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The impact of information that reduces ambiguity on the use of pesticides: a theoretical and experimental approach

Stéphane Couture Stéphane Lemarié Sabrina Teyssier
Pascal Toquebeuf

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Very preliminary version

1 Introduction

The harmful effects of pesticides on the health of users and on the environment are increasingly recognized and publicized (Aubertot *et al.*, [1]). In this context, the public authorities have the desire to increase their control for reducing pesticide uses. In this sense, the French government launched in 2008 the "*Ecophyto 2018*" plan to reduce the use of these products by 50% over a ten-year period, compared to the practice benchmark in 2008. Despite the substantial resources implemented, the overall results of this plan remain disappointing (Guichard *et al.*, [9]). Farmers use pesticides to protect their crops against plant disease risk. Such a risk is often misunderstood and underestimated by farmers, and its assessment is uncertain at the moment. Therefore farmers choose their level of pesticides using their knowledge and information mostly available by early-warning systems for plant disease. Indeed, such systems provide farmers with information about the phytosanitary risk assessment and the immediate and delayed consequences of farmers' practices (Reboud *et al.*, [15]). However it is presently observed that the impact of plant disease early-warning system on farmers' crop protection practices is not straightforward.

The objective of this paper is to study the effect of the information provided by a plant disease early-warning system about the assessment of phytosanitary risk on the farmers' pesticide use in an ambiguous context. Such an impact is theoretically and empirically analyzed. Indeed, a farmer can achieve a costly phytosanitary treatment aiming at reducing the crop damage due to a pest. Such an attack is likely to occur with a not necessarily well known probability. An early-warning system allows a better knowledge of this probability.

In agricultural economics, rich literature already exists, but rather to highlight the farmers' behavior in terms of input and output choices under risk taking into account their risk preferences (Chavas *et al.*, [5]). More specifically, many studies have focused on the assessment of farmers' risk preferences, using econometric (Couture *et al.* [6]) or experimental (Brunette *et al.*, [3]) approaches. It is generally found that farmers are risk averse. In an ambiguous context with not precisely known risk, ambiguity preferences in addition to risk preferences may explain farmers' decisions (Gassmann, [7]). However, few studies aim at quantifying farmers' ambiguity preferences (Gassmann, [7]). This work will contribute to filling this shortcoming.

Moreover, in an ambiguous situation, it is possible to provide information reducing the existing uncertainty on the definition of risk, and affecting farmers' choices. Therefore this information has value. In a theoretical and general framework, some authors (Nocetti, [13]; Hoy *et al.*, [11]; Snow, [16]) analyze the information value that decision makers place on the acquisition of information that partially or completely resolves uncertainty over the correct distribution of outcomes, and study how attitude towards ambiguity affect the value of information, in an ambiguous context (see Gollier, [8] for a risk context). They use the model of smooth ambiguity preferences proposed by Klibanoff *et al.* [12], according to the instrumental value of information acquisition (liked or disliked value). An important unanswered question is whether these theoretical conclusions are empirically verified. To our knowledge, only Peysakhovich and Karmarkar [14] explore empirically how the addition of partial information affects decision taken under ambiguity, using experiments in both gain and loss domains. They find that when information supporting a favorable (unfavorable) result increases (has less impact on) valuation of an ambiguous prospect.

The evaluation of such an information has been yet realized for sequential agricultural decision problem under risk (Carpentier, [4]; Bontems et Thomas, [2]; Williams and Johnson, [17]), but to our knowledge, no study has the purpose of assessing the value of information reducing ambiguity. Our work is the first such attempt that aims to answer the following question: does providing information reducing the ambiguity on plant

disease risk affect farmers' pesticide use?

Our approach chosen to achieve this goal is based first on a theoretical model allowing to analyze the farmer's behavior in an ambiguous context, and second on an experiment with farmers. First, we have proposed a theoretical self-insurance model in which a farmer may use a costly pesticide reducing the damage in case of pest attack. The farmer has both risk and ambiguity preferences (α -MEU functional). We show that if the farmer has ambiguity aversion, then providing information reduces the interest in using pesticide. This type of behavior could not be described by the expected utility model. An advantage of our theoretical approach is therefore to provide a theoretical rational basis for the intuition that an improvement in the pest risk knowledge could lead to a reduction in the pesticide use. More the α -MEU model allows us to assess information value that is the amount the farmer is willing to pay to improve the knowledge of the plant disease probability. We show that information that reduces ambiguity has a positive (negative) value for a ambiguity-averse (ambiguity-prone) farmer.

Second, we have conducted framed field experiments with 84 farmers and agricultural students, familiar with the phytosanitary problem. The main interest of such experiments is to have a strong control of the conditions allowing a perfect analysis of farmers' behavior and more precisely a robust study of the impact of additional information on this behavior. It is then possible to assess the effect of additional information on the optimal pesticide use, other things being equal (the same input and output prices, the same technical context). In this experiment, farmers and agricultural students faced a simple but real phytosanitary problem such as the brown rust in soft wheat and the rape winter stem weevil. All the participants faced six situations, three risky situations and three ambiguous ones. For each situation, they had to decide to use or not pesticide for fourteen levels of costs associated to the corresponding economic gains. Such an experiment allows us to quantify the risk and ambiguity preferences of the participants, to know their optimal choice in the different situations, and to quantify the value of the information reducing ambiguity. Our first analysis shows that participants have heterogeneous risk and ambiguity preferences.

We observe that a reduction in ambiguity due to a good (bad) information (i.e. a decrease (increase) in the pest attack probability) leads to a decrease (increase) in the pesticide use. However, we find that an additional information has globally little effect on the frequency of treatment, but at a more disaggregated level, this induces additional pesticide use for some groups and contrary a reduction in pesticide use for other groups.

The paper is organized as follows. Section 2 presents the theoretical

model and the expected impact of information reducing ambiguity on farmers' pesticide use as well as the value of information. The experimental design is detailed in section 3. Section 4 gives the results, while section 5 concludes.

2 Theoretical model

The decision criterion Consider a farmer with initial wealth w_0 who faces the risk of losing an amount $\ell \in (0, w_0)$ in the event A , which may occur with probability p . She may reduce this loss with a self-insurance activity by an amount $I < \ell$, but she then has to pay a cost c whatever happens. Overall, decisions are \bar{T} , which is "doing nothing", and T , that is "to apply a phytosanitary treatment". Consecutively, the farmer's final wealth is a risky asset, whose outcome depends on the realization of event A or its complementary, N . Mathematically, \bar{T} is the random variable that yields a final wealth $w_f(\bar{T}, A) = w_0 - \ell$ in the accident state and $w_f(\bar{T}, N) = w_0$ in the no accident state, and T is the r.v. that yields $w_f(T, A) = w_0 - \ell + I - c$ in the first case and $w_f(T, N) = w_0 - c$ in the second case.

