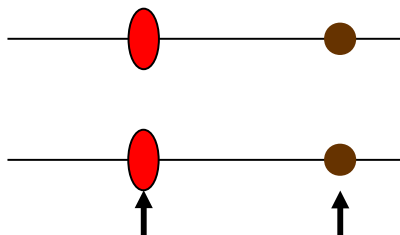
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QTL rare allele

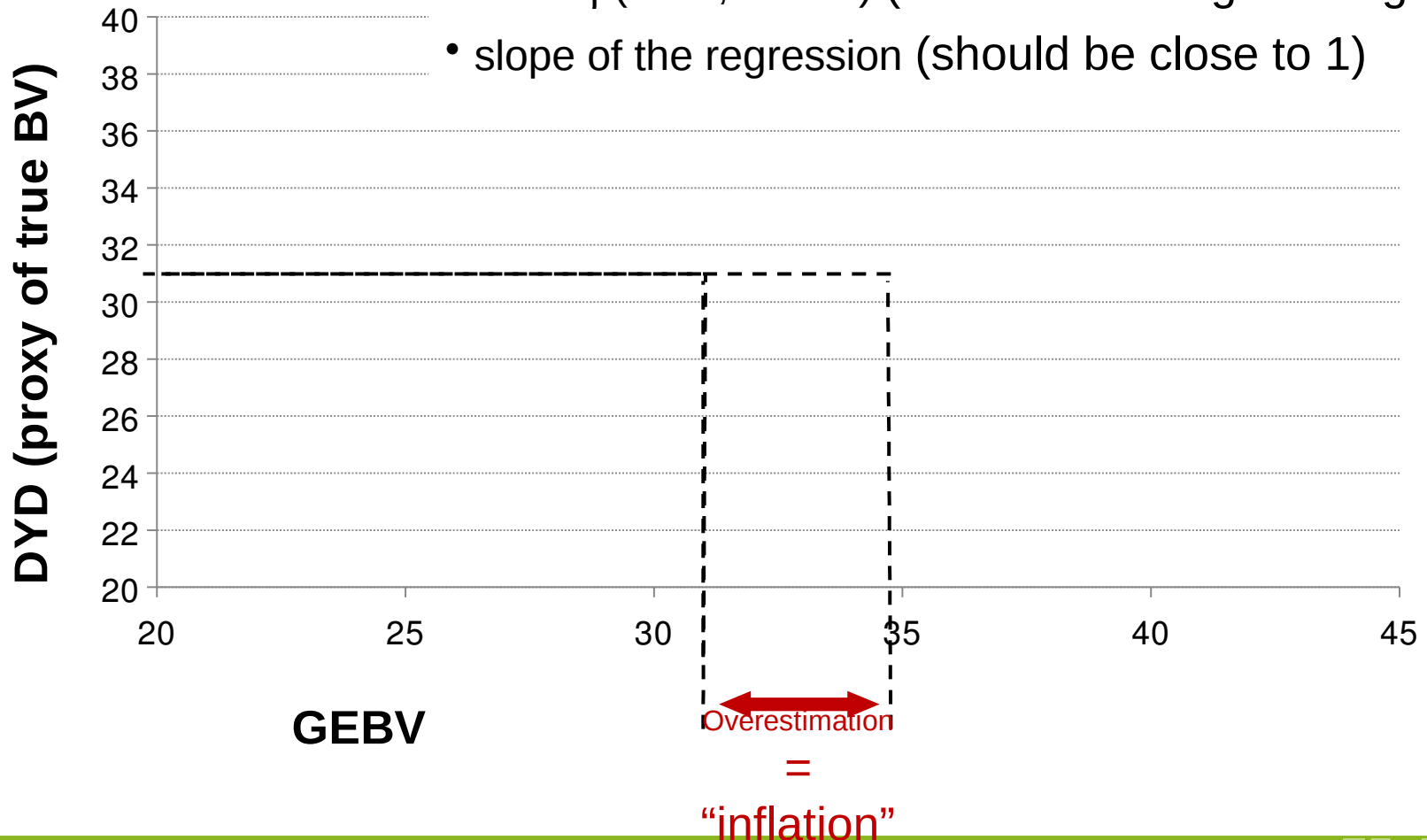


SNP

Validation test

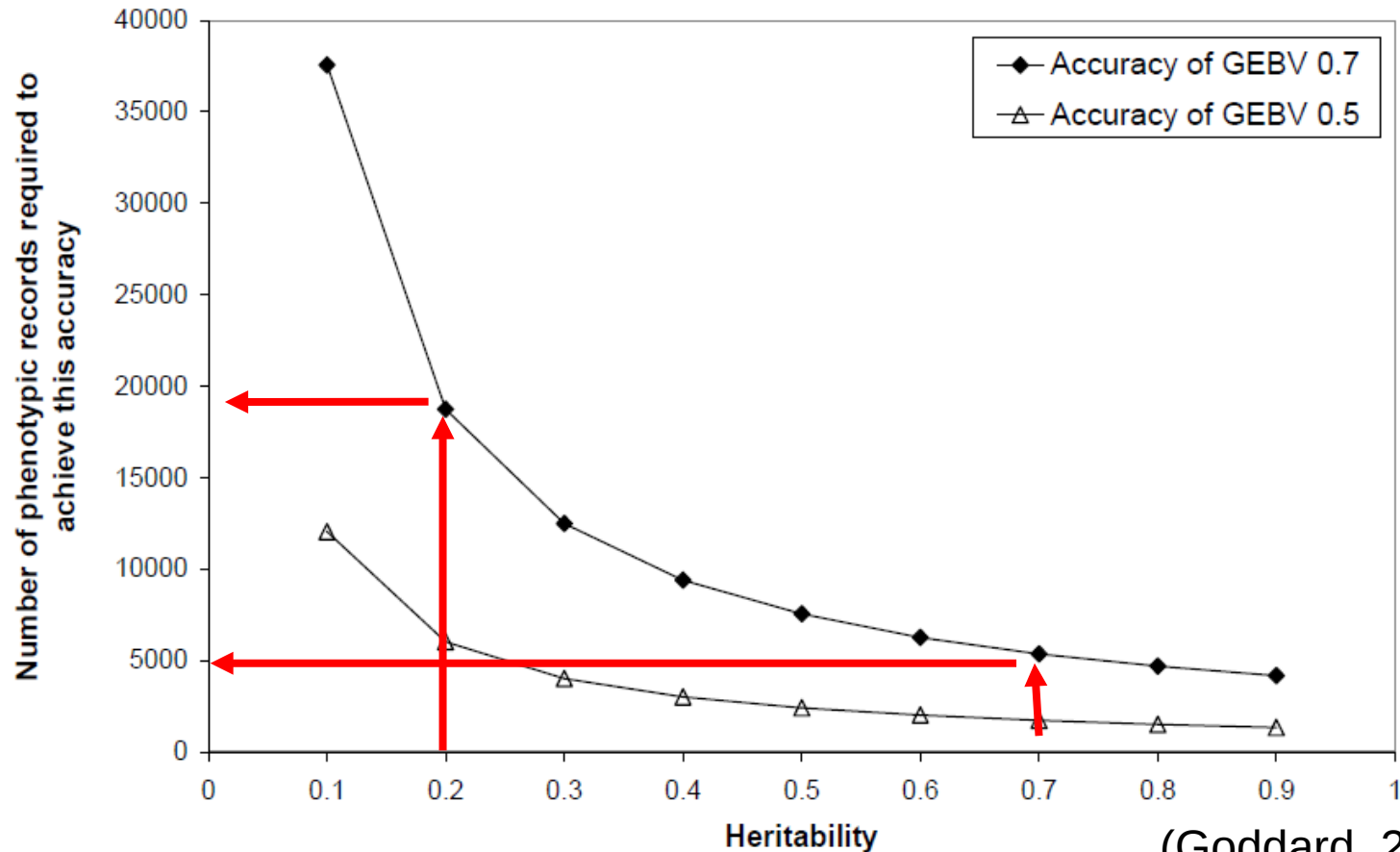
Two important parameters :

