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Coordination problems and the control of epidemics affecting fruit trees

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A complex management problem

- production by private owners distributed within a landscape
- economic losses due to the infection outbreak
- diffusion of pathogens intra and inter-patch
- finite horizon, multi-year production
- treatment by (partially inefficient) detection and removal of infected trees, (discrete binary choice)

Figure: Sharka example
Objective: Understand the decentralized problem

Problem often studied under the centralized perspective.

Our objective: understand better the decentralized behavior.
Emerging literature: [Atallah et al., 2017], [Fenichel et al., 2014], [Costello et al., 2017]

We analyze classical questions... coordination issues, inefficiency characterization... with specific modeling constraints
Modeling: infection diffusion within a period

Management options: $\rho_i \in \{0, \rho_{\text{max}}\}; \ 0 < \rho_{\text{max}} < 1$

State variables:
- $I_i$ Quantity of infected in patch $i$.
- $S_i$ Quantity of uninfected trees.

Growth and diffusion of the infection: $r_{ij}$

Evolutionary law (discrete time model), with $I << S$:

\[
(I_i^{t+1}, S_i^{t+1}) = f(S^t, I^t, \rho^t)
\]

\[
I_i^{t+1} = I_i^t (1 - \rho_i) + \sum_{j=1}^{N} I_j^t (1 - \rho_j) r_{ji}
\]

\[
S_i^{t+1} = S_i^t - \sum_{j=1}^{N} I_j^t (1 - \rho_j) r_{ji}
\]
Diffusion in a two patches model

\[ I_{i}^{t+1} = I_{i}^{t}(1 - \rho_{i}) + \sum_{j=1}^{N} I_{j}^{t}(1 - \rho_{j})r_{ji} \]

patch 1

\[ r_{11}I_{1}^{t}(1 - \rho_{1}^{t}) \]

\[ r_{12}I_{1}^{t}(1 - \rho_{1}^{t}) \]

\[ \rightarrow \]

partch 2

\[ r_{21}I_{2}^{t}(1 - \rho_{2}^{t}) \]

\[ r_{22}I_{2}^{t}(1 - \rho_{2}^{t}) \]
Economic model: profit function

\[ \pi_i^t(l^t, S^t, \rho^t) = \left( S_i^{t+1} v_i + l_i^{t+1} u_i - \frac{\rho_i^t}{\rho_{\text{max}}} (c_a + c_h A_i) \right) \]

subject to:

\[ (l_i^{t+1}, S_i^{t+1}) = f(S^t, l^t, \rho^t). \]

\( v_i \): production value by an uninfected tree in patch \( i \)
\( u_i \): production value by an infected tree \( i \)
\( c_a \): access cost
\( c_h \): per ha\(^{-1} \) inspection cost
\( A_i \): patch \( i \) surface
Resolution for the closed loop feedback-Nash equilibrium concept.

Comparison with the Pareto optimum.
Impact of the initial condition in the 2 patches 2 steps model:

- An example of analytical result: zone where \((\rho_{max}, \rho_{max}, \rho_{max}, \rho_{max})\) is the unique FNE
- Multiplicity of FNE
- Characterization of inefficiency
Maximal effort as a FNE

**Proposition:** Within the initial condition state space, there is a zone where initial infection is sufficiently high so that both players do maximal effort:

\[(\rho_{\text{max}}, \rho_{\text{max}}, \rho_{\text{max}}, \rho_{\text{max}}) \text{ is the unique Nash equilibrium if and only if } (I_1^0, I_2^0) \in \Delta_{\text{max}}, \text{ where } \Delta_{\text{max}} \text{ is defined by the set of inequalities:} \]

\[
\begin{align*}
I_2^0 &> \frac{\alpha_1 - I_1^0 (1 - \rho_{\text{max}}) (1 + r_{11})}{(1 - \rho_{\text{max}})^2 r_{21}} \\
I_2^0 &> \frac{\alpha_2 - I_1^0 (1 - \rho_{\text{max}}) r_{12}}{(1 - \rho_{\text{max}})(1 + r_{22})} \\
I_2^0 &> k_2 \\
I_1^0 &> k_1
\end{align*}
\]