In a purely risky set-up, in which the farmer knows the probability p of incurring the loss ℓ , her preferences are represented by the expected utility functional $E_p U(\cdot)$ defined as:

$$E_p U(X) = pU[w_f(X, A)] + (1 - p)U[w_f(X, N)], \quad (1)$$

for $X \in \{\bar{T}, T\}$. In our set-up, introducing ambiguity means that the farmer only knows that the probability p lies in the interval $P = [p_{min}, p_{max}]$. We assume her preferences to be represented by an α -MEU functional¹ $V(\cdot)$ defined by:

$$V(X) = \alpha E_{p_{max}} U(X) + (1 - \alpha) E_{p_{min}} U(X), \quad (2)$$

where $E_{p_{max}} U(X)$ (resp. $E_{p_{min}} U(X)$) is the minimal (resp. maximal) expected utility of X . In this model, U is strictly increasing and depicts risk attitude as in the classical expected utility model. $\alpha \in [0, 1]$ depicts the decision maker ambiguity aversion. Specifically, $\alpha = 1$ displays pure pessimism whereas $\alpha = 0$ represents pure optimism.

We assume that, under risk, the unique probability p is equal to $\bar{p} = (p_{min} + p_{max})/2$. Decisions are then taken according to the center of the interval P . This unbiasedness condition is also assumed by Snow (2010, 2011) and Alary et al. (2013) in the context of the smooth ambiguity model. It ensures that the objective probability \bar{p} is unaffected by the presence of ambiguity. It is then natural to define ambiguity as the diameter of the

¹See Ghirardato et al. (2004) for an axiomatic foundation.

interval, $a = p_{max} - p_{min}$. It has a simple interpretation, since it measures how important is the deviation from the risky situation.

When $a = 0$ or, equivalently, $\alpha = 1/2$, decision criteria (1) and (2) coincide. We then set this value of α as ambiguity neutrality, since expected utility maximizers are ambiguity neutral. Therefore, $\alpha > (<)1/2$, which implies the utility decreases (increases) comparatively to the expected utility case, is interpreted as strict ambiguity aversion (loving). With the introduction of ambiguity, the DM places a supplementary weight on the worst (best) outcome, comparatively to an ambiguity neutral farmer. Overall, the parameter α supplies a complete characterization of ambiguity attitudes.

2.1 Self-insurance under ambiguity

Let \tilde{I} denotes the random variable T when the cost is $c = 0$:

$$w_f(\tilde{I}, A) = w_0 - \ell + I, \quad w_f(\tilde{I}, N) = w_0$$

Let c^* be the cost level for which the farmer is indifferent between apply a phytosanitary treatment and doing nothing. Hence c^* is the WTP for an increase I of the wealth in the accident state when ambiguity attitude is α and ambiguity is a , that is $V(T^*) = V(\bar{T})$.

Since $U' > 0$, an increase in $V(\tilde{I}) - V(\bar{T})$ implies that c^* must increase to maintain the equality. We then wish to know how the difference $V(\tilde{I}) - V(\bar{T})$ changes when a varies. Therefore, given ambiguity aversion, one should expect that an increase in ambiguity yields an increase in c^* as well. The partial derivative of $V(\tilde{I}) - V(\bar{T})$ with respect to a is

$$\left(\frac{1}{2} - \alpha\right)[U(w_0 - \ell) - U(w_0 - \ell + I)], \quad (3)$$

which is positive when $\alpha > 1/2$, negative when $\alpha < 1/2$ and null otherwise. It implies that c^* is increasing in a in the first case and decreasing in a in the second case.

Proposition 1. *For ambiguity averse (loving) decision makers, the willingness to pay for self-insurance increases (decreases) with greater ambiguity.*

Proposition 1 means that ambiguity averse farmers should be more willing to apply a phytosanitary treatment when ambiguity increases. Therefore, if we consider that most of farmers are ambiguity averse, one should expect that if we give information that reduces ambiguity to farmers, they should be less willing to use phytosanitary products.

Notice that, given a positive level of ambiguity, the difference $V(\tilde{I}) - V(\bar{T})$ is increasing in α , too. This implies that an increase in ambiguity aversion increases the willingness to pay for self-insurance.

2.2 The value of information

The value of information that reduces ambiguity Following Snow (2010), we define the value of information as the DM's willingness to pay for a reduction/elimination of ambiguity. Intuitively, an ambiguity averse farmer should be ready to pay a positive amount to reduce or eliminate ambiguity.

Let $a_1 > a_2$, such that the situation with ambiguity a_1 is more ambiguous than situation with ambiguity a_2 . Given ambiguity a_j , $j = 1, 2$, the resulting probabilities interval is $P_j = [\bar{p} - a_j/2, \bar{p} + a_j/2]$ and we write as V_j the corresponding α -MEU.

Let X_j be the optimal choice given criterion (2) when ambiguity is a_j , that is $X_j = \max_{X=T, \bar{T}} V_j(X)$.

The value of information that reduces ambiguity, denoted as r , is the willingness to pay for information that reduces ambiguity from a_1 to a_2 : $V_2(X_2 - r) = V_1(X_1)$.

When the decision maker is ambiguity neutral, her preferences are represented by $E_{\bar{p}}U$ for any ambiguity level a . Therefore, information that reduces ambiguity has a null value, as it is intuitive. Otherwise, this information may have a positive or a negative value, depending on if the decision maker is ambiguity averse or ambiguity loving.

Proposition 2. *Information that reduces ambiguity has a positive (negative) value for decision makers who are ambiguity averse (ambiguity loving).*

The value of information that eliminates ambiguity Given ambiguity a_j , the value of information that eliminates ambiguity e is defined as the individual's willingness to pay for an ambiguity elimination: $E_p(X_0 - e) = V_j(X_j)$, where $X_0 = \max_{X=\bar{T}, T} E_{\bar{p}}U(X)$.

Under ambiguity neutrality, introducing ambiguity does not change anything. However, when $\alpha > 1/2$, the value of information that eliminates ambiguity is positive and increases when ambiguity increases. On the contrary, when $\alpha < 1/2$, this value is negative and decreases when ambiguity increases.

Proposition 3. *When the farmer is not ambiguity neutral, the absolute value of willingness to pay for information that resolves ambiguity increases with greater ambiguity.*

The same implication holds when, given a positive level of ambiguity, ambiguity attitude changes. Specifically, the value that eliminates ambiguity is increasing in α .

