

Dynamic games applied to common resources: modeling and experimentation - preliminary analysis

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Motivation

- Without regulation, Common Pool Resources (CPR) are subject to overexploitation (Hardin, 1968)
- Ex : forest, earth, groundwater, fish stocks.
- To correctly anticipate the effect of regulation, we need to understand how agents take decisions

Objectives

- Clarify some ambiguities between discrete and continuous time, and the time horizon chosen for lab experiments
- What type of behavior will the experimental subjects exhibit : feedback, myopic, open-loop or social optimum?
- Continuous time can be approached by discrete time \Rightarrow confront theory with experimentation

Outline

- 1 Introduction
 - Motivation
 - Literature
- 2 The theoretical model
 - Infinite horizon modeling
 - The optimal control
 - The game
- 3 Theory and experimentation
- 4 Econometric analysis in continuous time
 - Preliminary results - continuous time
- 5 Discussion and conclusion
- 6 Further works

Literature

- Theoretical article : Rubio & Casino (2003) \Rightarrow continuous time, infinite horizon
- Lab experiment : Janssen & al. (2010) \Rightarrow spatial aspect
- Theoretical with lab experiment :
 - Herr & al. (1997) \Rightarrow discrete time, finite horizon
 - Oprea & al. (2014) \Rightarrow compares continuous and discrete time
 - Tasneem & al. (2017) \Rightarrow continuous time, infinite horizon

Model

- Infinite horizon framework
- Study the exploitation behavior of a renewable groundwater table by 2 identical and symmetrical farmers \Rightarrow optimal control and game
- the **continuous time** problem :

$$\max_{w_i(t)} \int_0^{\infty} e^{-rt} \left[\underbrace{aw_i(t) - \frac{b}{2} w_i(t)^2}_{\text{Gross profit}} - \underbrace{\overbrace{c_t(H(t))}^{\text{Unitary cost}} w_i(t)}_{\text{Total cost}} \right] dt \quad (1)$$

$$\text{st } \begin{cases} \dot{H}(t) = R - \alpha w_i(t) : \text{the optimal control} \\ \dot{H}(t) = R - \alpha \sum w_i(t) : \text{the game} \\ w_i(t) \geq 0 \\ H(t) \geq 0 \end{cases}$$

$$H(0) = H_0, \text{ and } H_0 \text{ given}$$

Model

- Calibration :

$$a = 2.5; b = 1.8; \alpha = 1; R = 0.56; c_0 = 2; c_1 = 0.1; r = 0.005; H_0 = 15$$

- The unitary cost is such that :

$$c_t(H(t)) = \begin{cases} (c_0 - c_1 H(t)) & \text{if } 0 \leq H(t) < 20 \\ 0 & \text{if } H(t) \geq 20 \end{cases} \Rightarrow \begin{cases} (2 - 0.1 H(t)) & \text{if } 0 \leq H(t) < 20 \\ 0 & \text{if } H(t) \geq 20 \end{cases}$$

illustrations : extraction behaviors

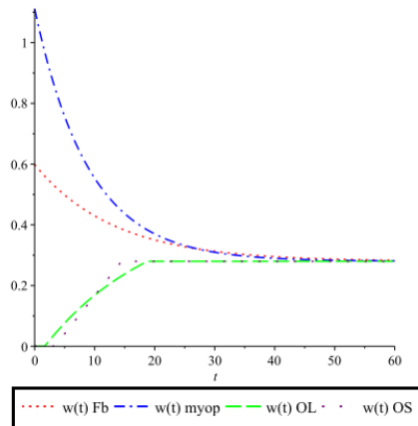


FIGURE – The game : feedback, myopic, open-loop and social optimum

Model

- the **discrete time** problem :

$$\max_{w_{i \ n}} \sum_{n=0}^{\infty} \underbrace{(1 - r\tau)^n}_{\beta^n} \left[a w_{i \ n} - \frac{b}{2} w_{i \ n}^2 - c_n(H_n) w_{i \ n} \right] \tau \quad (2)$$

$$\text{st} \begin{cases} H_{n+1} = H_n + \tau (R - \alpha w_{i \ n}) : \text{the optimal control} \\ H_{n+1} = H_n + \tau (R - \alpha \sum w_{i \ n}) : \text{the game} \\ w_i(t) \geq 0 \\ H(t) \geq 0 \end{cases}$$

$$H(0) = H_0, \text{ and } H_0 \text{ given}$$

- Continuous & discrete time : availabilty of all formulas for the optimal control and the game

illustrations : continuous and discrete time comparison

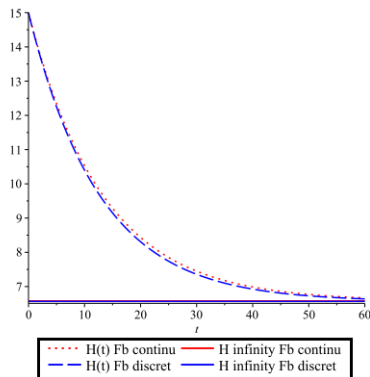
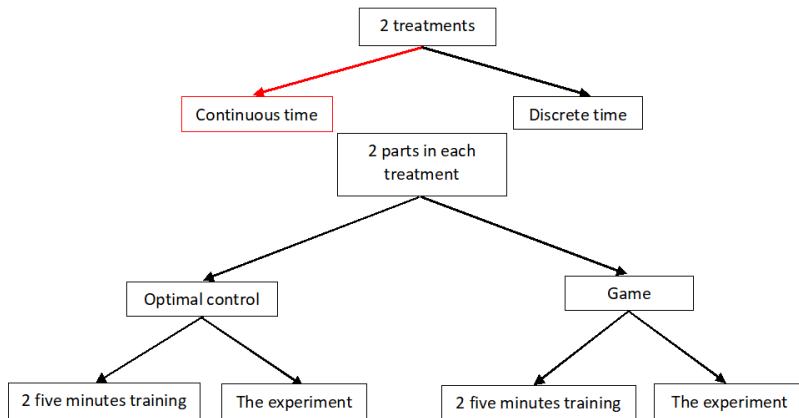


FIGURE – Feedback : groundwater table $H(t)$ convergence for $\tau = 1$

Experimental design



Experimental design

- No contextualization
- common knowledge experimentation with full information

Subject were informed that...

« You initially have 15 resource units. At any time, you can take a quantity between 0 and 2.8 resource units, with a precision of two decimal places. You are free to choose the quantity you want to take, namely 0, 0.01, 0.02. . . 2.79, 2.8 »

Preliminary results - continuous time

		Game			
	Type	Feedback	Myopic	Optimal	Total
-----+-----+-----					
	Other	0	0	1	1
Optimal	Myopic	19	2	5	26
control	Optimal	7	1	5	13
-----+-----+-----					
	Total	26	3	11	40

FIGURE – Behavior in the optimal control and the game

Discussion and conclusion

We found that :

- Continuous time model \equiv discrete time model when $\tau \rightarrow 0$
- But $\tau = 1$ also works \Rightarrow easy to understand in experimentation

The question is...

Which model best represents the reality?

- Subjects who were myopic in optimal control mostly played feedback in the game
- Econometric analysis not complete \Rightarrow correct time-series treatments

Further works

First of all :

- Discrete time lab experiment
- Experimentation : continuous time **vs** discrete time model

Then :

- Test the game without the optimal control
- Modify the given information \Rightarrow dynamics of the resource **vs** dynamics of costs

Thank you for your attention!!