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Coordination problems and the control of epidemics affecting fruit trees

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A complex management problem

- production by private owners distributed within a landscape
- economic losses due to the infection outbreak
- diffusion of pathogens intra and inter-patch
- finite horizon, multi-year production
- treatment by (partially inefficient) detection and removal of infected trees, (discrete binary choice)

Figure: Sharka example
Objective: Understand the decentralized problem

Problem often studied under the centralized perspective.

Our objective: understand better the decentralized behavior.
Emerging literature: [Atallah et al., 2017], [Fenichel et al., 2014], [Costello et al., 2017]

We analyze classical questions... 
coordination issues, inefficiency characterization...  
with specific modeling constraints
Modeling: infection diffusion within a period

Management options: \( \rho_i \in \{0, \rho_{\text{max}}\}; \; 0 < \rho_{\text{max}} < 1 \)

State variables:
\( l_i \) Quantity of infected in patch \( i \).
\( S_i \) Quantity of uninfected trees.

Growth and diffusion of the infection: \( r_{ij} \)

Evolutionary law (discrete time model), with \( I \ll S \):

\[
(l_{i}^{t+1}, S_{i}^{t+1}) = f(S^{t}, l^{t}, \rho^{t})
\]

\[
l_{i}^{t+1} = l_{i}^{t}(1 - \rho_{i}) + \sum_{j=1}^{N} l_{j}^{t}(1 - \rho_{j})r_{ji}
\]

\[
S_{i}^{t+1} = S_{i}^{t} - \sum_{j=1}^{N} l_{j}^{t}(1 - \rho_{j})r_{ji}
\]
Modeling: Infection diffusion, two patches model

Diffusion in a two patches model

\[ I_{i}^{t+1} = I_{i}^{t}(1 - \rho_{i}) + \sum_{j=1}^{N} I_{j}^{t}(1 - \rho_{j}) r_{ji} \]

patch 1

\[ r_{11} I_{1}^{t}(1 - \rho_{1}^{t}) \]

\[ r_{12} I_{1}^{t}(1 - \rho_{2}^{t}) \]

\[ \rightarrow \]

partch 2

\[ r_{21} I_{2}^{t}(1 - \rho_{1}^{t}) \]

\[ r_{22} I_{2}^{t}(1 - \rho_{2}^{t}) \]
Economic model: profit function

\[ \pi_i^t(I^t, S^t, \rho^t) = \left( S_i^{t+1} v_i + I_i^{t+1} u_i - \frac{\rho_i^t}{\rho_{\text{max}}} (c_a + c_h A_i) \right) \]

subject to:

\[ (I^{t+1}, S^{t+1}) = f(S^t, I^t, \rho^t). \]

- \( v_i \): production value by an uninfected tree in patch \( i \)
- \( u_i \): production value by an infected tree \( i \)
- \( c_a \): access cost
- \( c_h \): per ha\(^{-1} \) inspection cost
- \( A_i \): patch \( i \) surface
Conceptual framework

Resolution for the closed loop feedback-Nash equilibrium concept. Comparison with the Pareto optimum.

\[ V_i^T(\rho^0, \rho^1, I^0, S^0) = \pi^0_i(I^0, S^0, \rho^0) + \beta \pi^1_i(I^1, S^1, \rho^1) \]

\[ L^{t+1}, S^{t+1} = f(L^t, S^t, \rho^t). \]
Impact of the initial condition in the 2 patches 2 steps model:

- An example of analytical result: zone where \((\rho_{\text{max}}, \rho_{\text{max}}, \rho_{\text{max}}, \rho_{\text{max}})\) is the unique FNE
- Multiplicity of FNE
- Characterization of inefficiency
Maximal effort as a FNE