Proposition 4. *The willingness to pay for information that resolves ambiguity increases with greater ambiguity aversion.*

The value of a early-warning system (EWS) for plants disease In most of economic situations, ambiguity is not resolved by means of a reduction in the radius of probabilities intervals. Rather, the decision maker learns an information from an experiment and this information tells us which are probabilities are relevant for the decision maker. This is the usual notion of information in decision theory. For a EWS, we model information as a state space $\{H, L\}$, whose realization learns to the decision maker if the true probability of incurring a loss is high or low. If it is high, the farmer knows that the true probability of the accident state lies in $H = [\bar{p}, \bar{p} + a/2]$. If it is low, the farmer knows that this probability lies in $L = [\bar{p} - a/2, \bar{p}]$. She has an equal chance to learn if the true probability of the accident state is more or less than the "mean" probability \bar{p} : H and L have an equal probability to occur.

Conditionally to states H and L , individual preferences are represented by the following conditional α -MEU:

$$\begin{aligned} V[X|H] &= \alpha E_{p_{max}} U(X) + (1 - \alpha) E_{\bar{p}} U(X) \\ V(X|L) &= \alpha E_{\bar{p}} U(X) + (1 - \alpha) E_{p_{min}} U(X) \end{aligned}$$

for $X = T, \bar{T}$. The updating theory underlying our approach is described in appendix A3. Let us define X_H and X_L as:

$$\begin{aligned} X_H &= \max_{X=T, \bar{T}} V(X|H) \\ X_L &= \max_{X=T, \bar{T}} V(X|L) \end{aligned}$$

The value of the EWS, denoted as v , is defined as the decision maker's willingness to pay for information processed by $\{H, L\}$:

$$\frac{1}{2} V(X_H - v|H) + \frac{1}{2} V(X_L - v|L).$$

Proposition 5. *Let $a_2 = a_1/2$. Then the willingness to pay for the information $\{H, L\}$ is equal or superior to the willingness to pay for information that reduces ambiguity from a_1 to a_2 .*

As it is shown in appendix A3, when the decision maker makes the same choice in situation with ambiguity $a_2 = a_1/2$ and consecutively to the experiment $\{H, L\}$, then the value of information coming from this experiment is equal to the value of information that reduces ambiguity. Indeed, in this case, information processed by $\{H, L\}$ is equivalent to information that reduces ambiguity and there is no choice reconsideration.

Proposition 5 has interesting implications about the link between ambiguity attitude and the value of the experiment $\{H, L\}$. Indeed, it implies that this value is strictly positive if the decision maker is strictly ambiguity averse. In addition, it may be negative for an ambiguity lover decision maker.

Furthermore, the information $\{H, L\}$ offers a DM an opportunity to reconsider her ex-ante choice. New information give better opportunities and that is why, independently of her ambiguity attitude, the DM may take some advantage of new information. Nevertheless, if information results in an increase of the knowledge of the true probability, i.e. if it results in a reduction of ambiguity, an ambiguity loving DM may reject it. The difference $(v - r)$ may then be interpreted as the value of a choice reconsideration. That is why, even for a Bayesian decision maker holding additive beliefs, that is $\alpha = 1/2$, information processed by $\{H, L\}$ may have a strictly positive value, if it results in a change in decisions. Nonetheless, under risk, that is $P = \{\bar{p}\}$, information processed by $\{H, L\}$ has a null value, since the experiment $\{H, L\}$ does not improve the decision maker knowledge.

Insurance cost and the value of information Let us resume the link between the insurance cost and the value of information. We need to fix ambiguity attitude for study the role risk attitude. We shall assume that the decision maker is ambiguity averse.

Figure 1 shows the link between v , r and c . We get a step function for both $v(c)$ and $r(c)$. [HERE INSERT FIGURE 1]

Let us consider the link between r and c .

At the first step, $c \leq c_2^*$, the DM choose to be insured under both ambiguity a_1 and a_2 . In this case, the value of information that reduces ambiguity increases (decreases) with c for a risk-averse (loving) DM. It does not depend on c for a risk neutral decision maker.

At the second step, $c_2^* < c \leq c_1^*$, r increases along with c , regardless risk-attitude. The explanation is the following. An increase in c decreases the value of $V_1(\tilde{x}_1)$ while leaving $V_2(\tilde{x}_2)$ unchanged, hence it decreases the difference $V(\tilde{x}_2) - V(\tilde{x}_1)$. But c is an increasing function of this difference. In this case, risk-aversion reinforces the effect of ambiguity aversion on r .

At the third step, $c > c_1^*$, the decision maker never chooses to be insured, and thus r does not depend on c .

Notice that, for $a_2 = a_1/2$, v and r coincide when $c \leq c_2^*$ and $c > c_1^*$.

For $c_L^* < c \leq c_1^*$, the farmer chooses to apply a phytosanitary treatment when she has no information and in the event where the probability of a plant disease is high. v is greater than r since the EWS leads her to change her decisions in the event H .

When $c_1^* < c \leq c_H^*$, we have, again, $v > r$. In this case, the farmer treats only if event H occurs, but does not treat otherwise. Since the EWS entails a change a decision in this event, its value is greater than the value of information that reduces ambiguity.

3 Experiment

In order to test the theoretical model described above, we conducted a framed field axeperiment (Harrison and List, [10]) where farmers and students in farming disciplines familiar with the phytosanitary problem make decisions in a highly controlled environment contextualized in farming terms. They faced a simple but real phytosanitary problem such as the brown rust in soft wheat and the rape winter stem weevil. The experimental design is such that participants choose between applying a phytosanitary treatment or not under sequential conditions.

3.1 Decisions

Participants in the experiment had to make choices in six various situations that differ by the degree of risk and ambiguity regarding the potential phytosanitary attack. For each degree of risk and ambiguity, participants had to choose between two options, “do not treat” and “treat”, for different costs of treatment. Without treatment, the gain without attack and the gain with attack are constant across treatment costs. With treatment, the gain without attack and the gain with attack are reduced by the same amount as the treatment costs increase. Gains without attack are higher without treatment rather than with treatment whereas gains with attack are lower without treatment rather than with treatment. The treatment option is a self-insurance regarding the phytosanitary attack.

Parameters Because we wanted to have realistic options for farmers and students in farming disciplines, we calculated the gains under the different options based on realistic values. We set up two contexts and applied one or the other to the participants depending on the usual culture of the farmer. We randomized the context among students. For soft wheat, we assumed an average return of 6 tons per hectare, 700 Euros per hectar of loads and, an average wheat price of 180 Euros per ton. For rape winter, we assumed an average return of 3 tons per hectare, 700 Euros per hectar of loads and, an average rape price of 360 Euros per ton. For both cultures, we assume that the loss of return in case of attack without treatment is 25% while it is

only 5% if there is a treatment that is set up. Amounts are identical under the various situations for both contexts. Indeed, without treatment, the gain is 380 Euros per hectare if there is no attack and 110 Euros per hectare if there is attack. With treatment, the maximum gain is for the lowest cost of treatment that we stated between 20 Euros and 150 Euros per hectare.² When the treatment cost is 20 Euros, the gain is 360 Euros per hectare if there is no attack and 306 Euros per hectare if there is attack. The treatment cost increases with a 10 Euro path and gains are therefore decreased by the same amount. For each situation, participants had to choose between “do not treat” and “treat” in table 1.