- R^2 or $\rho(\text{DYD}, \text{GEBV})$ (should be « large enough »)
- slope of the regression (should be close to 1)



Size of the reference population

- Greatly influences the accuracy of genomic evaluations



(Goddard, 2008)

Increase size of reference populations

- Genotype as many progeny tested bulls as possible
- International collaborations
 - **Holstein**: 2 big consortia
 - **USA + Canada** ~>25000 bulls (?) + Italy + GB
 - **Eurogenomics** France, The Netherlands, (Germany), Nordic countries, Spain, Poland
~34000 bulls
- **For small breeds**: not enough AI bulls
 - combine with many genotyped cows



Information from bulls and COWS

Number of cows with 1 own record
equivalent to a bull with performances on daughters

	h2				
CD*	0.1	0.2	0.3	0.4	0.5
0.40	6.0	2.7	1.6	1.0	0.7
0.50	9.0	4.0	2.3	1.5	1.0
0.60	13.5	6.0	3.5	2.3	1.5
0.70	21.0	9.3	5.4	3.5	2.3
0.80	36.0	16.0	9.3	6.0	4.0
0.90	81.0	36.0	21.0	13.5	9.0

* CD : coefficient of determination
= reliability of EBV based on progeny performance only (Boichard, 2015)

Basic data for genomic evaluation

- $y = YD$ (Yield deviation) = Individual record corrected for all fixed and non genetic random effects

or

- $y = DYD$ (Daughter yield deviation) = (2 x) average for a bull of all YD of their daughters corrected for $\frac{1}{2}$ genetic effect of their dam (with associated weight = EDC (Equivalent Daughter Contribution))

or

- $y = \text{deregressed proofs}$ (= DYD reconstructed from EBV, EDC and relationships: a genetic evaluation based on deregressed proofs will give back the bulls' EBV))

or

- $y = EBV$ (~~~« proxy » of DYD, valid only if reliabilities are high~~)

Different genomic evaluation methods

Can be grouped into 4 families:

1. Derived from BLUP: **GBLUP**

2. General multidimension methods when the number of unknowns (p) is much larger than the number of observations (n) : the « $p \gg n$ problem » (PLS, LASSO, Elastic Net...)

3. **Bayesian methods** A, B, C, $C\pi$, D, R ...

4. ~~« Machine learning »~~ methods (big black box)

To be applied on individual **SNP** or **haplotypes**

GBLUP: Basic « genomic » model

Hypotheses :

- 1) QTLs (quantitative Trait loci) have a purely additive effect
- 2) All the variability at QTLs is explained the markers (!)

N biallelic markers

2 alleles A/B but only one effect defined = substitution effect m_i

$$x_i m_i = \quad \quad x_i m_i =$$

$$w_i = (x_i - 2p_i) m_i$$

example:	AA	-1 m_i	or 0	or	- 2 $p_i m_i$
	AB	0	+ m_i		(1 - 2 p_i) m_i
	BB	+1 m_i	+2 m_i		(2 - 2 p_i) m_i

where p_i is the allelic frequency of allele B of marker i

Distribution of marker effects

$$g = BV = \sum_{i=1}^N x_i m_i$$

$$\text{? GEBV} = \sum_{i=1}^N x_i \hat{m}_i$$

With the third form, and assuming that each marker contributes the same way to the variance of the trait, we have (Van Raden, 2009):

$$m_i \sim N(0, \sigma_m^2) \text{ with } \sigma_m^2 = \frac{\sigma_g^2}{2 \sum_j p_j (1 - p_j)}$$

Why this formula?: for consistency with the additive genetic variance. But what allelic frequency should be used? Quite a few were proposed (observed ones/ in the base population..)

