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Coordination problems and the control of epidemics affecting fruit trees

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A complex management problem

- production by private owners distributed within a landscape
- economic losses due to the infection outbreak
- diffusion of pathogens intra and inter-patch
- finite horizon, multi-year production
- treatment by (partially inefficient) detection and removal of infected trees, (discrete binary choice)

Figure: Sharka example
Introduction

Objective: Understand the decentralized problem

Problem often studied under the centralized perspective.

**Our objective:** understand better the decentralized behavior.
Emerging literature: [Atallah et al., 2017], [Fenichel et al., 2014], [Costello et al., 2017]

We analyze classical questions... 
**coordination issues, inefficiency characterization**... 
with specific modeling constraints
Modeling: infection diffusion within a period

Management options: \( \rho_i \in \{0, \rho_{\text{max}}\}; \ 0 < \rho_{\text{max}} < 1 \)

State variables:
\( I_i \) Quantity of infected in patch \( i \).
\( S_i \) Quantity of uninfected trees.

Growth and diffusion of the infection: \( r_{ij} \)

Evolutionary law (discrete time model), with \( I \ll S \):

\[
(I_i^{t+1}, S_i^{t+1}) = f(S^t, I^t, \rho^t)
\]

\[
I_i^{t+1} = I_i^t (1 - \rho_i) + \sum_{j=1}^{N} I_j^t (1 - \rho_j) r_{ji}
\]

\[
S_i^{t+1} = S_i^t - \sum_{j=1}^{N} I_j^t (1 - \rho_j) r_{ji}
\]
Modeling: Infection diffusion, two patches model

Diffusion in a two patches model

\[ I^{t+1}_i = I^t_i(1 - \rho_i) + \sum_{j=1}^{N} I^t_j(1 - \rho_j)r_{ji} \]

- **patch 1**
  - \( r_{11}I^t_1(1 - \rho^t_1) \)
  - \( r_{12}I^t_1(1 - \rho^t_1) \rightarrow \)

- **patch 2**
  - \( r_{21}I^t_2(1 - \rho^t_2) \leftarrow \)
  - \( r_{22}I^t_2(1 - \rho^t_2) \)
Economic model: profit function

\[ \pi^t_i(I^t, S^t, \rho^t) = \left( S_i^{t+1} v_i + I_i^{t+1} u_i - \frac{\rho_i^t}{\rho_{max}} (c_a + c_h A_i) \right) \]

subject to:

\[ (I^{t+1}, S^{t+1}) = f(S^t, I^t, \rho^t). \]

- \( v_i \): production value by an uninfected tree in patch \( i \)
- \( u_i \): production value by an infected tree \( i \)
- \( c_a \): access cost
- \( c_h \): per ha\(^{-1} \) inspection cost
- \( A_i \): patch \( i \) surface
Conceptual framework

Framework

\[ V_i^T (\rho^0, \rho^1, I^0, S^0) = \pi_i^0 (I^0, S^0, \rho^0) + \beta \pi_i^1 (I^1, S^1, \rho^1) \]

\[ I^{t+1}, S^{t+1} = f(I^t, S^t, \rho^t). \]

Resolution for the closed loop feedback-Nash equilibrium concept.
Comparison with the Pareto optimum.
Results

Impact of the initial condition in the 2 patches 2 steps model:

- An example of analytical result: zone where \((\rho_{max}, \rho_{max}, \rho_{max}, \rho_{max})\) is the unique FNE
- Multiplicity of FNE
- Characterization of inefficiency
Maximal effort as a FNE