Table 1: Set of decisions for each situation

Without treatment		Your decision		With treatment	
Gain if no attack	Gain if attack	Do not treat	Treat	Gain if no attack	Gain if attack
€ 380	€ 110			€ 360	€ 306
€ 380	€ 110			€ 350	€ 296
€ 380	€ 110			€ 340	€ 286
€ 380	€ 110			€ 330	€ 276
€ 380	€ 110			€ 320	€ 266
€ 380	€ 110			€ 310	€ 256
€ 380	€ 110			€ 300	€ 246
€ 380	€ 110			€ 290	€ 236
€ 380	€ 110			€ 280	€ 226
€ 380	€ 110			€ 270	€ 216
€ 380	€ 110			€ 260	€ 206
€ 380	€ 110			€ 250	€ 196
€ 380	€ 110			€ 240	€ 186
€ 380	€ 110			€ 230	€ 176

Situations In total, participants had to make decisions in six situations: three under risk and three under risk and ambiguity. The table participants had to fill in is table 1 that is identical for all situations. The only parameter that varies between situations is the probability of attack. For situations under risk, the probability of attack is 0.1 in situation 1 (*Risk10*), 0.3 in situation 2 (*Risk30*) and 0.5 in situation 3 (*Risk50*). For situations under ambiguity,

²We are aware that some values of treatment costs are extreme as compared to real values but this range of amounts allows us to observe for which treatment cost participants modify their behavior. A sufficient range was needed to observe such changes.

the probability of attack is between 0.1 and 0.5 in situation 4 (*Ambiguity10-50*). Situations 5 and 6 correspond to situations that add some information regarding the interval of the probability of attack to participants. In situation 5, participants had to fill in two tables: in table A, the probability of attack is between 0.1 and 0.3 (good information, *Ambiguity10-30*) and in table B, the probability of attack is between 0.3 and 0.5 (bad information, *Ambiguity30-50*) (*Ambiguity10-30-50*). In situation 6, the probability of attack is between 0.2 and 0.4 (*Ambiguity20-40*).

Decisions to treat in the various situations allow us to determine how the participants' WTP for self-insurance is affected by the level of ambiguity. Our basic case is the situation with the probability of attack is between 0.1 and 0.5 (*Ambiguity10-50*). A plant disease early-warning system may strongly reduce ambiguity and provide farmers an unambiguous probability of attack: 0.1 (*Risk10*), 0.3 (*Risk30*) or, 0.5 (*Risk50*). If the probability of attack is uniformly distributed in the interval, 0.3 is the probability of attack when ambiguity is eliminated. Although we do not give participants any information regarding the distribution of the probability of attack in the interval, we assume as more likely that participants estimate this type of distribution rather than a more optimistic or pessimistic one. Comparing participants' decisions to treat between *Ambiguity10-50* and *Risk10* give the change in decisions when ambiguity is eliminated. Probabilities of 0.1 and 0.5 correspond to control cases in the optimistic and pessimistic cases, respectively.

Nevertheless, a plant disease early-warning system is more likely to reduce ambiguity instead of eliminating it. The ambiguity is reduced when the interval of the probability of attack changes to 0.2 to 0.4 (*Ambiguity20-40*). Comparing participants' decisions in *Ambiguity10-50* and *Ambiguity20-40* give the change in decisions when ambiguity is reduced. Another type of ambiguity reductions may be that farmers receive information regarding the interval of probability of attack comes. They may receive a good information that informs them that the probability of attack is between 0.1 and 0.3 or a bad information that informs them that the probability of attack is between 0.3 and 0.5. Comparing decisions in *Ambiguity10-50* and *Ambiguity10-30-50* captures the impact of such an information. *Ambiguity10-30-50* and *Ambiguity20-40* are constant in ambiguity interval (interval of 0.2) but differ only by the type of information farmers receive. Comparing decisions if these two situations gives the impact of such information provision should not impact decisions to treat as the level of ambiguity is identical. Nevertheless, it is possible that participants do not symmetrically evaluate positive and negative information. This is this effect that the comparison of *Ambiguity10-30-50* and *Ambiguity20-40* captures.

Our design also allows us to estimate the participants' value of information based on their elicited risk and ambiguity aversion. We then can determine how the value of information is affected by the level of ambiguity. Following proposition 1 of the theoretical section, predictions relating to the impact of a reduction of ambiguity on participants' decisions to treat depends on ambiguity preferences of participants. An ambiguity neutral participant should not change her behavior when ambiguity is reduced. Following proposition 2, we expect that the value of information is positive for ambiguity averse participants and negative for ambiguity loving ones. Propositions 3 and 5 define how ambiguity preferences affect the value of information. These theoretical predictions are summarized in table 2.

Table 2: Predictions on the impact of a reduction of ambiguity on decision to treat and value of information

	Ambiguity averse	Ambiguity loving
WTP for self-insurance	Positive or negative, depends on risk preferences	
<i>Risk30 - Ambiguity10-50</i>	Negative	Positive
<i>Ambiguity20-40 - Ambiguity10-50</i>	Negative	Positive
<i>Ambiguity10-30-50 - Ambiguity10-50</i>	Negative	Positive
<i>Ambiguity20-40 - Ambiguity10-30-50</i>	Null	Null
Value of information	Positive	Negative
(<i>Ambiguity20-40 - Ambiguity10-50</i>) - (<i>Risk30 - Ambiguity10-50</i>)	Negative	Positive
(<i>Ambiguity10-30-50 - Ambiguity10-50</i>) - (<i>Risk30 - Ambiguity10-50</i>)	Negative	Positive
(<i>Ambiguity20-40 - Ambiguity10-50</i>) - (<i>Ambiguity10-30-50 - Ambiguity10-50</i>)	Negative	Positive

3.2 Procedures

The experiments have been conducted with farmers and agricultural students. The experiment is incentive compatible as participants receive real money in cash at the end of the experiment depending on their own decisions and on the eventuality of an attack or not. In total, 84 participants answered our experiment: 25 farmers and 59 agricultural students. 48 participants took their decisions in the soft wheat context and 36 in the rape winter.³ The experiments were conducted at the chamber of agriculture close to the farmers exploitation and in the high school of agricultural students.

³Because no significant differences are observed nor between farmers and agricultural students neither between type of crop, we present the results for the whole sample.