Evaluation model = BLUP on markers

If all animals with records are genotyped, and if we take allelic frequencies into account by centering x_i :

$w_i = x_i - 2 p_i$ (grouped in a matrix **W**) , we can write:

The corresponding Mixed Model Equations are:

$$\begin{bmatrix} X'X & X'W \\ W'X & WW' \end{bmatrix} \begin{bmatrix} \hat{\mu} \\ \hat{m} \end{bmatrix} + \frac{\sigma_e^2}{\sigma_m^2} \begin{bmatrix} X'y \\ W'y \end{bmatrix} = \begin{bmatrix} X'y \\ W'y \end{bmatrix}$$

For any new animal * without performance: $GEBV = \sum_{i=1}^N w_i^* \hat{m}_i$

An equivalent model

- We can also write: $\mathbf{g} = \mathbf{W}\mathbf{m}$

But, since we have $\mathbf{m}_i \sim N(\mu, \sigma_m^2)$, we also have:

$$\mathbf{g} \sim N(\mu_g, \mathbf{G})$$

$$\begin{aligned} \text{with } \mathbf{G} &= \text{var}(\mathbf{W}\mathbf{m}) = \mathbf{W} \text{var}(\mathbf{m}) \mathbf{W}' \\ &= \mathbf{W}\mathbf{W}'/k \end{aligned}$$

where k is a standardisation factor connecting the marker effects variance to the additive genetic variance

$$k = 2 \sum_j p_j (1 - p_j)$$

An equivalent model

- **G** plays the same role as the classical relationship matrix **A**

G is called « **genomic relationship matrix** »

- So, reparameterizing the model

if all recorded animals are genotyped:

or more generally (adding columns of 0 to **Z** if no phenotype):

GBLUP

The Mixed Model Equations are the same as in the classical case except that the relationship matrix **A** (=« expectation » of relationship coefficients) is replaced by the genomic relationship matrix **G**:

$$\begin{array}{c} \diamond x'x \\ \diamond ? \\ \diamond ? \\ \diamond ? \\ \diamond ? \end{array} \quad \begin{array}{c} x'z \\ \\ \\ \\ \end{array} \quad \begin{array}{c} \diamond ? \\ \diamond ? \\ \diamond ? \\ \diamond ? \\ \diamond ? \end{array} \quad \begin{array}{c} \diamond \hat{\beta} \\ \diamond ? \\ \diamond ? \\ \diamond ? \\ \diamond ? \end{array} \quad \begin{array}{c} \diamond x'y \\ \diamond ? \\ \diamond ? \\ \diamond ? \\ \diamond ? \end{array} \quad \begin{array}{c} \diamond ? \\ \diamond ? \\ \diamond ? \\ \diamond ? \\ \diamond ? \end{array}$$

$$\begin{array}{c} \diamond z'x \\ \diamond ? \\ \diamond ? \\ \diamond ? \\ \diamond ? \end{array} \quad \begin{array}{c} z'z \\ \\ \\ \\ \end{array} + \frac{\sigma_e^2}{\sigma_g^2} \mathbf{G}^{-1} \quad \begin{array}{c} \diamond ? \\ \diamond ? \\ \diamond ? \\ \diamond ? \\ \diamond ? \end{array} \quad \begin{array}{c} \diamond ? \\ \diamond ? \\ \diamond ? \\ \diamond ? \\ \diamond ? \end{array} = \quad \begin{array}{c} \diamond x'y \\ \diamond ? \\ \diamond ? \\ \diamond ? \\ \diamond ? \end{array} \quad \begin{array}{c} \diamond ? \\ \diamond ? \\ \diamond ? \\ \diamond ? \\ \diamond ? \end{array}$$

(some) limits of GBLUP

- Assuming that **G** is correct, how to compute it efficiently?

$$\mathbf{G} = \mathbf{W}\mathbf{W}'/k \quad \mathbf{W} \text{ is dense!}$$

Very long to compute, can be too big to store and to invert

□ iterative solutions

- How to compute reliabilities □ approximations
- A marker model seems more attractive.... (at least, its complexity does not increase too much when the number of genotyped animals increases!)

(some) limits of GBLUP

- It is not realistic to think that all the genetic variation at QTLs is captured by markers. This leads to genomic values which show «inflation » □ requires an additional « residual » polygenic effect:

Depending on the trait, $\omega = 5$ to 30%

(some) limits of GBLUP

- How to combine genomic and classical (e.g., phenotypes from progeny) information?