**Proposition:** Within the initial condition state space, there is a zone where initial infection is sufficiently high so that both players do maximal effort:

\[(\rho_{\text{max}}, \rho_{\text{max}}, \rho_{\text{max}}, \rho_{\text{max}})\] is the unique Nash equilibrium if and only if \((l_1^0, l_2^0) \in \Delta_{\text{max}}\), where \(\Delta_{\text{max}}\) is defined by the set of inequalities:

\[
\begin{align*}
\begin{cases}
l_2^0 &> \frac{\alpha_1 - l_1^0 (1 - \rho_{\text{max}})(1 + r_{11})}{(1 - \rho_{\text{max}})r_{21}} \\
l_2^0 &> \frac{\alpha_2 - l_1^0 (1 - \rho_{\text{max}})r_{12}}{(1 - \rho_{\text{max}})(1 + r_{22})} \\
l_2^0 &> k_2 \\
l_1^0 &> k_1
\end{cases}
\end{align*}
\]

where \(\alpha_i \equiv \frac{1}{F_i} (c_a + c_h \frac{1}{\rho_{\text{max}}} A_i)\) where \(F_i \equiv (v_i - u_i) r_{ii} - u_i\), and \(k_1\) and \(k_2\) are some constants.
Private efficiency in the case of maximal effort

\[
\alpha_i = \frac{1}{F_i} = \frac{v_i - u_i}{r_{ii} - u_i}
\]

\[
\left(\rho^0_1, \rho^0_2, \rho^1_1, \rho^1_2\right)^* = (\rho_{\text{max}}, \rho_{\text{max}}, \rho_{\text{max}}, \rho_{\text{max}})
\]

\[
\left(\rho^0_1, \rho^0_2, \rho^1_1, \rho^1_2\right)^* = (\rho_{\text{max}}, 0, \rho_{\text{max}}, \rho_{\text{max}})
\]
Illustration for $\rho_{\text{max}}$ as a unique Nash equilibrium

\[
\beta_i \equiv \frac{1}{D_i} (c_a + c_h \frac{1}{\rho_{\text{max}}} A_i) \text{ where } D_i \equiv (v_i - u_i) r_{ii} + (v_j - u_j) r_{ij} - u_i
\]

\[
\alpha_i \equiv \frac{1}{F_i} (c_a + c_h \frac{1}{\rho_{\text{max}}} A_i) \text{ where } F_i \equiv (v_i - u_i) r_{ii} - u_i
\]
Number of Nash equilibria

**Proposition:** Multiplicity might arise... even in a symmetric case (proof using an example).
Example, Nash equilibria according to the initial condition
Symmetric example, inefficiency

Figure: Pareto optimum, symmetric example

Figure: Nash equilibria, symmetric example
Symmetric example, inefficiency
Main results
When infection is still small, \((I << S)\), and detection imperfect, and given parameters \((T, R, U, V...\))

- Nash feedback resolution of the game shows equilibria depending on the initial infection level
- Geometric characterization of efficiency and inefficiency zones as a function of the initial infectious state
- Coordination issues: multiplicity of equilibria for some \((I_1^0, I_2^0)\)
Conclusion

Perspectives

- Introduce asymmetry in the case study, look at the impact of other parameters
- Study de-synchronization of production cycles and longer time horizons
- Apply this framework to analyze real life problems (find some data); question large scale management programs using known parameters
- Work on the modeling: $SI$ model, probabilistic framework...
Thanks for listening!


Temporal structure

BEGINNING OF THE GAME

\[ \text{decision} \]
\[ (\rho_0^1, \rho_0^2) \]
\[ \downarrow \]
\[ t = 0 \]

\[ \text{evolution:} \]
\[ (I_1^0, I_2^0) = f(I_1^0, I_2^0, \rho_0^1, \rho_0^2) \]

END OF THE GAME

\[ \text{decision} \]
\[ (\rho_1^1, \rho_1^2) \]
\[ \downarrow \]
\[ t = 1 \]

\[ \text{evolution:} \]
\[ (I_1^1, I_2^1) = f(I_1^1, I_2^1, \rho_1^1, \rho_1^2) \]

\[ \text{first season payoffs} \]
\[ \pi_1^0(I_1^0, \rho_0^1) = g(I_1^1) \]
\[ \pi_2^0(I_2^0, \rho_0^2) = g(I_2^1) \]

\[ \text{second season payoffs} \]
\[ \pi_1^1(I_1^1, \rho_1^1) = g(I_1^2) \]
\[ \pi_2^1(I_2^1, \rho_1^2) = g(I_2^2) \]