When all participants were seated in the room, we starting reading the instructions aloud (instructions are presented in Appendix XX). Before starting the experiment, the participants had to fill in a situation with other numbers and values to make sure that they understood the mechanism. Then, we explained the three risk situations and let the participants fill in successively the three risk situations. We then explained the three ambiguity situations and let the participants fill in successively the three ambiguity situations. The participants always started by the risk situations before the ambiguity situations as they were more easily understandable. The ambiguity situations added some complexity in the decision process. To indicate clearly the probabilities of phytosanitary attacks, we physically presented them with urns and red and blue balls. At the end of the experiment, a ball was drawn in the urn. If the ball was red, it meant that there was an attack while if the ball was blue, it meant that there was no attack. For the risk situations, there was an urn with 10 blue and red balls. There was 1 red ball in *Risk10*, 3 red balls in *Risk30* and, 5 red balls in *Risk50*. For the ambiguity situations, there was 10 blue, red and, uncolored balls. In the uncolored balls, we placed a paper with a red or blue circle that could not be seen by the participants. There was 1 red ball, 5 blue balls and 4 uncolored balls in *Ambiguity10-50*. In *Ambiguity10-30-50*, there was two urns (one for the good information and one for the bad information): in *Ambiguity10-30*, there was 1 red ball, 7 blue balls and 2 uncolored balls and, in *Ambiguity30-50*, there was 3 red balls, 5 blue balls and 2 uncolored balls. In *Ambiguity20-40*, there was one urn with 2 red balls, 6 blue balls and 2 uncolored balls.

To determine participants' payment, one situation was randomly selected with a dice, then the urn corresponding to the selected situation was constituted and a ball was drawn from it. A treatment cost was also randomly selected: we drawn one paper from a bag containing 14 papers with a value of each treatment cost written on it. Participants' gain was the gain corresponding to their decision to treat or not in the selected situation and treatment cost depending on whether an attack occurred (red ball) or not (blue ball). The payment participants received in cash was their gain divided by ten.

4 Results

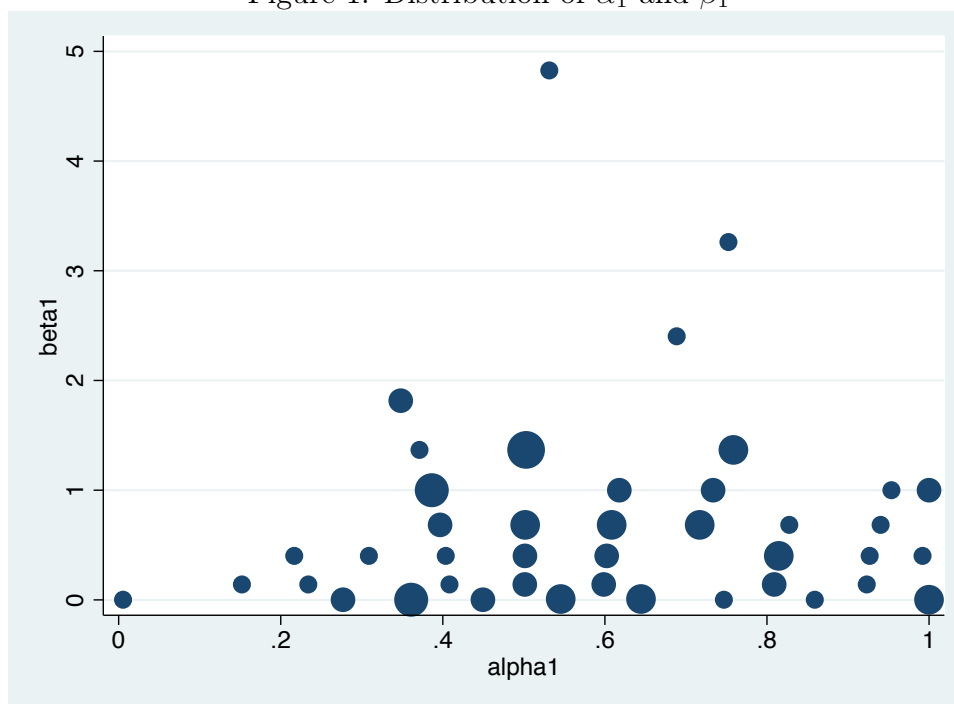
We first estimate participants' individual risk and ambiguity preferences. Second, we investigate the impact of a reduction of ambiguity on the participants' likelihood to treat. Third, we determine the value participants attribute to an information reducing ambiguity and compare the value of the

different types of information.

4.1 Ambiguity preferences

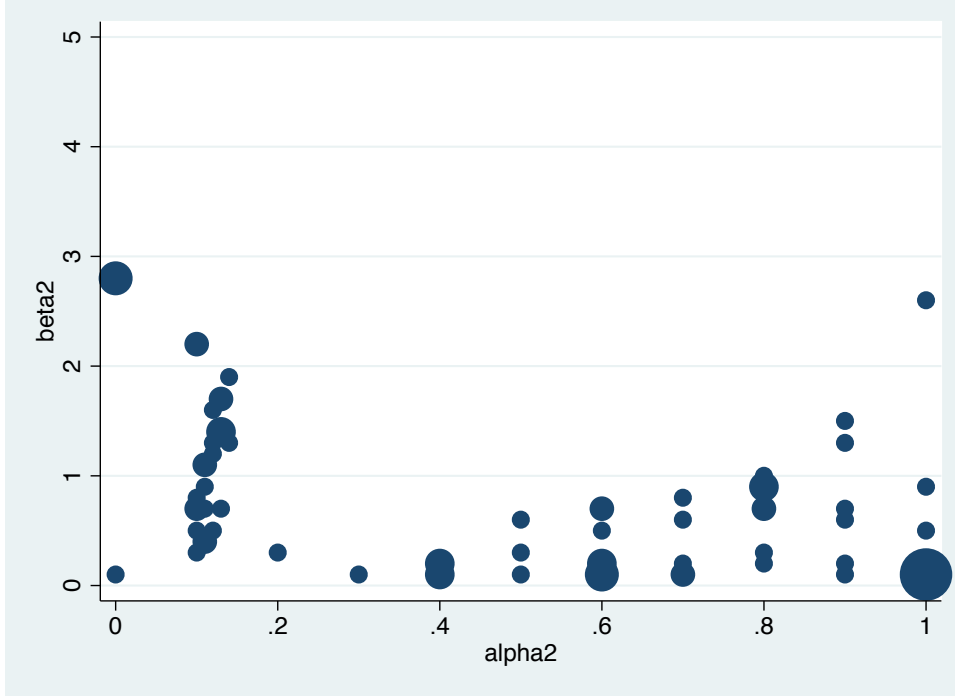
We estimate individual ambiguity aversion and risk aversion based on the number of decisions to treat in each situation. We define two criteria. The first criterium is based on the participant's decisions to treat in *Risk30* to estimate her risk preferences β_1 and in *Ambiguity10-50* to estimate her ambiguity aversion α_1 conditionnally on the participant's estimated risk preferences. These twop situations are the situations that serve as references. A second criterium is to consider the participant's decision to treat in all situations and determine the values of risk and ambiguity aversion that maximize the number of decisions to treat they explain. We then estimate β_2 and α_2 . According to the two types of estimations, we obtain that around 30% of participants are ambiguity loving and then 70% are ambiguity neutral or ambiguity averse. Figures 1 and 2 display the distribution of ambiguity aversion depending on risk aversion according to these two criteria, respectively.