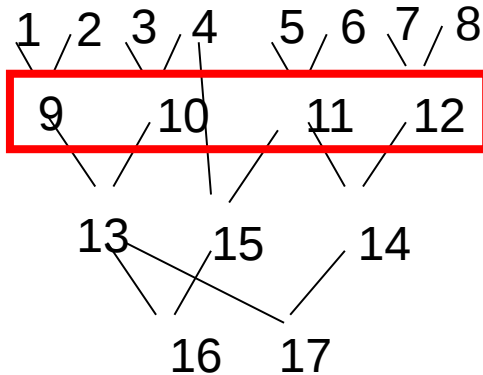
Van Raden's (2009) approach:

- 1/ run a classical genetic evaluation with all animals
 - 2/ run a genomic evaluation with all genotyped animals
 - 3/ rerun the same (classical) evaluation with genotyped animals
- ...(note: BLUP hypotheses are not fulfilled!)
- 4/ combine the 3 sources of information (more or less: $1 + 2 - 3$) with selection index theory, for each animal

- How to make non genotyped animals benefit from genomic information of relatives?

An example

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]	[,9]	[,10]	[,11]	[,12]	[,13]	[,14]	[,15]	[,16]	[,17]
[1,]	1.00									0.50				0.25			0.12 0.12
[2,]		1.00								0.50				0.25			0.12 0.12
[3,]			1.00								0.50			0.25			0.12 0.12
[4,]				1.00							0.50			0.25			0.12 0.12
[5,]					1.00							0.50		0.25	0.25	0.12 0.12	
[6,]						1.00						0.50		0.25	0.25	0.12 0.12	
[7,]							1.00						0.50	0.25			0.12 0.12
[8,]								1.00						0.25			0.12 0.12
[9,]	0.50	0.50							1.00					0.50			0.25 0.25
[10,]			0.50	0.50						1.00				0.50			0.25 0.25
[11,]					0.50	0.50					1.00				0.50	0.50	0.25 0.25
[12,]							0.50	0.50				1.00					0.25 0.25
[13,]	0.25	0.25	0.25	0.25					0.50	0.50			1.00				0.25 0.25
[14,]					0.25	0.25	0.25	0.25			0.50	0.50		1.00			0.25 0.25
[15,]					0.50	0.25	0.25			0.25	0.50			0.12	0.25	1.00	0.56 0.50
[16,]	0.12	0.12	0.12	0.38	0.12	0.12			0.25	0.38	0.25			0.56	0.12	0.56	1.06 0.34
[17,]	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.25	0.25	0.25	0.25	0.50	0.50	0.19	0.34	1.00



?

?

G

1.00 0.50 0.50 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25
0.50 1.00 0.50 0.50 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25
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0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 0.25 1.00

Transmission of information (to non genotyped animals)

Let **g2** be the genetic (genomic) values of **genotyped** animals and **g1** the genetic values of **non genotyped** animals

$$\text{Let } \mathbf{A} = \begin{pmatrix} \text{?} & A_{12} & \text{?} \\ \text{?} & A_{22} & \text{?} \\ \text{?} & \text{?} & \text{?} \end{pmatrix} \begin{matrix} A^{11} \\ A^{12} \\ A^{21} \\ A^{22} \end{matrix} \text{ and } p(\mathbf{g}_2) \sim N(\mathbf{0}, \mathbf{G}\sigma_g^2)$$

By regression of **g1** on **g2**:

$$p(\mathbf{g}_1 | \mathbf{g}_2) \sim N(\mathbf{A}_{12} \mathbf{A}_{22}^{-1} \mathbf{g}_2, \mathbf{V} \sigma_g^2)$$

$$\text{with } \mathbf{V} = (\mathbf{A}^{11})^{-1} = \mathbf{A}_{11} + \mathbf{A}_{12} \mathbf{A}_{22}^{-1} \mathbf{A}_{21}$$