where \(\alpha_i \equiv \frac{1}{F_i} (c_a + c_h \frac{1}{\rho_{\text{max}}} A_i)\) where \(F_i \equiv (v_i - u_i) r_{ii} - u_i\), and \(k_1\) and \(k_2\) are some constants.
Private efficiency in the case of maximal effort

\[ \alpha_i \equiv \frac{F_i(c_a + c_h)}{\rho_{\text{max}} A_i} \]

\[ (\rho_1^0, \rho_2^0, \rho_1^1, \rho_2^1)^* = (\rho_{\text{max}}, \rho_{\text{max}}, \rho_{\text{max}}, \rho_{\text{max}}) \]
Illustration for $\rho_{\text{max}}$ as a unique Nash equilibrium

$$\beta_i \equiv \frac{1}{D_i}(c_a + c_h \frac{1}{\rho_{\text{max}}} A_i) \text{ where } D_i \equiv (v_i - u_i)r_{ii} + (v_j - u_j)r_{ij} - u_i$$

$$\alpha_i \equiv \frac{1}{F_i}(c_a + c_h \frac{1}{\rho_{\text{max}}} A_i) \text{ where } F_i \equiv (v_i - u_i)r_{ii} - u_i$$
Number of Nash equilibria

**Proposition:** Multiplicity might arise even in a symmetric case (proof using an example).
Example, Nash equilibria according to the initial condition

Figure: Map of Nash equilibria according to initial state variables values. The black area indicates initial conditions leading to multiple Nash equilibria. Simulations parameters correspond to the symmetric case (see ??).
Symmetric example, inefficiency

Figure: Pareto optimum, symmetric example

Figure: Nash equilibria, symmetric example
Symmetric example, inefficiency
Conclusion

Main results
When infection is still small, \((I << S)\), and detection imperfect, and given parameters \((T, R, U, V...)\)

- Nash feedback resolution of the game shows equilibria depending on the initial infection level
- Geometric characterization of efficiency and inefficiency zones as a function of the initial infectious state
- Coordination issues: multiplicity of equilibria for some \((I_1^0, I_2^0)\)
Conclusion

Perspectives

- Introduce asymmetry in the case study, look at the impact of other parameters
- Study de-synchronization of production cycles and longer time horizons
- Apply this framework to analyze real life problems (find some data); question large scale management programs using known parameters
- Work on the modeling: $SI$ model, probabilistic framework...
Thanks for listening!


Temporal structure

BEGINNING OF THE GAME

\[
\begin{align*}
\text{decision} & \quad (\rho_1^0, \rho_1^0) \\
\downarrow & \\
t = 0 & \\
\end{align*}
\]

\[
(\rho_1^1, \rho_2^0) = f(\rho_1^0, \rho_2^0, \rho_1^0, \rho_2^0)
\]

\[
\begin{align*}
\text{first season payoffs} & \\
\pi_1^0(\rho_1^0, \rho_2^0) &= g(\rho_1^1) \\
\pi_2^0(\rho_1^0, \rho_2^0) &= g(\rho_2^0)
\end{align*}
\]

END OF THE GAME

\[
\begin{align*}
\text{decision} & \quad (\rho_1^1, \rho_2^1) \\
\downarrow & \\
t = 1 & \\
\end{align*}
\]

\[
(\rho_1^2, \rho_2^0) = f(\rho_1^1, \rho_2^1, \rho_1^1, \rho_2^1)
\]

\[
\begin{align*}
\text{second season payoffs} & \\
\pi_1^1(\rho_1^1, \rho_2^1) &= g(\rho_1^2) \\
\pi_2^1(\rho_1^1, \rho_2^1) &= g(\rho_2^0)
\end{align*}
\]