Figure 1: Distribution of α_1 and β_1



We are able to conjointly estimate risk aversion and ambiguity aversion according to the α -MEU functional. Following Harrison and Ruström (2005),

Figure 2: Distribution of α_2 and β_2



we specify likelihoods conditional on the model where we calculate individual expected utility for each lottery, which means for each treatment cost in each situation. Participants have made individually 98 choices. Each expected utility was defined for a candidate estimate of risk preferences (β) and a candidate estimate of ambiguity preferences following the α -MEU functional. We calculate the difference between individual expected utility when he chooses not to treat and when he chooses to treat: $\Delta V = V_T - V_{\bar{T}}$. We assume that this latent index is linked to the participants' choices between doing nothing and applying a phytosanitary treatment through a standard cumulative normal distribution function ($\Phi(\Delta V)$). The conditional log-likelihood based on the α -MEU functional and estimates of β and α is then:

$$\ln L^V(\beta, \alpha, X_i) = \sum_i l_i^V = \sum_i [X_i \ln(\Phi(\Delta V)) + (1 - X_i) \ln(-\Phi(\Delta V))] \quad (4)$$

where $X_i = 1$ if the participant chooses to treat in decision i , i.e. for a treatment cost in a situation and, $X_i = 0$ if the participant chooses not to treat. Table 3 gives the maximum likelihood estimation of the risk and ambiguity preferences parameters of the α -MEU functional. We used clusters at the participants' level. In model (1), we consider the decisions of the

participants only in situations *Risk30* and *Ambiguity10-50* while in model(3) we consider all decisions of the participants. Model (2) and Model (4) include the status (farmer of agricultural student) and the type of crop (soft wheat or rape winter) of the participant. We find that the status and type of crop of the participants do not significantly affect their decision to treat or not and then their risk and ambiguity preferences are not conditional on these two variables.

Table 3: Risk and ambiguity preferences estimations

	Model (1)	Model (2)	Model (3)	Model (4)
β	0.4586*** (0.0153)	0.4726*** (0.0237)	0.4465*** (0.0136)	0.4610*** (0.0206)
Farmer		-0.0193 (0.0343)		-0.0030 (0.0301)
Soft wheat		-0.0139 (0.0330)		-0.0231 (0.0285)
α	0.4586*** (0.0385)	0.5927*** (0.0577)	0.5761*** (0.0534)	0.5747*** (0.779)
Farmer		-0.1135 (0.1008)		-0.1236 (0.1322)
Soft wheat		0.0279 (0.0777)		0.0683 (0.1110)

4.2 Self-insurance

Table 4 presents the marginal effects of a probit estimation of participants' decision to treat in the different situations. To control for correlations between standard errors at the individual level we use clusters at the subject level. The reference situation is the situation with the highest ambiguity level, i.e. *Ambiguity10-50*. We observe that when we consider all participants, there is no significant effect of an information reducing ambiguity on the decision to treat. This phenomenon is explained by the fact that our sample consists in both ambiguity averse and ambiguity loving individuals. For both ambiguity preferences estimation, we find that ambiguity averse individuals treat significantly less when they receive information reducing ambiguity while ambiguity loving participants treat significantly more. This is when the reduction of ambiguity should not change the average probability. When the probability of attack is decreased to 0.1 or increased to 0.5, the likelihood of choosing to treat respectively decreases and increases for the whole sample. We also ran a regression in which we differentiate the good

and bad information in *Ambiguity10-30-50*. We find that, for both ambiguity averse and ambiguity loving participants are less likely to treat when they receive a good information and more likely to treat when they receive a bad information. This corresponds to the fact that this information, on top of reducing ambiguity, it changes the participants' expectations regarding the average probability of attack.

Table 4: Probit regression explaining the decision to treat - marginal effects

	Model (1)	Model (2) $\alpha_1 < 0.5$	Model (3) $\alpha_1 \geq 0.5$	Model (4) $\alpha_2 < 0.5$	Model (5) $\alpha_2 \geq 0.5$
<i>Risk10</i>	-0.398*** (0.0320)	-0.257*** (0.0529)	-0.462*** (0.0362)	-0.246*** (0.0383)	-0.481*** (0.0383)
<i>Risk30</i>	-0.0218 (0.0278)	0.225*** (0.0440)	-0.138*** (0.0255)	0.161*** (0.0479)	-0.109*** (0.0311)
<i>Risk50</i>	0.255*** (0.0304)	0.402*** (0.0575)	0.187*** (0.0322)	0.385*** (0.0632)	0.201*** (0.0323)
<i>Ambiguity20-40</i>	-0.0285 (0.0242)	0.0969** (0.0378)	-0.0862*** (0.0301)	0.0201 (0.0279)	-0.0518 (0.0355)
<i>Ambiguity10-30-50</i>	-0.0318 (0.0216)	0.0945*** (0.0268)	-0.0905*** (0.0284)	0.0569** (0.0231)	-0.0759** (0.0312)
treatmentcost	-0.00808*** (0.000922)	-0.00695*** (0.00198)	-0.00870*** (0.000977)	-0.00637*** (0.00181)	-0.00899*** (0.00103)
<i>N</i>	8232	2450	5782	2450	5782
Clusters	84	84	84	84	84
pseudo R^2	0.285	0.251	0.312	0.241	0.328

Marginal effects; Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

4.3 Value of information

Using α_1 and β_1 , we obtain the average value of information for each treatment cost ranging from 20 to 150. We estimate the value of information based on two methods: one that considers theoretical decisions and one that is based on real decisions. Table 5 presents the average value of information

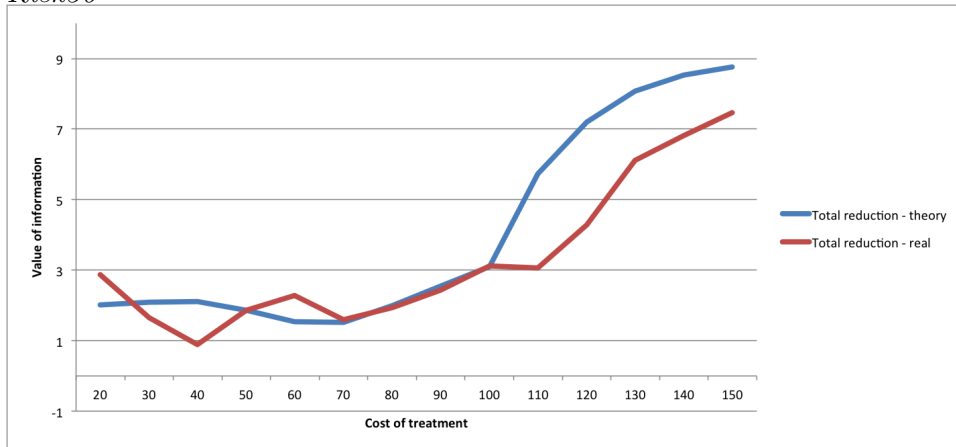
for each type of ambiguity reduction and depending on participants ambiguity aversion.