Transmission of information (to non genotyped animals)

Let **g2** be the genetic (genomic) values of **genotyped** animals and **g1** the genetic values of **non genotyped** animals

An estimate of **g1** based on genomic information is obtained
by regression of **g1** on **g2**

This information is added to regular BLUP equations

□ «Single-Step» approach

Legarra, Misztal, Aguilar; Christensen & Lund; D. Johnson

- **H** = Variance of 

«Single-step» approach

non genotyped

genotyped

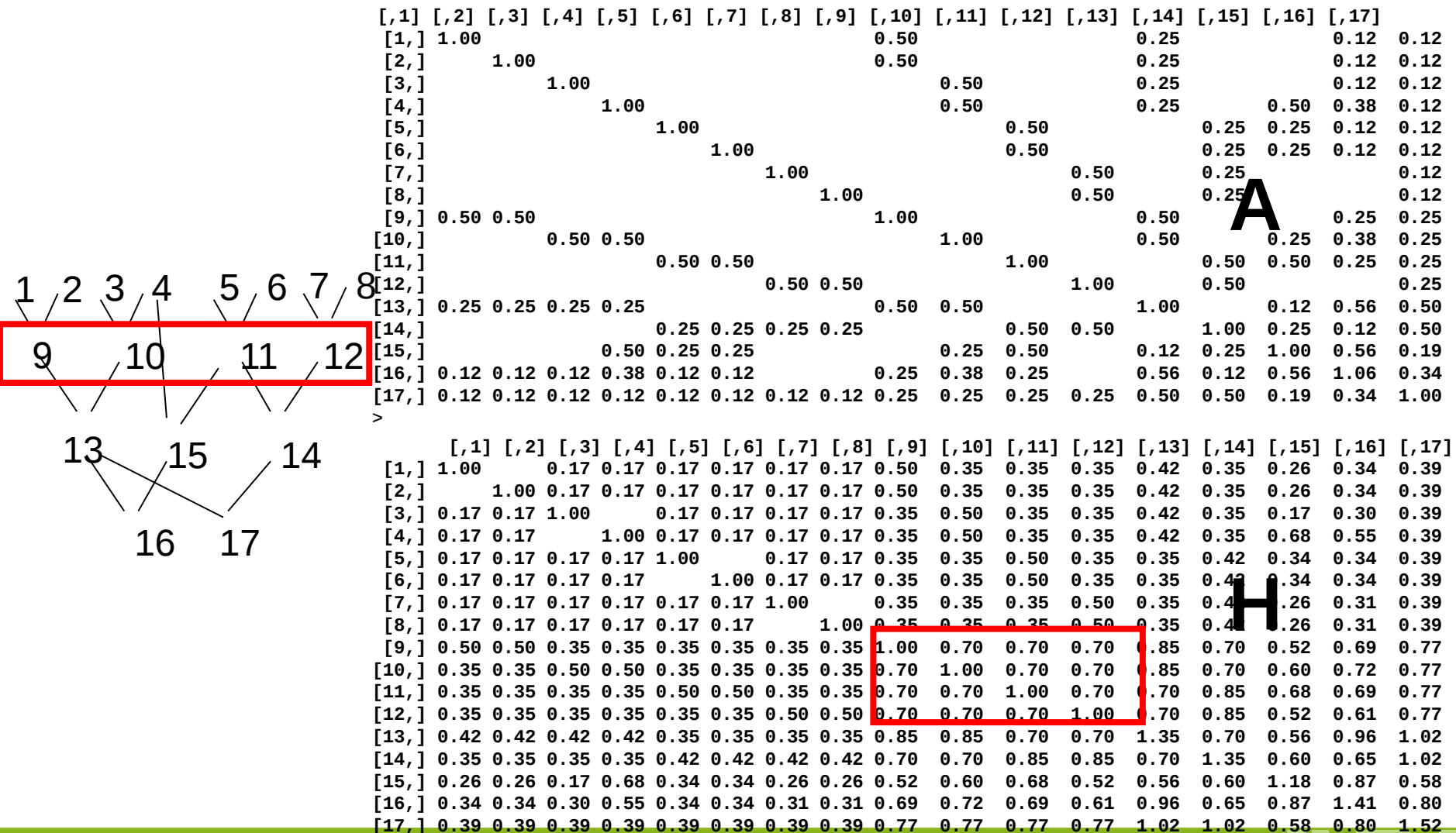
$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} \\ \mathbf{H}_{21} & \mathbf{H}_{22} \end{bmatrix} + \mathbf{A}_{12} \mathbf{A}_{22}^{-1} (\mathbf{G} - \mathbf{A}_{22}) \mathbf{A}_{22}^{-1} \mathbf{A}_{21} \quad \mathbf{A}_{12} \mathbf{A}_{22}^{-1} \mathbf{G}$$