**Proposition:** Within the initial condition state space, there is a zone where initial infection is sufficiently high so that both players do maximal effort:

\[(\rho_{\text{max}}, \rho_{\text{max}}, \rho_{\text{max}}, \rho_{\text{max}}) \text{ is the unique Nash equilibrium if and only if } (I_1^0, I_2^0) \in \Delta_{\text{max}}, \text{ where } \Delta_{\text{max}} \text{ is defined by the set of inequalities:} \]

\[
\begin{align*}
I_2^0 &> \frac{\alpha_1 - I_1^0(1 - \rho_{\text{max}})(1 + r_{11})}{(1 - \rho_{\text{max}})r_{21}} \\
I_2^0 &> \frac{\alpha_2 - I_1^0(1 - \rho_{\text{max}})r_{12}}{(1 - \rho_{\text{max}})(1 + r_{22})} \\
I_2^0 &> k_2 \\
I_1^0 &> k_1
\end{align*}
\]

where \(\alpha_i \equiv \frac{1}{F_i} \left( c_a + c_h \frac{1}{\rho_{\text{max}}} A_i \right) \) where \(F_i \equiv (v_i - u_i) r_{ii} - u_i\), and \(k_1\) and \(k_2\) are some constants.
Private efficiency in the case of maximal effort

\[
\frac{\alpha_1}{(1 - \rho_{max})^2}
\]

\[
\frac{\alpha_2}{(1 - \rho_{max})(1 + r_{22})}
\]

\[
k_2
\]

\[
k_1
\]

\[
\rho^0_1, \rho^0_2, \rho^1_1, \rho^1_2 \Rightarrow (\rho_{max}, \rho_{max}, \rho_{max}, \rho_{max})
\]

\[
(\rho^0_1, \rho^0_2, \rho^1_1, \rho^1_2)^* = (\rho_{max}, \rho_{max}, \rho_{max}, \rho_{max})
\]

\[
(\rho^0_1, \rho^0_2, \rho^1_1, \rho^1_2)^* = (\rho_{max}, 0, \rho_{max}, \rho_{max})
\]
Illustration for $\rho_{\text{max}}$ as a unique Nash equilibrium

\[ \beta_i \equiv \frac{1}{D_i} \left( c_a + c_h \frac{1}{\rho_{\text{max}}} A_i \right) \text{ where } D_i \equiv (v_i - u_i)r_{ii} + (v_j - u_j)r_{ij} - u_i \]

\[ \alpha_i \equiv \frac{1}{F_i} \left( c_a + c_h \frac{1}{\rho_{\text{max}}} A_i \right) \text{ where } F_i \equiv (v_i - u_i)r_{ii} - u_i \]
Number of Nash equilibria

**Proposition:** Multiplicity might arise... even in a symmetric case (proof using an example).
Example, Nash equilibria according to the initial condition
Symmetric example, inefficiency

Figure: Pareto optimum, symmetric example

Figure: Nash equilibria, symmetric example
Symmetric example, inefficiency
Conclusion

Main results
When infection is still small, \((I \ll S)\), and detection imperfect, and given parameters \((T, R, U, V...)\)

- Nash feedback resolution of the game shows equilibria depending on the initial infection level
- Geometric characterization of efficiency and inefficiency zones as a function of the initial infectious state
- Coordination issues: multiplicity of equilibria for some \((I_1^0, I_2^0)\)
Conclusion

Perspectives

- Introduce asymmetry in the case study, look at the impact of other parameters
- Study de-synchronization of production cycles and longer time horizons
- Apply this framework to analyze real life problems (find some data); question large scale management programs using known parameters
- Work on the modeling: SI model, probabilistic framework...
Thanks for listening !


Temporal structure

BEGINNING OF THE GAME

decision
$(\rho_0^1, \rho_0^1)$

$\downarrow$

$t = 0$

evolution :
$(I_1^0, I_2^0) = f(I_1^0, I_2^0, \rho_0^0, \rho_0^0)$

$\downarrow$

$0$

decision
$(\rho_1^1, \rho_1^1)$

$\downarrow$

$t = 1$

evolution :
$(I_1^1, I_2^1) = f(I_1^1, I_2^1, \rho_1^1, \rho_1^1)$

$\downarrow$

$1$

first season payoffs

$\pi_1^0(I_0^0, \rho_0^0) = g(I_1^0)$

$\pi_2^0(I_0^0, \rho_0^0) = g(I_2^0)$

periods

$\rightarrow$

$2$

second season payoffs

$\pi_1^1(I_1^1, \rho_1^1) = g(I_1^1)$

$\pi_2^1(I_1^1, \rho_1^1) = g(I_2^1)$

END OF THE GAME

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