Table 5: Average value of information

	Theoretical decisions		
	All sample	$\alpha_1 < 0.5$	$\alpha_1 \geq 0.5$
<i>Risk30</i>	4.076	-12.307	11.019
<i>Ambiguity20-40</i>	1.563	-6.588	5.017
<i>Ambiguity10-30-50</i>	3.541	-4.372	6.894
	Real decisions		
	All sample	$\alpha_1 < 0.5$	$\alpha_1 \geq 0.5$
<i>Risk30</i>	3.312	-15.169	11.142
<i>Ambiguity20-40</i>	0.932	-7.311	4.426
<i>Ambiguity10-30-50</i>	2.222	-5.518	5.501

We represent in figures 4, ?? and 5 the value of information depending on the cost of treatment when the ambiguity changes from *Ambiguity10-50* to *Risk30*, to *Ambiguity20-40* and to *Ambiguity10-30-50*, respectively.

Figure 3: Evolution of the average value of information from *Ambiguity10-50* to *Risk30*



We observe three main levels of treatment costs: low from 20 to 50, medium from 60 to 110 and high from 120 to 150. We run an OLS regression to determine how the value of information varies between the different types of reduction of ambiguity and we control for the level of treatment costs. We use as reference the change from *Ambiguity10-50* to *Risk30* that corresponds to complete reduction of ambiguity. The low class of treatment costs serves as a reference. Tables 6 and 7 present coefficients of OLS regressions explaining

Figure 4: Evolution of the average value of information from *Ambiguity10-50* to from *Ambiguity20-40*

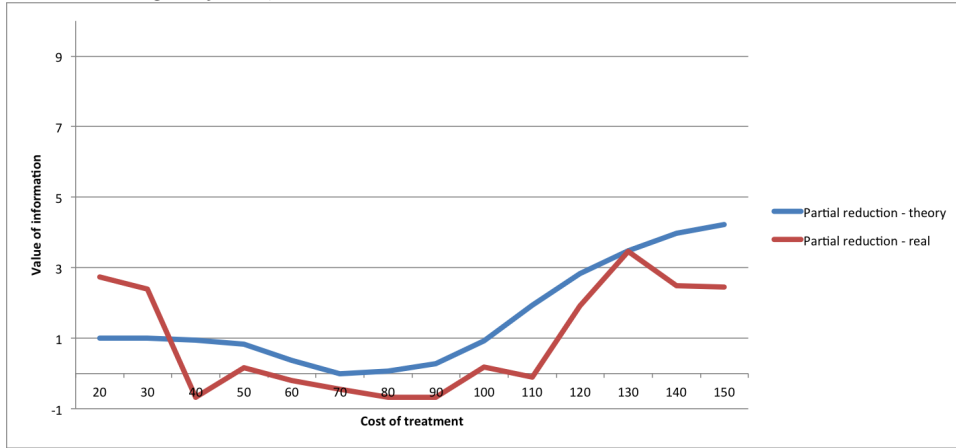
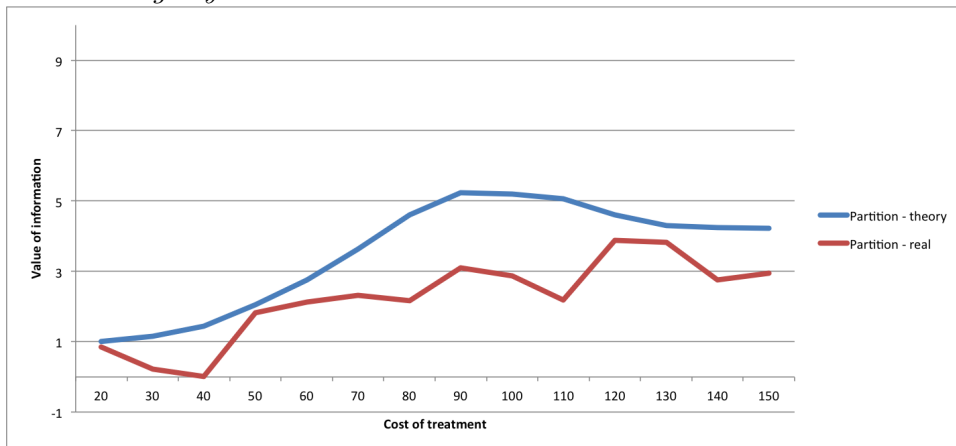


Figure 5: Evolution of the average value of information from *Ambiguity10-50* to from *Ambiguity10-30-50*



how the value of information is affected by the type of reduction of information (from *Ambiguity10-50* to *Ambiguity20-40* or to *Ambiguity10-30-50*), based on theoretical and real estimations, respectively.

The experimental results are in line with theoretical predictions showing that the value of information is positive for ambiguity averse individuals and that they attribute a positive value to a reduction of the ambiguity level.

Table 6: OLS regression explaining the value of information - theoretical estimates

	Model (1)	Model (2)	Model (3)	Model (4)	Model (5)	Model (6)
		$\alpha_1 < 0.5$	$\alpha_1 \geq 0.5$		$\alpha_1 < 0.5$	$\alpha_1 \geq 0.5$
<i>Ambiguity20-40</i>	-2.513*** (0.775)	5.719*** (0.769)	-6.002*** (0.643)	-3.126** (1.237)	10.15*** (1.526)	-8.751*** (0.935)
<i>Ambiguity10-30-50</i>	-0.536 (0.795)	7.935*** (0.807)	-4.125*** (0.653)	-2.657** (1.274)	11.03*** (1.564)	-8.456*** (0.965)
<i>Ambiguity20-40</i> x medium cost				-0.357 (0.532)	-5.517*** (1.061)	1.829*** (0.320)
<i>Ambiguity10-30-50</i> x medium cost				2.998*** (0.572)	-1.834 (1.177)	5.046*** (0.426)
<i>Ambiguity20-40</i> x high cost				2.678*** (0.944)	-7.236*** (1.118)	6.880*** (0.757)
<i>Ambiguity10-30-50</i> x high cost				2.929*** (1.043)	-8.075*** (1.122)	7.592*** (0.857)
Intercept	4.076*** (1.521)	-12.31*** (1.828)	11.02*** (1.155)	4.076*** (1.522)	-12.31*** (1.831)	11.02*** (1.156)
<i>N</i>	3528	1050	2478	3528	1050	2478
<i>R</i> ²	0.009	0.120	0.076	0.017	0.187	0.141