$$\mathbf{G} \mathbf{A}_{22}^{-1} \mathbf{A}_{21} \quad \mathbf{G}$$

- Incredible but true ! **H-1** has a relatively simple form:

$$\mathbf{H}^{-1} = \mathbf{A}^{-1} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{G}^{-1} - \mathbf{A}_{22}^{-1} \end{bmatrix}$$

Back to the example



«Single-step» approach

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}\mathbf{a} + \mathbf{e} \quad \square \text{ (G)BLUP on all data}$$

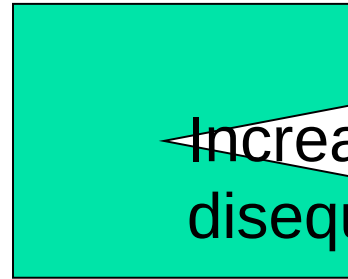
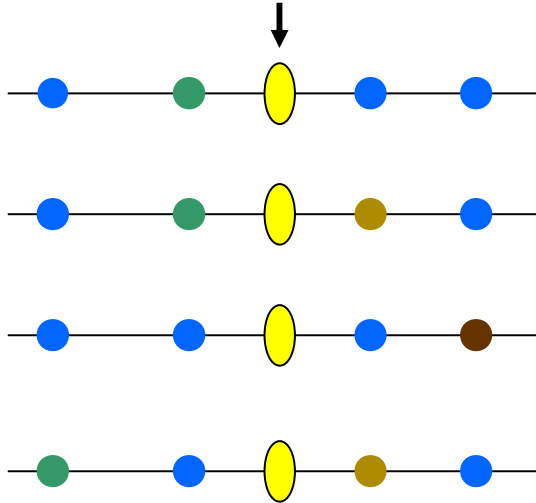
$$\begin{array}{ccccc} \diamond \mathbf{X}'\mathbf{X} & & \mathbf{X}'\mathbf{Z} & & \diamond \mathbf{Z}'\mathbf{y} \diamond \\ \diamond & & & & \diamond \\ \diamond \mathbf{Z}'\mathbf{X} & & \mathbf{Z}'\mathbf{Z} + \alpha \mathbf{H}^{-1} & & \diamond \mathbf{Z}'\mathbf{y} \diamond \\ \diamond & & & & \diamond \end{array}$$

- Huge (and relatively dense) system but with some tricks, can be applied to large data sets (Legarra)
- **Advantages:**
 - Conceptually very simple (=BLUP)
 - quasi-optimal use of information from genotyped animals
 - automatically accounts for (genomic) selection

«Single-step» approach: current issues

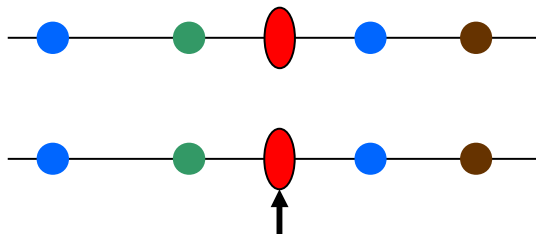
Haplotypes vs SNP

QTL frequent allele



Increase linkage disequilibrium marker(s)-QTL

- Requires marker phasing (but ☐ imputation)
- more effects to estimate



QTL rare allele

The French genomic model (since April 2015)

QTL size	Genomic evaluation
Large	traced with markers □ haplotype effects
Moderate	
Small	
Tiny	Consider their sum only: $\hat{u} = \sum_{j'} \hat{m}_{j'} \sim N(0, \text{pedigree relationship matrix})$ $\sim N(0, \text{pedigree relationship matrix})$ $\sim N(0, \text{genomic relationship matrix})$

$$\square \text{ DGV} = \sum_{j=1, \dots, K}^+ \hat{h}_j$$

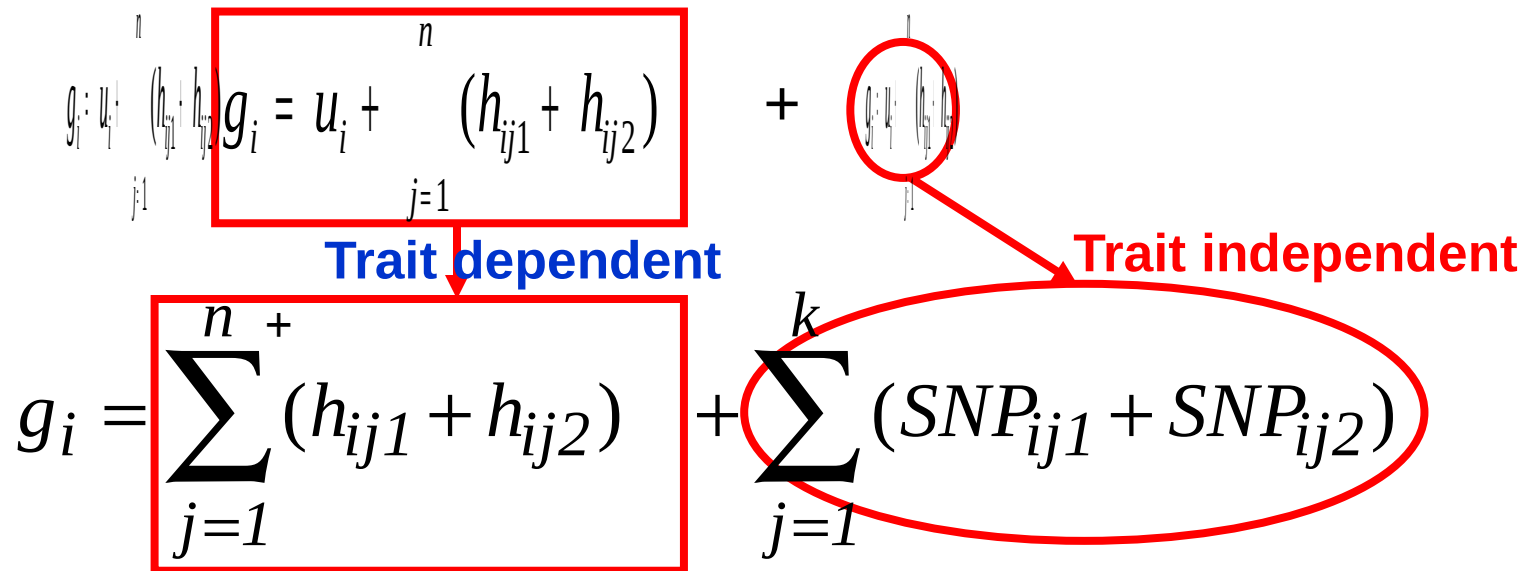
Consider their sum only:

$$\hat{u} = \sum_{j'} \hat{m}_{j'} \sim N(0, \text{pedigree relationship matrix})$$
~~$$\sim N(0, \text{pedigree relationship matrix})$$~~

$$\sim N(0, \text{genomic relationship matrix})$$

SNP & Haplotypes

$$g_i = u_i + \sum_{j=1}^n (h_{ij1} + h_{ij2}) + \sum_{j=1}^k (SNP_{ij1} + SNP_{ij2})$$



Trait dependent

Trait independent

Comparison of genomic evaluation methods

- correlation between GEBV and DYD in validation population