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

5 Conclusion

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Table 7: OLS regression explaining the value of information - real estimates

	Model (1)	Model (2)	Model (3)	Model (4)	Model (5)	Model (6)
		$\alpha_1 < 0.5$	$\alpha_1 \geq 0.5$		$\alpha_1 < 0.5$	$\alpha_1 \geq 0.5$
<i>Ambiguity20-40</i>	-2.379** (1.093)	7.857*** (1.579)	-6.717*** (0.951)	-2.156 (1.821)	12.08*** (2.576)	-8.188*** (1.867)
<i>Ambiguity10-30-50</i>	-1.090 (1.055)	9.651*** (1.646)	-5.641*** (0.770)	-2.581 (1.622)	12.33*** (2.291)	-8.900*** (1.459)
<i>Ambiguity20-40</i> x medium cost				-1.471 (1.357)	-5.467*** (1.626)	0.223 (1.768)
<i>Ambiguity10-30-50</i> x medium cost				1.728 (1.071)	-2.525** (1.209)	3.530** (1.376)
<i>Ambiguity20-40</i> x high cost				1.425 (1.853)	-6.574 (4.019)	4.815** (1.873)
<i>Ambiguity10-30-50</i> x high cost				2.627 (1.584)	-5.597** (2.469)	6.111*** (1.829)
Intercept	3.312* (1.787)	-15.17*** (2.506)	11.14*** (1.361)	3.312* (1.788)	-15.17*** (2.511)	11.14*** (1.362)
<i>N</i>	3528	1050	2478	3528	1050	2478
<i>R</i> ²	0.004	0.083	0.051	0.007	0.102	0.071

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

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Appendix A: Mathematical appendix and proofs

First notice that the preference function $V(\cdot)$ may be rewritten as:

$$V(X) = E_p U(X) + a\left(\frac{1}{2} - \alpha\right)[U[w_f(X, N)]_U[w_f(X, A)]] \quad (5)$$

for $X = T, \bar{T}$.

Our mathematical results deal with changes in the value function $V(\cdot)$ due to changes in ambiguity attitude α and in ambiguity a . Hence it is inconvenient to write $V_{ij}(X)$ to denote the individual valuation of decision X when ambiguity attitude is α_i in situation with ambiguity a_j .

We denote as X_{ij} the optimal decision under ambiguity a_j and ambiguity attitude α_i :

$$X_{ij} = \max_{X=T, \bar{T}} V_{ij}(X).$$

When left unspecified, X_{ij} is not necessarily unique. Furthermore, for $\alpha_i = 1/2$ and/or $a_j = 0$, preferences are represented by $E_{\bar{p}}U$ and we denote by X_0 the optimal choice given this criterion.

Appendix A1: Proof of proposition 2

Let a_1 and a_2 be two ambiguity degrees such that $a_1 > a_2 \geq 0$. Let r_i be the willingness to pay for information that reduces ambiguity from a_1 to a_2 when ambiguity attitude is α_i . We wish prove that $\alpha_i > (<)1/2$ implies $r_i > (<)0$.

Let $\alpha_i > 1/2$. Assume, *per absurdum*, $r_i \leq 0$. In this case, $V_{i1}(X_{i1}) \geq V_{i2}(X_{i2})$, which implies $V_{i1}(X_{i1}) \geq V_{i2}(X_{i1})$. Since $\alpha_i > 1/2$, this is equivalent to $a_1 \leq a_2$, which is impossible.

The case with $\alpha_i < 1/2$ may be proved with the same argument.

Appendix A2 : Proof of propositions 3 and 4

Proof of proposition 3 Let e_{ij} be the value of information eliminating ambiguity a_j when ambiguity attitude is α_i .

First notice that, by proposition 2 with $a_1 = a_j$ and $a_2 = 0$, $\alpha_i > (<)1/2$ implies $e_{ij} > (<)0$.

Furthermore, $\alpha_i > 1/2$ and $a_1 > a_2$ implies $e_{i1} > e_{i2} > 0$: $\alpha_i > 1/2$ implies $V_{i1}(X_{i1}) < V_{i2}(X_{i1})$. Since $V_{i2}(X_{i1}) \leq V_{i2}(X_{i2})$, we get $V_{i1}(X_{i1}) < V_{i2}(X_{i2})$ hence $e_{i1} > e_{i2}$.

The case with $\alpha_i < 1/2$ may be proved with the same argument.

Proof of proposition 4 $\alpha_1 > \alpha_2$ implies $V_{1j}(X_{1j}) < V_{2j}(X_{1j})$. Since $V_{2j}(X_{1j}) \leq V_{2j}(X_{2j})$, we get $V_{1j}(X_{1j}) < V_{2j}(X_{2j})$ and then $e_{1j} > e_{2j}$.

Appendix A3 : Proof of proposition 5

Let $a_2 = a_1/2$ and let v be the value of the experiment $\{H, L\}$. Let $V_{ij}(\cdot|E)$ be the alpha-MEU representation of the preference conditional to information $E = H, L$, for an initial level of ambiguity a_j and ambiguity attitude α_j . We denote as X_{ij}^E the corresponding optimal choice:

$$X_{ij}^E = \max_X V_{ij}(X|E)$$

We wish prove that if $X_{i2} = X_{i1}^H = X_{i1}^L$, then $r_i = v$ and $v \geq r_i$ otherwise.

We have:

$$\begin{aligned} & \frac{1}{2}V_{i1}(X_{i2}|H) + \frac{1}{2}V_{i1}^1(X_{i2}|L) \\ &= \frac{1}{2} \left[E_{\bar{p}}U(X_{i2}) + \frac{a_1}{2} \left(\frac{1}{2} - \alpha_i \right) [U(X_N) - U(X_A)] + E_{\bar{p}}U(X_{i2}) + \frac{a_1}{2} \left(\frac{1}{2} - \alpha_i \right) [U(X_A) - U(X_N)] \right] \\ &= E_{\bar{p}}U(X_{i2}) + \frac{1}{2} \left[a_1 \left(\frac{1}{2} - \alpha_i \right) [U(X_N) - U(X_A)] \right] \\ &= V_{i2}(X_{i2}) \end{aligned}$$

Hence $v = r_i$. Furthermore, since $V_{i1}(X_{i1}^L|L) \geq V_{i1}(X_{i2}|L)$ and $V_{i1}(X_{i1}^H|H) \geq V_{i1}(X_{i2}|H)$, we must have:

$$\frac{1}{2}V_{i1}(X_{i1}^L|L) + \frac{1}{2}V_{i1}(X_{i1}^H|H) \geq V_{i2}(X)$$

which implies $v \geq r_i$ in the general case. Finally, when X_{i1}^L and/or X_{i2}^H is unique, the previous inequality is strict.